A Poisson Probability Model of Protected Species Take Risk

Author
Stephen M. Stohs
National Marine Fisheries Service
8604 La Jolla Shores Drive
La Jolla, CA 92037-1508
Stephen.Stohs@noaa.gov

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Introduction

A variety of economic production activities pose the risk of harm to threatened or endangered species. Examples include Pacific Northwest logging activities which destroy spotted owl habitat, economic development which compromises wetland habitat, and commercial marine fishing which results in the incidental take of threatened or endangered species. Fish and game managers who are responsible for upholding the requirements of federal laws governing protected species face the challenge of monitoring and controlling the risk posed by economic production activities. As the populations of these species are low by definition, a small size reduction can potentially result in a significant negative impact to survival prospects. Risk management in such settings generally requires obtaining data on protected animals which are injured or killed and drawing statistical inference from the (preferably) small counts involved.

Bycatch of protected species is an important policy issue for marine fisheries management. The Endangered Species Act (ESA) prohibits the incidental take of species which are determined to be endangered. The ESA creates a challenge for fisheries management, since protected species bycatch is an inherent risk of fishing which is difficult to fully eliminate. Restricting fishing in favor of species protection imposes costs on commercial and recreational fishermen in the form of lost fishing opportunities. Hence it is important to quantify the relationship between the level of fishing effort and the risk of protected species bycatch. Because it is not possible to predict the relationship with certainty, a probability-based approach is warranted.

One example of protected species bycatch is that of leatherback turtle (Dermochelys coriacea) take in the large-mesh drift gillnet (California DGN) fishery for swordfish and thresher shark off the west coast of the USA. A drift gillnet fishing trip consists of a number of sets, typically on the range from 1 to 20, where each set involves lowering a net into the water for approximately twelve hours then hauling it up to retrieve the day's catch. The sets are roughly homogenous with respect to duration and gear type and are each regarded as one day's worth of fishing effort. If an endangered leatherback turtle is entangled more than one hour before the end of a set, it is virtually certain to die of suffocation, due to a biological limit on the time a turtle can hold its breath underwater.

Incidental take of leatherback turtles in the California DGN fishery is a rare event: There were a total of twenty-three leatherback takes in 7,221 observed California DGN sets over the seasons from 1990-2005. However, a small number of leatherback takes can have large economic implications for the responsible fisheries. In the case of the California DGN fishery, a large area was closed to DGN fishing for the period from August 15 through November 15 after it was deemed the risk of leatherback take was excessive, even though the overall number of leatherback takes appeared to be extremely small.

The empirical context calls for a probability model suitable to characterize a small risk of leatherback bycatch which is sparsely distributed over a large amount of fishing effort. The Poisson distribution is the standard probability model for rare event counts. The model may be specified in a natural way to reflect the stochastic relationship between the number of DGN sets, leatherback catch per unit effort (CPUE), and leatherback take. Once estimated, the model may be used to construct prediction intervals for future leatherback take conditional on the number of DGN sets. The Poisson probability model may be extended to include covariates which explain the variation in CPUE across observation units.

I specify and estimate a Poisson probability model of leatherback take using a Bayesian approach to inference and prediction. I use historical data for fishing in the area north of Pt. Conception over the years from 1990-2005 to fit a specification with a noninformative gamma prior distribution and a Poisson likelihood function which assumes that leatherback take risk scales linearly with fishing effort. A Bayesian version of Pearson's chi square goodness of fit test is employed to test the fit of the model to existing observer data. I next show how the fitted Poisson model may be used to make probability statements or construct prediction intervals for numeric measures of interest to fishery managers.

The Poisson Probability Model

The statistician I.J. Good argued that the Poisson distribution should have been named the von Bortkiewicz distribution, after the economist Vladislav Bortkiewicz (Anonymous n.d.). Bortkiewicz published a work, "The Law of Small Numbers," in 1898 which summarized his results on the Poisson distribution. In this he was the first to note that events with low frequency in a large population followed a Poisson distribution even when the probabilities of the events varied. E. J. Gumbel writes in the *International Encyclopedia of the Social Sciences*,

"A striking example was the number of soldiers killed by horse kicks per year per Prussian army corps. Fourteen corps were examined, each for twenty years. For over half the corps-year combinations there were no deaths from horse kicks; for the other combinations the number of deaths ranged up to four. Presumably the risk of lethal horse kicks varied over years and corps, yet the over-all distribution was remarkably well fitted by a Poisson distribution[1]."

[1] Applied econometricians are always on the lookout for probability models which provide a good fit to observational data samples from inhomogeneous sampling units, as real

The Poisson distribution is applicable for modeling a series of Bernoulli trials, or two-outcome experiments $\{B_1, B_2, \ldots, B_n,\ i = 1, 2, \ldots, n\}$, where each B_i is either equal to 1 or 0, the Bernoulli outcomes are exchangeable[2], the probability of success on each trial θ is small, and the number of trials n is very large. The standard model for the number of "successes" in a fixed number of exchangable Bernoulli trials is the Binomial model:

$$f(y) = \binom{n}{y} \theta^y (1-\theta)^{n-y}, \tag{1}$$

where $Y = \sum_{i=1}^{n} B_i$ is a Binomial random variable. It is straightforward to demonstrate that with $\lambda = n\theta$ held fixed,

$$\lim_{\theta \to 0} f(y) = e^{-\lambda} \frac{\lambda^y}{y!}. \tag{2}$$

This gives rise to the so-called *Law of Rare Events*, which states that for large n and small θ, we may closely approximate a $Binomial(n, \theta)$ random variable by the Poisson model, taking

$$p(y|\theta, n) = e^{-n\theta} \frac{(n\theta)^y}{y!} \tag{3}$$

We write $Y \sim Pois(\lambda)$ to indicate that Y follows a Poisson distribution

world applications seldom offer the luxury of controlled experimental trials.

[2] A sequence of probabilistic outcomes is exchangeable if permutation of the labels does not affect the joint distribution of the outcomes.

with rate parameter λ. Straightforward calculations show that

$$E(Y) = Var(Y) = \lambda. \tag{4}$$

To specify a model of incidental protected species take, we let y_i denote the take in observation unit[3] i, for $i = 1, \ldots, n$, and assume the i^{th} observation unit is subject to a Poisson rate of λ_i. The Poisson model of protected species take for observation unit i conditional on the rate parameter becomes

$$p(y_i|\lambda_i) = e^{-\lambda_i}\frac{\lambda_i^{y_i}}{y_i!}. \tag{5}$$

A restricted version takes θ as the constant, homogeneous level of CPUE across all observation units, and n_i as the units of effort for observation unit i. The Poisson rate parameter for the i^{th} observation unit is assumed proportional to effort:

$$\lambda_i = n_i\theta. \tag{6}$$

Subject to this restriction, the probability model for the number of protected species takes in observation unit i becomes

$$p(y_i|\theta, n_i) = e^{-n_i\theta}\frac{(n_i\theta)^{y_i}}{y_i!}. \tag{7}$$

[3]Generally we could consider breaking up observations across time periods, spatial regions, or both.

Classical versus Bayesian Inference

The classical approach to estimation and prediction differs fundamentally from the Bayesian approach by how it construes the roles of model parameters and data. Under the classical paradigm, the parameters are viewed as unobserved, nonrandom quantities that govern the probability distribution which generated the data. A data set is typically assumed to represent a random sample drawn from the underlying population distribution. Any particular data set is a subset of an infinite number of different possible realizations of the random variables which comprise the data generating process, and the analyst's job is to best use the data to estimate the unknown but fixed parameters which enter the model.

Under the Bayesian view, both data and parameters are described by probability distributions, expressing logically coherent beliefs about these quantities of interest which are consistent with the observed data, rather than describing the relative frequencies of occurrence over an infinite sequence of hypothetical repetitions of a controlled random experiment. The Bayesian view has intrinsic appeal to researchers whose disciplines constantly face a paucity of real world data which can be reasonably interpreted as arising from identical trials under controlled experimental conditions. In the case of managing economic production subject to the risk of harm to protected species, the Bayesian approach seems more appropriate, as it does not rely on a counterfactual assumption of controlled experimental trials.

Estimation and Prediction

The key elements of the Bayesian approach are a prior distribution, generically notated $p(\theta)$, which summarizes prior beliefs about the parameter or parameters in question, and a likelihood function, represented $p(y|\theta)$, which may be interpreted as the probability distribution for the data y conditional on the parameter(s) θ. Estimation and inference in the Bayesian approach is based on the application of Bayes' Rule, which provides an algorithm for using the prior distribution and the observed data to obtain a posterior parameter distribution which represents updated beliefs about the parameters in light of the empirical evidence:

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}, \tag{8}$$

where $p(y) = \int p(y \,|\, \theta)p(\theta)d\theta$ is the marginal distribution[4] of y. The data represent a given set of observations and hence may be regarded as constant in the formulation, and thus the marginal distribution $p(y)$ represents a scale factor which makes the posterior density integrate to 1. In light of this, Bayes' rule is often expressed more simply in proportionality form as

$$p(\theta|y) \propto p(y|\theta)p(\theta), \tag{9}$$

[4]Typical notation for describing Bayes' rule involves an abuse of notation, as $p(\cdot)$ is used to describe the the prior, the posterior, the marginal, and the posterior predictive distributions, with the different interpretations indicated by the arguments of $p(\cdot)$.

with the understanding that $p(y)$ may be recovered by integration, as shown above.

Estimation

Suppose we have on hand data for N exchangeable observation units on the number of sets fished n_i and count data y_i for the bycatch on the i^{th} set, for $i = 1, 2, \ldots, N$. For the Poisson model with homogeneous bycatch risk per unit of effort[5], θ, the likelihood function is formally identical to the corresponding classical likelihood:

$$p(y \,|\, \theta, n) \propto e^{-n\theta}\theta^{y}, \tag{10}$$

where $n = \sum_{i=1}^{N} n_i$ is the total exposure and $y = \sum_{i=1}^{N} y_i$ is the total bycatch count.

The Poisson distribution represents a case where the form of the likelihood gives rise to what is known as a conjugate prior, a parametric probability distribution which may be used to quantify available information before reflecting the likelihood of the observed data sample, and which combines in a natural manner with the likelihood to form the posterior probability distribution. The conjugate prior for the Poisson distribution is

$$p(\theta) \propto e^{-\beta\theta}\theta^{\alpha-1}, \tag{11}$$

[5]The hypothesis that a model with homogeneous Poisson bycatch risk per unit of effort provides an adequate fit to the observed data is tested in a later section.

which (with the addition of a normalizing factor) is known as the $\Gamma(\alpha, \beta)$ distribution. The distribution has mean and variance parameters given by

$$E(\theta) = \frac{\alpha}{\beta} \tag{12}$$

and

$$Var(\theta) = \frac{\alpha}{\beta^2}, \tag{13}$$

and for suitable choice of the location parameter α and shape parameter β, the distribution can reflect a wide range of prior beliefs about the rate parameter θ. Applying Bayes' rule with a Gamma prior distribution to the case of my model of turtle bycatch yields results in the following posterior density:

$$p(\theta \,|\, n, y) \propto e^{-(\beta+n)\theta}\theta^{\alpha+y-1}, \tag{14}$$

which is a $\Gamma\left(\alpha + y, \beta + n\right)$ distribution. The form of the posterior suggests that the roles of α and β are analogous to the prior number of takes and the prior number of sets, respectively.

For simplicity and comparability with maximum likelihood estimation results, I consider the case $\alpha = 0$ and $\beta = 0$, which gives rise to an improper prior[6]. The prior distribution in this case has form

$$p(\theta) = \theta^{-1},\ 0 < \theta < \infty \tag{15}$$

[6] An improper prior is one which does not integrate over the support, but which gives rise to an integrable function after multiplication by the likelihood.

which can be interpreted as a diffuse prior which reflects ignorance about the precise value of θ before observing the data, and places the highest weight on the smallest values of $\theta > 0$.

The posterior is given by Bayes' rule as

$$p(\theta \,|\, n, y) \propto e^{-\theta n} \theta^{y-1}, \tag{16}$$

which bears formal similarity to the likelihood function, but which is interpreted to summarize the uncertainty about the value of the homogeneous rate parameter θ in light of the observed bycatch count y and exposure n. The posterior distribution assumes the form of a gamma distribution with mean mean $\mu_\theta = \frac{y}{n}$ and variance $\sigma^2_\theta = \frac{y}{n^2}$, which are formally identical to the mean and variance of the maximum likelihood estimator for the classical model, but subject to different interpretation in the Bayesian case. The Bayesian view holds that the posterior distribution gives a complete summary of the inference about the rate parameter θ in light of the observed data and the probability model in use, which may be subsequently used to obtain summary statistics such as the mean and variance of the posterior, or prediction intervals.

Prediction

The posterior distribution of θ may be used in conjunction with the likelihood function to derive the predictive distribution of $\tilde{y}$, denoted $p(\tilde{y} \,|\, \tilde{n}, n, y)$, for subsequent prediction of the number of takes. The predictive distribution is defined as the integral of the likelihood for a new observation conditional on effort, integrated over the posterior predictive density:

$$p(\tilde{y} \,|\, \tilde{n}, n, y) = \int p(\tilde{y} \,|\, \theta, \tilde{n}) p(\theta \,|\, n, y) d\theta, \tag{17}$$

where $\tilde{y}$ is the (stochastic) future predicted bycatch count conditional on future effort level $\tilde{n}$.

The predictive distribution may be interpreted as a mixture of the likelihood for a future count observation $\tilde{y}$ conditional on effort $\tilde{n}$ with the posterior distribution of the rate parameter θ. The predictive distribution reflects posterior uncertainty in the rate parameter θ and stochastic variation in future experience in a coherent manner. It is shown in the appendix that the posterior distribution for the Poisson model subject to homogeneous risk per unit of effort with gamma prior is a negative binomial distribution, $Negbin(y, \frac{n}{\tilde{n}})$.

Application to Leatherback Turtles

I next apply the model to observer data from a fishery where leatherback take is an ongoing concern. The data were extracted from the California Drift Gillnet Observer Database, and are a representative sample of approximately 20% of the fishing effort which took place for the portion of California DGN fishery North of Pt. Conception[7] over the period from 1990-2004.

The number of sets fished over the period were $n = 2876$ and the number of leatherback takes were $y = 21$. For this data and the prior discussed above, the posterior is

$$p(\theta \,|\, n, y) \propto e^{-n\theta}\theta^{y-1}, \tag{18}$$

which is easily recognized to be in the form of a gamma distribution with $\beta = n = 2876$ and $\alpha = y = 21$.

The mean and variance of the posterior distribution for θ are

$$E(\theta) = \frac{\alpha}{\beta} = \frac{21}{2876} \approx 7.302 \times 10^{-3}, \tag{19}$$

and the variance of θ is

$$\sigma_\theta^2 = Var(\theta) = \frac{\alpha}{\beta^2} = \frac{21}{2876^2} \approx 2.539 \times 10^{-6}, \tag{20}$$

with corresponding standard deviation given by

$$\sigma_\theta = \sqrt{\sigma_\theta^2} = 1.593 \times 10^{-3}. \tag{21}$$

For sufficiently large values of β, the Gamma distribution is approximately normally distributed. The graph below compares the gamma poste-

[7]Pt. Conception lies at 37°27′ North Latitude, and represents a dividing line between the geographically and ecologically distinct southern and northern ranges of the DGN fishery. I restrict my attention to the northern portion of the DGN fishery because it is a region where leatherback bycatch is known to be a problem, and where it may be reasonable to assume the risk is homogeneous with respect to time and area.

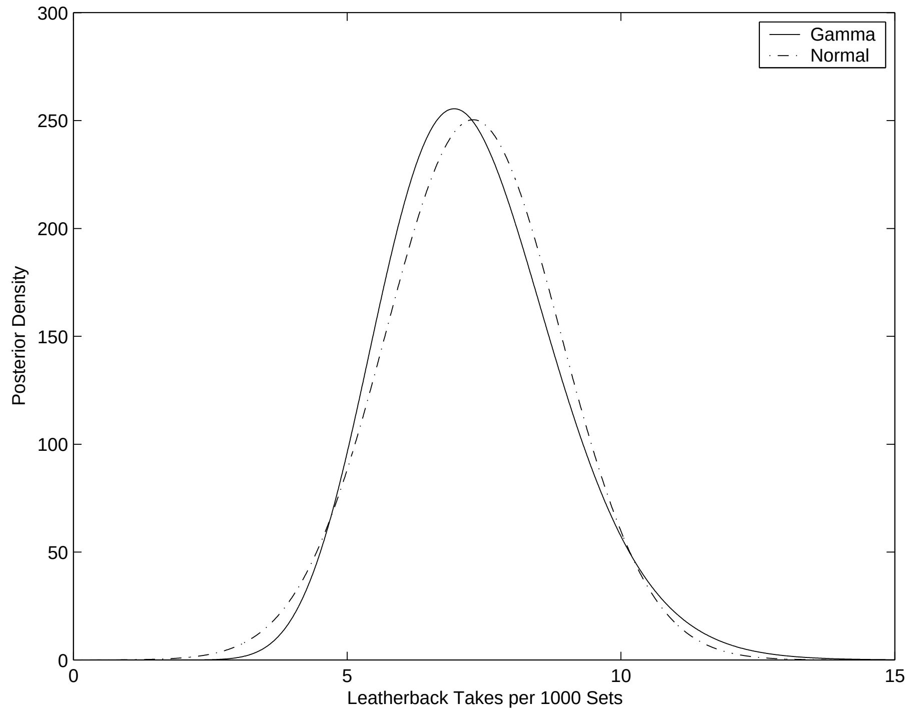

Figure 1: Gamma posterior and approximating normal distribution

rior distribution to a normal approximation with the same mean and variance.

Assessing Goodness of Fit

The model used above implicitly assumes a sufficiently homogeneous level of leatherback take risk to justify pooling observations across seasons and geographic location within the area North of Point Conception. I employed two approaches to assessing whether the model is consistent with the data. First I created graphs which compare the expected number of leatherback

Season	Sets	Takes
1990	94	1
1991	210	1
1992	423	4
1993	445	2
1994	265	1
1995	282	5
1996	236	2
1997	292	4
1998	235	0
1999	153	1
2000	142	0
2001	35	0
2002	46	0
2003	13	0
2004	5	0
Totals	2876	21

Table 1: DGN sets and leatherback takes by season

takes across seasons and across latitudes compared to the observed number in each case. Next I tested the fit numerically using a Chi square goodness of fit test adapted to a Bayesian context.

Graphical Assessment of Model Fit

To assess intertemporal agreement of the model with data, I consider observed DGN sets and leatherback takes broken out by season, as shown in the Table 1.

Because of the small number of sets fished north of Pt. Conception after the 1999 season, I aggregated effort and leatherback takes for those years[8]. Using the fitted gamma posterior distribution and the assumption of homogeneous Poisson rate parameter across seasons, the expected number of takes[9] in season $i = 1990, 1991, \ldots, 1999, 2000 - 2004$ may be calculated as

$$\begin{aligned} E(\tilde{y}_i \,|\, y, n) &= E[E(\tilde{y}_i \,|\, n_i, \theta) \,|\, y, n] \\ &= E(n_i \theta \,|\, y, n) \\ &= n_i E(\theta \,|\, y, n), \end{aligned} \qquad (22)$$

where $\tilde{y}_i$ is the predicted number of leatherback takes in season i, n_i is the observed number of DGN sets and $E(\theta \,|\, y, n)$ is the mean of the posterior distribution for θ.

Figure 2 shows the expected and observed numbers of leatherback takes by seasons, with the rightmost points (labeled 2000) corresponding to aggregate values for the 2000-2004 seasons. The graph suggests that the observed numbers of leatherback takes by season are in reasonably close agreement with the expected numbers under the hypothesis of a homogeneous Poisson rate parameter which was used to calculate the expected numbers of takes.

A similar approach was used to check for agreement between the fitted model and the observed numbers of leatherback takes at different degrees of

[8]The validity of the asymptotic Poisson approximation to the binomial distribution becomes questionable for small numbers of observed sets.

[9]The outer expectation in the iterated expectation on the first line is over the posterior distribution of θ, and the inner expectation is conditional on θ, as shown.

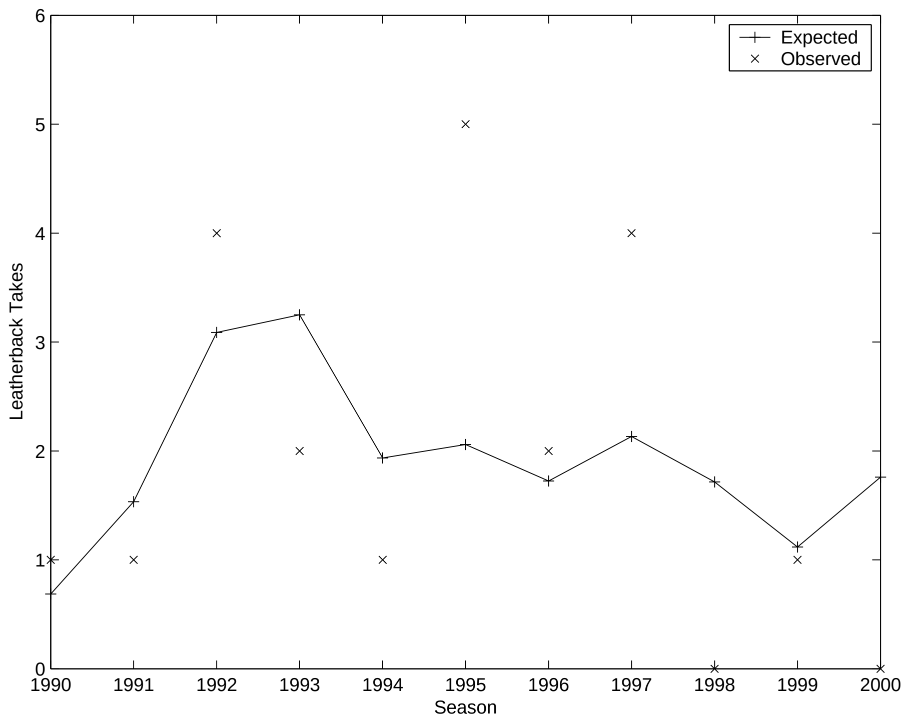

Figure 2: Expected and observed leatherback takes by season

latitude. Figure 3 shows the geographic distribution of the 23 leatherback takes which were observed in the California DGN fishery over the period from 1990-2004. The graph includes a horizontal line at the latitude for Point Conception (34.45° N), which may regarded as a separation boundary between the northern and southern portions of the fishery. 21 leatherback takes are shown to the north of this boundary, and only 2 below. Of the two leatherback takes for the southern portion of the fishery, one was located very near the boundary. Overall the 23 leatherback takes exhibit wide geographic dispersion, indicating that leatherback take risk is not narrowly concentrated over any particular portion of the range where DGN fishing occurs.

Leatherback take risk is believed to vary with respect to latitude; for instance, only two leatherback takes were observed over 4291 sets of effort south of Pt. Conception over the same fishing seasons (1990-2004) that 21 leatherback takes occurred over 2876 sets fished north of Pt. Conception. Table 2 shows the distribution of leatherback takes by degrees latitude[10] for the range of latitudes north of Pt. Conception.

In order to maintain the applicability of the asymptotic Poisson distribution assumption, I grouped the data from 44 degrees latitude north. I computed expected numbers of takes in a similar matter for the latitude classification as I did above for the classification by season. Figure 4 shows the expected and observed numbers of takes graphed on degrees of latitude

[10]For convenience, the classification was based on the largest whole degrees latitude less than or equal to the exact latitude for each observation.

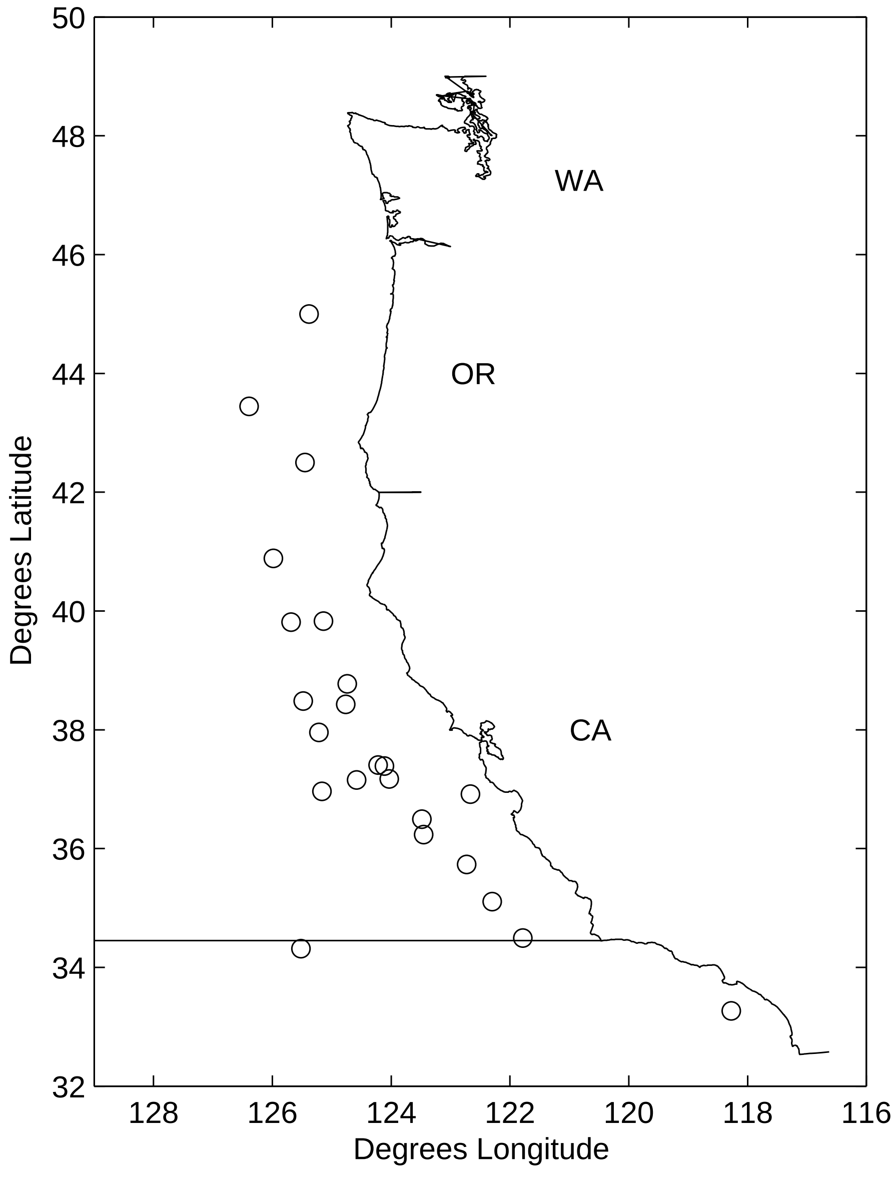

Figure 3: Observed leatherback takes by geographic location

Latitude	Sets	Takes
37	1739	12
38	235	3
39	136	2
40	170	1
41	250	0
42	86	1
43	145	1
44	83	0
45	21	1
46	11	0
Total	2876	21

Table 2: DGN sets and leatherback takes by degrees latitude

where fishing occurred. The fit of the observations to the predicted numbers of leatherback takes is remarkably close. In particular, there is a clear indication that the large number of takes over time at 37 degrees of latitude is well explained by the concentration of fishing effort at that latitude, rather than by a higher level of leatherback bycatch risk relative to the other latitudes north of Pt. Conception.

Using Bayesian *p*-values to Assess Model Fit

The graphical comparisons shown above provide indication that the Poisson model with homogeneous bycatch rate parameter and uninformative gamma prior distribution fits the observed data reasonably well across seasons and across latitudes. I used a Bayesian p-value approach to assess whether the agreement between the expected and observed takes would be unlikely to

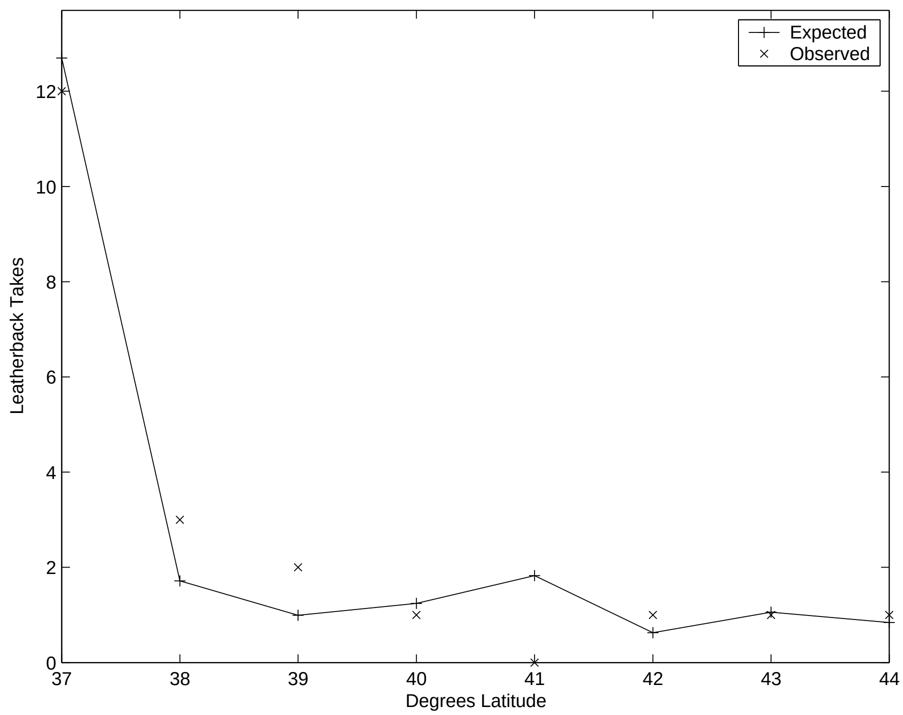

Figure 4: Expected and observed leatherback takes by degrees latitude

occur under the maintained hypothesis of a homogeneous Poisson rate parameter over the ranges of seasons and degrees of latitude represented in the data.

Following Gelman et al. (Gelman, Carlin, Stern & Rubin 2004), I used the χ^2 discrepancy statistic $T(y,\theta) = \sum_{i=1}^{N} \frac{(y_i - E(y_i \,|\, \theta, n_i))^2}{var(y_i \,|\, \theta, n_i)}$, where the observed numbers of leatherback takes y_i and DGN sets n_i are partitioned either by fishing season or by degrees of longitude, and used simulation based on the posterior distribution of θ and replications from the distribution of y_i conditional on n_i and the simulated values of θ to compute Bayesian p-values, defined as the probability the χ^2 statistic calculated from the replicated data could be more extreme than that calculated from the observed data:

$$p_B = \Pr(T(y^{\mathrm{rep}}, \theta) > T(y, \theta) \,|\, n_i, y_i,\, i = 1, 2, \ldots, N). \tag{23}$$

The probability is taken over the posterior distribution of θ and the posterior predictive distribution of y^{rep}. Unlike the Classical case, where the value of the test statistic computed from the observed data is held fixed in computing a p-value, the Bayesian p-value calculation reflects independent variation in θ over its posterior distribution and variation in y_i over the conditional distribution; hence no degrees of freedom adjustment is necessary.

Based on 100,000 simulated draws, the p-values which result are 0.36 for the test of homogeneous Poisson rate across fishing seasons, and 0.70 for the test of homogeneous Poisson rate across degrees of latitude. These p-

values may be interpreted as the probability that a random draw of θ from its posterior distribution and $\tilde{y}$ from the posterior predictive distribution conditional on θ would give rise to a value of the Chi square discrepancy statistic in excess of the one calculated for the observed sample of values y_i and n_i for $i = 1, 2, \ldots, N$.

A p-value near 0 (say less that 0.05) would indicate a lack of fit between the data and the model, as the Chi square discrepancy computed for the observed y_i would indicate a worse fit of the data to the hypothesized model than the same measure applied to all but a small percentage of replicated data samples. Similarly, a p-value close to 1 (say greater than 0.95) would indicate a model which overfits the observed data, as the Chi square discrepancy measure for more than 95% of the replicated data samples would then indicate a poorer fit to the model than that of the observed data sample. In both the case of annual variation and latitudinal variation in the observation units, the Bayesian p-values lie squarely in the center of the range from 0.05 to 0.95, providing no significant evidence that the observed data lie outside the normal range of variation which would occur under the hypothesis of a homogeneous Poisson leatherback bycatch rate per unit of fishing effort.

Predicting the Number of Leatherback Takes Conditional on Effort

A standard approach to controlling bycatch in marine fisheries is to utilize some combination of a limit to fishing effort and a cap on allowable bycatch. All three possible approaches have drawbacks; if only an effort cap is used, there is a risk that unacceptable levels of bycatch will result. With a stringent bycatch limit, there is a chance that allowable fishing effort will be curtailed to a level where fishing is no longer economically viable. Layering a bycatch cap on top of a given effort limit provides an extra layer of precaution against the risk of an unacceptable bycatch level, but results in an even lower expected level of allowable fishing opportunity, and still less chance the fishery will remain economically viable given the regulatory constraint.

If bycatch is regulated using only an effort limit, a question of interest concerns the number of leatherback takes that would occur at a given level of allowable effort. The posterior predictive distribution for the number of leatherback takes conditional on fishing effort provides a tool for computing various estimates of the predicted level of leatherback take, including the expected level of take conditional on effort, and posterior prediction intervals.

The posterior predictive distribution for the number of leatherback takes for effort of 400 DGN sets is displayed in the bar graph of Figure 5. The distribution is seen to have a mode of 2, but is skewed to the right, with more than a 5% probability of six or more leatherback takes.

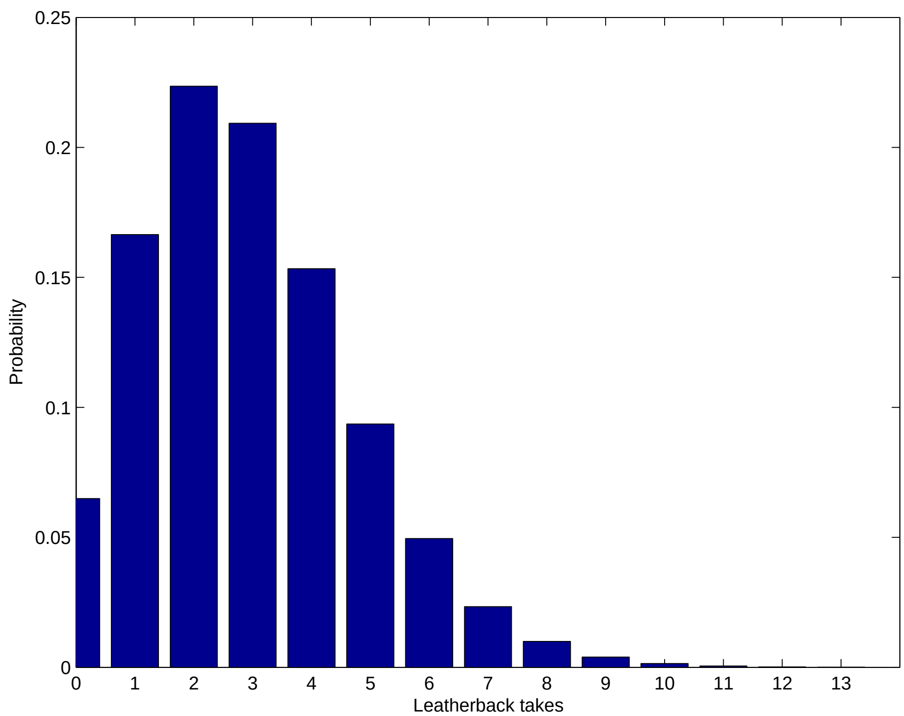

Figure 5: Posterior predictive distribution for 400 DGN sets

A 95% prediction interval may be defined[11] as the smallest range of values of $\tilde{y}$ with probability greater than or equal to 95%. The interpretation is that under the assumptions of the model, there is a probability of 95% or more that the number of leatherback takes will lie on this range, provided that effort reaches the assumed level. I constructed 95% prediction intervals for the cases of 200, 300, or 400 sets of fishing effort. Due to the high probability of a small number of leatherback takes, the lower bound of the prediction interval was 0 in each case, but the upper bound varied from 4 to 5 to 6 as the assumed level of effort increased from 200 to 300 to 400, reflecting the increased probability of higher bycatch levels with an increase in effort.

Has Leatherback Take Risk Recently Declined?

One question of interest which the predictive distribution may be used to address is whether the lack of any leatherback takes from the 2000 season on represents significant evidence that the leatherback take risk has dropped. The predictive distribution may be used to compute the probability of zero leatherback takes given that there were 99 sets of fishing over this period[12]. A very low value for the probability of observing zero takes would suggest that the take rate had dropped, while a higher value would show that the

[11] Exact 95% prediction bounds are generally not obtainable for discrete distributions.

[12] A classical statistician might claim that it is meaningless to compute the probability of an event which has already taken place – the probability that it occurred is now either 1 or 0 but not both. To a Bayesian, the probability may lie between 0 and 1 provided the outcome is unknown to the researcher.

observation of zero takes would not have been unlikely, even if the take risk remained as high as its previous level.

Based on the posterior predictive distribution, the probability of zero leatherback takes in 99 sets is calculated as

$$Pr\{\tilde{y} = 0 \,|\, \tilde{n}, n, y\} = 0.491, \tag{24}$$

using $\tilde{y} \sim Negbin(y, \frac{n}{\tilde{n}})$ with $\tilde{n} = 99, n = 2876$, and $y = 21$. This calculation indicates that the absence of leatherback takes in fishing seasons from 2000 on does not represent strong evidence of a drop in leatherback take risk, as there would have been a high probability of zero takes even if the risk remained at its previous level, given that only 99 sets of fishing occurred. Under traditional rules of thumb for hypothesis testing[13], the data are consistent with a null hypothesis that take leatherback take risk has not decreased.

Conclusion

The need to balance fishing opportunities against the risk of endangered species take represents a challenge for fisheries management which depends on reasonable predictions of endangered species take as a function of fishing effort. Regulating protected species bycatch typically involves limiting production contingent on a small and random number of protected species

[13]Traditionally, predicted probabilities below 5% are considered weak evidence that a null hypothesis is inconsistent with the data, while predicted probabilities below 1% are considered strong evidence.

interactions.

Ignoring the stochastic nature of protected species risk may lead to suboptimal policy choices. The Poisson model potentially offers a more realistic description in both the narrow context of fisheries bycatch management described in this paper, as well as the broader context of managing other economic production activities which pose risk to protected species. The approach described herein may offer a more effective tool for evaluating alternative management strategies for controlling protected species risk.

A missing ingredient from the approach taken in this paper was any explicit conditioning on detailed scientific information concerning leatherback turtle habitat, migration patterns, or variation in seasonal abundance. Instead, leatherback take risk was treated as a black box, and evidence was sought for a significant departure from a maintained hypothesis of homogeneous risk per unit of fishing effort. While the results presented here point in the direction of homogeneous risk over the time and geographic range of the data for the portion of the California DGN fishery north of Pt. Conception, introduction of available scientific information could potentially identify risk variation at finer scales of time or geography which would be useful in risk management. Future research will explore the possibility of using extant scientific information to refine estimates of leatherback take risk.

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Appendix

The negative binomial posterior predictive distribution is derived from the gamma posterior distribution and Poisson likelihood function as follows:

$$
\begin{aligned}
p(\tilde{y} \,|\, \tilde{n}, n, y) &= \int p(\tilde{y} \,|\, \tilde{n}, \theta) p(\theta \,|\, n, y) d\theta \\
&= \frac{n^y}{\Gamma(y)} \int \frac{\exp(-\tilde{n}\theta)(\tilde{n}\theta)^{\tilde{y}}}{\tilde{y}\,!} \exp(-n\theta)\theta^{y-1} d\theta \\
&= \frac{n^y \tilde{n}^{\tilde{y}}}{\Gamma(y)\Gamma(\tilde{y}+1)} \int e^{-(\tilde{n}+n)\theta} \theta^{(\tilde{y}+y)-1} d\theta \\
&= \frac{n^y \tilde{n}^{\tilde{y}}}{(y-1)!\tilde{y}!} \cdot \frac{(\tilde{y}+y-1)!}{(\tilde{n}+n)^{\tilde{y}+y}} \\
&= \binom{\tilde{y}+y-1}{\tilde{y}} \left(\frac{\tilde{n}}{\tilde{n}+n}\right)^{\tilde{y}} \left(\frac{n}{\tilde{n}+n}\right)^{y} \\
&= \binom{\alpha+\tilde{y}-1}{\tilde{y}} \left(\frac{\beta}{\beta+1}\right)^{\alpha} \left(\frac{1}{\beta+1}\right)^{\tilde{y}},
\end{aligned} \tag{25}
$$

with $\alpha = y$ and $\beta = n/\tilde{n}$.

Taking $p = \frac{\tilde{n}}{\tilde{n}+n}$ and $N = \tilde{y} + y - 1$, we may alternately express the posterior predictive distribution using

$$p(\tilde{y} \,|\, \tilde{n}, n, y) = binom(\tilde{y}, N, p) \cdot (1-p), \tag{26}$$

where

$$binom(\tilde{y}, N, p) = \binom{N}{\tilde{y}} p^{\tilde{y}} (1-p)^{N-\tilde{y}} \tag{27}$$

is the binomial probability mass function with parameters N and p.