Groundwater Use under Incomplete Information

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Abstract

In this paper, we introduce a game theoretic model of groundwater extraction in a two-cell aquifer under incomplete information. A novel assumption is that individual users have incomplete knowledge of the speed of lateral flows in the aquifer: although a user is aware that his neighbor's water use has some influence on their future water stock, they are uncertain about the degree of this impact. We find that the lack of information may either increase or decrease the rate of water use and welfare. In a two-period framework, the relevant characteristic is the ratio of the periodic marginal benefits of water use. Depending on whether this ratio is convex or concave, the average speed with which the aquifer is depleted decreases or increases when users learn more about the local hydrologic properties of groundwater. We also show that the effect of better information on the welfare of the average producer may be negative even in the situations when, on average, groundwater is allocated more efficiently across irrigation seasons.

Keywords: common property resource, groundwater, information

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1. Introduction

In a seminal article, Gisser and Sanchez (GS) found that the welfare gain from groundwater management policies is likely to be negligible. In an infinite-horizon model of an openly accessed aquifer, they analytically determined the trajectory of water use from competitive (unregulated) pumping, as well as the optimal control trajectory that maximizes the net present value of users' welfare. These trajectories were then computed using parameter values representing the Pecos Basin in New Mexico. The computed difference in the net present values of welfare between the two trajectories, representing the potential gain from optimal management policies, was on the order of 0.01%.

Intrigued by these results, several researchers applied the GS model to other regions and explored new model refinements. A typical such study was that of Kim et al., who applied the basic model to the Texas High Plains based on empirically estimated water demand functions, and also modified the model to allow users to plant different irrigated crops as the aquifer declines. The estimated welfare gains from optimal management were quite small (less than 3.7%). Similarly small gains were found by several other authors including Allen and Gisser, Nieswiadomy, and Fienerman and Knapp. More recently, Holland and Moore found somewhat larger benefits from groundwater management in the presence of other market distortions. Yet in general, Gisser and Sanchez's basic result appears to be robust to changes in many model parameters and assumptions. See Koundouri for a more comprehensive review of this literature.

Another line of research explored whether the GS model's results are sensitive to its more basic assumptions about hydrology and access to the resource. A key variable in groundwater models is the distance between the land surface and the water table, also known as pumping lift. In the GS model, pumping lift can grow without bound as time progresses, implicitly assuming a "bottomless" aquifer that initially holds an infinite volume of water (Brill and Burness). This assumption is potentially important because it ignores the stock externality arising from common property water use (Provencher and Burt). If the supply of water is finite, consumption by one user imposes a cost on other users because they will have less water available in the future. In the GS model, this cost is implicitly set to zero because of the infinite water stock. The GS model considers only the pumping cost externality, which arises because consumption by one user lowers the water table and increases pumping costs for other users.

As noted above, the GS model also assumes an openly accessed aquifer. Competitive pumping is then characterized by complete rent dissipation, where new users enter until the marginal net benefits of water use reach zero each period. In most aquifers, however, access is restricted due to institutional and physical factors (Negri). Extraction always requires ownership of land overlying the aquifer and, usually, acquisition of one of a limited number of water rights. Negri, Dixon, and Provencher and Burt developed dynamic game-theoretic models groundwater use under restricted access. In these models, the number of users is fixed and competitive pumping levels are a Nash equilibrium in closed-loop strategies, where each user takes the decision rules of his rivals as given.[1] These models were useful in identifying the strategic aspects of individual behavior leading to welfare losses. For example, Provencher and Burt were able to analytically decompose the stock and pumping cost externalities from their equilibrium conditions.

Both the GS and the game theoretic models that followed essentially assume a "bathtub aquifer." Groundwater is assumed to flow horizontally at an instantaneous rate, so that consumption by one user has an immediate and equal impact on the water available to other users. The modeling advantage of this assumption is that the spatial distribution of water use can be ignored; all that is relevant to an individual user is the aggregate water stock in period t, which depends only on the aggregate stock and water use in period $t - 1$. In reality, the speed of lateral flow of groundwater (known as aquifer transmissivity) is quite slow on average, but depends on a number of spatially variable features of the aquifer. Thus, at any well location, the change in saturated thickness over time depends on the withdrawals from the well in question as well as the withdrawals from nearby wells—but the influence of neighboring wells varies throughout the aquifer.

[1] As a referee noted, Brooks et al. proved that under plausible conditions for most common property resources, the restricted access equilibrium converges to the rent dissipation outcome as the number of users approaches infinity. Thus, the GS model will approximate the restricted access equilibrium if the number of users is large.

This paper departs from the literature by relaxing the bathtub assumption while accounting for both the stock and depletion externalities. In particular, we introduce a game theoretic model with two users, under the key assumption that individual users have incomplete knowledge of the transmissivity of the aquifer: although a user is aware that his neighbor's water use has some influence on his future water stock, he is uncertain about the degree of this impact. At the extremes, the aquifer becomes a "bathtub" if the transmissivity is instantaneous, or it consists of two independent aquifers (cells), if there are no lateral flows. Given the geologic variability in most aquifers, it is difficult for an individual user to infer the lateral flow parameter from the pumping rates of nearby wells, since these pumping rates are private information.

To focus our analysis on the effect of information, we employ the simplest possible environment with two periods and symmetric users. Under these assumptions, the socially efficient solution is independent of aquifer transmissivity because efficient water use is identical across farmers and lateral flow is zero. This provides a common benchmark to which we can compare the equilibrium outcomes under different information regimes, which in turn allows us to isolate the role of information on the welfare gains from optimal management. After obtaining the equilibrium solutions under incomplete as well as complete hydrologic information, we find that better information may either enlarge or diminish the welfare gains, and the concomitant reduction in water use, resulting from optimal management.

Under the reasonable assumption that information is incomplete in real aquifers, this result implies that models assuming complete information could either under- or over-estimate the hydrologic and welfare effects of optimal management. We establish that the direction of these errors depend on a specific property of users' net benefit functions. Under a plausible specification of the model, this property reduces to a curvature condition on the Arrow-Pratt coefficient of absolute risk aversion, which, in principle, is an empirically testable property. Simple simulations of our model suggest that assuming complete information is more likely to under-estimate the change in water use from optimal management, but is about equally likely to under- or over-estimate the welfare gains.

More generally, it is well known that more information may not improve welfare if it decreases the scope of risk-sharing opportunities among the agents in the economy.[2] This happens because under better (private or public) information about the environment, not only does decision-making becomes better tailored to circumstances, but the set of feasible choices may also become constrained. Although this paper presents a model of groundwater use, our basic result would apply to other cases where users of a spatially distributed resource (such as fish, wildlife, or oil) are uncertain about the degree to which the resource is non-exclusive. In the case of groundwater, exclusivity depends on aquifer transmissivity, while for fisheries it is determined by the rates of biomass dispersal across space (e.g., Sanchirico and Wilen). In general, better public information amplifies (or lessens) the tragedy of the commons and decreases (increases) social welfare when, upon observing the signal, the agents revise upward (downward) the extent to which the resource is non-exclusive and is shared among them. The goal of this paper is to formalize this trade-off and establish conditions under which the effect of better public information about the resource on the (average) intertemporal allocation efficiency and social welfare is unambiguous.

2. Model

There are two periods, $t = 1,2$, and two identical users (farmers), $i = 1,2$. The model of the aquifer is depicted in Figure 1. In the beginning of period 1, the stocks of groundwater on each farm, $x_{i,1}$, $i = 1,2$ are the same. In what follows, in the doubly subscripted variables, the first symbol identifies the farm and the second identifies the period, while the subscripts on functions denote differentiation with respect to the lettered arguments. For concreteness, we normalize the initial stocks of groundwater to unity, $x_{1,1} = x_{2,1} = 1$. Let $u_{i,t}$ denote the amount of groundwater pumped in period t on farm i. The amount that can be used for irrigation on each farm cannot exceed that farm's groundwater stock:

[2] There is a large literature that investigates the value of information in various environments. An important early contribution is Hirschleifer (1971) who demonstrated that in an exchange economy the value of information may be negative. Eckwert and Zilcha (2001) show that sufficiently risk-averse agents may become worse off under better information in production economies. Stiglitz (1984) analyzes the efficiency and welfare in screening models with endogenous information acquisition.

(1) $u_{i,t} \le x_{i,t}$ for $t = 1,2$ and $i = 1,2$.

For simplicity, we assume no aquifer recharge, although recharge could easily be incorporated in the analysis and would not change the qualitative nature of our results.

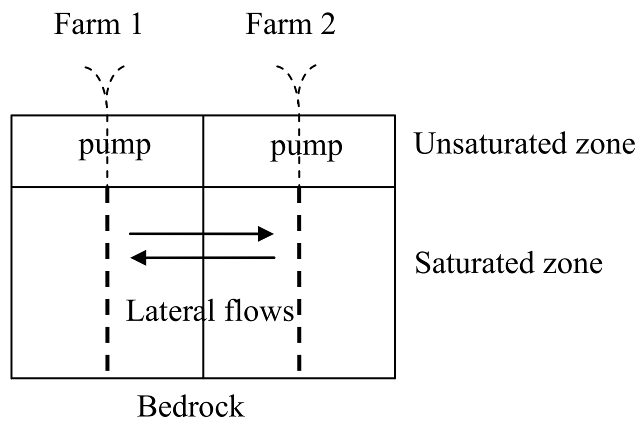

Figure 1. Hydrology of groundwater

2.1 Lateral groundwater flows

Condition (1) above distinguishes our model from the standard common property setting, as it assumes groundwater is essentially a private resource *within* each irrigation season: farmer *i* cannot access the groundwater lying beneath farm *j* within a given period. This assumption reflects the spatial separation of the wells and the notion that groundwater flows too slowly for the extractions to interact within seasons.[3] *Between* periods 1 and 2, however, groundwater will flow toward the well with the greater extraction in period 1. In particular, the inter-period flow of groundwater from farm 2 to farm 1 is given by Darcy's law:

$$Q = -\alpha((x_{1,1} - u_{1,1}) - (x_{2,1} - u_{2,1})) = \alpha(u_{1,1} - u_{2,1}),$$

[3] This assumption is motivated by the fact that a typical irrigation season is brief in comparison to the time elapsed between seasons. In the region overlaying the central High Plains aquifer, for example, the irrigation season for summer crops is about 75 days (mid-June to late August), leaving about 9.5 months between seasons. In reality, extraction at each well creates a "cone of depression" in the groundwater surface that grows wider and deeper as more water is extracted. Lateral flow of groundwater then eliminates these depressions between seasons. Our assumption of intra-seasonally private use presumes either that neighboring cones of depression do not overlap or that the resulting intra-seasonal flow between farms is negligible.

where $\alpha \in [0,0.5]$ summarizes the hydrological properties of the region, $(x_{1,1} - u_{1,1}) - (x_{2,1} - u_{2,1}) = u_{1,1} - u_{2,1}$ is the hydraulic gradient (the difference in hydraulic head between wells).[4] The flow of groundwater from farm 1 to farm 2 is $-Q$. The stocks of groundwater available in period 2 are

(2) $x_{1,2} = x_{1,1} - u_{1,1} + Q = (1-\alpha)(1-u_{1,1}) + \alpha(1-u_{2,1})$,

$x_{2,2} = x_{2,1} - u_{2,1} - Q = \alpha(1-u_{1,1}) + (1-\alpha)(1-u_{2,1})$.

And so, while groundwater is always an intra-seasonally private property resource, $\alpha = 0.5$ corresponds to the inter-seasonally common property resource because it implies that groundwater levels are equalized across farms in period 2, $x_{1,2} = x_{2,2}$, for any pumping in period 1, while $\alpha = 0$ corresponds to the purely private resource.

2.2 Benefits of groundwater use

The net benefits of water use on each farm is given by

(3) $g(u,x)$.

Function g summarizes the benefits due to irrigation net of all costs including the cost of pumping groundwater. For simplicity, the rainfall is the same on each farm in both periods. Net benefits depend on *x* because a larger stock leads to smaller pumping costs as well as higher crop yields due to a faster rate of water delivery. g is assumed to be strictly increasing, strictly concave, and twice differentiable over the relevant domain, and also has the property that $g(0,\cdot) = 0$ and $g_u(0,\cdot) = \infty$.[5] For example, (3) can take the following form:

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[4] $\alpha = kS/L$, where *k* is hydraulic conductivity, *S* is the cross-sectional area of flow, *L* is the distance between wells on each farm (e.g., see Freeze and Cherry).

[5] The assumption that $g(u,x)$ is strictly increasing in u for all $u \in [0,x]$ reflects a situation of absolute water scarcity; all of the water remaining in period 2 will be consumed. If water is not scarce in this sense, so that $g(u,x)$ is decreasing in u for $u \in [\hat{u},x]$, the analysis needs only minor modifications. Also, note that all our results can be obtained under a weaker technical condition than the joint concavity of $g(u,x)$ on $[0,1]\times[0,1]$ such as $g_{uu}(.,x) < 0$ and $0.5[g_{uu}(x,x) + g_{xx}(x,x)] + g_{ux}(x,x) < 0 \quad \forall x \in (0,1)$. To interpret this condition, suppose the current groundwater stock is *x* and that all of this water is consumed in the current period ($u = x$). Under standard assumptions on the irrigation technology (see Peterson and Ding), the cross-partial derivative term is positive, reflecting the fact that the marginal benefits of water use, g_u, will increase with respect to *x* (e.g., due to more rapid water delivery or a smaller pumping lift). And so,

(4) $g(u,x) = v\left(py(u) - c(u,x) - k \right)$,

where p is the per unit price of the crop, y is yield, c is the cost of pumping groundwater, k is the cost of other farming inputs, and v is a utility-of-income function. An empirically estimated specification of (4) is provided in Peterson and Ding.

2.3 Information about the hydrology of the region

We distinguish between two information regimes. Under *complete* information, in period 1 farmers know with certainty the "speed" of lateral groundwater flow, α. Under *incomplete* information, in period 1 farmers view $\tilde{\alpha}$ as a random variable and only know its probability distribution, $\Pr(\tilde{\alpha} \le \alpha) = H(\alpha)$, where H represents the variation in geologic conditions parameters throughout the aquifer.[6] In the latter case, information is assumed to be symmetric across farmers, so that their subjective probabilities, H, are identical.

Farmers maximize the sum of discounted per period profits:

$g(u_{i,1},1) + \beta g(u_{i,2}, x_{i,2})$ subject to (1) and (2),

where $\beta \le 1$ is the discount factor. Let $\pi_i(\alpha)$ and $\pi_i(E[\tilde{\alpha}])$ denote the maximum expected profits attained ex ante by the non-cooperating farmers, $i = 1,2$, respectively under complete and incomplete information about the hydrology of the region.

3. Social planner

Before we turn to the analysis of the equilibrium groundwater intertemporal allocation decisions, we characterize the efficient allocation. We assume that social planner has perfect information about α. Conditional on α, the planner chooses $u^s_{i,t}$, $i = 1,2$, $t = 1,2$, to maximize joint profits (producer welfare)

(5) $W(\alpha) = \max_{u^s_{1,1}, u^s_{2,1}, u^s_{1,2}, u^s_{2,2}} \sum_{i=1,2} g(u^s_{i,1},1) + \beta g(u^s_{i,2}, x_{i,2})$ subject to (1) and (2).

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this condition states that g must be "sufficiently" concave in u and x, so that the cross-partial term does not exceed half of the bracketed term in absolute value.

[6] There is a large variation in local hydrologic properties such as the aquifer's storativity and transmissivity values as well as well-spacing requirements that vary from 4 miles in parts of Kansas to less than 300 feet in Texas (e.g., Brozovic et al., Kaiser and Skiller).

The following result shows that the efficient allocation of groundwater is independent of the speed of groundwater lateral flows (the extent to which the resource is public or private).

Proposition 1. (Efficient pumping) *Efficient intertemporal allocation of groundwater is independent of the speed of lateral flows, and is given by*

(6) $u^s_{i,1}(\alpha) = f^{-1}(1) \;\; \forall \alpha \in [0,0.5]$, *where* $f^{-1}(\cdot)$ *is the inverse function of*

$$f(u) = \frac{g_u(u,1)}{\beta(g_u(1-u,1-u)+g_x(1-u,1-u))}.$$

Proof: First, note that in period 2, the planner optimally exhausts the remaining stock on each farm because *g* is strictly increasing in *u*. This implies that constraint (1) binds for *t* = 2 (i.e., $u_{i,2} = x_{i,2}$), so that (5) can be written

(7) $W(\alpha) = \max_{u^s_{1,1},u^s_{2,1}} \sum_{i=1,2} g(u^s_{i,1},1) + \beta g(x_{i,2}, x_{i,2})$ subject to (1) and (2).

Because (7) is symmetric and concave in $u^s_{1,1}$ and $u^s_{2,1}$, optimality requires that $u^s_{1,1} = u^s_{2,1}$. Because farms are identical, this implies that there are no lateral groundwater flows, $Q = 0$. Additionally, corner solutions are ruled out because g is increasing and concave in each argument, $g(0,.) = 0$, and $g_u(0,.) = \infty$. Substituting the law of motion (2), $x_{i,2} = 1 - u_{i,1}$ for $i = 1,2$, into the objective function and differentiating, the first-order conditions for a maximum are

(8) $g_u(u^s_{i,1},1) - \beta(g_u(1-u^s_{i,1},1-u^s_{i,1}) + g_x(1-u^s_{i,1},1-u^s_{i,1})) = 0$, or

$$f(u^s_{i,1}) = 1.$$

By the concavity of *g*, *f* is a strictly decreasing function, implying the existence of an inverse function, f^{-1}. Applying this inverse function to (8) completes the proof. ■

Even though the lateral flows are possible, Proposition 1 states that it is efficient to pump the same amount of groundwater on each farm because farms are symmetric. This implies that detailed data on α is not needed to obtain the efficient solution. And so, the average efficient pumping of water on each farm in period 1 and the maximum attainable profit is

(9a) $E[u^s_{i,1}(\tilde{\alpha})] = \int_0^{0.5} u^s_{i,1} dH(\alpha) = f^{-1}(1)$, $i = 1,2$,

(9b) $E[\pi^s_i(\tilde{\alpha})] = g(f^{-1}(1),1) + \beta g(1 - f^{-1}(1), 1 - f^{-1}(1))$, $i = 1,2$.

The function *f* in equation (6) can be interpreted as a farmer's intertemporal rate of substitution for water. The numerator represents the benefit of extracting and consuming the marginal unit in period 1, while the denominator is the discounted benefit of saving the marginal unit until period 2. The two terms in the parentheses of the denominator reflect the two types of benefits from saving: g_u is the consumption value of the marginal unit extracted in period 2 and g_x reflects the marginal reduction in pumping costs. Given this interpretation of *f*, condition (8) can be seen as an instance of a well known-result from consumption-savings problems: efficiency requires that agents' intertemporal rate of substitution be set equal to the gross return on savings (i.e., 1 + *r* where *r* is the expected rate of return). From the planner's perspective, the gross return on water saved in the aquifer is exactly one—a unit saved in period 1 is a unit available in period 2.

In what follows, the inverse function f^{-1} and its properties will play important roles. In the context of the planner's problem, this function maps the gross rate of return on groundwater savings into the efficient amount of water consumed in period 1. The easiest case to interpret is when $\beta = 1$ and the water benefits depend only on water use, *u*, and are independent of the groundwater stock. Under these assumptions, $f(u) = g_u(u)/g_u(1–u)$. Function *f* then attains a value of 1 when $u = 0.5$, or equivalently, $f^{-1}(1) = 0.5$. Thus, in the case where pumping costs are unaffected by the stock level and $\beta = 1$, the efficient solution distributes the available water equally across the two periods.

As will be shown below, the same function f^{-1} emerges from farmers' individual decision problems in an unregulated equilibrium. In particular, a farmer's individually optimal consumption in period 1 can be computed by evaluating f^{-1} at the farmer's rationally expected gross return on water saved in the aquifer.

4. Equilibrium

We proceed by first charactering equilibrium allocation under incomplete information. The information regime where farmers have complete knowledge of the realized value of $\tilde{\alpha}$ is a special case of the incomplete information regime. Therefore, we obtain the characterization of equilibrium under complete information as a special case.

4.1. Incomplete information

In this section, we determine equilibrium pumping by both farmers when they know the probability distribution of the lateral flow speed, $H(\alpha)$, but not the local realization of $\tilde{\alpha} \in [0,0.5]$.

In period 2, both farmers optimally exhaust the available stocks of underground water because g is increasing—i.e., $u_{i,2}^n = x_{i,2}(\tilde{\alpha})$ for $i = 1,2$. Here superscript "*n*" stands for "no information". Farmer *i*'s net benefits depend on decisions made in period 1 by virtue of the binding constraint (2); i.e., $x_{i,2}(\tilde{\alpha}) = (1-\tilde{\alpha})(1-u_{i,1}^n) + \tilde{\alpha}(1-u_{j,1}^n)$. The dependence of the groundwater stocks in period 2 on the speed of lateral flows introduces uncertainty into the farmers' decisions problems. And so, in period 1, farmer i chooses $u_{i,1}^n$ to maximize

(10) $\pi_i(E[\tilde{\alpha}]) = \max_{u_{i,1}^n} E[g(u_{i,1}^n,1) + \beta g(x_{i,2}(\tilde{\alpha}), x_{i,2}(\tilde{\alpha}))]$ subject to (1) and (2).

Because the speed of lateral flows only affects period 2 profit, for a given $u_{j,1}^n$, the best response, $u_{i,1}^n$, by farmer $i \neq j$ satisfies

(11) $g_u(u_{i,1}^n,1) - \beta E[(1-\tilde{\alpha})(g_u(x_{i,2}(\tilde{\alpha}), x_{i,2}(\tilde{\alpha})) + g_x(x_{i,2}(\tilde{\alpha}), x_{i,2}(\tilde{\alpha})))] = 0$.

The Nash equilibrium is a pair of pumping levels, $u_{i,1}^n$ (i = 1, 2), which simultaneously satisfy equation (11). This is given in the following proposition.

Proposition 2. (Equilibrium pumping) *If farmers' expected rate of lateral flow is* $E[\tilde{\alpha}]$, *the equilibrium allocation of groundwater is symmetric (i.e., identical across farms) and unique, and is given by*

$$u_{i,1}^n(E[\tilde{\alpha}]) = f^{-1}(1-E[\tilde{\alpha}]),\ i = 1,2.$$

Proof: We will first show that the equilibrium is symmetric. The best response conditions, equation (11), can be written

$$g_u(u_{1,1}^n,1) - \beta E[(1-\tilde{\alpha})q(1-(1-\tilde{\alpha})u_{1,1}^n - \tilde{\alpha}u_{2,1}^n)] = 0$$

$$g_u(u_{2,1}^n,1) - \beta E[(1-\tilde{\alpha})q(1-(1-\tilde{\alpha})u_{2,1}^n - \tilde{\alpha}u_{1,1}^n)] = 0,$$

where $q(x) = g_u(x,x) + g_x(x,x)$.Without loss of generality, suppose the equilibrium were asymmetric with $u_{1,1}^n > u_{2,1}^n$. Then we have

$$g_u(u_{1,1}^n,1) = \beta E[(1-\tilde{\alpha})q(1-(1-\tilde{\alpha})u_{1,1}^n - \tilde{\alpha}u_{2,1}^n)] \geq \beta E[(1-\tilde{\alpha})q(1-(1-\tilde{\alpha})u_{2,1}^n - \tilde{\alpha}u_{1,1}^n)] = g_u(u_{2,1}^n,1)$$

The first inequality follows by assumption, $u_{1,1}^n > u_{2,1}^n$, and the concavity of q (see footnote 5) . Note that $u_{1,1}^n > u_{2,1}^n$ implies that, for any $\alpha \in [0,0.5)$, $1-(1-\alpha)u_{1,1}^n - \alpha u_{2,1}^n$ $< 1-(1-\alpha)u_{2,1}^n - \alpha u_{1,1}^n$, so that $q(1-(1-\alpha)u_{1,1}^n - \alpha u_{2,1}^n) \geq q(1-(1-\alpha)u_{2,1}^n - \alpha u_{1,1}^n)$, and hence $(1-\alpha)q(1-(1-\alpha)u_{1,1}^n - \alpha u_{2,1}^n) \geq (1-\alpha)q(1-(1-\alpha)u_{2,1}^n - \alpha u_{1,1}^n)$. Because the last inequality holds for any $\alpha \in [0,0.5)$, taking the expectation over $\tilde{\alpha}$ does not change the sign of the inequality. And so, we obtained a contradiction as the concavity of g implies that $g_u(u_{1,1}^n,1) < g_u(u_{2,1}^n,1)$ when $u_{1,1}^n > u_{2,1}^n$. Therefore, in any Nash equilibrium $u_{1,1}^n = u_{2,1}^n$. Symmetry implies that the stock in period 2 simplifies to $x_{i,2}(\tilde{\alpha}) = (1 - \tilde{\alpha})(1 - u_{i,1}^n)$ $+ \tilde{\alpha}(1 - u_{i,1}^n) = 1 - u_{i,1}^n$. Substituting this relationship into (11), the best response condition becomes

(12) $$g_u(u_{i,1}^n,1) - (1-E[\tilde{\alpha}])\beta(g_u(1-u_{i,1}^n,1-u_{i,1}^n) + g_x(1-u_{i,1}^n,1-u_{i,1}^n)) = 0.$$

By concavity of g, the left-hand side of (12) is decreasing in $u_{i,1}^n$. And so, equation (12) has a unique solution, $u_{i,1}^n$. By the definition of *f*, this unique solution also satisfies $f(u_{i,1}^n) = 1 - E[\tilde{\alpha}]$; applying the inverse function, f^{-1}, to this equation completes the proof. ■

The intuition for Proposition 2 is that each farmer anticipates a fraction, $E[\tilde{\alpha}]$, of each unit of water he saves in period 1 will "escape" to beneath his rival's farm by period

2. His rationally expected gross return on water saved is then $1-E[\tilde{\alpha}]$, giving him an incentive to increase period 1 consumption above the efficient level in equation (8).

In sum, the groundwater pumped in period 1 and welfare attained by non-cooperative farmers under incomplete information about the speed of lateral flow between the neighboring farms are

(13a) $u_{i,1}^n = f^{-1}(1-E[\tilde{\alpha}])$,

(13b) $\pi_i(E[\tilde{\alpha}]) = g(f^{-1}(1-E[\tilde{\alpha}]),1) + \beta g(1-f^{-1}(1-E[\tilde{\alpha}]),1-f^{-1}(1-E[\tilde{\alpha}]))$.

Next we consider equilibrium under complete information.

4.2. Complete information

Equilibrium pumping under complete information about the speed of groundwater lateral flow, α, can be obtained as a special case of equilibrium characterized above by setting $\Pr(\tilde{\alpha}=\alpha)=1$ for some $\alpha \in [0,0.5]$. Then in period 1, after observing α, farmer i solves

(14) $\pi_i(\alpha) = \max_{u_{i,1}^c} g(u_{i,1}^c,1) + \beta g(x_{i,2}(\alpha), x_{i,2}(\alpha))$ subject to (1) and (2).

Here superscript "*c*" stands for "complete information". By (12) and (13a), we have

(15) $g_u(u_{i,1}^c,1) - (1-\alpha)\beta(g_u(1-u_{i,1}^c,1-u_{i,1}^c) + g_x(1-u_{i,1}^c,1-u_{i,1}^c)) = 0$, or

(16) $u_{i,1}^c(\alpha) = f^{-1}(1-\alpha)$.

The case of $\alpha = 0.5$ is the extreme situation when it is common knowledge that the aquifer is a pure public resource. Because *f* is strictly decreasing, this situation yields the maximum possible equilibrium pumping levels. However, even in this case, users would not exhaust the resource in period 1 and some water would be saved for period 2 (i.e., $u_{i,1}^c(0.5) < 1$).[7]

[7] $u_{i,1} = 1$ would correspond to complete rent dissipation, in the sense that all available resource rents would be captured in period 1. This equilibrium cannot arise because $f^{-1}(0.5) < 1$ if $g_u(0,.) = \infty$. In the case of instantaneous inter-seasonal lateral flow with *n* users, the equilibrium pumping rate converges to $u_{i,1}^c = 1$ as $n \to \infty$ (a demonstration of this is available from the authors). Thus, rents would be fully dissipated in the open access limit. As such, the incomplete rent dissipation in our 2-user model is purely due to restricted access and not because groundwater is a private resource within each season (see footnote 3). We are grateful to a referee for raising this issue.

To an observer who knows only the probability distribution of $\tilde{\alpha}$ over the aquifer, the average water pumped in period 1 and welfare attained by non-cooperative farmers with complete hydrologic information are

(17a) $E[u_{i,1}^{c}(\alpha)] = \int_{0}^{0.5} f^{-1}(1-\alpha)dH(\alpha)$,

(17b) $E[\pi_i(\alpha)] = \int_{0}^{0.5} [g(f^{-1}(1-\alpha),1) + \beta g(1-f^{-1}(1-\alpha),1-f^{-1}(1-\alpha))]dH(\alpha)$.

Next we compare the non-cooperative equilibrium under both information regimes to the efficient solution.

4.3. Equilibrium and efficient allocations

From (13), it is evident that the magnitude of $u_{i,1}^{n}$, and hence farmer profits, only depends on the expected value of $\tilde{\alpha}$. As a result, it is possible to establish upper and lower bounds on $u_{i,1}^{n}$. Regardless of the properties of *H*, its expectation must lie within its support; i.e., $E[\tilde{\alpha}] \in [0,0.5]$. Recall that, by concavity of g, the ratio of marginal benefits of irrigation in period 1 and 2, f, is strictly decreasing in the amount of groundwater used in period 1. It follows that $u_{i,1}^{n}$ is bounded between $f^{-1}(1)$ and $f^{-1}(0.5)$, since f^{-1} is a strictly decreasing function. A comparison of conditions (6), (13a), and (16) reveals that

$$u_{i,1}^{s} = f^{-1}(1) \leq u_{i,1}^{n}(E[\tilde{\alpha}]) = f^{-1}(1-E[\tilde{\alpha}]) \leq f^{-1}(0.5) = u_{i,1}^{c}(0.5).$$

Therefore, farmers with incomplete hydrologic information pump more than the efficient amount in period 1 ($u_{i,1}^{n} \geq u_{i,1}^{s}$). Non-cooperative water use in period 1 is socially efficient in the unique case of a perfectly private resource, when it is common knowledge that $\alpha = 0$ throughout the aquifer. Otherwise the inequality is strict and non-cooperative behavior leads to excessive pumping. We also know that they pump less in period 1 than in the case when it is known that $\alpha = 0.5$ throughout the aquifer ($u_{i,1}^{n}(E[\tilde{\alpha}]) \leq u_{i,1}^{c}(0.5)$). So, although hydrologic uncertainty results in less pumping than the "bathtub" case of $\alpha = 0.5$, it still results in excessive pumping relative to the efficient amount.

These results are depicted in Figure 2 for $g(u,x) = u^{0.8}$ and $\beta = 1$ (see equation (4)). Intuitively, each farmer perceives the marginal benefit of saving the groundwater resource for use in period 2 to equal $1 - E[\tilde{\alpha}] < 1$. This results in "too much" pumping in period 1. When the resource is, at least, to some extent common, $E[\tilde{\alpha}] > 0$, each farmer finds it privately optimal to pump more in order to prevent "stealing" by her neighbor.

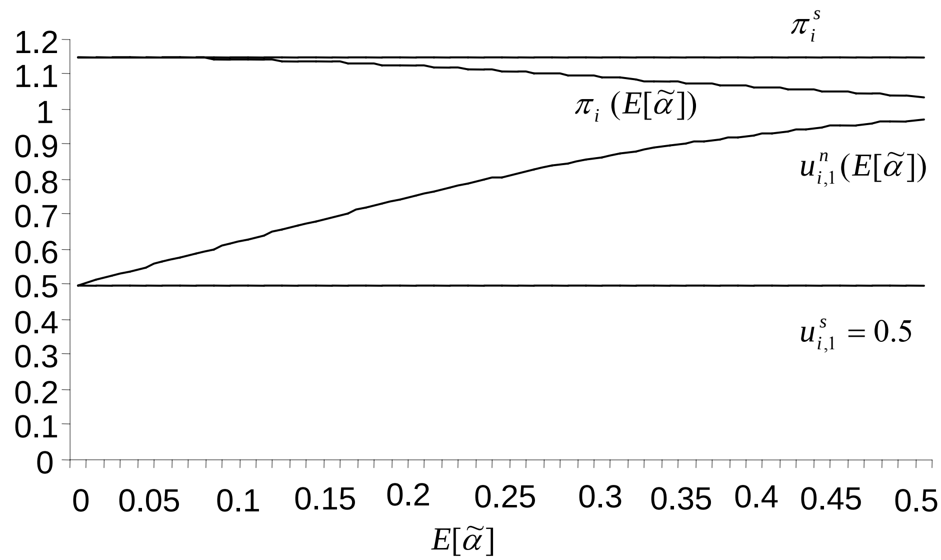


Figure 2. Equilibrium groundwater pumping and profits

The divergence between the equilibrium and efficient outcomes reflects the two types of externalities identified in the literature. This is most easily demonstrated in the complete information case, where the aquifer is characterized by a known value of α. It is instructive to rewrite the equilibrium condition for this case, (15), as follows

(18) $$g_u(u,1) - \beta[g_u(1-u,1-u) + g_x(1-u,1-u)] = -\alpha\beta[\underbrace{g_u(1-u,1-u)}_{\text{stock externality}} + \underbrace{g_x(1-u,1-u)}_{\text{pumping cost externality}}].$$

The left side of this equation can be interpreted as the net marginal benefits of water use to society; economic efficiency is obtained if this is set to zero (condition (8)). As written, however, the right side is always negative provided that $\alpha\beta > 0$, implying that u exceeds the socially efficient level. The two terms in brackets reflect the two externalities. First, users consume more in period 1 because they correctly anticipate that less water will be available for beneficial use in period 2, compared to the efficient solution. This is the stock externality. Second, additional pumping occurs in period 1 because users see that their pumping costs will rise in period 2 due to a diminished groundwater stock. The latter effect is the pumping cost externality. An intuitive property of equation (18) is that the effects of both externalities vary proportionately with the degree to which the resource is inter-seasonally public, α .

An identical analysis will show that the same externalities are present in the incomplete information equilibrium, which is characterized by equation (12). While these results establish that pumping rates exceed the efficient amount in both information regimes, it does not tell us how the pumping rates in the two regimes compare to each other. To be precise, consider an aquifer where α varies according to the known distribution *H*. If farmers are initially uninformed about the hydrology, we seek to understand how pumping rates and welfare would be affected if they all learned the precise local values of α. In analyzing this question, we obtain the main result of this paper: the average speed with which the aquifer is depleted may either increase or decrease when users have better information about local hydrologic properties of groundwater.

5. Complete versus incomplete information

5.1. Average pumping rates

The foregoing analysis demonstrates that, on the one hand, the uncertainty about the speed of lateral groundwater flow (i.e., whether the resource is private or common) may provide a private incentive to pump less to safeguard against a possibly smaller stock in period 2. On the other hand, it may also provide a private incentive to increase pumping to capture more of the common stock. The following proposition provides conditions under which farmers pump more (less) groundwater under incomplete information about the extent to which the resource is public.

Proposition 3. (Information and pumping) *Suppose that the ratio of the marginal benefits of groundwater in period 1 and 2 is concave (convex) in period 1 pumping,* $f'' \leq (\geq) 0$. *Then non-cooperative farmers pump, on average, more groundwater in period 1 under incomplete information about the speed of lateral flows,* $u_{i,1}^{n}(E[\tilde{\alpha}]) \geq (\leq) E[u_{i,1}^{c}(\tilde{\alpha})]$.

Proof: Using the inverse function theorem and differentiating twice yields, $\partial^2 f^{-1}(x)/\partial x^2 = -f''(f^{-1}(x))/(f'(f^{-1}(x)))^3 \leq 0$, because $f' < 0$, and by assumption,

$f'' \leq 0$, where prime and double prime denote first and second derivatives. Then, using (13a) and (17a), by Jensen's inequality,

$$u_{i,1}^{n}(E[\tilde{\alpha}]) = f^{-1}(1-E[\tilde{\alpha}]) \geq (\leq) E[f^{-1}(1-\tilde{\alpha})] = E[u_{i,1}^{c}(\tilde{\alpha})]$$

depending on whether $f''(z) \leq (\geq) 0 \;\; \forall z \in (0.5,1)$. ■

And so, the curvature of the intertemporal marginal rate of substitution, $f(u) = g_u(u,1) / \beta(g_u(1-u,1-u) + g_x(1-u,1-u))$, determines whether non-cooperative farmers pump, on average, more or less groundwater under incomplete information about the speed of lateral flows.

As discussed above, farmers choose pumping levels in period 1 by comparing the intertemporal rate of substitution, f, a strictly decreasing function, to either the expected ($1-E[\tilde{\alpha}]$) or realized ($1-\alpha$) gross return on water saved, depending on the information regime. As also shown above, both regimes result in inefficiently large pumping levels because farmers' perceived return on water savings are smaller than that perceived by the planner, who sets $f = 1$. Proposition 3 fully characterizes how the pumping levels in the two information regimes compare to each other. Under incomplete information (where farmers set $f = 1-E[\tilde{\alpha}]$), the pumping level will be only "a little larger" than the efficient amount if *f* is convex, while pumping will be "much larger" than the efficient quantity if *f* is concave. In the complete information case, in contrast, there is a distinct equilibrium point for each realization of α. By Jensen's inequality, the average of these points will geometrically lie above *f* if it is convex and below *f* it is concave. Complete information therefore yields more (less) pumping than incomplete information when *f* is convex (concave).

5.2. Welfare comparison

In the previous section, we showed that the effect of uncertainty about the hydrologic properties of groundwater on the average pumping in period 1 depends on the curvature of the ratio of marginal benefits of irrigation. The effect of uncertainty about the environment on the expected welfare is somewhat more subtle because welfare depends not only on the average deviation of equilibrium pumping from the efficient allocation but also on the higher moments of its distribution. The following result shows that welfare is unambiguously higher under incomplete information when the ratio of the marginal benefits is convex. By Proposition 3, this assures that poorly informed farmers, on average, pump groundwater more efficiently compared with the pumping under complete information. The proof of the next result is in the Appendix.

Proposition 4. (Information and welfare) *Suppose that the ratio of the marginal benefits of groundwater in period 1 and 2 is convex in period 1 pumping,* $f'' \geq 0$. *Then non-cooperative farmers attain higher expected welfare under incomplete information about the speed of lateral flows,* $E[\pi_i(\tilde{\alpha})] \leq \pi_i(E[\tilde{\alpha}])$.

In contrast with Proposition 3, the concavity of f is not sufficient to guarantee that farmers are better off under complete information. Note that, by Proposition 3, farmers, on average, allocate groundwater more efficiently (i.e., pump less in period 1) under complete information if $f'' \leq 0$. However, the benefits associated with the stability of pumping rates arising due to the lack of information about local conditions may outweigh the average gain in efficiency.

This is illustrated in Figure 3, where $g(u) = u^{\gamma}$, $\gamma \in (0,1)$, β=1, and the speed of lateral flows takes values $\alpha = 0$ and $\alpha = 0.5$ with equal probability.[8] In this example, farmers' utility levels are defined over their water use quantities, *u*, and function *g* reflects risk preferences characterized by constant relative risk aversion, where $1-\gamma$ is the relative risk aversion parameter. Farmers are better off under incomplete information, $E[\pi_i(\tilde{\alpha})] \leq \pi_i(E[\tilde{\alpha}])$, if they are sufficiently risk averse, $\gamma \leq 0.8615$. On the other hand, if the marginal benefits of irrigation are approximately constant, farmers are, on average, better off when they are able to adjust pumping rates in response to the local hydrological properties, $E[\pi_i(\tilde{\alpha})] \geq \pi_i(E[\tilde{\alpha}])$.

Correspondingly, the returns to water management vary depending on the accuracy of farmers' information and their attitudes to risk: $\pi_i^s - E[\pi_i(\tilde{\alpha})]$

[8] Gollier discusses plausible upper bounds on the measure of relative risk aversion, 1-γ (note that γ=1 corresponds to risk-neutrality).

$\geq (\leq) \pi_i^s - \pi_i(E[\tilde{\alpha}])$ as $\gamma \leq (\geq) 0.8615$, where $\pi_i^s = (1/2)^\gamma + (1/2)^\gamma$. The returns to water management are the highest when farmers are knowledgeable and sufficiently risk-averse, or when farmers are uninformed about local hydrologic properties and approximately risk-neutral.

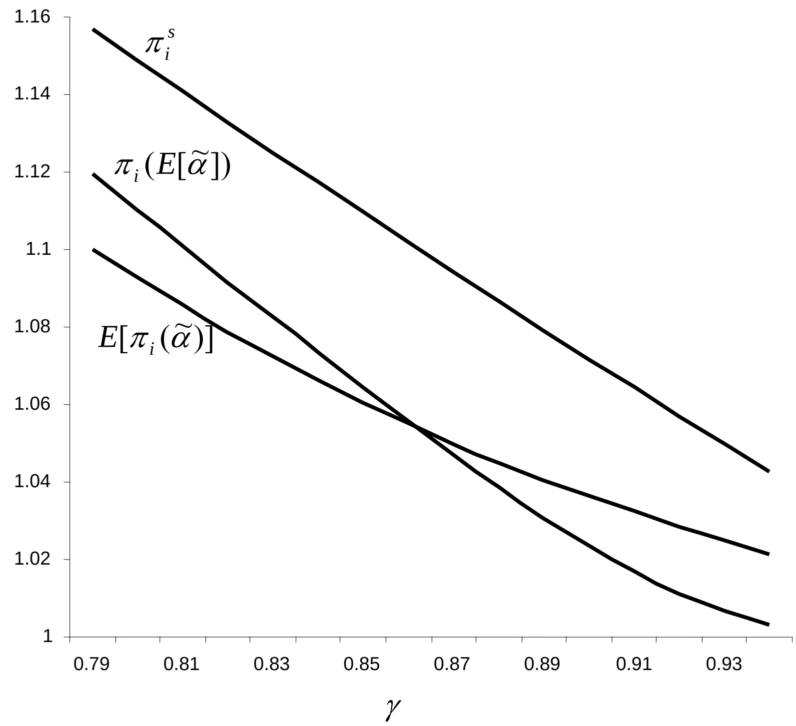


Figure 3. Welfare and information about the hydrology

Next we discuss conditions under which farmers pump more or less in the presence of incomplete information about the hydrologic properties of the region in greater detail.

5.3. Convexity/concavity of the ratio of marginal per period benefits of irrigation

While Propositions 3 and 4 fully characterizes circumstances when incomplete information about the environment may either moderate or amplify the dynamic inefficiency of groundwater use, it is not apparent what determines the curvature of the ratio of the marginal per period benefits, f. To this end, consider the following specification (see (4)):

(19) $g(u,x) = v(py(u,x) - c(u,x) - k)$,

where $y = \bar{y} + u$ is the linear production function, and $c(u,x) = u(c_0 + c_1(1-x))$ is the per unit cost of pumping groundwater: $c_0 > 0$ is the per unit cost of irrigation and $c_1 > 0$ is the per unit lifting cost associated with the height of the water table.

A little work shows that the ratio of the marginal benefits is convex (concave), $f'' \geq (\leq) 0$, if

(20) $m_1^2 A'(r_1) - m_2^2 A'(r_2)$

$$\leq (\geq) 4c_1(A(r_1)m_1 / m_2 + A(r_2)) + (m_1 A(r_1) + m_2 A(r_2))^2$$

$$\forall u \in [0.5,1],$$

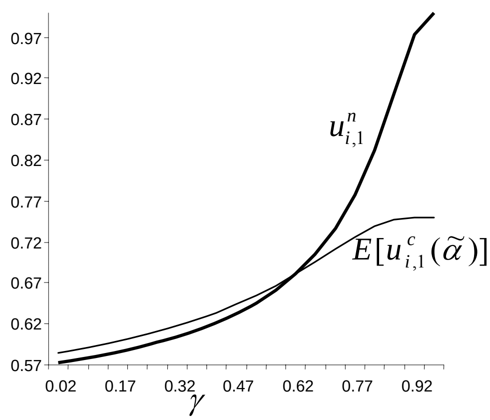
where $A(u) = -v''(.)/v'(.)$ is the Arrow-Pratt measure of absolute risk-aversion, $r_1 = p\bar{y} + (p - c_0)u - k$ and $r_2 = p\bar{y} + (1-u)(p - c_0 - c_1 u) - k$ are per period net revenues, and $m_1 = p - c_0$, $m_2 = p - c_0 - c_1(1-2u)$ are the marginal products of irrigation in periods 1 and 2. And so, keeping everything else equal, the smaller are the savings due to a higher stock of groundwater, c_1, the more likely it is that (20) holds with "$\geq$". However, (20) must hold with "$\leq$" for preferences that are characterized by either constant or "sufficiently" large absolute risk aversion.

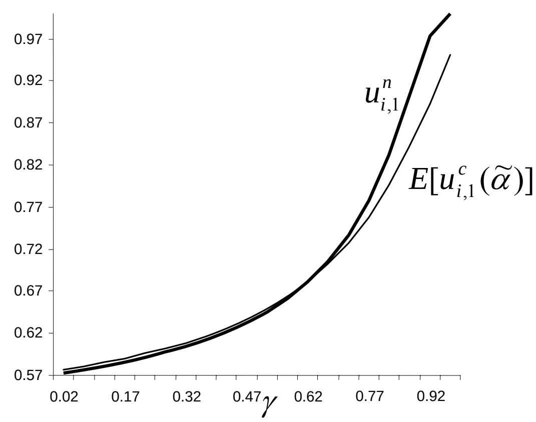
To provide a better sense as to what determines the curvature of the ratio of marginal benefits, consider the case where there is no "pumping cost externality" associated with the level of the groundwater stock, $g_x(u,x) \equiv 0$, or $c_1 = 0$. Also, for simplicity, let $p - c_0 = 1$, $\bar{y} = k = 0$. Then (20) reduces to

(21) $A'(u) - A'(1-u) \leq (\geq)(A(u) + A(1-u))^2 \ \ \forall u \in [0.5,1]$,

It is natural to assume that the coefficient of absolute risk aversion is decreasing in wealth. If the Arrow-Pratt measure of risk-aversion is "sufficiently" convex, farmers, on average, pump *more* when they are not sure about the hydrology of the region (i.e., to what extent their ground water resource is private). On the other hand, if A is not too convex (or is concave), (21) cannot hold with "$\geq$" sign, and farmers, on average, pump *less* when they are have less precise information about the environment.

For example, (21) is satisfied with "≤" sign for some commonly used utility functions such as $g(u,x) = u - (a/2)u^2$, $g(u,x) = -e^{-u}$, or $g(u,x) = \ln(u)$. On the other hand, for preferences that are characterized by constant relative risk aversion (CRRA): $v(u) = u^{\gamma}$, $\gamma \in (0,1)$, (21) may hold with either sign. In that case, absolute risk aversion is decreasing and convex in wealth, $A'(u) \geq A'(1-u)$ $\forall u \in [0.5,1]$, and (21) holds with "≥" for $u \geq 1 - \gamma/2$, and "≤" for $u \leq 1 - \gamma/2$. And so, it appears plausible that non-cooperative farmers with CRRA preferences may pump *more* as a result of incomplete information about the speed of groundwater flows. Nonetheless, there is no reason to assume that even preferences characterized by CRRA exhibit the required degree of convexity of absolute risk aversion. The following example illustrates.





a) Two-point probability distribution of $\tilde{\alpha}$ b) Uniform probability distribution of $\tilde{\alpha}$

Figure 4. Average pumping under complete and incomplete information

Example. Let $g(u) = u^{\gamma}$, $\gamma \in (0,1)$, and $\beta=1$. In case (a), the speed of lateral flows takes values $\alpha = 0$ and $\alpha = 0.5$ with equal probability. In case (b), the speed of lateral flows is uniformly distributed on $[0,0.5]$. The average pumping under complete and incomplete information as a function of γ are plotted in Figure 4. Of course, pumping rates under incomplete information are identical in case (a) and (b) because the average speed of lateral flows is the same, $E[\tilde{\alpha}] = 0.25$. Also, the efficient pumping rate is $u_i^s = 0.5$.

Farmers pump less under incomplete information, if the farmers are sufficiently risk-averse, $\gamma \leq 0.615$. However, the extent of the discrepancy due to the private information is relatively small. If the farmers are weakly risk-averse (γ is close to 1), they pump more under incomplete information and the extent of the discrepancy can be *large*. Figure 5 summarizes the comparison of pumping rates and farmer welfare by combining the results in Figures 3 and 4(a).

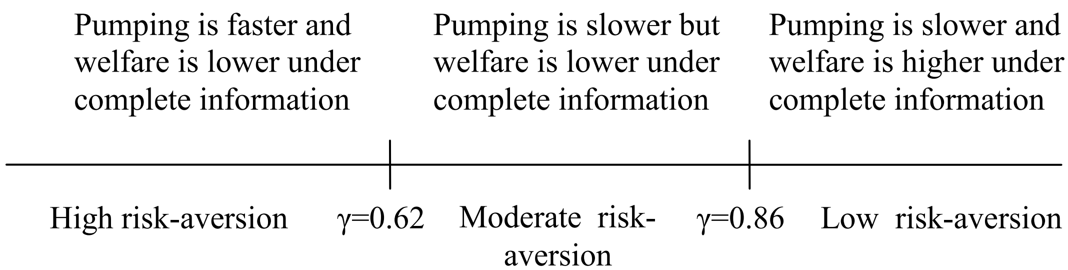


Figure 5. Comparison of the average pumping rates and welfare

6. Model Extensions and Discussion

To isolate the role of information about the local hydrologic properties, we consider the simplest possible dynamic spatial setting with (i) a two-cell aquifer, (ii) two periods, (iii) identical producers, and (iv) either complete or no information. The derived conclusions are robust to some but not all of these simplifying assumptions. For example, the results can be easily extended to the case when farmers observe a signal that provides partial but not complete information about the local hydrologic properties of the aquifer such as porosity and storativity. Then the informativeness of signals can be ranked based on the sensitivity of the conditional expected value of the likely speed of lateral flows to the observed signal. This criterion is less restrictive than the Blackwell's sufficient statistic approach. Also, the analysis carries over with only slight modifications to the case of multiple users as long as they are symmetric as in Eswaran and Lewis.

The analysis of the effect of better information about the environment on equilibrium allocation in a more realistic setting, where one or more of conditions (i) – (iii) are relaxed, is more complicated. In a multi-cell aquifer, it is likely that equilibrium pumping rates differ across, even otherwise identical, farmers. In particular, it can be shown that farmers that are closer to the center of the aquifer pump groundwater faster than farmers that are farther away (however, this relationship may not be monotone). Therefore, incompleteness of information has potentially important distributional consequences across farms.

In a multi-period setting, information about the hydrology of the region impacts not only the speed of pumping but also the "useful" lifetime of the aquifer. Consider an environment with infinite time-horizon, limited entry, diminishing marginal product of water, per period fixed cost, $g(0,x) = -F < 0$, and no recharge. Then it can be shown that the lifetime of the aquifer may increase or decrease when producers are better informed about the speed of lateral flows depending on the properties of the water benefits and the discount factor. Allowing for a rechargeable aquifer does not change the results as long as the statistic of interest is the time before the level of groundwater in the aquifer falls below a certain level.

The result that the better information about the extent of interconnectedness among the users has an ambiguous effect on the efficiency also extends to renewable resources such as fisheries.[9] In the simplest setting where the rate of growth does not depend on the density of biomass (fish population) and the biomass dispersal is proportional to the difference in biomasses across locations, the equations of motion between the two locations (patches), (2), become:

$$x_{1,2} = a(x_{1,1} - u_{1,1} + Q) = a((1-\alpha)(1-u_{1,1}) + \alpha(1-u_{2,1})),$$

$$x_{2,2} = a(x_{2,1} - u_{2,1} - Q) = a(\alpha(1-u_{1,1}) + (1-\alpha)(1-u_{2,1})),$$

where $a - 1 \geq 0$ is the rate of growth of a renewable resource (e.g., fish population). This formulation presumes that the time during which the resource is extracted is short compared with the time that elapses between the episodes of extraction. Then all of our results continue to hold if we redefine the intertemporal marginal rate of substitution as[10]

$$f(u;a) = \frac{g_u(u,1)}{\beta[g_u(a(1-u),a(1-u)) + g_x(a(1-u),a(1-u))]}.$$

[9] See Sanchirico and Wilen (and the references cited therein) for a state-of-the-art analysis of spatial management of renewable resources under the assumption of perfect information.
[10] Of course, the result that the optimal allocation does not depend on the degree of user interconnectedness (the properties of spatial dispersal) will not hold when users are asymmetric.

Finally, suppose that the users that are located in area overlying the aquifer are heterogeneous in their derived demand for groundwater due to the differences in acreage, soil types, availability of surface water, environmental regulations, etc. Then they likely have different incentives to achieve a more dynamically efficient allocation and to learn more about the hydrologic properties of the groundwater resource. Another important possibility not explored in this paper is that farmers have asymmetric information: some producers may be better informed about the local hydrologic properties than others. Understanding the effect of private and public information on natural resource exploitation in the framework that incorporates these and other realistic features is left for future research.

7. Conclusions and Implications

This paper departs from the existing literature in that it explicitly relaxes the "bathtub" aquifer assumption routinely used in the models of groundwater use. Because it is patently unrealistic to assume that producers have perfect knowledge of the local hydrologic properties, we ask the question: What is the effect of incomplete information about the speed of lateral groundwater flows on equilibrium pumping rates and producer welfare? We find that the answer depends on rather subtle properties of the production technology as well as the nature of uncertainty about the speed of lateral flows.

A somewhat counter-intuitive finding is that the effect of information on water use and welfare may be of opposite signs. This can be understood as a consequence of spatial variability. If better information becomes available, users in different locations will either increase or decrease extraction rates, depending on whether the newly learned spatial interactions are stronger or weaker than initially believed. In the areas where resource extraction increases dramatically, welfare will fall sharply. Due to the concavity of the net benefits function, even if extraction rates decrease on average, thereby bringing the system as a whole closer to the efficient path, expected welfare may fall. This implies that (unobservable) changes in welfare cannot be directly inferred from (observable) changes in resource extraction rates.

Users' beliefs about spatial externalities are a sort of self-fulfilling prophecy: if users believe externalities are small or non-existent, they will behave accordingly, and resource use will be approximately efficient. It is in the cases where users believe externalities are significant where policies must be designed with care. Educating and informing users about the true resource dynamics in such cases may actually accelerate their extraction rates. That is, better information is not a substitute for correcting the underlying externalities. As in other cases where incomplete information aggravates other distortions, better information will be welfare-improving only if the distortions are eliminated first.

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Appendix

Proof of Proposition 3:

By (13b) and (17b), the effect of incomplete information on the expected welfare depends on the curvature of $\pi_i(\alpha) = g(f^{-1}(1-\alpha),1) + \beta g(1-f^{-1}(1-\alpha),1-f^{-1}(1-\alpha))$.

Differentiation yields $\pi_i'(\alpha) = -\beta(g_u(f^{-1}(1-\alpha),1) - \beta[g_u(1-f^{-1}(1-\alpha),1-f^{-1}(1-\alpha)) + g_x(1-f^{-1}(1-\alpha),1-f^{-1}(1-\alpha))]) / f'(f^{-1}(1-\alpha)) = \alpha\beta[g_u(1-f^{-1}(1-\alpha), 1-f^{-1}(1-\alpha)) + g_x(1-f^{-1}(1-\alpha),1-f^{-1}(1-\alpha))] / f'(f^{-1}(1-\alpha)) \leq 0$, where the first-order condition (12) is used to obtain the last equality. Differentiating twice yields $\pi_i''(\alpha) = \beta\pi_i'(\alpha)/\alpha + \alpha\beta[g_{uu}(1-f^{-1}(1-\alpha),\ 1-f^{-1}(1-\alpha)) + g_{xx}(1-f^{-1}(1-\alpha), 1-f^{-1}(1-\alpha)) + 2g_{ux}(1-f^{-1}(1-\alpha),1-f^{-1}(1-\alpha)) + (g_u(1-f^{-1}(1-\alpha),\ 1-f^{-1}(1-\alpha)) + g_x(1-f^{-1}(1-\alpha),1-f^{-1}(1-\alpha)))\ f''(f^{-1}(1-\alpha))/f'(f^{-1}(1-\alpha))]/(f'(f^{-1}(1-\alpha)))^2 \leq 0$. The inequality follows because each term is non-positive since $\pi_i'(\alpha) < 0$, g is concave, $f' < 0$, and by the maintained hypothesis, $f'' \geq 0$. Hence, the result follows by Jensen's inequality. ■