Quarterly Storage Model of U.S. Cotton Market: Estimation of the Basis under Rational Expectations

Oleksiy Tokovenko[1]
Lewell F. Gunter

Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Orlando, FL, July 27 - 29, 2008

[1]Oleksiy Tokovenko (leksa@uga.edu) is a graduate research assistant and Lewell F. Gunter (lgunter@agecon.uga.edu) is Professor in the Department of Agricultural and Applied Economics, College of Agricultural and Environmental Sciences, University of Georgia

Monthly Storage Model of U.S. Cotton Market: Estimation of the Basis under Rational Expectations

Oleksiy Tokovenko Lewell F. Gunter

Abstract

The paper outlines an approach to estimation and analysis of the futures basis in the U.S. cotton market under weakly rational expectations. Given the model specification derived from the underlying dynamic profit optimization problem of the dealers, the intermediary market model is estimated using the self-organizing state-space (SOSS) approach. Estimation results are used to evaluate the prediction power of the method and test the main assumptions about the existence and consistency of the subjective rational expectations incorporated in the model.

Research in progress. Do not quote without authors' permission.

Introduction

The contemporary theory of commodity markets attempts to model the behavior of commodity prices in order to explain the factors that generate the price fluctuations and thus to make predictions of future prices, basis and market response. Assumptions about rationality of price expectations have been widely used in empirical studies in order to provide dynamic links and close the market model. Although the rational expectations of the market prices are often efficiently approximated through the observed futures prices on the relevant commodities, this approach is more appropriate to studying of contemporaneous or past market history as well as to making short period predictions based on the current information. An alternative, endogenous modeling of market expectations allows one to estimate the effects of structural changes in the model and thus analyze market performance under alternative scenarios (e.g. Miranda and Helmberger (1988)). Applications of endogenous rational expectations models to the analysis of agricultural commodity markets in a fully stochastic-dynamic setting can be found, for example, in Miranda and Glauber (1993) and Peterson and Tomek (2005) . The main issue with this class of models is using parameterized expectations as a function of the current value of state variables, such as carryover of commodity. We propose to treat the values of future prices as unobserved market expectations applying the idea behind the state-space approach to time-series analysis. In such a framework expected values of prices and basis risk at a future period can be learned through the information available up to the current period. We suggest to impose the weaker condition for rationality of the model behavior (such as consistency of price expectations or asymptotic rationality) that will serve as an important argument for the model identification.

Objectives

The objective of this paper is to develop an alternative estimation algorithm for the commodity storage market model with nonlinear rational expectations and to use the underlying structural model to obtain accurate estimates and forecasts of the futures basis at different points of time that can be used to support the marketing decisions made under uncertainty (see, e.g. Taylor, Dhuyvetter and Kastens (2006) and Lai, Myers and Hanson (2003)).

Model

In the case of storable commodity markets the analysis of futures markets can be focused on the decisions of the dealers who serve as intermediaries between the farmers and consumers. Consider risk-averse dealers with their risk preferences represented by an increasing and concave von Neumann-Morgenstern utility function of profit, $U(\pi_t)$. Assume that at any time period t the intermediates face demand, output and relative price uncertainty in the absence of input price uncertainty. At the beginning of each decision period dealers choose an amount of the commodity s_t to purchase at the spot market at the current market price p_t that can be sold next period at the expected price $\tilde{p}_{t+h}$ or held for the future transaction if the higher value of inventory is expected. The dealers charge the sellers and buyers commissions $v(s_t)$ which establish the nonspeculative income of the intermediates. They also carry storage, financing and distributional costs $\phi(s_t)$ associated with the amount of commodity purchased. In order to reduce the risk associated with the spot price uncertainty the dealers take a position at the futures market by selling x_t futures contracts at price f_t for delivery at time T. At time $t+h$ the value of one contract will be defined by the expected price $\tilde{f}_{t+h}$ therefore the dealers can make profit by adjusting their futures position based on the expected difference in futures prices of two periods. With this assumptions the expected profit of dealers at time $t+h$ is defined by

$$\tilde{\pi}_{t+h} = (\tilde{p}_{t+h} - p_t)s_t + v(s_t) - \phi(s_t) + x_t(f_t - \tilde{f}_{t+h}) \tag{1}$$

At present, we are interested in the one period decisions therefore $h = 1$ is fixed. By recognizing the intertemporal arbitrage opportunities dealers seek to maximize the expected discounted stream of their utility of profits over the infinite horizon (we consider the rollover hedging using consequent overlapping contracts):

$$\max E_t \sum_{t=0}^{\infty} \beta^t U(\tilde{\pi}_t) \tag{2}$$

where β is the discount rate and E_t is the conditional expectation operator given information $\mathfrak{F}_t$ at time t. At each period $t = 0, 1, \ldots,$ the decisions of dealers are subject to the stochastic constraints arising from the optimal actions of their counterparts. Thus the spot market decisions are limited by the following transition equation that defines the supply of inventory investment as

$$s_{t+1} = s_t + g(p_{t+1}) + \epsilon_t \tag{3}$$

where $g(p_t)$ is the inverse function that maps current production, export and consumption levels into the equilibrium price on the positive half line, while ϵ_t combines the supply and demand shocks of time t. The inventory choice assumes $s_t \geq 0$ for all periods which introduces additional nonlinearities into the conditional expectations functions. Simultaneously, the choice of amount to hedge x_t bounds the behavior of the futures price through the weighted value of the expected spot price $\tilde{p}_{t+1}$ and the risk premium r resulting from the net hedging pressure

$$f_{t+1} = \alpha \tilde{p}_{t+1} + r x_t + \nu_t \tag{4}$$

To solve the stochastic optimization problem (2) subject to stochastic constraints (3) and (4) along the lines of Chow (1992) we introduce Lagrange multipliers λ_t and μ_t

and set to zero the derivatives of the Lagrangian expression

$$L = E_t \sum_{t=0}^{\infty} \Big[\beta^t U(\tilde{\pi}_t) - \beta^{t+1}\lambda_{t+1}(s_{t+1} - s_t - g(p_{t+1}) - \epsilon_t) \\ - \beta^{t+1}\mu_{t+1}(f_{t+1} - \alpha\tilde{p}_{t+1} - rx_t + \nu_t) \Big] \tag{5}$$

with respect to the action variables s_t and x_t and state variables p_t and f_t, given the expectations of the futures and spot prices are known. In this study we place an emphasis on the existence of the subjective expectations, formed by dealers conditional on the past and present information $\mathfrak{F}_t$ available to them. The subjective price expectations serve as the hidden states of the system that can be revealed once the system response is observed. To make a prediction given $\mathfrak{F}_t$ we need to bound the time path of $\tilde{p}_{t+1}$ and $\tilde{f}_{t+1}$ using the optimal conditions obtained from maximizing the Lagrangian function (5). Differentiating (5) with respect to s_t, p_t and p_{t+1} and simplifying yields

$$-E_t\beta\lambda_{t+1} = E_t U'(\tilde{\pi}_{t+1})[(\tilde{p}_{t+1} - p_t) + v'(s_t) - \phi'(s_t)] + \lambda_t \tag{6}$$

$$\lambda_t = -E_t U'(\tilde{\pi}_{t+1}) s_t / g'(p_t) \tag{7}$$

$$-E_t\beta\lambda_{t+1} = E_t[\beta U'(\tilde{\pi}_{t+2}) s_{t+1}/g'(p_{t+1})] \tag{8}$$

Now, by substituting (7) and (8) into (6) and collecting the terms we derive the intertemporal substitution condition that relates subjective spot price expectations of two consecutive periods

$$E_t \Big[\frac{s_{t+1}\beta U'(\tilde{\pi}_{t+2})}{g'(p_{t+1})U'(\tilde{\pi}_{t+1})} \Big] = (\tilde{p}_{t+1} - p_t) + v'(s_t) - \phi'(s_t) - s_t/g'(p_t) \tag{9}$$

which can be rewritten as

$$\frac{s_{t+1}\beta U'(\tilde{\pi}_{t+2})}{g'(p_{t+1})U'(\tilde{\pi}_{t+1})} = (\tilde{p}_{t+1} - p_t) + v'(s_t) - \phi'(s_t) - s_t/g'(p_t) + \eta_{t+1} \tag{10}$$

by introducing the error term η_{t+1}. By analogy, the second set of optimal conditions is obtained by differentiating (5) with respect to x_t and f_{t+1} and then simplifying to get

$$-E_t\beta\mu_{t+1} = E_t U'(\tilde{\pi}_{t+1})(f_t - \tilde{f}_{t+1})/r \tag{11}$$

$$-E_t\beta\mu_{t+1} = -E_t[\beta U'(\tilde{\pi}_{t+2})x_{t+1}] \tag{12}$$

Substituting (12) into (11) and collecting the terms yields the second intertemporal substitution condition that relates subjective futures price expectations of two consecutive periods

$$E_t\Big[\frac{rx_{t+1}\beta U'(\tilde{\pi}_{t+2})}{U'(\tilde{\pi}_{t+1})}\Big] = \tilde{f}_{t+1} - f_t \tag{13}$$

Again we introduce the error term ω_{t+1} and rewrite (13)

$$\frac{rx_{t+1}\beta U'(\tilde{\pi}_{t+2})}{U'(\tilde{\pi}_{t+1})} = \tilde{f}_{t+1} - f_t + \omega_{t+1} \tag{14}$$

The final optimality condition we need is precisely (4). When the corresponding spot price value p_{t+1} is subtracted from both sides of this constraint it provides useful decomposition of the forecast error

$$f_{t+1} - p_{t+1} = (\alpha\tilde{p}_{t+1} - p_{t+1}) + rx_t + \nu_t \tag{15}$$

where the deviation of the futures price from the objective market expectation, that would otherwise be rational in the sense of Muth (1961), can be explained by the existence of the endogenous risk premium rx_t, unavoidable error ν_t and the Bayesian error $\alpha\tilde{p}_{t+1} - p_{t+1}$. The last component characterizes the difference between the subjective and the objective price expectations which is the key argument for relaxing the perfect rational expectations assumption in a favor of it's asymptotic equivalent. For practical purposes we assume a constant relative risk aversion utility function such that

$$U(\tilde{\pi}_t) = \begin{cases} \tilde{\pi}_t^{1-\gamma}/(1-\gamma), & \text{if} \quad \gamma \neq 1 \quad ; \\ \log(\tilde{\pi}_t), & \text{if} \quad \gamma = 1 \quad . \end{cases} \tag{16}$$

where $\gamma > 0$ denotes a measure of relative risk aversion of dealers. This particular form of the utility function implies that $U'(\tilde{\pi}_t) = \tilde{\pi}_t^{-\gamma}$ for all admissible values of γ.

Estimation

Given the specification derived from the underlying dynamic optimization problem the market model is estimated using the self-organizing state-space (SOSS) method introduced in Kitagawa (1998) implemented through the genetic algorithm type resampling of non-linear particle filter suggested in Higuchi (1997). The general parametrized state-space model can be described as

$$\boldsymbol{k}_{t+1} = H(\boldsymbol{k}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{1t}) \tag{17}$$

$$\boldsymbol{y}_t = M(\boldsymbol{k}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{2t}) \tag{18}$$

where H and M are the parametrized state transition and measurement equations, $\boldsymbol{k}_t$, $\boldsymbol{u}_t$, $\boldsymbol{y}_t$ are the state, control and measurement vectors, and $\boldsymbol{\epsilon}_{1t}$ and $\boldsymbol{\epsilon}_{2t}$ are the process and measurement noise vectors, all at period t. Since the state-space systems in (17) and (18) are often non-linear and have non-Gaussian disturbances, the estimation is complicated since one have to solve computational problems involving numerical integration over multiple dimensions of the state space (Tanizaki (1996), Ristic, Arulampalam and Gordon (2004)). In this case the tool known as the particle filter (PF) based on Monte Carlo methods can be used for smoothing and filtering purposes. In particle filter algorithms arbitrary non-Gaussian densities are approximated by many particles that can be considered realizations from the corresponding distributions. Among the most popular PF algorithms are Monte Carlo filter introduced in Kitagawa (1993, 1996) and Tanizaki and Mariano (1998) and bootstrap filter (sampling importance resampling filter) developed in Gordon, Salmond and Smith (1993). Using relevant posterior densities and recurrent relations, it is possible to construct the simulated likelihood function of interest. However, unlike in the signal extraction applications the system parameters are often unknown and have to be estimated. Unfortunately, the simulated nature of the likelihood function makes conventional statistical approach maximum likelihood method almost impractical, especially in the case of high-dimensional problems. Kitagawa (1998) refers to two factors that are the sources of limitations. First, the non-Gaussian filtering and smoothing procedures are computationally intensive and thus it is extremely hard to use the iterated numerical optimization algorithms for maximizing the likelihood function effectively for practical purposes. Second, the particle filter likelihood function is approximated using only the finite sample of particles and therefore is the subject to the sampling error inherent in the Monte-Carlo approximation. In order to obtain precise maximum likelihood estimates and inference about them one should reduce the sampling error by using a very large number of particles or by parallel application of many particle filters, which increases the computational costs dramatically. Several approaches were proposed to deal with these difficulties by introducing the class of self-organized time series models, estimated in the framework of the genetic algorithm (GA) particle filter (Higuchi (1997)) and the self-organizing state-space model (Kitagawa (1998)). The GA filter is based on the strong parallelism between the Monte Carlo filter and the genetic algorithm. It replaces the prediction step in the MC filter with the mutation and crossover steps in GA to avoid the estimation of parameters of the transition equation (17). In latter approach, the unknown parameters of the model are treated as the additional state variables so that both the state and the parameters are estimated simultaneously using filtering and smoothing. Instead of estimating the parameters of the model, Kitagawa (1998) suggests to implement a Bayesian estimation by augmenting the state vector with the vector of model parameters $\boldsymbol{\theta}$ as $\boldsymbol{z}_t = [\boldsymbol{k}_t, \boldsymbol{\theta}_t]^T$. Given the augmented state vector $\boldsymbol{z}_t$ the self-organizing form of the original state-space model is

$$\boldsymbol{z}_{t+1} = H^*(\boldsymbol{z}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{1t}) \tag{19}$$

$$\boldsymbol{y}_t = M^*(\boldsymbol{z}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{2t}) \tag{20}$$

where $H^*(\boldsymbol{z}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{1t}) = [H(\boldsymbol{k}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{1t}), \boldsymbol{\theta}_t]^T$ and $M^*(\boldsymbol{z}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{2t}) = M(\boldsymbol{k}_t, \boldsymbol{u}_t, \boldsymbol{\epsilon}_{2t})$. Given the particular form of utility function we accepted, the Euler equations derived in (10) and (14) reduce to the transition equations of the state space model for the original problem as follows

$$\tilde{\pi}_{t+2}^{-\gamma} = \frac{\tilde{\pi}_{t+1}^{-\gamma} g'(p_{t+1})[(\tilde{p}_{t+1} - p_t) + v'(s_t) - \phi'(s_t) - s_t/g'(p_t) + \eta_{t+1}]}{s_{t+1}\beta} \tag{21}$$

$$\tilde{\pi}_{t+2}^{-\gamma} = \frac{\tilde{\pi}_{t+1}^{-\gamma}[\tilde{f}_{t+1} - f_t + \omega_{t+1}]}{\beta r x_{t+1}} \tag{22}$$

where the error terms η_{t+1} and ω_{t+1} are assumed to be random shocks that follow some bivariate distribution with zero means and covariance matrix $\boldsymbol{P}$. Further transformation of transition equation into the general state-space representation of (17) requires raising both sides of (21) and (22) to the power $-1/\gamma$ and rearranging the terms to get

$$\tilde{p}_{t+2} = \frac{p_{t+1}s_{t+1} - v(s_{t+1}) + \phi(s_{t+1}) - x_{t+1}(f_{t+1} - \tilde{f}_{t+2})}{s_{t+1}} + \frac{\tilde{\pi}_{t+1}g'(p_{t+1})^{-1/\gamma}[(\tilde{p}_{t+1} - p_t) + v'(s_t) - \phi'(s_t) - s_t/g'(p_t) + \eta_{t+1}]^{-1/\gamma}}{s_{t+1}^{\gamma-1/\gamma}\beta^{-1/\gamma}} \tag{23}$$

$$\tilde{f}_{t+2} = \frac{(\tilde{p}_{t+2} - p_{t+1})s_{t+1} + v(s_{t+1}) - \phi(s_{t+1}) + x_{t+1}f_{t+1})}{x_{t+1}} - \frac{\tilde{\pi}_{t+1}[\tilde{f}_{t+1} - f_t + \omega_{t+1}]^{-1/\gamma}}{\beta^{-1/\gamma}r^{-1/\gamma}x_{t+1}^{\gamma-1/\gamma}} \tag{24}$$

where the vector of state variables $\boldsymbol{k}_t = \{\tilde{p}_{t+1}, \tilde{f}_{t+1}\}$. The corresponding measurement equation is defined by (4). Equations (23), (24) and (4) describe the state-space model with nonlinear transition equations and multiplicative errors, that governs first order dynamics of the unobserved states of the system by incorporating information from the current and the past decision periods. Any information from the time past two lags is unnecessary as it does not affect the transition functions. The SOSS approach assumes the simultaneous estimation of unobserved state variables and the model parameters in sequential manner using the Bayesian update as the new information comes into the market (which allow the use of it in the "on-line" decision support systems). The algorithm provides an optimal statistical inference about the model components and naturally allows for a time-varying specification which is useful in high frequency and seasonal data analysis.

Computation

All computations are done on Pentium 4 2.8 GHz IBM PC computer using Mathworks MatLab R2006b programming environment. The estimation algorithm can be described by the following pseudocode

Step 0a: Initialization Set the number of particles n, number of time periods T and GA algorithm parameters and set prior distributions for $\boldsymbol{\theta}_t$, $\tilde{p}_t$ and $\tilde{f}_t$.

Step 0b: Initialization Set $t = 1$ and simulate vectors $\boldsymbol{q}_t^i$, $i = \overline{1, n}$, containing independent realizations of $\boldsymbol{\theta}_t$, $\tilde{p}_t$ and $\tilde{f}_t$ from the corresponding prior distributions.

Step 1: Prediction Generate the n proposed values of $\boldsymbol{\theta}_{t+1}$, $\tilde{p}_{t+1}$ and $\tilde{f}_{t+1}$ from $\boldsymbol{\theta}_t$, $\tilde{p}_t$ and $\tilde{f}_t$ using the corresponding state transition equations and store the results in n vectors $\boldsymbol{q}_{t+1}^i$.

Step 2: Update Form n vectors $\boldsymbol{q}_{t+1}^i$ containing independent realizations of $\boldsymbol{\theta}_{t+1}$, $\tilde{p}_{t+1}$ from their respective marginal posterior distributions using GA resampling scheme where the fit of $\boldsymbol{q}_{t+1}^i$ is evaluated using the likelihood function of measurement equation.

Step 3: Counter check If $t < T$ set $t = t + 1$ and go to **Step 1**. Otherwise Stop.

Step 1 is implemented in blocks. First, for each i at iteration t a set of model parameters $\boldsymbol{\theta}_t$ is sampled from the posterior density. Second, given the values of generated parameters the pair of price expectations $\tilde{p}_t$ and $\tilde{f}_t$ is sampled using the Gibbs' algorithm, starting with the initial guess of $\tilde{f}_t$ (if $\tilde{p}_t$ is drawn first). The sampling blocks are repeated until n vectors $\boldsymbol{q}_t^i$ are obtained. The Step 3 requires evaluating the likelihood function of measurement equation at each of $\boldsymbol{q}_t^i$ to get the $n \times 1$ vector $\boldsymbol{\xi}_t > 0$ that describes fitness of each possible combination of states examined. The elements of $\boldsymbol{\xi}_t$ are then normalized to sum to one and used as the vector of probability masses to resample the states in a nonparametric bootstrap manner. In this case the combinations of elements in $\boldsymbol{q}_t^i$ that have a better fit are more likely to be chosen for the next iteration. In addition, GA resampling allows for "mutations", i.e. perturbation of the state space up to a chosen degree to improve the global search for the optimal path and avoid the local maxima.

Data

The data used for the study are quarterly time-series from 1989 to 2006. The relevant data have been collected from the Cotton and Wool Yearbook and Cotton and Wool Outlook published by the USDA Economic Research Service. The futures data is used for the cotton futures contracts traded at the New York Board of Trade (NYBOT) through the ICE (IntercontinentalExchange (NYSE: ICE)). Both monthly average futures prices and volume traded have been collected from the Commodity Research Bureau.

Expected Results

At the time of this writing we have run the simulations while correcting model specification and improving the estimation algorithm in terms of efficiency. The proposed method is designed to provide an optimal prediction for unobserved components of the model (price expectations and basis) by using all the information available in the market at any given moment. For each period t the estimation algorithm will generate the simulated distributions of the subjectively expected futures and spot prices. Using bootstrap techniques we will construct the distribution for the deviation of these two expectations and compute the appropriate point estimate of the basis. In order to justify the assumptions we made for the persistency of the forecast error generated by using the subjective expectations, we will test the hypothesis of the null difference between $\tilde{p}_t$ and $\tilde{f}_t$ using the simulated distributions for such expectations. The results of two other tests will be provided to measure the forecast power of the model, both for the out-of-sample forecast and in comparison with the conventional methods such as moving average smoothing.

References

1. Chow, G. C. 1992. "Dynamic Optimization without Dynamic Programming." *Economic Modelling* 9(1) : 3 - 9

2. Gordon, N., D.J. Salmond and C. Ewing. 1995. "Bayesian State Estimation for Tracking and Guidance Using the Bootstrap Filter". *Journal of Guidance, Control and Dynamics* 18 (6): 1434 - 1443.

3. Higuchi, T. 1997. "Monte Carlo Filter Using the Genetic Algorithm Operators." *Journal of Statistical Computation and Simulation* 59 : 1 - 23

4. Kitagawa, G. 1998. "A Self-Organizing State-Space Model." *Journal of the American Statistical Association* 93(443): 1203 - 1215

5. Lai, J.-Y., R.J. Myers and S.D. Hanson. 2003. "Optimal On-Farm Grain Storage by Risk-Averse Farmers." *Journal of Agricultural and Resource Economics* 28(3) : 558 - 579

6. Miranda, M.J. and J.W. Glauber. 1993. "Estimation of dynamic nonlinear rational expectations models of primary commodity markets with private and government stockholding." *Review of Economics and Statistics* 75 : 463 - 470

7. Miranda, M.J. and P.G. Helmberger. 1988. "The Effects of Price Band Buffer Stock Programs." *American Economic Review* 78 : 46 - 58

8. Muth, J.M. 1961. "Rational Expectations and the Theory of Price Movements." *Econometrica* 315 - 335.

9. Peterson, H.H. and Tomek W.G. 2005. "How Much of Commodity Price Behavior Can a Rational Expectations Storage Model Explain?" *Agricultural Economics* 33 : 289 - 303

10. Ristic, B., S. Arulampalam and N. Gordon. 2004. *Beyond the Kalman Filter: Particle Filters for Tracking Applications.* Artech House, Boston, MA.

11. Tanizaki, H. 1996. *Nonlinear Filters.* Springer-Verlag, Berlin.

12. Tanizaki, H. and R.S. Mariano. 1998. "Nonlinear and Non-Gaussian State-Space Modeling with Monte-Carlo Simulations." *Journal of Econometrics*, 83 : 263 - 290.

13. Taylor, M.R., K.C. Dhuyvetter and T.L. Kastens. 2006. "Forecasting Crop Basis Using Historical Averages Supplemented with Current Market Information." *Journal of Agricultural and Resource Economics* 31(3) : 549 - 567

14. Williams, J.C. and B.D. Wright. 2005. *Storage and Commodity Markets*. New ed. Cambridge University Press,