AN OPTIMAL CONTROL FRAMEWORK FOR INTER-REGIONAL DAIRY POLICY ANALYSIS

Rudolfo V. Tanjuakio, Conrado M. Gempesaw II, and G. Joachim Elterich

Abstract

An eleven-region stochastic coefficient econometric model was estimated and used in an optimal control framework to evaluate the effectiveness of the dairy price support program and marketing orders in reducing and stabilizing government purchases of dairy products. The results showed significant pressure on the reduction of the support price both in the presence and absence of Class I differentials. The optimal control model also showed that the drop in price support levels did not dramatically alter the regional distribution of milk production.

Key words: dairy policy, optimal control, stochastic coefficients

Government intervention in agriculture has long been a major area of interest in policy research. Concern about the growing federal budget deficit has drawn more attention to the role of the public sector in agricultural markets. In the dairy industry, the merits of regulation or deregulation have been extensively discussed. While the debate continues, the consensus is that total deregulation is "an unlikely option" for the near term (Novakovic).

U.S. government dairy policies are shaped by the objective of assuring adequate supplies of milk and milk products at reasonable prices. A corollary, if not an overriding, objective is to provide a fair return to milk producers. Two major policy programs-the federal and state marketing orders and the price support program-have been instituted to support these objectives. With the emergence of the domestic milk surplus problem and the huge federal budget deficits in the 1980s, the price support program took an additional objective of reducing the excess milk supply. The effectiveness of these two dairy programs with respect to the excess milk supply problem and their impacts on regional milk production were analyzed using an interregional optimal control-stochastic coefficient approach.

Initially applied to engineering problems, optimal control theory has been used in economics to determine values for decision variables that optimize an objective performance measure given a set of constraints. The works of Pindyck, Chow, Aoki, and Kendrick serve as basic references on the use of optimal control in macroeconomics. In farm commodity policy research, optimal control has been used by Taylor and Talpaz, Burt et al., and Arzac and Wilkinson. Richardson used optimal control for a comprehensive analysis of U.S. farm policy. In the dairy industry, optimal control was applied by McGuckin and Ghosh and Tauer and Kaiser to evaluate the effects of bovine somatotropin (bST). Chang and Stefanou used a similar type of analysis in their research on supply growth and deregulation in the dairy industry. In all these studies, an aggregate national model of the dairy industry was used. This study builds on past research in two respects: first, the regional optimal control model of the dairy industry was developed to measure the differential impacts of the price support program on individual regions and to incorporate the effects of the marketing orders, and second, the stochastic coefficient regression procedure was used in the econometric model estimation to capture the variability of the effects of exogenous factors on endogenous variables.

CONCEPTUAL FRAMEWORK AND METHODOLOGY

Optimal control follows a mathematical programming framework specifically defined to determine a set of variables that optimizes a given objective function under a set of constraints in a dynamic or multi-period environment. The optimal control problem in this study, defined in a quadratic tracking form, is as follows:

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$$(1)\ \underset{U,X}{\text{minimize}} \sum_{t=t_o}^{t=T} \left((X_t - \tilde{X}_t)'Q(X_t - \tilde{X}_t) + (U_t - \tilde{U}_t)'R(U_t - \tilde{U}_t) \right)$$

subject to

$$(2)\ X_t = AX_{t-1} + BU_{t-1} + CZ_t$$

$$(3)\ X_o = \zeta$$

$$(4)\ X_T = \Phi$$

where

- X = vector of state variables
- $\tilde{X}$ = vector of desired levels of state variables
- U = vector of control variables
- $\tilde{U}$ = vector of desired levels of control variables
- Z = vector of exogenous variables
- Q = penalty matrix for deviations of state variables
- R = penalty matrix for deviations of control variables
- X_o = initial values of state variables
- X_T = terminal values of state variables.

The objective function represents the minimum of the squared deviations of specified state and control variables from pre-defined desired target levels. This form of the objective function is known as the quadratic tracking criterion (Pindyck). The state variables are the endogenous elements in the econometric model used to define the objective function and form the set of constraints in the optimal control model. The control variables are the means or instruments affecting both the objective function and the state variables. The solution, which includes optimal levels of the control variables, is obtained by applying the minimum principle to the Hamiltonian function, the equivalent of the Lagrangian in the optimal control framework. The Hamiltonian is defined as:

$$(5)\quad H = \frac{1}{2}(X - \hat{X})'Q(X - \hat{X}) + \frac{1}{2}(U - \hat{U})'R(U - \hat{U}) + \lambda(AX + BU + CZ).$$

The corresponding first order conditions are:

$$(6)\ \frac{\partial H}{\partial \lambda} = AX + BU + CZ$$

$$(7)\ \frac{\partial H}{\partial x} = -\dot{\lambda} = Q(X - \hat{X}) + A'\lambda$$

$$(8)\ \frac{\partial H}{\partial u} = O = R(U - \hat{U}) + B'\lambda$$

$$(9)\ X_o = \zeta$$

$$(10)\ \lambda_T = Q(X_T - \hat{X}_N)$$

The first set of equations (6) contains the econometric model constraints in state-space form. The second set (7) states that a change in the marginal value of each of the state variables, $\dot{\lambda}$, is equal to its contribution to the objective function as measured by the first term, $Q(X - \hat{X})$, plus its incremental effect on its own rate of change evaluated at its marginal value, λ. The third set (8), which is the partial differential of the Hamiltonian with respect to each of the control variables, imposes the condition that the contribution of a change in the control variable to the objective function (first term, right side) must be equal to its effect on the state variables evaluated at their marginal values (second term, right side). The fourth set (9) spells out the initial conditions or values of the state variables. The last set (10) results from the transversality condition which states that the marginal value of each state variable at the terminal period is equal to its respective contribution to the objective function. The optimal solution consists of the state and control variable levels with a minimized objective function.

In this study, the objective was to minimize the weighted squared deviations of government purchases of dairy products (collectively expressed in milk equivalent form), the support price, and the Class I differentials from their exogenously specified desired levels. In the quadratic form defined for the study, the objective function was similar to that specified by McGuckin and Ghosh except for the inclusion of the support price and the Class I differentials in this study. These control variables were included in the objective function to provide a realistic bound in the Class I differentials in the optimal control solution. The studies by Chang and Stefanou and Tauer and Kaiser specified the maximization of a net economic surplus measure in the objective function. While maximizing economic surplus is a more comprehensive objective, the objective of reducing and stabilizing CCC purchases as spelled out in the 1990 Farm Bill was chosen in this study.

The state variables (X) were regional yield per cow, number of cows by region, regional production, regional all-milk price, and total CCC purchases. These are the endogenous variables in the econometric model. The control variables (U) were support price and Class I differentials. The control variable was limited to the support price in dairy industry optimal control studies other than the Tauer and Kaiser study, which used cow removals. The exogenous variables (Z) in the optimal control problem were demand for fluid and manufacturing milk at the national level, price of feed, wage rate, price of milk cows, and milk used on the farm for each region.

The penalty matrices Q and R were derived following Kendrick's procedure which essentially involves "a normalization scheme so that deviations of each state and control variable from their desired tracks have roughly the same penalty" (Kendrick, p. 156). Using the initial weights based on this procedure did not have an effect on the CCC purchases because the control variables-support price and Class I differentials-remained unchanged. However, doubling the penalty weights on CCC purchases resulted in reduction of CCC purchases to desired levels. It has to be emphasized that although penalty weights affect model results, they have to be interpreted as statistical measures designed to capture the importance of the underlying objective of the optimal control model. In this study, because stabilization and reduction of CCC purchases were emphasized, the penalty weights on this variable were increased until the desired level of CCC purchases was achieved. Furthermore, the utility of the model can be examined based on the resulting production shares and levels of support price and Class I differentials. A complete description of the optimal control formulation is provided in Tanjuakio.

ECONOMETRIC MODEL

To operationalize the optimal control problem, an econometric model that determines government purchases under the price support program was estimated based on national demand for and regional supply of milk. Dairy demand was specified at the national level mainly because data on regional sales were not available. On the other hand, supply was specified at the regional level to explicitly capture the effects of the support price and of the Class I differentials, which, in the optimal control model, constitute the set of control variables. Dairy demand was defined in terms of two major categories: fluid milk consumption and manufactured dairy product consumption. Assuming utility maximization and incorporating habit formation, the econometric specification for the market demand for milk was:

$$(11)\ USFL = f(PRFLUID, PRFRUIT, PRMEAT, INCAP, USFL_{t-1}, T)$$

$$(12)\ USMF = f(PRMFG, PRFOOD, INCAP, USMF_{t-1}, T)$$

where
USFL = Class I (Fluid) Milk Demand (million pounds)
PRFLUID = Retail Price Index of Fluid Milk (1982-1984=100)
PRFRUIT = Consumer Price Index for Fruits (1982-1984=100)
PRMEAT = Consumer Price Index for Meats (1982-1984=100)
INCAP = Personal Income per capita (1982 dollars)
T = Time (t=0 in 1970)
USMF = Manufacturing Milk Demand (million pounds)
PRMFG = Retail Price Index of Manufactured Milk (1982-1984=100)
PRFOOD = Consumer Price Index for Food (1982-1984=100)
t - 1 = one-year lag.

Habit formation was captured by the inclusion of consumption in the past year ($USFL_{t-1}$ and $USMF_{t-1}$). The use of lagged demand to capture the effect of habit formation or persistence is discussed in Intriligator (pp. 476-477) and Kmenta (pp. 238-241). In the dairy industry, Kaiser, Streeter and Liu specified these variables in estimating the demand for fluid and manufactured milk.

On the supply side, a three-equation model leading to a supply equation for each region was specified as follows (without regional subscripts):

$$(13)\ YIELD = f(PALL_{t-1}, FEEDP, WAGER, YIELD_{t-1}, T)$$

$$(14)\ NUMB = f(PALL_{t-1}, FEEDP, WAGER, COWP, YIELD_{t-1}, NUMB_{t-1})$$

$$(15)\ PROD = \frac{YIELD * NUMB}{1000}$$

where:
YIELD = production per cow (pounds per year)
NUMB = number of dairy cows (thousands)
PROD = milk production (million pounds)
PALL = weighted all-milk price ($/cwt)

FEEDP = price of feed (16% dairy ration, \$/ton)
WAGER = wage rate (\$/hour)
COWP = price of milk cows (\$/head)
T = time (1971=0).

The trend variable in the yield equation was included to capture autonomous technical change.

Price variables were deflated by the overall consumer price index. The all-milk price was specified as:

(16) $PALL_i = (PRSP, DIF_i)$

where:
PRSP = support price (\$/cwt)
DIF = Class I price differential (\$/cwt).

The variable CCC (total government purchases) was defined as:

(17) $CCC = \sum_{i} PROD_i - (USFL + USMF + FUSE)$

where
CCC = net removals (million pounds)
FUSE = milk used on the farm (million pounds).

The equations were estimated in linear form.

DATA

The USDA regional delineation *(Dairy Situation and Outlook Report)* was used in this study with two slight modifications. The Midwest was redefined to consist only of Minnesota and Wisconsin with Michigan being included in the Corn Belt. This is due to the use of the Minnesota-Wisconsin dairy area as the primary base for federal order pricing. The other modification was the specification of California as an entirely separate region instead of being part of the Pacific region. California is one of the major dairy producing states in the U.S. and has its own marketing order. The eleven regional divisions were comprised of the Northeast, Corn Belt, Upper Midwest, Northern Plains, Appalachia, Delta, Southeast, Southern Plains, Mountain, Northwest, and California. Table 1 presents the regional delineation of individual states used in this study.

State and marketing order data on the variables specified in the model were aggregated to conform with the regional delineations. Regional milk quantity data were derived by simple aggregation across states or marketing order areas. Regional price data were estimated as arithmetic weighted averages using state milk production shares as weights. Annual data from 1970 to 1988 were collected from various USDA statistical publications including *Milk Production, Disposition and Income Statistics, Federal Milk Order Market Statistics, Agricultural Statistics,* and *Agricultural Prices.*

Table 1. Delineation of States into Regional Divisions

Region	State
1 - Northeast	Connecticut Delaware Maine Maryland Massachusetts New Hampshire New Jersey New York Pennsylvania Rhode Island Vermont
2 - Corn Belt	Illinois Indiana Iowa Michigan Missouri Ohio
3 - Upper Midwest	Minnesota Wisconsin
4 - Northern Plains	Kansas Nebraska North Dakota South Dakota
5 - Appalachia	Kentucky North Carolina Tennessee Virginia West Virginia
6 - Southeast	Alabama Florida Georgia South Carolina
7 - Delta States	Arkansas Louisiana Mississippi
8 - Southern Plains	Oklahoma Texas
9 - Mountain States	Arizona Colorado Idaho Montana Nevada New Mexico Utah Wyoming
10 - Northwest	Oregon Washington
11 - California	California

ESTIMATION PROCEDURES

The econometric model was estimated using the SWAMSLEY algorithm which provides for stochastic coefficient estimates (Swamy and Tinsley). The individual equations were specified to follow a first-order variant of the generalized ARIMA stochastic coefficient model defined as:

$$(18)\ Y_t = X_t'B_t$$

$$(19)\ B_t - B = \theta(B_{t-1} - B) + \alpha_t$$

where Y_t is the dependent variable, X_t is the vector of explanatory variables, and the stochastic parameter vector (B_t) is assumed to follow a first-order autoregressive process with mean vector (B). The error term α_t is assumed to follow a sequence of uncorrelated vector random variables with zero first moment and constant covariance matrix (Δ_a). The correlation matrix θ is a matrix of fixed but unknown correlation coefficients. The SWAMSLEY algorithm provides for a data-based iterative estimation method for estimating efficient and consistent estimates of Δ_a, θ, and B_t. The advantages of the stochastic coefficient estimator were outlined in Conway et al. In addition, the root mean square errors (RMSE) of equations estimated using the stochastic coefficient method have been shown to be significantly lower than the RMSEs based on ordinary least squares estimates (Swamy et al.).

The optimal control solution was determined using the MINOS nonlinear programming solver operating within the GAMS environment (Brooke et al.). The solution algorithm for a quadratic optimal tracking control problem consists of the matrix recursion formulae as derived from the necessary conditions dictated by the minimum principle from the Hamiltonian function. These necessary conditions can likewise be derived from the Kuhn-Tucker conditions in nonlinear programming problems. The MINOS nonlinear programming solver can easily accommodate new constraints in addition to the econometric model itself compared to the more tedious and cumbersome matrix recursion algorithm. GAMS was used because of its advantage in keeping optimization programs more understandable and tractable.

EMPIRICAL RESULTS

The SWAMSLEY algorithm produces as many groups of parameter estimates as the number of iterations specified. Following Narasimham et al., the choice of which iteration (set of parameters) to use was based primarily on low RMSE and the conformity of the parameter signs with theoretical expectations. For brevity's sake, the parameter estimates are not reported because they cover eleven regional econometric models involving over 130 parameters. However, the regional production elasticities are presented in Table 2, which summarizes the information derived from the parameter estimates.

Table 2. Comparative All-Milk Price Elasticities for Milk Production, by Region

Region	This Study (1980-1988 Average)	Buxton (1985)	Weersink and Tauer (1990)
1. Northeast	0.161	0.607	0.11
2. Corn Belt	0.060	0.501	*
3. Upper Midwest	0.168	0.599	0.16**
4. Northern Plains	0.037	0.343	*
5. Appalachia	0.229	0.923	*
6. Southeast	0.290	0.573	*
7. Delta States	0.079	0.651	*
8. Southern Plains	0.133	0.710	*
9. Mountain States	0.265	0.523	*
10. Northwest	0.216	0.398	***
11. California	0.016	0.222	0.43***

* - grouped as 'all other' with an elasticity of .32.
** - refers to the Lake States which includes Michigan.
*** - refers to the Pacific region.

On the demand side, the own-price elasticities for fluid milk and manufactured milk products were estimated at -.19 and -.47 at the 1980-1988 mean values. Bailey et al. estimated these elasticities at -.48 and -.37. The own-price elasticity estimates of Kaiser and Tauer were -.05 and -.43. The income elasticity estimates were .26 for fluid milk and .42 for manufactured dairy products. Kaiser and Tauer estimated these elasticities at .48 and .32. The price elasticities of supply for regions, with comparable estimates from Buxton and from Weersink and Tauer are shown in Table 2. The previously reported supply elasticities are generally higher than the estimates in this study although in both cases they are inelastic over all regions.

The use of a fixed-coefficient estimating technique assumes constancy in the marginal contribution of milk price to milk production over the data series. Stochastic coefficient regression (SCR), on the other hand, allows for the marginal effect to vary over the data. This means that for each year, a marginal effect (beta) is estimated. In this study, the average regional beta was computed from the annual regional parameter estimates and used in estimating the regional milk supply elasticity. The use of SCR captures the "short and intermediate-term" impact of milk price on milk production following the terminology of Chavas and Klemme. This is due to the fact that the "average beta" used to measure the supply elasticity can be considered as the mean cumulative effect of milk price on milk production over the 1980-1988 data period.

The estimated supply elasticities in this study ranged from .01 to .29. These are consistent with the -.08 short-run and .14 long-run supply elasticity estimates of Howard and Shumway based on 1951-1982 data. Weersink and Tauer, using a similar data period (1950-1985) estimated the supply elasticity of milk as ranging from .11 to .43 for short-run and .25 to .46 for long-run. These estimates are also within the range of the "short- and intermediate-term" elasticities estimated by Chavas and Klemme for year zero (.11) to year four (.48). They reported that intermediate-run elasticities start at year one up to year ten, and year 15 to 30 cover the long-run estimates. Incidentally, their long-run supply estimates (around 3.9 to 6.7) were significantly higher than what has been reported in other studies.

In the optimal control model, the national demand for fluid milk and manufactured dairy products was assumed exogenous. This effectively requires that adjustments of the control variables in the optimal control solution be solely based on their impact on supply because the reduced form of the supply model provides for a direct relationship between production and the support price and differentials. The assumed exogeneity of demand is supported by the estimated parameters in the demand equations that were shown earlier to have resulted in inelastic own price effects and small income responses of both fluid and manufactured milk products.

For comparative purposes, the optimal control problem was solved for two time periods. The first period provides a historical perspective by specifying the optimal control problem in terms of industry and policy parameters in effect operating from 1980-1988. The second period covers 1988-1995 and involves projecting the optimal control problem under two production scenarios. The first assumes production behavior with no effective influence of new technology such as bST. The second scenario considers the effects of bST by revising the regional milk yields by the projected bST-induced annual productivity increase of 1800 pounds per cow (Fallert et al.) adjusted by adoption rates based on the study by Lesser et al.

In analyzing the effectiveness of the support price and the differentials in each time period, the optimal control solutions for two policy scenarios were estimated. The first scenario assumed fixed regional price differentials and allowed the support price to fluctuate freely. The optimal solution in this case was identical to an alternative scenario where both the support price and the differentials were flexible. The optimal control solver, in its attempt to minimize the sum of the squared deviations of all the variables being tracked, which included the regional Class I differentials, yielded a solution effectively fixing the differentials at the desired levels. Excluding the differentials from the objective function by defining them as control variables with zero penalty weights associated with their deviations resulted in a solution with severe fluctuations and unrealistic levels for the control variables. The second scenario eliminated the Class I differentials altogether, essentially defining the support price as the sole control variable. The optimal control solutions for these two policy scenarios were compared to the solution of the base scenario which assumed a fixed support price and Class I differentials.

HISTORICAL ANALYSIS: 1980-1988

The historical optimal control problem was defined in terms of the prevailing policies in the 1980s. The period averages were selected as the target or desired levels for the control variables. Setting the target levels at their average values provides a pattern for the optimal control model that prevents excessive and unrealistic deviation from the levels set in the past. The target level for CCC purchases was set at 3.75 billion pounds, a level between the trigger points for the adjustment of the support price. The initial conditions were defined by 1980 levels of the state and control variables.

When the support price was not constrained, the overriding objective of maintaining government dairy purchases at 3.75 billion pounds per year coming from the 1980 level of 8.8 billion pounds, was achieved (Table 3). However, a decrease in the support price from $12.04 in 1980 to an average of $10.07 from 1981-1988 was required. This is $1.97 lower than the actual average of $11.76 for the same period. With zero Class I differentials, government purchases were reduced and maintained at 3.75 billion pounds with a less drastic cut in the support price to an average of $ 11.13. When both the support price and the differentials were fixed at their actual averages for 1981-1988 period, the yearly average CCC purchases were 7.8 billion pounds.

Some discernible trends in the regional shares to total milk production for the historical period can be observed (Table 4). Regions with decreasing shares are the Northeast, Corn Belt, Midwest, Appalachia, and the Delta States. Regions with increasing shares are the Northern Plains, Mountain States, Northwest, and California. The Southern Plains and the Southeast have maintained stable shares over the years. Due to the inelastic short-run response to milk prices and the effect of the other exogenous variables on

Table 3. 1980-1988 Optimal Control Solution: Value of Objective Function, Average CCC Purchases, Average Levels of the Support Price and the Class I Differentials

		Policy Scenario (1981-1988 Average)		
Price Support Class I Differentials	1980 Actual	fixed fixed	flexible fixed	flexible zero
Value of Objective Function	-	71.74	.27	.09
Total CCC Purchases (million lbs.)	8735	7828	3763	3755
Price Support Level ($/cwt)	12.04	11.76	10.07	11.13
Class I Differentials ($/cwt)				
Northeast	2.53	2.95	2.95	0.0
Corn Belt	1.34	1.63	1.63	0.0
Midwest	0.91	1.15	1.15	0.0
Northern Plains	1.46	1.70	1.70	0.0
Appalachia	1.70	2.00	2.00	0.0
Southeast	2.52	2.96	2.96	0.0
Delta States	2.34	2.82	2.82	0.0
Southern Plains	2.05	2.53	2.53	0.0
Mountain States	2.03	2.29	2.29	0.0
Northwest	1.70	1.88	1.88	0.0
California	2.17	1.00	1.00	0.0

regional milk production, these trends are uniformly observed in all three policy scenarios.

PROJECTED OPTIMAL CONTROL: 1988-1995

The 1990 Farm Bill served as the basis for specifying the desired levels of CCC purchases and control variables. Desired government purchases were set at 5.85 billion pounds. This is in consideration of the provision that requires the use of the total solids basis in determining the milk equivalent of CCC purchases. The total solids basis milk equivalent is the weighted average of the milk equivalents of CCC purchases computed using both the milkfat and solids-not-fat (SNF) bases. The desired level of the support price was set at $10.10 per cwt while the target Class I differentials were set at their current levels.

Assuming that milk yields behave within historical trend patterns with no adjustments made due to bST, the optimal control results in all policy scenarios showed the support price well below the U.S. average milk production cost of $13.62 per cwt (*Economic Indicators of the Farm Sector: Costs of Production—-Livestock and Dairy*) and CCC purchases at or below the target level of 5.85 billion pounds (Table 5). However, it must be noted that due

Table 4. 1980-1988 Optimal Control Solution: 1980 Actual and 1988 "Projected" Regional Percent Shares of Total Milk Production

		1988 "Projected" Level		
Price Support Class I Differentials	1980 Actual	fixed fixed	flexible fixed	flexible zero
Northeast	20.4	19.6	19.7	19.5
Corn Belt	16.3	15.2	15.5	15.6
Midwest	24.9	22.3	22.0	22.4
Northern Plains	4.1	4.3	4.5	4.5
Appalachia	6.6	6.2	6.1	5.6
Southeast	3.5	3.4	3.2	2.9
Delta States	2.0	1.7	1.7	1.7
Southern Plains	3.7	4.1	4.2	4.2
Mountain States	4.8	5.8	5.2	5.5
Northwest	3.2	3.9	3.8	3.9
California	10.6	13.5	14.1	14.1

to the introduction of a new technological factor such as bST, there is a possibility of a decline in average milk production cost. With the productivity adjustments due to the adoption of bST, CCC target purchases were met, but with even lower support prices (Table 5). In the policy where the differentials were fixed, the average support price was $9.41 per cwt.

That the support price is below the cost of production required to achieve CCC target purchases presents a major economic and political dilemma. It cuts across the issue of the survival of dairy farms amidst the tight federal financial situation and the movement to reduce the economic protection traditionally accorded to agriculture. With zero differentials, the support price averaged $10.54 per cwt over the 1988-1995 period. With both the support price and the Class I differentials fixed, CCC purchases averaged 7.5 billion pounds per year during the period.

Maintaining CCC purchases at a predetermined target volume generally required production to be similar across policy scenarios, with or without bST. Controlling milk production was more difficult with the adoption of bST where yield increases had to be matched by significant decreases in the number of cows (Table 7). As in the 1980-1988 results, the regional distribution of total production generally follows the trends observed in the historical analysis despite the dramatic drop in the support price and the Class I differentials (Table 6).

SUMMARY AND CONCLUSIONS

An optimization model for evaluating public sector pricing policies in the dairy industry using a stochastic coefficient econometric model and a quadratic objective function was specified and estimated. Us-

Table 5. 1988-1995 Optimal Control Solution: Value of Objective Function, Average CCC Purchases, Average Levels of Support Price and the Class I Differentials

Price Support Class I Differentials	1988 Actual	Without bST (1989-1995 Average) fixed fixed	 flexible fixed	 flexible zero	With bST (1985-1995 Average) fixed fixed	 flexible fixed	 flexible zero
Value of Objective Function	-	2.01	.02	82.04	5.09	.06	81.94
Total CCC Purchases (million lbs.)	8900	4792	5843	5815	7480	5863	5836
Price Support Level ($/cwt)	10.33	10.10	10.55	11.68	10.10	9.41	10.54
Class I Differentials ($/cwt)							
Northeast	2.95	2.95	2.95	0.0	2.95	2.95	0.0
Corn Belt	1.63	1.63	1.63	0.0	1.63	1.63	0.0
Midwest	1.03	1.03	1.03	0.0	1.03	1.03	0.0
Northern Plains	1.66	1.66	1.66	0.0	1.66	1.66	0.0
Appalachia	2.38	2.38	2.38	0.0	2.38	2.38	0.0
Southeast	3.34	3.34	3.34	0.0	3.34	3.34	0.0
Delta States	3.23	3.23	3.23	0.0	3.23	3.23	0.0
Southern Plains	2.93	2.93	2.93	0.0	2.93	2.93	0.0
Mountain States	2.21	2.21	2.21	0.0	2.21	2.21	0.0
Northwest	1.70	1.70	1.70	0.0	1.70	1.70	0.0
California	1.00	1.00	1.00	0.0	1.00	1.00	0.0

Table 6. 1988-1995 Optimal Control Solution: 1988 Actual and 1995 Projected Regional Percent Shares to Total Milk Production, Without and With bST

Price Support Class I Differentials	1988 Actual	Without bST (1995 Projected) fixed fixed	 flexible fixed	 flexible zero	With bST (1995 Projected) fixed fixed	 flexible fixed	 flexible zero
Northeast	19.3	19.0	19.0	18.8	18.8	18.9	18.7
Corn Belt	15.2	13.8	13.8	13.9	13.5	13.6	13.8
Midwest	24.4	20.2	20.3	20.7	19.7	19.6	20.0
Northern Plains	3.8	4.3	4.3	4.3	4.4	4.5	4.5
Appalachia	5.8	5.8	5.8	5.3	5.7	5.7	5.1
Southeast	3.2	3.2	3.3	2.9	3.2	3.1	2.8
Delta States	1.7	1.3	1.3	1.3	1.2	1.2	1.1
Southern Plains	4.2	4.1	4.1	4.1	4.2	4.3	4.2
Mountain States	5.8	7.5	7.7	8.0	7.7	7.5	7.8
Northwest	3.8	4.7	4.7	4.9	5.1	5.1	5.2
California	12.8	15.9	15.7	15.8	16.4	16.7	16.8

ing the stochastic coefficient algorithm developed by Swamy and Tinsley, an econometric model that provided regional parameter estimates was formulated and estimated. The estimates confirm the generally price-inelastic nature of milk demand and production as reported in other studies. The optimization results provided several useful insights on the appropriate formulation of dairy policies, particularly with respect to the price support program and the marketing orders. However, it should also be noted that there are other policy alternatives (e.g., milk production quota) which could be pursued to stabilize CCC purchases. The empirical model developed in this study focused on the current policy environment which is based primarily on the support price and the Class I differentials. The effect of other policy alternatives on CCC purchases can be addressed in future research projects.

Under a tracking objective which emphasizes the reduction and stabilization of CCC purchases, the support price level required to meet this objective under different policy and production scenarios was

Table 7. Milk Yield, Number of Cows and Milk Production, by Policy Scenario, without and with bST, 1988-1995

	Without bST			With bST		
Price Support Class I Differentials	fixed fixed	flexible fixed	fixed zero	fixed fixed	flexible fixed	fixed zero
A. Number of Cows (000)						
1988 (actual)	10251	10251	10251	10251	10251	10251
1989	10232	10214	10207	10232	10213	10206
1990	10207	10218	10211	10205	10205	10197
1991	10180	10214	10207	10170	10162	10156
1992	10154	10208	10202	10124	10068	10062
1993	10127	10204	10198	10067	9938	9932
1994	10104	10203	10197	10009	9835	9828
1995	10081	10204	10196	9958	9774	9766
B. Milk Yield (pounds per cow)						
1988(actual)	13770	13770	13770	13770	13770	13770
1989	14159	14147	14154	14550	14147	14154
1990	14338	14349	14357	14363	14366	14375
1991	14552	14570	14578	14647	14643	14650
1992	14774	14800	14807	15039	15006	15013
1993	15001	15032	15039	15501	15440	15446
1994	15227	15265	15272	15912	15842	15851
1995	15456	15499	15506	16249	16190	16198
c. Milk Production (million pounds)						
1988(actual)	141152	141152	141152	141152	141152	141152
1989	144871	144494	144470	148871	144482	144459
1990	146349	146619	146595	146570	146604	146580
1991	148137	148823	148800	148956	148805	148782
1992	150013	151080	151058	152254	151082	151060
1993	151916	153389	153366	156046	153438	153405
1994	153853	155752	155726	159263	155805	155781
1995	155813	158150	158100	161812	158239	158191

estimated. Lagging demand and continued increases in production combined to exert significant downward pressure on the support price. From 1988-1995, with or without bST and in all policy scenarios, including the elimination of the Class I differentials, the support price fell below the average milk production cost.

On the regional distribution of total milk production, the Northern Plains, Mountain States, Northwest, and California exhibited increasing shares at the expense of the Northeast, Corn Belt, Midwest, Appalachia, and the Delta States. The production shares of the Southern Plains and the Southeast were stable. These trends which can be observed uniformly across the three dairy policy scenarios can be attributed to the inelastic (short-run) response of the individual regions to the milk price and the effects of input prices on regional production.

Past dairy policy objectives focused on the need to support the incomes of milk producers and assure an adequate supply of milk for consumers. In the past decade, as milk surpluses mounted and the government deficit soared, the pressure to reduce government expenditures on agricultural price support programs increased. This analysis illustrates the difficult trade-offs between the traditional objectives of the dairy industry and its supporters and the more pressing concern to alleviate the fiscal burdens of agricultural programs expressed by many. The emergence and potential widespread adoption of new technology such as bST is likely to create further pressures in controlling total milk production but not the pattern of regional production shares.

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