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« Voting Rules in Bargaining with Costly Persistent Recognition »

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Voting Rules in Bargaining with Costly Persistent Recognition*

Abstract

In this paper, we consider a model of multilateral bargaining where homogeneous agents may exert effort before negotiations in order to influence their chances of becoming the proposer. Effort levels have a permanent effect on the recognition process (persistent recognition). We prove three main results. First, voting rules are equivalent (that is, they yield the same social cost) when recognition becomes persistent. Secondly, an equilibrium may fail to exist, because players may have more incentives to reduce their effort level (in order to be included in winning coalitions) than to increase it (in order to increase their proposal power). Thirdly, we prove that the existence problem is driven by the intensity of competition at the recognition stage. Another definition of this process enables to fix this problem.

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1 Introduction

Negotiations are common in many important economic problems, such as legislative bargaining (Baron and Ferejohn 1989, Snyder et al. 2005), international environmental agreements, litigation processes, issues of corporate governance. Agents taking part in such processes have incentives to gain power in order to influence the outcome of the process. There are plenty of real life situations where agents exert (costly) efforts to promote their most preferred alternative. For instance, agents can provide services and contributions to the functioning of their organization in order to increase their chances of being elected to the executive committee. This in turn will enable them to influence the system of decision-making.

The agents' incentives to buy influence have been studied in different contexts (Evans 1997, Anbarci et al. 2002, Grossman and Helpman 2002, or Board and Zwiebel 2005). Unlike all these contributions, the main goal of this paper is to compare different voting rules with respect to the social cost resulting from influence activities. As such, the closest references are Yildirim (2007, 2010), where the author analyses a sequential bargaining situation in which agents compete in order to influence their chances of becoming the proposer (agents may be heterogeneous). Competition can take place at a pre-bargaining stage (persistent recognition) or at each stage of the negotiations (transitory recognition). In Yildirim (2007) the author characterizes unanimity as the voting rule minimizing the social cost resulting from influence activities when agents are identical and recognition is transitory. Then, in Yildirim (2010), he compares both recognition systems for a given rule (unanimity). Another close reference is Cardona and Polanski (2012), who analyse a similar type of problem when agents bargain over a unidimensional issue. The object of bargaining is very different from ours, and so are their results.

The present paper focuses on the case of persistent recognition, where agents exert effort to influence their chances of becoming proposers at the beginning of the process, i.e before the first round of negotiation. This is mainly because this type of recognition seems to be the most appropriate one when considering many important real world processes, such as legislative bargaining or executive committees in organizations.

Yildirim (2007, Section 6) provides a preliminary analysis of the case of persistent recognition. Specifically, he suggests two definitions of persistent recognition. In the first one, the recognition probability is assumed to be a mix between a transitory and an exogenous component. The second definition is the one used in the present paper, where agents exert efforts once-and-for-all before the negotiation process. In both cases, he compares the incentives of agents to exert effort when recognition is either transitory or persistent in a two-player example. The present contribution complements Yildirim (2007) by comparing the efficiency of the different voting rules when recognition is persistent.

The first contribution is an equivalence result regarding the social cost resulting from voting rules. While unanimity is the unique voting rule minimizing the social cost when recognition is transitory (Yildirim 2007), voting rules are equivalent (in that they yield the same social cost) when agents are identical and exert effort only once at the beginning of the process. Specifically, we focus on the (symmetric) stationary subgame perfect equilibria (SSPE) of the game (as in Yildirim 2007, 2010) and characterize the (unique) potential candidate by backward induction. We compute the resulting social costs and we prove that they coincide for all voting rules.

Our second contribution relates to the analysis of incentives to deviate from the (unique) candidate for a symmetric equilibrium. We show that, when the (symmetric) equilibrium fails to exist, this is *not* because players have incentives to increase their chances of becoming the proposer (through an increase of effort), but rather because they prefer to lower their effort level in order to be included in the winning coalition. In other words, under strict k-majority rules, the incentives to be included in winning coalitions by the other players dominate individual incentives and create a "race to the bottom", which eventually destabilises the unique (symmetric) equilibrium candidate. Thus, a qualitative property of the present model is that the ability to propose is less important than the ability to be included in winning coalitions. This differs notably from one of the main conclusions of the literature on bargaining, which highlights the dominance of proposal rights to define political power (see Eraslan 2002, or Kalandrakis 2006).

The third contribution is to show that the existence problem is driven by the intensity of competition at the recognition stage. When recognition probabilities are defined using Tullock (1980) contest success functions, this intensity is low, all agents exert the same level of effort, and the race to the bottom occurs. We show that defining the probabilities to win recognition as a function of the percentage mark-up of the highest effort level (à la Hirshleifer)[1] enables one to push the intensity of competition up, and to fix this issue.

Before moving on to the formal description of the model, there are two important points that have to be stressed. First, in most of the paper we use a specific form of recognition function, which is yet the most widely used form in the literature on rent

[1] This idea has been suggested by Hirshleifer in the contest literature and formalized by Alcalde and Dahm (2007).

seeking (see Tullock 1980). Secondly, except in Section 5, we do not analyse asymmetric equilibria. The reasons are as follows. The main goal of the analysis is to highlight the fact that the case of persistent recognition is specific, since voting rules are shown to be equivalent in terms of the resulting social cost, and that there are cases where a race to the bottom emerges at the recognition stage. Compared to Yildirim (2007), the persistent nature of the recognition process makes it much more difficult to derive analytically tractable expressions that enable us to compare the different voting rules.

The remainder of the paper is organized as follows. The model is introduced in Section 2 and an illustrative example highlights the main difference between transitory recognition and persistent recognition. The unique equilibrium candidate is characterised in Section 3. The equivalence result prevailing in the homogeneous case is provided in the same section. The (non) existence problem is analysed in Section 4, and fixed in Section 5. Section 6 concludes. Proofs that do not appear in the body of the paper are relegated to an appendix.

2 The model

2.1 Description of the model

We consider the problem introduced by Yildirim (2007, 2010) where $n \geq 2$ agents belonging to a set $N = \{1, ..., n\}$ bargain over the allocation of a surplus of fixed size (normalized to one) under a fixed k-majority rule (i.e., the approval of k players among the n is needed). Agents negotiate according to a bargaining protocol *a la* Rubinstein (1982), except that their recognition probabilities are endogenous.

The game has two stages. In the first stage, each agent exerts effort at the beginning of the process, and relative efforts determine each agent's bargaining power (their recognition probability) for all periods. We assume that, provided agent i exerts effort $x_i \geq 0$ at the beginning of the process, his recognition probability is given by $p_i \equiv p(x_i, x_{-i})$, where x_{-i} is the vector of efforts of the $n-1$ other players. Let $\boldsymbol{x}$ denote the vector of efforts of the n players. Effort is costly and, in order to keep the analysis as simple as possible, we assume that the cost of effort is linear (and that all agents have the same marginal cost of effort), and this cost is denoted by the positive parameter $c < 1$.

In the second stage, players bargain over a pie of size 1. They convene in periods $t = 1, 2, ...$ to divide the pie. In each period, player i is recognized with probability p_i and receives the right to make a proposal to the other players. A proposal $\mathbf{s}$ is an element of the set $S \equiv \left\{(s_1, s_2, ..., s_n) | \forall i \in N, s_i \geq 0 \text{ and } \sum_{i=1,...,n} s_i = 1\right\}$ which is the set of feasible allocations, where s_i is the share agent i receives. The n players then choose whether to accept or reject the proposal. If at least k players choose to accept the proposal, the pie is shared according to the proposal.

Each player i derives von Neuman-Morgenstern stage utility $u_i(\boldsymbol{s}) = s_i$. Players discount the future by an identical discount factor $0 < \delta < 1$. Thus, player i's payoff from a decision $\mathbf{s} \in S$ reached in period $t \geq 1$ is given by $\delta^{t-1} s_i - c x_i$. In the event of perpetual disagreement, this player receives $-c x_i$.

The main goal of the present paper is to compare the different voting rules according to their social costs. Yildirim (2007) provides this comparison in the case of transitory recognition only when agents are identical. This is due to the complexity of the underlying analysis. As a result, we too will focus on the homogeneous case where agents are identical (same time preferences, same marginal cost of effort).

We will have to impose more structure on the recognition probabilities, especially to characterize the social cost. Until Section 5, we will use a Tullock contest success function (TCSF):

Assumption (TCSF): *Let the recognition probability be such that, for* $\boldsymbol{x} \geq 0$,

$$p(x_i, x_{-i}) = \begin{cases} \frac{f(x_i)}{\sum_{l=1}^{n} f(x_l)} \text{ if } \boldsymbol{x} \neq \mathbf{0}, \\ \frac{1}{n} \text{ if } \boldsymbol{x} = \mathbf{0}, \end{cases} , \tag{1}$$

with $f \geq 0$, $f' > 0$ *and* $f'' \leq 0$.

This function has been introduced by Tullock and it has been widely used in the literature on contests. This is the simplest form of contest success functions with an axiomatic foundation (Skaperdas, 1996). In order to fix the existence problem that will be discussed in Section 4, we will use (in Section 5) another form of success function where recognition probabilities depend on the percentage mark-up of the highest effort level. Specifically:

Assumption (HCSF): If one considers agents' effort levels $x_1 \geq x_2 \geq ... \geq x_n$ then the probability that agent i be recognized as the proposer is:

$$p(x_i, x_{-i}) = \sum_{j=i}^{n} \frac{x_j - x_{j+1}}{j x_1}, \tag{2}$$

where it is assumed that $x_{n+1} = 0$. This form has been suggested by Hirshleifer in the contest literature and studied in Alcalde and Dahm (2007).

To be consistent with Yildirim (2007, 2010), we will focus on stationary subgame perfect equilibria (SSPE) in the remainder of the paper. Specifically, we use the following definition:

Definition 1 *A strategy profile is a SSPE if it constitutes a Nash equilibrium in each period, and is both stationary (time and history independent) and subgame perfect.*

Thus, a SSPE specifies identical actions in each continuation game following the rejection of an offer. The main reason for using this concept is that it solves the multiple equilibrium problem that arises in multilateral bargaining. With this equilibrium notion, it is easily checked that (since $\delta < 1$) any agent has incentives to make an offer that is immediately accepted.

In the next sections, we will use backward reasoning to characterize the SSPE and more specifically the agents' equilibrium payoffs, expected shares of the surplus, and levels of effort. We will focus on symmetric equilibria, i.e. equilibria where two identical players are treated the same (same share of the pie, same effort). In Section 3 the optimal strategies of the negotiation stage are characterized. In order to rule out cases where agents might be indifferent between certain strategies, we will have to rely on a tie breaking rule that will be described in this section. In Section 4, we will analyse the initial stage of recognition, and we will characterize the equilibrium candidate. Then we will complete the analysis by ruling out the potentially beneficial unilateral deviations. We will then show that the social cost resulting from the equilibrium is the same for any voting rule.

2.2 Comparison of transitory and persistent recognition: an illustrative two-player case

The present section illustrates how the ranking of voting rules (in terms of social costs) is affected when recognition is assumed to be persistent instead of transitory in the case of two players. The case of transitory recognition differs from the model presented above because instead of choosing an effort x_i once-and-for-all before the negotiation phase, as in the case of persistent recognition (as considered in Yildirim 2010 and in the present paper), players exert effort at each step of the negotiation process (which is the case analysed in Yildirim 2007).

With two players, there are only two possible voting rules, unanimity and dictatorship. In the case of unanimity, a proposal needs the approval of the other player to be implemented and in the case of dictatorship, the proposer can share the pie without the agreement of the other player. Both the transitory recognition model and the persistent recognition model coincide in the case of dictatorship. Indeed, when the proposer does not need the agreement of the other player, he keeps the whole pie. Both cases coincide with the standard rent-seeking model. Hence, in the case of two homogeneous players, the payoff of player $i = 1, 2$ is given by:

$$v_i = p\left(x_1, x_2\right) - cx_i. \tag{3}$$

Assuming that (TCSF) holds with $f(x) \equiv x$, it is easy to check that the unique equilibrium is such that $x_1^* = x_2^* = \frac{1}{4c}$. The social cost is then $SC = c\left(x_1^* + x_2^*\right) = \frac{1}{2}$.

Let us now compare transitory and persistent recognition under the unanimity rule:
Transitory recognition with two players: Under unanimity, the expected equilibrium payoff of player 1 satisfies (see Yildirim 2007):

$$v_1 = \max_{x_1 \geq 0} \left\{p_1\left[1 - \delta v_2\right] + \left(1 - p_1\right)\delta v_1 - cx_1\right\}. \tag{4}$$

With probability p_1, player 1 becomes the proposer and under unanimity he needs to compensate player 2 (by offering him δv_2), and player 2 accepts. With probability $1 - p_1$, player 2 is the proposer. Player 1 accepts the offer and receives δv_1. Similarly, the expected equilibrium payoff of player 2 satisfies

$$v_2 = \max_{x_2 \geq 0} \left\{p_2\left[1 - \delta v_1\right] + \left(1 - p_2\right)\delta v_2 - cx_2\right\}. \tag{5}$$

The equilibrium effort of player $i = 1, 2$ satisfies:

$$\frac{\partial p_i}{\partial x_i} \times \left[1 - \delta v_1 - \delta v_2\right] - c \leq 0 \text{ (} = 0 \text{ if } x_i > 0). \tag{6}$$

Assuming that (TCSF) holds with $f(x) \equiv x$, one can easily show that both players exert a positive effort, $x_i^* = \frac{1}{4c}\left(1 - \delta\left(1 - c\right)\right)$ and the social cost is $SC = c\left(x_1^* + x_2^*\right) = \frac{1}{2}\left(1 - \delta\left(1 - c\right)\right)$. Since $\delta > 0$ and $c < 1$, the social cost is strictly lower under unanimity than under dictatorship.
Persistent recognition with two players: the game has two stages; in the first stage players choose their effort, and players bargain in the second stage. We use backward induction to solve the game. In the second stage, given the efforts x_1 and x_2, the shares of the players, s_1 and s_2 satisfy:

$$s_1 = p_1\left[1 - \delta s_2\right] + \left(1 - p_1\right)\delta s_1, \tag{7}$$

and,

$$s_2 = p_2 \left[1 - \delta s_1\right] + (1 - p_2)\, \delta s_2. \tag{8}$$

Solving this set of two equations, we obtain $s_i = p_i$ for $i = 1, 2$.

Now consider the first stage where the players choose their efforts. The expected payoff of player $i = 1, 2$ is given by:

$$v_i = s_i - cx_i = p(x_i, x_{-i}) - cx_i \tag{9}$$

Hence, as in the case of dictatorship, the game coincides with the standard rent-seeking model.

At this stage it might be useful to comment on an interesting implication of the above example. In the transitory case, the equilibrium shares depend on the discount factor δ, while this is not true when recognition is persistent. The intuition is reasonably simple if one interprets the above two processes as rent seeking games. Actually, in the case of transitory recognition, it can be checked that the first stage is equivalent to a one-shot rent-seeking game with (endogenous) prize. For instance, in the case of unanimity, a simple inspection of the first order conditions enables us to conclude that this prize is equal to $1 - \delta(v_1 + v_2)$, which is a function of the discount factor. A similar conclusion holds for a strict k-majority rule (Yildirim 2007, condition (11), pp.176). This implies that any agent's equilibrium effort level depends on the discount factor, which leads to the conclusion that the equilibrium share is a function of this parameter as well. In the case of persistent recognition, by contrast, Yildirim (2010, Lemma 1) shows that, when agents have the same discount factor, the first stage is equivalent to a one-shot rent-seeking game in which the prize is equal to 1, and thus does not depend on the discount factor. This implies then that the equilibrium share does not depend on δ either. This illustrative example highlights an important difference between the two models. Whereas unanimity is the voting rule that (strictly) minimizes social costs in the case of transitory recognition, the voting rule does not affect social costs in the case of persistent recognition.

In the rest of the paper, we focus on the persistent recognition model where players choose an effort in the first stage and then the negotiation process takes place. In order to solve for the SSPE of the present two stage game, we use backward induction. We will first analyse the final stage of the game where agents negotiate in order to allocate the surplus.

3 Symmetric equilibrium

3.1 The negotiation stage

Let us first introduce some notations and definitions: $\overline{\psi}_{ij}$ denotes the probability that player i includes player j in his winning coalition, and $\overline{\boldsymbol{\psi}}_i = \left(\overline{\psi}_{ij}\right)_{j \in N \setminus \{i\}}$ denotes the vector of such probabilities for player i. It is immediately checked that, under a given k-majority rule, we must have $\overline{\psi}_{ij} \in [0,1]$ for all $i, j \in N$, $i \neq j$ and $\sum_{j \in N \setminus \{i\}} \overline{\psi}_{ij} = k - 1$ for all $i \in N$. As an illustration, consider the case of three players $(1, 2$ and $3)$. Under the 2-majority rule, we must have $\overline{\psi}_{12} + \overline{\psi}_{13} = 1$, i.e., player 1 includes either player 2 or player 3 in his winning coalition.

It might be convenient to define $\overline{\boldsymbol{\psi}}_{-i} = \left(\overline{\boldsymbol{\psi}}_1, \overline{\boldsymbol{\psi}}_2, ..., \overline{\boldsymbol{\psi}}_{i-1}, \overline{\boldsymbol{\psi}}_{i+1}, ..., , \overline{\boldsymbol{\psi}}_n\right)$ too. The shares $\overline{\boldsymbol{s}} = (\overline{s}_1, ..., \overline{s}_n)$ are characterized by:

$$\overline{s}_i = p_i \left(1 - \overline{w}_i\right) + \delta \overline{\mu}_i \overline{s}_i, \text{ for } i = 1, ..., n. \tag{10}$$

where,

$$\overline{w}_i = \sum_{j \neq i} \overline{\psi}_{ij} \delta \overline{s}_j \text{ and } \overline{\mu}_i = \sum_{j \neq i} p_j \overline{\psi}_{ji}. \tag{11}$$

In words, when agent i is the proposer (which happens with probability p_i) he offers to all members of his winning coalition their continuation values. When he is not the proposer, he is included in agent j's winning coalition (when agent j becomes the proposer, which happens with probability p_j) with probability ψ_{ji} and then receives his continuation value δs_i.

We now characterize the agents' optimal strategies during the negotiation process (taking into account that their recognition probabilities are fixed). At this stage of the analysis we will introduce a tie breaking rule that will explain the type of behavior that is assumed from identical agents. It will be helpful to avoid cases where agents might be indifferent between two alternative strategies. We now proceed with the analysis. Let us consider $\boldsymbol{p} = (p_1, ..., p_n)$ and $\boldsymbol{s} = (s_1, ..., s_n)$. Without loss of generality, we relabel the players in increasing share. The second stage equilibrium is characterized by $\boldsymbol{\psi} = (\boldsymbol{\psi}_1, ..., \boldsymbol{\psi}_n)$ such that:

$$\boldsymbol{\psi}_i = Arg \max_{\overline{\boldsymbol{\psi}}_i} \left\{ p_i [1 - \delta \sum_{j \neq i} \overline{\psi}_{ij} \overline{s}_j] + \delta \sum_{j \neq i} p_j \psi_{ji} \overline{s}_i \right\}; \tag{12}$$

The best reply of player i in the second stage of the game is given by:

$$\forall i, \forall j \neq i, \psi_{ij} = \begin{cases} 1 \text{ if } \overline{s}_j < \overline{s}_k \\ \leq 1 \text{ if } \overline{s}_j = \overline{s}_k \\ 0 \text{ if } \overline{s}_k < \overline{s}_j \end{cases} \tag{13}$$

This characterization leads to the following preliminary result:

Lemma 1 *In the equilibrium of the second stage, the vector of probabilities of inclusion, $\psi = (\boldsymbol{\psi}_i)_{i \in N}$, and the vector of shares $s = (s_i)_{i \in N}$ are functions of $(\boldsymbol{x}, \delta)$, with $\psi_i = \psi_i(\boldsymbol{x}, \delta)$ and $s_i = s_i(\boldsymbol{x}, \delta)$ for all $i \in N$. The vector of shares s is the solution of*

$$s_i = p_i (1 - w_i) + \delta \mu_i s_i, \textit{ for } i = 1, ..., n, \tag{14}$$

where

$$w_i = \delta \sum_{j \neq i} \psi_{ij} s_j \textit{ and } \mu_i = \sum_{j \neq i} p_j \psi_{ji}, \tag{15}$$

and

$$\forall i, \forall j \neq i, \psi_{ij} = \begin{cases} 1 \textit{ if } s_j < s_k \\ \leq 1 \textit{ if } s_j = s_k \\ 0 \textit{ if } s_k < s_j \end{cases} \tag{16}$$

The second stage equilibrium strategies are characterized implicitly. The above result has a simple interpretation. Any agent will include the agents with the $(k-1)$ cheapest votes in his winning coalition with certainty, and any other agent is excluded from it.

Before deriving the first stage equilibrium, we will show an interesting property of the second stage equilibrium. To show the result, we need to specify a tie breaking rule (Anonymity) regarding situations where the votes of two players have the same cost:
Assumption (Anonymity): If $s_l = s_k = s_i$ with $i \neq l$, then $\psi_{ji} = \psi_{jl} = \psi_{jk}$ if $j \neq i, l, k$, $\psi_{ki} = \psi_{kl}$ and $\psi_{ik} = \psi_{lk}$ if $k \neq i, l$.

In other words, if two agents i and l (with shares identical to the k^{th} smallest share) are to be chosen by any other agent j to be part of his winning coalition, then they will be chosen with the same probability. Moreover, each of agents i, l and k must include (and be included by) the other two agents with the same probability. We use the following preliminary result:

Lemma 2 *Assume that assumption (Anonymity) holds. Then, in the second stage equilibrium, we have:*

$$p_i \leq p_l \Longleftrightarrow s_i \leq s_l.$$

The above result relies mainly on the characterization of the equilibrium shares (14). This lemma implies that the higher the probability of being the proposer, the larger the agent's share. It enables us to characterize the agents' optimal strategies at the last stage of the game. In order to solve for the SSPE, we go backward and analyse the initial stage of the game where agents exert effort in order to influence their recognition probabilities. In the rest of the paper, we assume that assumptions (TCSF) and (Anonymity) hold.

3.2 The recognition stage

The present sub-section characterizes the potential candidate for a (symmetric) SSPE of the two stage game, by characterizing the first order conditions of the investment game. The analysis of the existence of the equilibrium is left to the next section. Reasoning backward, the first stage equilibrium of the game is the equilibrium of a one-shot game where the payoff of player i is given by $v_i(\boldsymbol{x}, \delta) = s_i(\boldsymbol{x}, \delta) - cx_i$, with $x_i \geq 0$. The rest of the paper will focus on the symmetric equilibrium of this one shot game.

A symmetric equilibrium of the one shot game is characterized by $\boldsymbol{x}^* = (x^*, ..., x^*)$ such that

$$v_i(\boldsymbol{x}^*, \delta) \geq v_i\left(x_i, \boldsymbol{x}^*_{-i}, \delta\right), \text{ for all } x_i \geq 0 \text{ and all } i, \tag{17}$$

with,

$$s_i = p_i(1 - w_i) + \delta\mu_i s_i, \textit{ for } i = 1, ..., n. \tag{18}$$

Using Lemma 1 and relabelling the players in increasing share, we have:

$$w_i = \begin{cases} w_k + \delta(s_k - s_i) \text{ if } s_i \leq s_k \\ w_k \text{ if } s_k \leq s_i \end{cases}, \tag{19}$$

and,

$$\mu_i \begin{cases} = 1 - p_i \text{ if } s_i < s_k \\ \leq 1 - p_i \text{ if } s_i = s_k \\ 0 \text{ if } s_k < s_i \end{cases}, \tag{20}$$

This new expression of the equilibrium shares will be used in the next sub-section.

3.3 Voting rules and social cost

In the present section, we characterize the unique candidate for a symmetric equilibrium and compare the social cost for the different possible voting rules.

Proposition 1 *Whatever the voting rule ($1 \leq k \leq n$), the candidates for a symmetric* *equilibrium are the Nash equilibria of the contest game with contest success function p. Hence, the unique candidate for a symmetric equilibrium is $\boldsymbol{x}^* = (x^*, ..., x^*)$ such that:*

$$\frac{f(x^*)}{f'(x^*)} = \frac{1}{c}\frac{n-1}{n^2}. \tag{21}$$

Let us provide some intuition for the above result. First, the case of dictatorship is quite simple. The proposer gets everything at the bargaining stage. At the same time, an agent who is not the proposer gets nothing. This implies that the equilibrium share at the negotiation stage is simply the probability that the agent is the proposer, and the pre-bargaining stage is then equivalent to a contest where the size of the prize is equal to one. The corresponding level of effort is well known from the contest literature. Now, as the number of agents required for a share to be agreed upon increases, the (expected) share offered to the other agents (as an aggregate) should become larger, the higher the number of agents required. Thus, the share secured when an agent becomes the proposer decreases. Moreover, as the majority requirement increases, the chance that a given agent (who is not the proposer) becomes part of the winning coalition increases. This results in a higher payoff when this agent is not the proposer. Under the symmetry assumption, these two effects have the same magnitude, and the pre-bargaining stage is again equivalent to a contest with a prize of size one. Hence, we obtain the same levels of effort.

We have shown that $\boldsymbol{x}^*$ is the unique candidate for a symmetric equilibrium. As mentioned previously, the above result has an interesting implication. Endogenous recognition has different effects depending on whether it is transitory or persistent. Specifically, under transitory recognition the unanimity rule yields the lowest social cost resulting from influence activities. Under persistent recognition, all voting rules yield the same cost. This can be summarized as follows:

Corollary 1: *Provided that a symmetric SSPE exists, all voting rules yield the same social cost.*

It remains to analyse the conditions under which the symmetric SSPE exists. This is the aim of the next section.

4 Existence of the symmetric equilibrium

Let us begin the section by the following result:

Lemma 3 *Whatever the voting rule ($1 \leq k \leq n$), no player has an incentive to deviate* *from the symmetric equilibrium by increasing his effort.*

The intuition for this result can be stated as follows. When a player deviates from the symmetric equilibrium by increasing his effort by an infinitesimal amount, this translates into an infinitesimal increase in his probability of being the proposer, and a downward jump in his probability of being included in any other player's winning coalition (which falls to 0). Hence, the players are worse off when they increase their effort with respect to the symmetric equilibrium candidate.

Now it remains to analyse what happens if an agent decides to decrease his effort level (with respect to the candidate equilibrium level) for a strict k-majority, and if unanimity/dictatorship is required. In the case of unanimity and dictatorship, it is easy to check that the symmetric equilibrium always exists. Indeed, in both cases, the payoff of player i is given by $v_i(\boldsymbol{x}, \boldsymbol{\delta}) = \frac{f(x_i)}{\sum_j f(x_j)} - cx_i$ (when at least one of the players' effort levels is strictly positive) and it is concave in x_i.

This proves the first claim of the following proposition:

Proposition 2 *Under the unanimity rule and the dictatorship rule, the symmetric equilibrium always exists. Under the strict k-majority rule ($2 \leq k \leq n-1$), the symmetric equilibrium fails to exist because each player has incentives to lower his effort.*

The proof of Proposition 2 shows that players have incentives to deviate from the symmetric equilibrium candidate by reducing their effort. The main intuition is that the agents face a trade-off with endogenous recognition. On one side, as they increase their effort, the chances that they become the proposer increase, which might result in a higher payoff. On the other side, a higher probability of becoming the proposer makes an agent's vote more expensive. This decreases his chance to be included in a winning coalition (in case a strict k-majority rule is used) if he is not the proposer, and this might result in a lower payoff. At the symmetric equilibrium candidate, the second effect dominates. This effect is strong, since a very small decrease in the player's effort induces a small decrease in his probability of being the proposer; but this has a large effect on his vote, which now becomes the cheapest one. The agent is then included with certainty in all the winning coalitions at the negotiation stage, which increases notably the payoff received when the agent is not the proposer. This result also highlights an important difference between transitory and persistent recognition. Whereas an equilibrium generally exists when recognition is transitory, it fails to exist for majority rules in the case of persistent recognition. Moreover, the reason why the symmetric equilibrium does not exist is an interesting qualitative property of negotiations where recognition is persistent. Lemma

3 and Proposition 2 show that the equilibrium fails to exist only because of the race to the bottom described above. Players have no incentive to increase their effort from the symmetric equilibrium (candidate). This is a quite unexpected and interesting result in the bargaining literature because players usually benefit from being the proposer (see Eraslan (2002)).

5 Discussion and extension

Before concluding the paper, it might be useful to provide some additional interpretations for the results obtained, and to discuss some limitations. First, the equivalence result obtained in section 4 requires further explanation. Specifically, it has been demonstrated that the unanimity rule is equivalent to one specific k-majority rule, namely the 1-majority rule, or dictatorship. Symmetric SSPE exist in both cases and yield the same social cost. As such, a third party in charge of the organization of negotiations and interested in minimising the social cost resulting from influence activities would be indifferent between requiring unanimous consent or a 1-majority rule.

A second implication of Proposition 2 is that a symmetric SSPE fails to exist under any other majority requirement. If one thinks of symmetric equilibria as a simple form of strategies, then a possible interpretation of this non-existence result would be that one will not be able to predict the outcome of negotiations in a simple way (under strict k-majority rule with $2 \leq k \leq n-1$).

This raises an additional point. There are two potential ways to look at the non-existence result. First, one might consider it as an interesting result per se, since it is a specific feature of the case of persistent recognition. When recognition is transitory, equilibrium existence is not an issue. Moreover, if one interprets the ability to propose as a way to define bargaining power endogenously, this property highlights an important characteristic of situations where the parties' power during negotiations is persistent and endogenous. Specifically, it is not the agents' incentives to increase their chances to propose that create a problem, but rather their willingness to be included in the other parties' winning coalitions. By contrast, the usual conclusion of the literature assuming an exogenous recognition process is that a higher bargaining power is profitable. The present result suggests that this is not a robust feature of participative processes. We consider that these are interesting conclusions resulting from the analysis.

Secondly, one might be willing to know if this non-existence result is generic. The problem is that the intensity of competition at the first stage is sufficiently low so that the unique equilibrium candidate is entirely symmetric, that is, all agents exert the same positive level of effort. Due to this symmetry, a race to the bottom occurs and destabilizes the candidate.

We show in the next proposition that one way to fix this issue is to define the recognition process by using another form of recognition probabilities that pushes the degree of competition up (and makes the equilibrium candidate asymmetric). Specifically, we consider a three agent example, and we can state:

Proposition 3 *Consider that recognition functions are defined using Assumption (HCSF) with $n = 3$. There exist two values $\underline{\delta} < \overline{\delta}$ in $(0,1)$ such that for any $\delta \in (0, \underline{\delta})$ or $\delta \in \left(\overline{\delta}, 1\right)$, for any voting rule ($1 \leq k \leq 3$) the SSPE equilibrium exists and the effort levels are characterised as follows:*

- *Two players are active and one abstains from exerting effort;*
- *Each active player exerts an effort level equal to $x^* = \frac{1}{2c}$.*

A few remarks are worth noticing. There is no assumption relying specifically on the number of agents in the above proof. Thus, the result can be extended to more than three agents. Specifically, there would still be only two active agents (see Alcalde and Dahm 2007), and all remaining players would abstain from exerting efforts. The only difference is that the characterisation of the bounds of the intervals ensuring the existence of SSPE might vary.

Secondly, using this new definition of the recognition stage enables to fix the existence problem for k-majority rules with $1 < k \leq n - 1$. SSPE exist and are not strictly symmetric, but are all outcome equivalent (they all result in the same equilibrium efforts). This asymmetry in the agents' contributions at the first stage enables to fix the existence issue. By lowering his effort by a small amount it is not possible any more to increase an agent's probability to be included in winning coalitions to a sufficient level, and the race to the bottom disappears.

Thirdly, all voting rules (unanimity and strict k-majority rules) share the same equilibrium outcome. This implies that all voting rules are then equivalent, which reinforces the conclusion of Corollary 1.

6 Conclusion

The issue of buying influence in collective decision making is extremely important as it is prevalent in many real world economic situations (lobbying in legislative bargaining, international negotiations, composition of executive committees in economic organizations). There are many questions related to this issue. The present contribution analyses a multilateral bargaining situation where recognition is persistent and endogenous. Voting rules are compared with respect to the social cost resulting from them. It is demonstrated that this comparison differs notably depending on the type of recognition that is considered. While unanimity is the only voting rule minimizing the social cost when recognition is transitory, voting rules become equivalent as soon as recognition becomes persistent (provided a symmetric equilibrium exists). We also show that (unlike the case of transitory recognition) the symmetric equilibrium may fail to exist because of a race to the bottom. Indeed, players may have incentives to reduce their proposal power in order to be included in the winning coalition. Finally, we highlight that one way to fix the existence problem is to redefine the recognition process in order to push the intensity of competition (at the lobbying stage) up.

These conclusions stress the fact that one should be cautious when thinking about the choice of the appropriate voting rule in collective decision making situations, especially when influence activities might be used.

One promising avenue for further research is to analyse the problem when agents are heterogeneous in order to assess the impact of heterogeneity on the robustness of the present results. We hope to contribute to this line in the near future.

Appendix

Proof of Lemma 2: The share of player $t = i,j$ is given by:

$$\begin{aligned} s_t &= p_t\left(1 - w_k - \delta s_k + \delta s_t\right) + \mu_t \delta s_t \\ &= \frac{p_t}{1-\delta\left(p_t+\mu_t\right)}\left(1 - w_k - \delta s_k\right). \end{aligned} \tag{22}$$

To show that $p_i \leq p_j \Rightarrow s_i \leq s_j$, consider $\boldsymbol{p}$ and $\boldsymbol{s}$ with $p_i \leq p_j$ and $s_j < s_i$. Using (22), we have

$$\frac{p_j}{1-\delta\left(p_j+\mu_j\right)} < \frac{p_i}{1-\delta\left(p_i+\mu_i\right)}. \tag{23}$$

If $s_j < s_i \leq s_k$, we have $\mu_i \leq 1 - p_i$ and $\mu_j = 1 - p_j$, then (23) leads to:

$$\frac{p_j}{1-\delta} < \frac{p_i}{1-\delta\left(p_i+\mu_i\right)} \leq \frac{p_i}{1-\delta}, \tag{24}$$

a contradiction.

If $s_k \leq s_j < s_i$, we have $\mu_i = 0$ and $\mu_j \leq 1 - p_j$, then (23) leads to:

$$\frac{p_j}{1 - \delta\left(p_j + \mu_j\right)} < \frac{p_i}{1 - \delta\left(p_i + \mu_i\right)} = \frac{p_i}{1 - \delta p_i}. \tag{25}$$

Hence, we must have $p_j < p_i\left(1 - \delta\mu_j\right) \leq p_i$; a contradiction.

To show that $s_i \leq s_j \Rightarrow p_i \leq p_j$, consider $\boldsymbol{p}$ and $\boldsymbol{s}$ with $s_i \leq s_j$ and $p_j < p_i$. If $s_i < s_j$, the proof above can be used to obtain a contradiction. Assume that $s_i = s_j$ and $p_j < p_i$. If $s_i = s_j \neq s_k$, we have $\mu_i = \mu_j$, and using (22) we have $p_j = p_i$, a contradiction. Assume that $s_i = s_j = s_k$; notice that

$$\mu_i - \mu_j = p_j\psi_{ji} - p_i\psi_{ij}, \tag{26}$$

and using (Anonymity), we have $\psi_{ji} = \psi_{ij}$ and then $\mu_i = \mu_j + (p_j - p_i)\,\psi_{ij}$. Using (22) and $s_i = s_j$, we have

$$\frac{p_j}{1 - \delta\left(p_j + \mu_j\right)} = \frac{p_i}{1 - \delta\left(p_i + \mu_i\right)}, \tag{27}$$

or,

$$(p_j - p_i)\left(1 - \delta\left(p_j\psi_{ij} + \mu_j\right)\right) = 0. \tag{28}$$

Since $\psi_{tl} \leq 1$ for all $t \neq l$ and $\sum\limits_{t \in N} p_t = 1$, we have $1 - \delta\left(p_j\psi_{ij} + \mu_j\right) > 0$. We conclude that $p_j = p_i$, a contradiction.$\square$

Proof of Proposition 1: First consider the case of unanimity, $k = n$. The shares of the players are characterized by (see Yildirim 2010), for any i:

$$s_i = \frac{p_i}{\sum\limits_j p_j} = p_i. \tag{29}$$

Now assume that $k \leq n - 1$. In a symmetric equilibrium, players' efforts are the same, $x_i = x_j$ for any $i, j \in N$. According to Lemma 2, the players have the same share, $s_i = s_j$ for any $i, j \in N$. The share of player $i \in N$ is then given by:

$$s_i = p_i[1 - \frac{k-1}{n-1}\sum_{j \neq i}\delta s_j] + \frac{k-1}{n-1}\sum_{j \neq i} p_j \delta s_i; \tag{30}$$

Using the fact that $\sum_{j \neq i} p_j = 1 - p_i$ and rearranging terms, we have:

$$s_i = \frac{p_i}{1 - \frac{k-1}{n-1}\delta}[1 - \frac{k-1}{n-1}\delta\sum_j s_j]. \tag{31}$$

Summing over the set of agents, we obtain:

$$\sum_j s_j = 1. \tag{32}$$

Thus, the share of player i is given by:

$$s_i = p_i. \tag{33}$$

Hence, whatever the voting rule ($1 \leq k \leq n$), the candidates for a symmetric equilibrium are the solutions to, for any i:

$$\underset{x_i \geq 0}{Max} \ \left(p_i\left(x_i, x_{-i}\right) - cx_i\right). \tag{34}$$

Using (TCSF) and assuming $x^* \neq 0$, player i's equilibrium effort level is given by:

$$x^* \in \arg\max_{x_i \geq 0} \left(\frac{f\left(x_i\right)}{f\left(x_i\right) + (n-1)\, f\left(x^*\right)} - cx_i\right). \tag{35}$$

In an interior (symmetric) equilibrium, we have

$$(n-1)\, f\left(x^*\right) f'\left(x^*\right) = c\left(f\left(x^*\right) + (n-1)\, f\left(x^*\right)\right)^2. \tag{36}$$

Hence,

$$(n-1)\, f\left(x^*\right) f'\left(x^*\right) = c\left(f\left(x^*\right) + (n-1)\, f\left(x^*\right)\right)^2. \tag{37}$$

Since $x^* > 0$, we have $f\left(x^*\right) > 0$ and then:

$$\frac{f\left(x^*\right)}{f'\left(x^*\right)} = \frac{1}{c}\frac{n-1}{n^2}. \tag{38}$$

To complete the proof, notice that $\boldsymbol{x} = (0, ..., 0)$ cannot be an equilibrium since any player has an incentive to deviate from this situation and exert an infinitesimal effort.$\square$

Proof of Lemma 3: Assume that one agent (say 1) deviates from $\boldsymbol{x}^*$ by exerting effort $x_1 > x^*$. According to Lemma 2, this implies $s_1 > s$, where s_1 denotes the share of player 1 and s denotes the share of all agents $2, ..., n$ (those shares resulting from the vector of effort $(x_1, x^*, x^*, ..., x^*)$). This and assumption (Anonymity) imply that the new equilibrium strategies at the second period are:

$$\psi_{i1} = 0; \;\; \psi_{ij} = \psi_{ji} = \frac{k-1}{n-2}; \;\; \psi_{1i} = \frac{k-1}{n-1}, \tag{39}$$

where $i = 2, ..., n$ and $j \neq i$. This yields the following characterisation of the equilibrium shares:

$$s_1 = p_1[1 - \delta(k-1)s], \tag{40}$$

and

$$s = p[1 - \delta(k-1)s] + (n-2)p\frac{k-1}{n-2}\delta s + p_1\frac{k-1}{n-1}\delta s, \tag{41}$$

where p_1 is the probability that player 1 is the proposer and p is the probability of each agent $i \geq 2$. Solving the above set of equations, we obtain:

$$s_1 = p_1 \frac{n-1-\delta\left(k-1\right)}{n-1-\delta\left(k-1\right)p_1}. \tag{42}$$

Replacing $p_1 = \frac{f(x_1)}{(n-1)f(x^*)+f(x_1)}$, we have:

$$s_1\left(x_1, \boldsymbol{x}_{-1}^*\right) = [n-1-\delta(k-1)]\frac{f\left(x_1\right)}{(n-1)[(n-1)f\left(x^*\right)+f\left(x_1\right)] - \delta f\left(x_1\right)\left(k-1\right)}. \tag{43}$$

Then, coming back to the first agent's expected payoffs:

$$v_1\left(x_1, \boldsymbol{x}_{-1}^*\right) = [n-1-\delta(k-1)]\frac{f\left(x_1\right)}{(n-1)[(n-1)f\left(x^*\right)+f\left(x_1\right)] - \delta f\left(x_1\right)\left(k-1\right)} - cx_1. \tag{44}$$

The above function is easily checked to be strictly concave. Moreover, we obtain the following expression of marginal expected payoffs:

$$\begin{aligned} \frac{\partial v_1}{\partial x_1}\left(x_1, \boldsymbol{x}_{-1}^*\right) &= \frac{[n-1-\delta(k-1)]f'\left(x_1\right)\left(n-1\right)^2 f\left(x^*\right)}{[(n-1)((n-1)f\left(x^*\right)+f\left(x_1\right)) - \delta f\left(x_1\right)\left(k-1\right)]^2} - c \\ &= \frac{f'\left(x_1\right)}{f\left(x_1\right)}\frac{[n-1-\delta(k-1)](n-1)^2\frac{f(x^*)}{f(x_1)}}{[(n-1)((n-1)+\frac{f(x^*)}{f(x_1)}) - \delta\frac{f(x^*)}{f(x_1)}(k-1)]^2} - c. \end{aligned} \tag{45}$$

Hence, for $x_1 = x^*$, using (21) the value of this function is:

$$\begin{aligned} \frac{\partial v_1}{\partial x_1}\left(\boldsymbol{x}^*\right) &= \frac{[n-1-\delta(k-1)]n^2(n-1)}{[(n-1)n - \delta(k-1)]^2}c - c \\ &= -\frac{\left(n^2-3n+2\right) - \delta(k-1)}{[(n-1)n - \delta(k-1)]^2}c < 0. \end{aligned} \tag{46}$$

Since the payoff of player 1 is not continuous at point $\boldsymbol{x}^*$, we now have to compare $v_1(\boldsymbol{x}^*)$ and the equilibrium payoff of player 1, $v_1^*(\boldsymbol{x}^*) = \frac{1}{n} - cx^*$. We obtain:

$$v_1(\boldsymbol{x}^*) - v_1^*(\boldsymbol{x}^*) = \frac{n-1-\delta(k-1)}{(n-1)n-\delta(k-1)} - \frac{1}{n} < 0 \tag{47}$$

This rules out the possibility of a profitable deviation for values higher than x^*.□

Proof of Proposition 2: Under the unanimity rule and the dictatorship rule, the functional form of the shares does not change when one player deviates from the symmetric equilibrium, we still have $s_i = p_i$ even out of equilibrium. It is sufficient to notice that p is concave with respect to x_i (for any i) to show that the symmetric equilibrium always exists.

Now assume that $2 \leq k \leq n-1$. We know that $\boldsymbol{x}^*$ is the unique candidate for a symmetric equilibrium. At this point, the payoff of each player is given by

$$v_1^*(\boldsymbol{x}^*) = \frac{1}{n} - cx^*. \tag{48}$$

Now we study players' unilateral incentives to deviate from this candidate. Let us assume that agent 1 deviates by choosing $x_1 < x^*$. According to Lemma 2, this implies that $s_1 < s$, where s denotes the equilibrium share of agents $2, ..., n$ and s_1 denotes the equilibrium share of agent 1. This implies that the new equilibrium strategies at the second period are:

$$\psi_{i1} = \psi_{j1} = 1; \;\; \psi_{ij} = \psi_{ji} = \frac{k-2}{n-2}; \;\; \psi_{1i} = \frac{k-1}{n-1}, \tag{49}$$

where $i \neq j$ and $i = 2, ..., n$. This yields the following characterisation of the equilibrium shares:

$$s_1 = p_1[1-\delta(k-1)s] + (n-1)p\delta s_1, \tag{50}$$

and

$$s = p[1-\delta s_1 - \delta(k-2)s] + (n-2)p\frac{k-2}{n-2}\delta s + p_1\frac{k-1}{n-1}\delta s. \tag{51}$$

Solving the above set of equations, we obtain:

$$s_1(x_1, \boldsymbol{x}_{-1}^*) = \frac{[n-1-\delta(k-1)]f(x_1)}{(n-1)(1-\delta)[(n-1)f(x^*) + f(x_1)] + \delta f(x_1)(n-k)}. \tag{52}$$

Then, coming back to the first agent's expected payoffs:

$$v_1\left(x_1, \boldsymbol{x}_{-1}^*\right) = \frac{[n-1-\delta(k-1)]f\left(x_1\right)}{(n-1)(1-\delta)[(n-1)f\left(x^*\right)+f\left(x_1\right)]+\delta f\left(x_1\right)(n-k)} - cx_1. \qquad (53)$$

For $x_1 = x^*$, the difference with the equilibrium payoff is

$$\begin{aligned} \Delta &\equiv v_1\left(\boldsymbol{x}^*\right) - v_1^*\left(\boldsymbol{x}^*\right) \\ &= \frac{n-1-\delta(k-1)}{(n-1)(1-\delta)n+\delta(n-k)} - \frac{1}{n} \end{aligned} \qquad (54)$$

One can easily check that Δ has the same sign than $n^2 - 2n + 1 > 0$.□

Proof of Proposition 3:

First, for the case of unanimity and dictatorship, a reasoning similar to the one used in the main part of the analysis enables us to conclude that $s_i = p_i$ for any agent i at the negotiation stage. Thus, at the first stage, the problem is equivalent to a one-shot rent seeking game where the contest function is defined by (**??**), and the prize is equal to one. The result then follows from Alcalde and Dahm (2007, Theorem 2.1).

We now prove the result for the case of $k = 2$ by focusing on the situation where agents 1 and 2 are active. Thus, we will prove that $x_1^* = x_2^* = x^*$ and $x_3^* = 0$ is sustained as a SSPE outcome. Let us first consider that agent 1 deviates by exerting effort $x_1 > x_2^* = \frac{1}{2c}$. This implies that agent 3 is still the agent with the cheapest vote at the second stage. Using the characterisation of the optimal shares, it is easily checked that this implies

$$s_1 = p_1, \;\; s_2 = p_2, \;\; s_3 = 0, \qquad (55)$$

which in turn implies that the first stage is equivalent to a one-shot rent seeking game between agents 1 and 2, where the contest function is defined by (**??**) and the prize is equal to one. Using the proof of Alcalde and Dahm (2007, Theorem 3.1) enables us to conclude that x_1 is not a profitable deviation.

Now, let us consider the deviation where agent 1 exerts effort $0 < x_1 < x_2^*$ while agent 3 abstains from entering the recognition stage. Once again, this implies that agent 3 is still the agent with the cheapest vote at the second stage, which results in the same characterisation of the optimal shares (than in the first sub-case). The same conclusion applies, which rules out x_1 as a profitable deviation. If $0 = x_1$, player 1's vote is then equal to that of player 3. One can easily show that the resulting payoff for player 1 is 0, which is not a profitable deviation.

A symmetric reasoning enables to conclude that agent 2 does not have a profitable deviation. It remains to prove that this is the case for the third agent as well. Let us consider that this agent decides to exert effort $0 < x_3 < x^* = \frac{1}{2c}$. Then the characterisation of the optimal shares yields:

$$s_i = \frac{p_i(1-\delta s_3)}{1-\delta \frac{p_3}{2}} \text{ for } i = 1, 2 \tag{56}$$

and

$$s_3 = (1-\delta)\frac{(p_3)^2}{(1-\delta(1-p_3))(2-\delta p_3) - p_3(1-p_3)}. \tag{57}$$

Now, coming back to the first stage of the game, agent 3 chooses x_3 such that it maximises his expected payoff, which is equal to:

$$E\Pi_3 = (1-\delta)\frac{(p_3)^2}{(1-\delta(1-p_3))(2-\delta p_3) - p_3(1-p_3)} - cx_3, \tag{58}$$

which, due to $p_3 = \frac{x_3}{3x^*} = \frac{2cx_3}{3}$ can be rewritten as:

$$E\Pi_3 = cx_3\{\frac{(1-\delta)2cx_3}{9(1-\delta) + 3cx_3(\delta^2+\delta-1) + 2(cx_3)^2(1-\delta^2)} - 1\}. \tag{59}$$

The present strategy does not constitute a profitable deviation if and only this agent's expected payoff is non positive, that is:

$$9(1-\delta) + 3cx_3(\delta^2+\delta-1) + 2(cx_3)^2(1-\delta^2) - (1-\delta)2cx_3 > 0, \tag{60}$$

or

$$9(1-\delta) + cx_3[3\delta^2+5\delta-5] + 2(cx_3)^2(1-\delta^2) > 0. \tag{61}$$

The first thing to notice is that this inequality is satisfied as long as $3\delta^2+5\delta-5$ is non negative, that is, provided that:

$$\delta \in]\frac{-5+\sqrt{85}}{6}, 1[\;=\;]\overline{\delta}, 1[. \tag{62}$$

In this case, the agent's strategy x_3 is not a profitable deviation.

Now, let us assume that this is not satisfied. Let us notice that condition (61) is a polynomial expression of cx_3 (of degree two), and introduce the following notation:

$$P(\delta) = \left(3\delta^2 + 5\delta - 5\right)^2 - 72(1-\delta)(1-\delta^2) = 9\delta^4 - 42\delta^3 + 67\delta^2 + 22\delta - 47. \tag{63}$$

Then, provided that $P(\delta) < 0$ we know that condition (61) will be satisfied necessarily. It is easily checked that $P(\delta)$ is an increasing function and $P(0) = -47$ and $P(1) = 9 > 0$, which implies that there exists a unique value $\underline{\delta} \in]0,1[$ such that:

$$P(\delta) < 0 \Longleftrightarrow \delta \in]0, \underline{\delta}[. \tag{64}$$

Thus, for any $\delta \in]0, \underline{\delta}[$ we know that condition (61) is satisfied too, which rules out x_3 as a profitable deviation. One can easily check that there is a profitable deviation for the remaining values of the discount factor.

Finally, let us consider that agent 3 deviates by choosing an effort level $x_3 \geq x^* = \frac{1}{2c}$. This implies that we have $s_3 \geq s_2 = s_1$ and that this agent's optimal share becomes:

$$\begin{aligned} s_3 &= p_3\left[1 - \frac{1}{2}\delta s_1 - \frac{1}{2}\delta s_2\right] && (65)\\ s_2 &= p_2\left[1 - \delta s_1\right] + p_1\delta s_2 + \frac{1}{2}p_3\delta s_2 \\ s_1 &= p_1\left[1 - \delta s_2\right] + p_2\delta s_1 + \frac{1}{2}p_3\delta s_1. \end{aligned}$$

The last two equations can be rewritten as:

$$s_2 = \frac{p_2}{1 - \delta\left(p_1 + \frac{1}{2}p_3\right)}\left[1 - \delta s_1\right] \tag{66}$$

$$s_1 = \frac{p_1}{1 - \delta\left(p_2 + \frac{1}{2}p_3\right)}\left[1 - \delta s_2\right]. \tag{67}$$

Substituting, we have

$$s_2 = \frac{p_2}{1 - \delta\left(p_1 + \frac{1}{2}p_3\right)}\left[1 - \delta\frac{p_1}{1 - \delta\left(p_2 + \frac{1}{2}p_3\right)}\left[1 - \delta s_2\right]\right] \tag{68}$$

or,

$$\begin{aligned}
&\left(1-\delta^2\frac{p_2}{1-\delta\left(p_1+\frac{1}{2}p_3\right)}\frac{p_1}{1-\delta\left(p_2+\frac{1}{2}p_3\right)}\right)s_2 \\
=&\quad \frac{p_2}{1-\delta\left(p_1+\frac{1}{2}p_3\right)}\left[1-\delta\frac{p_1}{1-\delta\left(p_2+\frac{1}{2}p_3\right)}\right]. \qquad (69)
\end{aligned}$$

Rewriting, we obtain:

$$\left(\frac{\left(1-\delta\left(p_1+\frac{1}{2}p_3\right)\right)\left(1-\delta\left(p_2+\frac{1}{2}p_3\right)\right)-\delta^2p_1p_2}{\left(1-\delta\left(p_1+\frac{1}{2}p_3\right)\right)\left(1-\delta\left(p_2+\frac{1}{2}p_3\right)\right)}\right)s_2 \qquad (70)$$

$$=\frac{p_2}{\left(1-\delta\left(p_1+\frac{1}{2}p_3\right)\right)\left(1-\delta\left(p_2+\frac{1}{2}p_3\right)\right)}\left[1-\delta\left(p_2+\frac{1}{2}p_3\right)-\delta p_1\right]$$

or,

$$s_2=\frac{p_2}{\left(1-\delta\left(p_1+\frac{1}{2}p_3\right)\right)\left(1-\delta\left(p_2+\frac{1}{2}p_3\right)\right)-\delta^2p_1p_2}\left[1-\delta+\delta\frac{1}{2}p_3\right]. \qquad (71)$$

By symmetry, we conclude that:

$$s_1=\frac{p_1}{\left(1-\delta\left(p_1+\frac{1}{2}p_3\right)\right)\left(1-\delta\left(p_2+\frac{1}{2}p_3\right)\right)-\delta^2p_1p_2}\left[1-\delta+\delta\frac{1}{2}p_3\right]. \qquad (72)$$

Hence, finally

$$\begin{aligned}
s_3 &= p_3\left[1-\frac{1}{2}\delta\frac{p_1+p_2}{\left(1-\delta\left(p_1+\frac{1}{2}p_3\right)\right)\left(1-\delta\left(p_2+\frac{1}{2}p_3\right)\right)-\delta^2p_1p_2}\left[1-\delta+\delta\frac{1}{2}p_3\right]\right] \\
&= p_3\left[1-\frac{1}{2}\delta\frac{(1-p_3)\left[1-\delta+\delta\frac{1}{2}p_3\right]}{\left(1-\delta\left(p_1+\frac{1}{2}p_3\right)\right)\left(1-\delta\left(p_2+\frac{1}{2}p_3\right)\right)-\delta^2p_1p_2}\right], \qquad (73)
\end{aligned}$$

and this agent's optimal problem at the first stage is:

$$\max_{x_3\geq\frac{1}{2c}} p_3[1-\frac{1}{2}\delta\frac{(1-p_3)[1-\delta+\delta\frac{1}{2}p_3]}{(1-\delta(p_1+\frac{1}{2}p_3))(1-\delta(p_2+\frac{1}{2}p_3))-\delta^2p_1p_2}]-cx_3. \qquad (74)$$

By definition of the recognition function, we deduce that $p_3=1-\frac{1}{3cx_3}$ and $p_1=p_2=p=\frac{1}{6cx_3}$, which in turn enables us to rewrite the agent's problem at the first stage as follows:

$$\max_{x_3 \geq \frac{1}{2c}} (1 - \frac{1}{3cx_3})\{1 - \frac{1}{2}\delta \frac{3cx_3(1-\frac{\delta}{2}) - \frac{\delta}{2}}{9(1-\frac{\delta}{2})^2(cx_3)^2 - \frac{\delta^2}{4}}\} - cx_3 \tag{75}$$

or

$$\max_{x_3 \geq \frac{1}{2c}} (1 - \frac{1}{3cx_3})\{1 - \frac{1}{2}\delta \frac{1}{3cx_3(1-\frac{\delta}{2}) + \frac{\delta}{2}}\} - cx_3. \tag{76}$$

The first order condition is:

$$6\frac{2-\delta}{\left(3\left(2-\delta\right)cx_3+\delta\right)^2} - c \leq 0. \tag{77}$$

If $x_3 > \frac{1}{2c}$, then

$$6\frac{2-\delta}{\left(3\left(2-\delta\right)cx_3+\delta\right)^2} = c, \tag{78}$$

or,

$$x_3 = \frac{1}{3\left(2-\delta\right)c}\left(\sqrt{\frac{6\left(2-\delta\right)}{c}} - \delta\right) \tag{79}$$

At this point, the expected payoff of player 3 is:

$$\begin{aligned} E\Pi_3 &= \left(\frac{\sqrt{\frac{6(2-\delta)}{c}} - 2}{\sqrt{\frac{6(2-\delta)}{c}}}\right) - \frac{1}{3\left(2-\delta\right)}\left(\sqrt{\frac{6\left(2-\delta\right)}{c}} - \delta\right) \\ &= 1 + \frac{\delta}{3\left(2-\delta\right)} - \frac{\sqrt{2}}{\sqrt{3c\left(2-\delta\right)}}\left(1+c\right). \end{aligned} \tag{80}$$

It increases with delta, and for $\delta = 1$, we have $1 + \frac{1}{3} - \frac{\sqrt{2}}{\sqrt{3c}}\left(1+c\right) < 0$.

If $x_3 = \frac{1}{2c}$, then the first order condition becomes:

$$6\frac{2-\delta}{\left(\frac{3}{2}\left(2-\delta\right)+\delta\right)^2} - c \leq 0. \tag{81}$$

At this point, the expected payoff of player 3 is:

$$\begin{aligned} E\Pi_3 &= \left(1 - \frac{2}{3}\right)\left(1 - \frac{1}{2}\delta\frac{2}{3(1-\frac{\delta}{2}) + \frac{\delta}{2}}\right) - \frac{1}{2} \\ &= -\frac{(\delta+3)}{6\left(3-\delta\right)} < 0. \end{aligned} \tag{82}$$

Hence there is no profitable deviation with $x_3 \geq x^*$.

To conclude the proof, it remains to notice that the other SSPE are symmetric to the above structure: two players are active and exert the same level of effort, while the remaining agent abstains from entering the recognition stage and does not exert efforts. □

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