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DEA Applied to a Gauteng Sample of South African Public Hospitals

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DEA APPLIED TO A GAUTENG SAMPLE OF SOUTH AFRICAN PUBLIC HOSPITALS†‡

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Abstract. The ability of the South African government to provide anti-retroviral medication to those in need will be determined by the ability of the public health services sector to efficiently provide that medication. If the delivery of other health services can be used as a guide, the goals of the anti-retroviral rollout will not be met. The research presented in this paper provides a preliminary analysis of the delivery of a few health care services by the public sector in Gauteng, South Africa. The data for the study was especially difficult to collect, suggesting the need for hospital level data information systems, as well as staff trained to analyse the information collected. The empirical results from the analysis suggest that services provided by small-scale medical facilities waste fewer resources, while medical centres offering more technical services, such as surgeries, also appear to deliver medical services more efficiently.

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1. Introduction

The provision of healthcare services represents a large component of the South African government's health budget, and, with the recent passage of legislation promising to deliver anti-retroviral medicines free of charge to anyone needing the medication, the healthcare services budget will represent an even larger component of South Africa's governmental expenditure.[1] If healthcare expenditures represented the only area in which the government purse was under pressure, then it might be possible, even though inappropriate, to be unconcerned with the efficient delivery of public healthcare services. However, healthcare services represent just one of many public sector service delivery concerns in the country. Other public sector projects competing with healthcare services include, but are not limited to: providing clean water and sanitation to a large swath of the population, improving the transportation and communication infrastructure, raising the standard and delivery of education at all levels, and reducing the level of crime across the country.

Given the large number of investment and current expenditure projects making up the public budget and remaining on the public's wish list, it is imperative that the public sector carefully examines whether or not the public budget is providing all it can. Recent municipality audit evidence suggests that much more can be done to improve the delivery of public sector services. Although auditing can provide very useful information regarding the exact allocation of inputs in the delivery of certain services, many audits cannot or do not assess the effectiveness of those input allocations. Furthermore, when audits are used to ascertain the effectiveness of inputs in the delivery of services, those audits are often one dimensional, and, therefore, an audit may be an incomplete approach to the measurement of service delivery effectiveness. For example, Hollingsworth & Parkin (1995) suggest that traditional efficiency indexes and performance indicators are subject to manipulation and other sorts of problems. Empirical approaches to measuring efficiency, on

[1] The health budget for 2005/06 is estimated at R9.8bn, and is set to rise to R10.7bn in 2006/07 and R11.2bn in 2007/08. Hospital services encompass more than 80% of that budget: estimated to be R7.4bn in 2005/06, R7.9bn in 2006/07, and R8.2bn in 2007/08, National Treasury (2005).

the other hand, are more difficult to manipulate, and can, therefore, provide more accurate measurements of efficiency.

In this paper, we present research into the efficiency of the delivery of public healthcare. The measurement approach that we use is Data Envelopment Analysis (DEA), which can be used to compare multiple multi-dimension service delivery outlets. DEA has often been used to examine the delivery of healthcare services. The primary reason for its popularity is the fact that it is one of the few empirical techniques capable of handling multiple inputs and multiple outputs in the same specification. Furthermore, one of its greatest advantages is the fact that DEA does not specify the function of interest to the analysis, which in this case is a production function, nor does it make an explicit assumption about the distribution of error terms, although there is an implicit assumption to be discussed below.

The presentation of research in the paper will continue, in Section 2, with a discussion of the most recent relevant research in the field. The theoretical model, as well as its associated advantages and pitfalls will be considered in Section 3. The data used in the analysis is presented in Section 4. Section 5 contains the results from the analysis. Finally, the presentation of the paper will be concluded with a few recommendations for future research and potential policy implications, in Section 6.

2. Relevant Literature

Efficiency, which occurs in various forms in economics, has a rich history in economics and underscores all of economic thinking. Despite the importance of efficiency in economics, the ability to measure it has only recently been developed. In production economics, efficiency takes on two forms, technical efficiency, where firms produce the most output possible with their current set of inputs, and allocative efficiency, where input prices determine the least costly mix of inputs capable of producing along the technically efficient frontier. Economically, profit-maximizing and cost-minimizing firms are assumed to achieve technical efficiency in the short run and both in the long run, as long as markets are unfettered. However, in the theory of production related to hospitals, cost-minimizing or profit-maximizing behaviour is not necessarily the *modus operandi*.[2]

Despite the fact that pure efficiency or absolute efficiency may not be the expected result when considering public hospital production, due, for example, to the fact that costs are covered by the national purse, eliciting more and better health care from available resources, or improving efficiency, is an important goal of the public.[3] For that reason, the relative efficiency of public hospital production has implications for public policy. Unfortunately, hospitals produce a multiple of intermediate goods, all of which go towards the improvement in final health, a final good that cannot be easily quantified. Due to the difficulty in measuring true hospital output, requiring the measurement of a multiplicity of intermediate outputs, the analysis of hospital production often focuses on the production of intermediate goods; see, for example, Grosskopf & Valdmanis (1987) and Sexton, Lieken, Nolan, Liss, Hogan & Silkman (1989).

DEA has been applied in a number of hospital efficiency studies. The various analyses in the literature include comparisons of efficiency across ownership types; Grosskopf & Valdmanis (1987) and Valdmanis (1992) compare public and not-for-profit hospital efficiency. Similarly, a number of studies have been conducted to determine the effect of financing on efficiency; Gruca & Nath (2001) and Steinmann & Zweifel (2003) represent two such examples. O'Niell (1998) and Grosskopf, Margaritis & Valdmanis (2001) compare teaching and non-teaching hospital performance, while Hofmarcher, Paterson & Riedel (2002) compare within and across hospital performance over different medical fields. Hospital congestion has been examined by Valdmanis, Kumanarayake & Lertiendumrong (2004). Dacosta-Claro & Lapierre (2003), amongst others, have examined returns to scale, while McCallion,

[2] Research by Newhouse (1970) and Evans (1971) represent early forays into alternative optimizing behaviour.
[3] Even if that goal is indirect, through, for example, the desire for lower taxes.

Glass, Jackson, Kerr & McKillop (2000), amongst others, have examined differences in performance based on hospital size. All of the preceding studies have been performed within one country or one area of a country; however, differences in efficiency across countries has been studied by Mobley & Magnussen (1998). Given the amount of data available for this study, which is very limited at this stage, the analysis in this paper focuses on the simpler comparisons surrounding returns to scale as well as efficiency differences between different types of hospitals.

Despite, or possibly because of, the popularity of DEA, uncovering robustness in the estimates is an analytical priority. In earlier research, validity relied upon simple dynamic and static comparisons. For example, Parkin & Hollingsworth (1997) examine whether or not efficiency scores change profoundly from one year to the next. In other analysis, O'Niell (1998) extends DEA to multifactor productivity indexes, which can then be compared to more aggregated DEA indexes. Furthermore, Steinmann & Zweifel (2003) examine whether or not the estimated scores are sensitive to the use of inpatient days as an input or as an output.[4] However, many validity issues in DEA are addressed through the introduction of probabilistic notions. For example, Cooper, Li, Seiford, Tone, Thrall & Zhu (2001) discuss the input and output variations required to move firms onto and off from the efficient frontier. Olesen & Pietersen (2002), in a similar vein, describe the measurement of probabilistic assurance regions in DEA. Unfortunately, the only validity analysis undertaken in this research is a comparison of efficiency scores across a wide range of input and output combinations; again, limited data makes it difficult to take the analysis too far.

Finally, it is important to note that the analysis undertaken and reported in this paper is not the first to consider South Africa, although it is the first to examine hospitals in Gauteng. Zere, McIntyre & Addison (2001) used data from the former Cape Province and the current Western Cape Province covering the years 1992 to 1998. The data they used was different from the data used in this research, which

[4] They argue that inpatient days, which are often used as an output in DEA, might better represent an input, since patients are using their days in the hospital to recuperate.

could explain why they measure average efficiency to be lower than the average estimates provided here. Importantly, since their data only covers the Western Cape, it is unclear whether or not their results are representative of healthcare delivery at a national level. Although the exact emphasis of the analysis presented in this paper is different from that presented by Zere et al. (2001), this research will help fill the gap in research that exists, regarding the effective delivery of healthcare services in South Africa.

3. Data Envelopment Analysis

Data Envelopment Analysis, based on the radial measure of efficiency originally developed by Farrell (1957) and extended by many others, including Charnes, Cooper & Rhodes (1978), Banker, Charnes & Cooper (1984), and Färe, Grosskopf & Lovell (1985), is the empirical model applied in this paper. Although the model is empirical, in the sense that observations determine the estimates, the model is non-parametric, in the sense that neither a functional form nor an empirical error distribution is assumed.[5] Although there are few specification assumptions, Newhouse (1994) argues that frontier estimation models should be treated cautiously because inputs and outputs are difficult to measure, certain strong and non-testable hypotheses regarding noise and inefficiency distributions must be made, and that limited degrees of freedom require too much aggregation of the data. Despite the Newhouse's (1994) concerns, it is possible that overarching tendencies can be uncovered in the empirical analysis, and, therefore, the analysis can provide some guidance for improvement.

An illustration of the intuition behind DEA is provided in Figure 1. In Figure 1, five combinations of weighted inputs and weighted outputs (see below) are illustrated as A through E. In the long run, under constant returns to scale (CRS) technology, combinations B and C are technically and scale efficient, while A, D and E are inefficient. On the other hand, under variable returns to scale (VRS)

[5] Accodring to Banker (1993), under certain assumptions, DEA is a maximum likelihood estimator, and, when error distributions are either half-normal or exponential, standard statistical tests can be conducted using the DEA estimates.

technology, in the short run, combinations A through D are technically efficient, while E remains inefficient. Therefore, combination E is inefficient in the short run as well as the long run, so that total inefficiency for E, denoted by the horizontal distance EF, can be explained by scale inefficiency, the horizontal distance FG, and technical inefficiency, the horizontal distance EG. Furthermore, the technical inefficiency of E is determined by a convex combination of C and D; therefore C and D represent technological peers of E.

In order to formalize the illustration, consider public hospitals, denoted by $i = \{1, 2, ..., I\}$, which produce outputs q_i^j, for $j = \{1, 2, ..., J\}$, using inputs x_i^k, for $k = \{1, 2, ..., K\}$. The preceding technology can be used for the creation of an index, determined by the ratio of a weighted sum of the inputs to a weighted sum of the outputs. DEA assumes this efficiency index or ratio, denoted by E, must lie in the unit simplex for all firms, so that[6]

$$E_i = \frac{\sum\limits_{j=1}^{J} \phi_i^j q_i^j}{\sum\limits_{k=1}^{K} \omega_i^k x_i^k} \in [0,1] \;\; \forall i \in \{1, 2, ..., I\}\,. \tag{1}$$

The output weights ϕ_i^j and the input weights ω_i^k, which both must be non-negative, are hospital specific. Assuming that all other hospitals must also meet the restriction in equation (1), the efficiency score for each hospital is chosen so that the relative weights allow for the most favourable view of the hospital. Although equation (1) is non-linear and the constraints, also given in equation (1) are non-linear, the efficiency score for each hospital can be determined by a linear program.

[6] An excellent intuitive description of the technique can be found in Parkin & Hollingsworth (1997), while a more technical, but readable description, can be found in Hollingsworth, Dawson & Maniadakis (1999). Equations (1), (2), and (3) are adapted from these two papers, amongst others.

Confining the consideration to relative weightings, such that either the relative input weights or the relative output weights determine the efficiency score, will lead to the following linear program, given in equation (2).[7]

$$
\begin{aligned}
&\text{Min: } E_0 = \theta_0 \\
\text{subject to: } &\sum_{i=1}^{I} x_i^k \lambda_i = x_i^0 \theta_0 - s_i^k \ \forall k \in \{1, 2, ..., K\} \\
&\sum_{i=1}^{I} \lambda_i q_i^j = .q_0^j + r_i^j \ \forall j \in \{1, 2, ..., J\} \\
&\lambda_i, s_i, r_i \geqslant 0 \ \forall i \in \{1, 2, ..., I\}
\end{aligned} \tag{2}
$$

Intuitively, θ_0 represents the smallest proportional reduction in inputs used by firm 0 to keep it on the frontier determined by a convex combination of the inputs used by all firms in the data set. Furthermore, the output produced by firm 0 cannot exceed the same convex combination of outputs produced by all the firms in the data set, where r and s measure slackness in the constraints. Program (2), which allows for the convex combinations to be chosen freely, is equivalent to an assumption of CRS, Charnes et al. (1978). However, if the convex combinations are further constrained, as in equation (3), VRS technology is assumed, Banker et al. (1984).[8]

$$
\sum_{i=1}^{I} \lambda_i = 1 \tag{3}
$$

Regardless of whether CRS or VRS is assumed, and both will be considered in this research, a public hospital is defined as efficient if and only if (i) $\bar{\theta}_i = 1$ and (ii) $\bar{r}_i^j = \bar{s}_i^j = 0$. One of the most useful features of the analysis is the fact that

[7] This program is actually the dual, although the primal problem is easily formulated.
[8] Although CRS technology results from the fact that output can only be doubled if inputs are doubled, which is a one-to-one relationship, suggesting that the restriction in equation (3) ought to relate to CRS, rather than VRS, that comparison is not correct. Instead, the restriction limits output expansion beyond the best firm and output contraction below the worst firm, given current input combinations. For a more thorough discussion see Valdmanis (1992).

the efficiency measure is invariant to the choice of measurement units, although it is not invariant to either the number of inputs or the number of outputs used in the analysis. Unfortunately, neither the input slacks, s, nor the output slacks, r, are invariant to the units of analysis, Steinmann & Zweifel (2003). As can be seen in Figure 1 and the discussion above, the input contraction required to make combination E technically efficient is a ratio of the horizontal distances G and E, and that ratio is unit free; however, the input slack for the same problem would be determined by the horizontal distance EG, and that total distance would depend upon the unit of measure along the input axis.[9]

For this research, the models represented in equations (2) and (3) are applied to different subsets of inputs and outputs. The goal of the analysis is to learn if public hospital efficiency is general, suggesting that certain public hospitals are more poorly managed than others, or if public hospital efficiency might be input-output specific, suggesting that certain hospitals undertake certain services or use certain inputs more efficiently than others. Additional non-parametric analysis of the DEA outcomes will be undertaken to determine if different types of hospitals are generally more or less efficient.

4. The Data

The data used in the analysis is primary data collected during 2004. All of the public hospitals in the province of Gauteng were contacted.[10] There are 29 public hospitals in the province, although one of them is a women's hospital, only, another one is long-term rehabilitation centre, while another is an academic hospital, so that none of the three were included in the analysis. Of the remaining 27 hospitals, only 14 provided data on some of the inputs and outputs desired for the investigation. However, not all of the hospitals could be used in all of the analyses, due to the

[9] The slacks can be made invariant to the choice of units via a reciprocal measure of efficiency, as well as the inclusion of upper bounds on the input and output weights, Steinmann & Zweifel (2003). Future research will apply the reciprocal efficiency measure.

[10] Gauteng is the wealthiest province in South Africa. It includes the business capital of the country, in Johannesburg and Sandton, as well as the executive branch of the national government, in Tshwane, formerly known as Pretoria.

fact that some hospitals did not provide complete information, e.g., some hospitals do not offer surgery or, if offered, data was not provided.

The participating hospitals provided monthly data on inputs and outputs, as far back as 1999, in a few cases; generally, though, the hospitals provided monthly data for the preceding year, 2003, and up to six months or more of the investigating year, 2004. The data provided by those 14 hospitals varied in detail, and therefore, the analysis was forced to focus on data commonalities. Three input variables were available from all of the hospitals: physicians (doctors and specialists), nurses, and active beds. In addition, up to four output variables were available from the hospitals: total admissions, inpatient visits, outpatient days and total surgeries. The most complete output information was available for admissions and inpatient days, although it was not available for all hospitals at all times.

As can be garnered from the 62% hospital response rate,[11] the willingness or ability to participate in the study was limited. In many cases, hospital CEOs or other administrators provided initial consent to the study, but were later forced to recant, because they did not have staff, who could provide us with the data, or because their hospital board had, in the meantime, rejected the research participation application. In many other cases, approval was granted, but data collection could not proceed, due to staff turnover. The average waiting time, between initial data request and final receipt of the data, was 4.35 months, Kibambe & Koch (2005). Unfortunately, as was clear from the data collection efforts, many of the hospitals lacked the necessary information systems or the staff to manage the information systems, and, therefore, data often had to be transcribed from numerous sources, if it was available. Despite the difficulties, some of the public hospitals in the province were able to provide data back to 1999, suggesting that some hospitals had adequate information systems, and the staff were adequately trained to work with the

[11] The calculated response rate was based on the fact that 18 hospitals, out of 29, responded positively to data requests. As already mentioned in the text, three of those hospitals were removed from the data, due to the specialist nature of their services, while one of the hospitals offered a single annual observation that turned out to be incorrect.

systems.[12] A further concern, raised by the poor response rate, is the potential that the sample is selective, e.g., only the hospitals efficient enough to collect their own data were willing to participate. If the observations were from a selected sample, then the results reported below would only be representative of the sample, rather than being representative of the entire province.

A summary of the data is provided in Table 1.[13] The monthly output data has been averaged over each year for all of the hospitals, for which at least part of the year's data was available. In addition, for hospitals, from which more than one year of data was available, each year's data is counted as a separate observation in the sample. Due to the restructuring of the monthly data, the 14 hospitals could be reorganized into 42 different observations. The data in the table is presented by size of hospital, as measured by the number of active beds, where 220 was chosen as the cut-off between large and small hospitals, because it was nearly the median value. The table includes the input and output variables averages and standard deviations as well as the number of non-zero responses for that particular input or output.

The presentation of the data in Table 1 highlights a number of important issues. As already mentioned, all hospitals were able to provide data on all of the inputs, but not for the outputs. Also, there is a notable difference between large and small hospitals in terms of input usage as well as production. For example, if you consider simple ratios of inputs to outputs, larger hospitals appear to use relatively more inputs than smaller hospitals in producing each of the outputs. For example, large hospitals use 5.0 (763/152) times more nurses than smaller hospitals, although total admissions is only 1.1 times larger (128/118), total outpatient days produced is only 4.6 (7763/1678) times larger; for inpatient days the ratio is 4.9 times (17164/3470). These results suggest that there may be important hospital scale effects, further supporting the comparison between CRS and VRS technologies.

[12] In related research, Kibambe & Koch (2005) find a strong positive relationship between the hospital's ability to provide data and certain measures of efficiency.
[13] Data is not presented by hospital, even under moniker, in order to prevent any single hospital from being singled out in the analysis.

Unfortunately, the available input data may not, necessarily, be the appropriate hospital production inputs, while the available output data may not accurately measure hospital production. Individuals expect a number of different services from hospitals; however, the individual's expectation for health improvement is likely to be a strong determinant of hospital usage.[14] For that reason, the best measure of hospital production is the amount of improvement obtained by the patient. However, data on health improvement does not exist. Despite the lack of data on one measure of output quality, it would still be possible to control for quality in other ways, if data on the medical centre's case-mix could be garnered. Given the difficulty in obtaining basic data on hospital outputs, however, it was decided that the efforts needed to obtain case-mix data or other measures of quality required more time. Therefore, the results presented below focus on the data that has been made available.

5. The Results

The main results from DEA applied to the Gauteng public hospital dataset are presented in Tables 2 through 7. Tables 2 through 6 present a comparison between efficiency measured against CRS to the efficiency calculated against VRS, for each of the possible input-output combinations, for which there is enough data, while Table 7 contains the results for non-parametric statistical tests of potential population differences. Due to the limited availability of data, as discussed in the previous section, it was impossible to provide DEA calculations for some output combinations; furthermore, some of the results in each of the tables are based on rather small samples, and, therefore, those results should be treated cautiously.

5.1. **Single Outputs.** Initially, the efficiency scores for public hospitals were separately computed for each output at the hospital level. The calculations were conducted assuming both CRS and VRS; the results are summarised in Tables 2

[14] Research by Leonard, Milga & Mariam (2003) shows that these quality of care perceptions are very important for determining health centre bypass behaviour, where individuals bypass a closer health facility in order to seek health care from farther away, in rural Africa.

and 3. The first column of each table lists the technology assumption used in the analysis as well as the number of public hospitals included in the analysis. In Table 2, the next group of columns, headed by "Inputs" and "Outputs", show, by means of an 'x' in the column, which inputs and outputs were included in the analysis. Finally, the last few columns provide the average relative efficiency[15] attained by the public hospitals in the analysis, the number of hospitals in the sample to have attained an efficiency score of 1, and the number of hospitals to be classified as operating under increasing returns to scale, constant returns to scale, and decreasing returns to scale, respectively.[16] In Table 3, however, the headings are slightly different. Table 3 provides information on the slacks,[17] which are calculated in the DEA.[18] Therefore, the second group of columns provides information on the number of hospitals in the sample, which required further input reductions, while the third group of columns provides the number of hospitals in the sample requiring output expansions. The last column in Table 3 reiterates the calculated average efficiency score in the sample.

The results in Table 2 show that the efficiency scores, as expected, depend upon the input combinations used to produce the output, as well as the choice of output. The results in the table also show that the efficiency score rises when the model specification is relaxed. In these models, the relaxation occurs in two dimensions. In the seventh and eighth rows of each eight-row block in the table, there are three inputs used to produce each output, as opposed to the two inputs assumed in the first six rows. In each block in the table, the average efficiency score is higher in the last two rows than in any of the first six rows. The other dimension along which the model can be relaxed allows for varying returns to scale, the results of which appear

[15] The efficiency score is out of a possible 100, as in per cent, as opposed to 1, as required in equation (1).
[16] Returns to scale calculations are only available under the VRS model assumption.
[17] Due to the non-invariance of input slacks in this model, the information provided is the total number of hospitals in the sample with observed positive slack values for each input and output used in the calculation.
[18] The input slacks represent additional reductions, beyond the proportional reduction calculated by the efficiency score, required to keep a firm's input on the convex combination of all firms' inputs. Output slacks are similarly calculated.

in the even rows of each block in the table. As expected, adjusting the model from CRS to VRS increases the number of efficient hospitals in the sample, which is part of the explanation for the increased average relative efficiency observed in the sample.[19] Average relative efficiency across CRS calculations varies from a low of 37.9% up to a maximum of 77.8%, while average relative efficiency in VRS models varies from a low of 63.6% up to a maximum of 90.3%.

From an economic perspective, the results presented in the table are less obvious. Essentially, there are two implications contained in Table 2, subsequently supported in Table 3. The first implication is that public hospitals in Gauteng, according to the analysis, are more likely to be operating under decreasing returns to scale than either increasing returns to scale or constant returns to scale. Such a result suggests that public hospitals have too many inputs; however, that would be a naïve analysis of the results. Due to the fact that trained doctors and nurses have become some of the most common emigrants from South Africa, it might be expected that public hospitals had too many active beds, given the population of doctors and nurses. Intuitively, returns to scale are determined by the fixed input, which is, in most of the calculations, active hospital beds.[20] Table 3, provides anecdotal evidence that, in fact, there may be too many beds relative to medical professionals. The second implication taken from the results in Table 2 is that hospitals providing inpatient services, and were able to provide inpatient day numbers, as well as surgeries, and were able to provide surgery numbers, are more similar to each other than the hospitals only able to provide data on admissions and outpatient days; however, neither group is necessarily more or less likely to be more efficient than the other. The output slacks in Table 3 further support the implied similarity between certain

[19] The rest of the increased average is due to the fact that all the remaining hospitals in the sample cannot have a lower efficiency score under VRS than CRS. Only some of the hospitals will actually rise to full efficiency, though; see Figure 1 for an illustration.

[20] Input and output slacks, residuals from the analysis, show that, with few exceptions, additional reductions in bed inputs are not required, once the efficiency score has been calculated in order to keep bed inputs on the convex combination; see equation (2). On the other hand, doctors, and especially nurses, are slack more often. In other words, active beds are driving the efficiency score, so that returns to scale are strongly influenced by the ability of beds to translate into output.

types of hospital production in the sample.[21] The number of hospitals with output slacks for inpatient days and surgeries are smaller than the number of hospitals with output slacks for admissions and outpatient days.

5.2. **Dual Outputs.** Tables 4 through 6 contain summary information for DE Analysis undertaken for multiple output combinations, using the same input combinations discussed in the preceding subsection. The information contained in Tables 4 and 6 is the same as the information contained in Table 2, and, therefore, their column headings follow the same pattern; Table 5 contains the same information as Table 3, and, therefore, the two tables have equivalent column headings. Although there are actually six potential two-output combinations, there were only 12 observations in the sample when outpatient days and surgeries were combined, for that reason, there are only five two-output combinations listed in Tables 4 and 5. Similarly, although there are three three-output combinations and one four-output combination available in the data, including both outpatient days and surgery in the output combinations resulted in two few observations; therefore, there are only two three-output combinations presented in Table 6.

The empirical results in each of the last three tables show, as expected and shown before, that increasing the model's flexibility cannot reduce the average efficiency score in the sample, because no single efficiency score can be lowered. For example, more public hospitals, regardless of the combination investigated, are determined to be efficient under VRS than CRS: CRS DEA averages range from 70.3% to 90.3%, while the VRS DEA averages range from 83.3% to 98.9%. Finally, efficiency averages are higher in Table 4 than in Table 2, due to the inclusion of an additional output in the mix. Once again, due to the fact these efficiencies are relative, the higher average does not absolutely imply a more efficient set of public hospitals. Rather, it could also imply a more uniform set of observations, which are actually less efficient, overall. As with the single output analysis, decreasing returns to scale is relatively more common than increasing returns to scale, although constant

[21] With only one output, the CRS model will not yield output slacks; rather there must be at least two outputs.

returns to scale is more common than either increasing or decreasing returns to scale. The most troublesome result appearing in the table is the fact that under VRS, too many hospitals are deemed to be efficient. In each case where there are 21 or fewer observations, no fewer than nine hospitals are deemed to be efficient, while up to 14 are calculated as efficient. Therefore, the results from this part of the analysis will have very limited interpretational value.

The inclusion of an additional input, as compared to the results in Tables 2 and 3, however, makes the interpretation of input and output slacks more difficult. The input and output slacks for the dual output DEA models are presented in Table 5. Unlike in the single output case, there are no obvious patterns. In the single output models, admissions and outpatient days were associated with a larger number of observed output slacks. When either admissions or outpatient days are combined with inpatient days, the same result holds; however, when either outpatient days or admissions are combined with another output, including each other, there are fewer observed output slacks. A similar story emerges regarding input slacks, also in Table 5. With few exceptions, as in Table 3, there are fewer positive active bed slacks than with other inputs, which could increase the count of decreasing returns to scale observations, especially when compared to increasing returns to scale. However, the second set of outcomes presented in each table, in particular, suggests very similar numbers of increasing returns and decreasing returns observations, despite the small number of observed active bed input slacks.

The final DEA table, Table 6, presents the three-output combination DEA results. A detailed discussion of the results will not be undertaken here, given the similarity with results already presented, as well as the fact that very few public hospitals in the sample were deemed to be inefficient. However, the table does, once again, continue to reveal the increase in calculated efficiency likely to result from the increase in model flexibility. Although there is some support for the continued presence of decreasing returns to scale over increasing returns to scale, the numerical differences are less pronounced than in Tables 2 and 4.

5.3. **Efficiency Differences.** The analysis of results concludes with an analysis of the differences in measured efficiency across hospital populations. Due to the fact that many of the calculations involved small numbers of observations, these final comparisons are based only on the sets of results for which there was a minimum of 30 observations. In other words, the comparison is for the first two single output DEA models (presented in Tables 2 and 3) as well as the first of the dual output DEA models (presented in Tables 4 and 5). Using the available sample data to distinguish between (i) large and small hospitals, (ii) hospitals offering outpatient services, and (iii) hospitals offering surgical services, a non-parametric test is used to statistically differentiate the populations, if they can be differentiated. Table 7 contains the χ_1^2-statistic associated with a Kruskal-Wallis non-parametric test, which, as its null hypothesis, assumes samples are from the same population.[22]

When the sample is split by hospital size, where more than 220 beds represents a large hospital, the results of the Kruskal-Wallis test suggest a failure to accept the null hypothesis in all but 9 of 24 cases. However, the opposite is true, when the sample is split based upon whether or not the medical centre offers outpatient services.[23] The null hypothesis was not rejected in all but one case out of the 24. If, on the other hand, hospitals are split according to whether or not they provided data on surgical services, the null hypothesis of equal populations was accepted in 8 of 24, and, therefore, not accepted in 16 of the 24. In conclusion, the organization and provision of services at large hospitals is often statistically different from the organization and provision of services at small hospitals. In fact, the average efficiency score is higher in small hospitals, suggesting that smaller hospitals more efficiently organize their production activities.[24] Furthermore, hospitals offering data on surgical procedures are often statistically different from those not offering

[22] The critical value for the test, using 5% confidence, is 3.84.

[23] In actual fact, it is not clear whether or outpatient services are or are not provided; rather it is only clear that the medical facility did not make data on outpatient visits available.

[24] Although a table of these averages is not provided, the average efficiencies for the sample of small hospitals using beds and doctors to produce admissions were 75.6 (CRS) and 91.9 (VRS), compared to the large hospital averages of 29.1 (CRS) and 47.6 (VRS). The difference in averages across many of the other model specifications is similarly large.

such services, where again, the tendency is towards improved efficiency.[25] However, there does not appear to be any difference between medical centres providing data on outpatient services compared to those centres providing that data.

6. Conclusion and Recommendations

An incomplete sample of Gauteng public hospitals was used for the purpose of generating efficiency scores using a linear programming technique referred to as Data Envelopment Analysis. The data was difficult to obtain due to participation reluctance as well as information system inadequacy. Although every attempt was made to include all public hospitals in the analysis, approximately 50% of the population could not be included in the analysis. Due to the limited participation, which could have been selective in nature, the results from the preceding analysis may not broadly represent the province or the country. For those reasons, the following conclusions should be treated as in need of further strengthening, with the exception of the need for increased data accessing capabilities within the province. The broadest empirical conclusions to be extracted from the analysis can only be extracted from a small set of the DE Analyses that were employed in the research. For a set of analyses, there is a statistical difference between small and large medical centres as well as between centres offering and not offering surgical procedures. The statistical difference between large and small hospitals is consistent with another broad observation that public hospitals in Gauteng more commonly operate under decreasing returns to scale than under increasing returns to scale. Decreasing returns to scale could be due to the emigration of qualified medical professionals, or it could be related to the need to hold excess capacity in case of a large-scale negative health event.

Regarding efficiency, according to the single output estimates, where there are a reasonably large number of observations, surgeries and inpatient days are more

[25] Medical centres providing data on surgeries, averaged 64.2 (CRS) and 71.1 (VRS) per cent efficiency, compared to 51.4 (CRS) and 67.0 (VRS), for those centres that did not, in the case of producing admissions using medical doctors and nurses. Similar results obtain in other model specifications.

efficiently produced than outpatient visits and admissions. However, the relatively improved efficiency could obtain because the medical centres providing surgeries and inpatient services are more uniform than the medical centres providing outpatient services and admissions; in particular, it is true that all medical centres admit patients, which suggests that if there are differences between centres, that heterogeneity will be most acute across total admissions.

The broadest conclusion to be extracted from the analysis relates to the data used in the analysis. The empirical approach used in this paper, as discussed in earlier sections, is not without flaws. Despite those flaws, DEA, when used carefully, can be used to guide resource allocation in multiple output production units, as long as the data used in the analysis is representative of the production process and can be compared to appropriate peer production units. The data used in this analysis suffers on both of the preceding points; therefore, it is absolutely necessary that medical centres across the country be encouraged, if not required, to develop and implement data warehousing systems, so that future research on this topic can be conducted. Furthermore, those warehousing systems should be equivalent across the entire public health delivery system.

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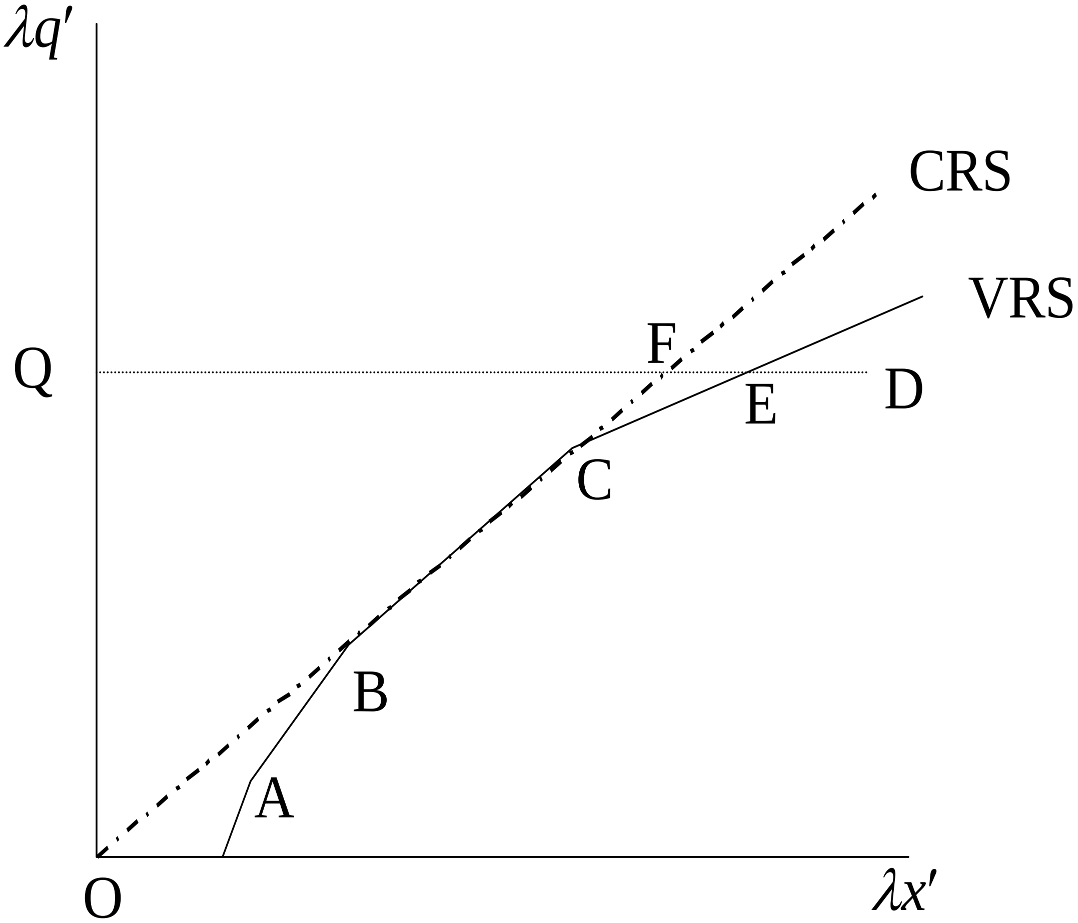

Figure 1. Constant Returns to Scale (CRS) Compared to Variable Returns to Scale (VRS) Calculated Via Data Envelopment Analysis

Table 1. Summary Statistics of Analysis Data

	Large Hospitals (n=20) Active Beds > 220			Small Hospitals (n=22) Active Beds < 221	
Input Variable	Average	Non-zero Observations		Average	Non-zero Observations
Active beds	762.94	20		151.90	22
	(304.4)			(35.7)	
Medical doctors & Specialists	192.28	20		12.51	22
	(114.4)			(3.9)	
Nurses	920.67	20		124.95	22
	(407.0)			(20.4)	
Output Variable					
Outpatient Visits	7763.48	12		1678.02	10
	(15926.9)			(2126.7)	
Total admissions	1572.62	18		944.43	22
	(1843.4)			(471.8)	
Inpatient days	17163.94	18		3470.05	22
	(9975.9)			(1295.0)	
Theater case/ Surgeries	127.53	6		117.66	15
	(327.6)			(95.6)	

Note: Standard errors are in parenthesis.

Source: Authors' calculations from primary data collected for a subset of public hospitals in Gauteng province from 1999 to 2004.

Table 2. Summary Results for Single Output DEA Models

	Inputs			Outputs						Retruns to Scale		
Assumed Model	Beds	Doctors	Nurses	Outpatient Visits	Inpatient Days	Admissions	Surgeries	Average Efficiency	Numbef of TEchnically Efficient Public Hospitals	Number of IRS	Number of CRS	Number of DRS
CRS (n=39)	x	x				x		54.1	3			
VRS (n=39)	x	x				x		71.4	10	5	9	25
CRS (n=39)	x		x			x		45.8	3			
VRS (n=39)	x		x			x		63.6	6	8	6	25
CRS (n=39)		x	x			x		57.6	3			
VRS (n=39)		x	x			x		69.5	10	11	10	18
CRS (n=39)	x	x	x			x		58.1	5			
VRS (n=39)	x	x	x			x		72.6	13	6	13	20
CRS (n=39)	x	x			x			77.6	3			
VRS (n=39)	x	x			x			88.8	11	8	10	21
CRS (n=39)	x		x		x			69.6	3			
VRS (n=39)	x		x		x			81.4	8	10	8	21
CRS (n=39)		x	x		x			62.0	1			
VRS (n=39)		x	x		x			83.9	8	12	8	19
CRS (n=39)	x	x	x		x			77.8	3			
VRS (n=39)	x	x	x		x			90.0	13	7	13	19
CRS (n=21)	x	x		x				52.0	2			
VRS (n=21)	x	x		x				70.0	8	6	3	12
CRS (n=21)	x		x	x				37.9	1			
VRS (n=21)	x		x	x				65.8	7	0	2	19
CRS (n=21)		x	x	x				51.8	2			
VRS (n=21)		x	x	x				67.3	3	5	3	13
CRS (n=21)	x	x	x	x				53.4	2			
VRS (n=21)	x	x	x	x				71.7	8	5	3	13
CRS (n=21)	x	x					x	68.6	2			
VRS (n=21)	x	x					x	89.1	5	2	5	14
CRS (n=21)	x		x				x	71.6	2			
VRS (n=21)	x		x				x	86.8	6	6	6	9
CRS (n=21)		x	x				x	68.6	2			
VRS (n=21)		x	x				x	84.4	5	2	5	14
CRS (n=21)	x	x	x				x	73.7	3			
VRS (n=21)	x	x	x				x	90.3	7	2	7	12

Source: Authors' calculations from DEA analysis on subset of Gauteng public hospitals.

Table 3. Summary of Single Output DEA Slack Estimates

	Number of Public Hospitals with Input Slacks				Number of Public Hosptials with Output Slacks				
Assumed Model	Beds	Doctors	Nurses		Outpatient Visits	Inpatient Days	Admissions	Surgeries	Average Efficiency
CRS (n=39)	6	11							54.1
VRS (n=39)	4	13					19		71.4
CRS (n=39)	23		9						45.8
VRS (n=39)	4		12				22		63.6
CRS (n=39)		6	15						57.6
VRS (n=39)		10	10				14		69.5
CRS (n=39)	17	9	16						58.1
VRS (n=39)	7	13	18				17		72.6
CRS (n=39)	2	5							77.6
VRS (n=39)	4	7				9			88.8
CRS (n=39)	3		5						69.6
VRS (n=39)	9		6			4			81.4
CRS (n=39)		28	8						62.0
VRS (n=39)		12	15			3			83.9
CRS (n=39)	3	10	31						77.8
VRS (n=39)	6	7	17			6			90.0
CRS (n=21)	3	1							52.0
VRS (n=21)	3	12			12				70.0
CRS (n=21)	17		3						37.9
VRS (n=21)	10		9		12				65.8
CRS (n=21)		1	6						51.8
VRS (n=21)		10	5		12				67.3
CRS (n=21)	11	1	11						53.4
VRS (n=21)	8	13	11		12				71.7
CRS (n=21)	1	8							68.6
VRS (n=21)	0	4						3	89.1
CRS (n=21)	0		1						71.6
VRS (n=21)	0		6					3	86.8
CRS (n=21)		5	8						68.6
VRS (n=21)		4	7					3	84.4
CRS (n=21)	1	8	5						73.7
VRS (n=21)	2	5	9					4	90.3

Source: Authors' summary of slack results from DEA applied to Gauteng public hospitals.

Table 4. Summary Results for Multiple Output DEA

	Inputs				Outputs						Returns to Scale		
Assumed Model	Beds	Doctors	Nurses		Outpatient Visits	Inpatient Days	Admissions	Surgeries	Average Efficiency	Numbef of Technically Efficient Public Hospitals	Number of IRS	Number of CRS	Number of DRS
CRS (n=37)	x	x				x	x		85.4	9			
VRS (n=37)	x	x				x	x		91.8	14	7	13	17
CRS (n=37)	x		x			x	x		75.0	5			
VRS (n=37)	x		x			x	x		84.8	9	8	9	20
CRS (n=37)		x	x			x	x		79.5	6			
VRS (n=37)		x	x			x	x		89.5	14	14	14	9
CRS (n=37)	x	x	x			x	x		87.0	12			
VRS (n=37)	x	x	x			x	x		92.7	16	7	16	14
CRS (n=21)	x	x			x	x			85.0	6			
VRS (n=21)	x	x			x	x			93.8	13	6	9	6
CRS (n=21)	x		x		x	x			83.7	3			
VRS (n=21)	x		x		x	x			94.8	10	8	6	7
CRS (n=21)		x	x		x	x			80.9	4			
VRS (n=21)		x	x		x	x			90.0	8	7	8	6
CRS (n=21)	x	x	x		x	x			87.5	6			
VRS (n=21)	x	x	x		x	x			95.7	14	5	10	6
CRS (n=20)	x	x				x		x	91.4	7			
VRS (n=20)	x	x				x		x	98.0	13	2	13	5
CRS (n=20)	x		x			x		x	92.3	8			
VRS (n=20)	x		x			x		x	97.8	12	3	12	5
CRS (n=20)		x	x			x		x	80.2	4			
VRS (n=20)		x	x			x		x	93.3	9	2	9	9
CRS (n=20)	x	x	x			x		x	93.0	8			
VRS (n=20)	x	x	x			x		x	98.9	14	2	14	4
CRS (n=19)	x	x			x		x		70.3	3			
VRS (n=19)	x	x			x		x		83.9	13	1	9	9
CRS (n=19)	x		x		x		x		69.3	4			
VRS (n=19)	x		x		x		x		83.7	9	5	5	9
CRS (n=19)		x	x		x		x		73.5	4			
VRS (n=19)		x	x		x		x		83.3	10	1	9	9
CRS (n=19)	x	x	x		x		x		74.1	4			
VRS (n=19)	x	x	x		x		x		85.6	13	1	9	9
CRS (n=19)	x	x					x	x	87.8	5			
VRS (n=19)	x	x					x	x	97.8	10	1	10	8
CRS (n=19)	x		x				x	x	82.9	4			
VRS (n=19)	x		x				x	x	96.7	10	4	10	5
CRS (n=19)		x	x				x	x	88.2	5			
VRS (n=19)		x	x				x	x	97.0	10	3	10	6
CRS (n=19)	x	x	x				x	x	90.3	7			
VRS (n=19)	x	x	x				x	x	98.9	12	1	12	6

Source: Authors’ calculations from DEA analysis on subset of Gauteng public hospitals.

Table 5. Summary of Multiple Output DEA Slack Estimates

	Number of Public Hospitals with Input Slacks				Number of Public Hosptials with Output Slacks				
Assumed Model	Beds	Doctors	Nurses		Outpatient Visits	Inpatient Days	Admissions	Surgeries	Average Efficiency
CRS (n=37)	2	6				1	6		85.4
VRS (n=37)	3	6				7	16		91.8
CRS (n=37)	12		6			5	12		75.0
VRS (n=37)	6		6			5	18		84.8
CRS (n=37)		11	7			4	10		79.5
VRS (n=37)		6	10			6	6		89.5
CRS (n=37)	4	5	15			0	6		87.0
VRS (n=37)	4	6	15			5	15		92.7
CRS (n=21)	0	1			11	4			85.0
VRS (n=21)	0	6			10	5			93.8
CRS (n=21)	6		3		16	0			83.7
VRS (n=21)	3		8		14	6			94.8
CRS (n=21)		12	4		12	4			80.9
VRS (n=21)		6	6		12	6			90.0
CRS (n=21)	5	8	7		11	4			87.5
VRS (n=21)	3	8	7		10	6			95.7
CRS (n=20)	0	5				0		3	91.4
VRS (n=20)	1	2				6		3	98.0
CRS (n=20)	0		5			0		4	92.3
VRS (n=20)	0		4			4		5	97.8
CRS (n=20)		8	7			4		8	80.2
VRS (n=20)		4	6			4		7	93.3
CRS (n=20)	0	7	8			0		4	93.0
VRS (n=20)	1	2	5			4		3	98.9
CRS (n=19)	6	8			9		0		70.3
VRS (n=19)	1	9			9		5		83.9
CRS (n=19)	8		5		8		0		69.3
VRS (n=19)	3		11		9		9		83.7
CRS (n=19)		2	5		8		0		73.5
VRS (n=19)		6	2		9		5		83.3
CRS (n=19)	9	6	9		8		0		74.1
VRS (n=19)	3	9	6		9		6		85.6
CRS (n=19)	4	4					3	7	87.8
VRS (n=19)	3	2					6	5	97.8
CRS (n=19)	0		6				5	6	82.9
VRS (n=19)	1		4				6	5	96.7
CRS (n=19)		4	7				3	8	88.2
VRS (n=19)		2	5				2	5	97.0
CRS (n=19)	4	4	6				3	6	90.3
VRS (n=19)	3	2	6				6	4	98.9

Source: Authors' summary of slack results from DEA applied to Gauteng public hospitals.

Table 6. Summary Results for Additional Multiple Output DEA

	Inputs				Outputs						Returns to Scale		
Assumed Model	Beds	Doctors	Nurses		Outpatient Visits	Inpatient Days	Admissions	Surgeries	Average Efficiency	Numbef of Technically Efficient Public Hospitals	Number of IRS	Number of CRS	Number of DRS
CRS (n=19)	x	x			x	x	x		92.2	7			
VRS (n=19)	x	x			x	x	x		97.3	14	5	10	4
CRS (n=19)	x		x		x	x	x		93.2	5			
VRS (n=19)	x		x		x	x	x		98.7	12	6	8	5
CRS (n=19)		x	x		x	x	x		89.5	6			
VRS (n=19)		x	x		x	x	x		94.9	10	5	9	5
CRS (n=19)	x	x	x		x	x	x		95.0	8			
VRS (n=19)	x	x	x		x	x	x		98.7	14	4	10	5
CRS (n=19)	x	x				x	x	x	95.8	12			
VRS (n=19)	x	x				x	x	x	98.7	14	1	14	4
CRS (n=19)	x		x			x	x	x	95.1	11			
VRS (n=19)	x		x			x	x	x	99.2	16	1	16	2
CRS (n=19)		x	x			x	x	x	93.5	9			
VRS (n=19)		x	x			x	x	x	97.8	13	3	13	3
CRS (n=19)	x	x	x			x	x	x	97.1	14			
VRS (n=19)	x	x	x			x	x	x	99.7	16	0	16	3

Source: Authors' calculations from DEA analysis on subset of Gauteng public hospitals.

Table 7. Summary of Non-parametric Tests of Distribution Equivalence Across a Selected Subsample of Gauteng Public Hospitals

	Inputs				Outputs				Kruskall-Wallis Chi-Sq Values		
Assumed Model	Beds	Doctors	Nurses		Outpatient Visits	Inpatient Days	Admissions	Surgeries	Large and Small Hospitals the Same?	Offer or not Offer Surgery, the same?	Offer Outpatient Services or not, the same?
CRS (n=39)	x	x					x		13.8	8.9	0.4
VRS (n=39)	x	x					x		6.1	6.7	0.5
CRS (n=39)	x		x				x		14.5	0.9	0.1
VRS (n=39)	x		x				x		17.0	1.1	0.0
CRS (n=39)		x	x				x		16.2	7.1	0.1
VRS (n=39)		x	x				x		6.3	5.2	0.0
CRS (n=39)	x	x	x				x		16.7	6.8	0.0
VRS (n=39)	x	x	x				x		8.1	5.1	0.1
CRS (n=39)	x	x				x			2.7	0.2	0.4
VRS (n=39)	x	x				x			4.0	5.3	0.0
CRS (n=39)	x		x			x			2.3	0.4	1.6
VRS (n=39)	x		x			x			9.6	1.2	1.8
CRS (n=39)		x	x			x			0.7	7.9	0.8
VRS (n=39)		x	x			x			0.3	9.0	0.5
CRS (n=39)	x	x	x			x			2.9	0.2	0.4
VRS (n=39)	x	x	x			x			3.9	4.1	0.0
CRS (n=37)	x	x				x	x		9.0	6.2	0.7
VRS (n=37)	x	x				x	x		1.1	3.7	0.5
CRS (n=37)	x		x			x	x		4.3	0.3	3.0
VRS (n=37)	x		x			x	x		2.9	1.9	0.2
CRS (n=37)		x	x			x	x		13.4	10.2	0.7
VRS (n=37)		x	x			x	x		1.7	6.1	0.0
CRS (n=37)	x	x	x			x	x		10.3	5.9	0.8
VRS (n=37)	x	x	x			x	x		2.4	4.9	0.0

Source: Chi-squared values computed via STATA 8.2 SE kwallis command. Data taken from DEA results summarized in Tables 2 and 4.