



# **Measuring the welfare cost of inflation in South Africa**

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# Measuring the welfare cost of inflation in South Africa

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## Abstract

In this paper, we estimate the long-run equilibrium relationship between money balance as a ratio of income and the Treasury bill rate for the period of 1965:02 to 2007:01, and, in turn, use the relationship to obtain welfare cost estimates of inflation. Using the Johansen (1991, 1995) technique, we estimate a log-log specification and a semi-log model of the above relationship. Based on the fits of the specifications, we decided to rely more on the welfare cost measure obtained under the log-log money demand model. Our estimates suggest that the welfare cost of inflation for South Africa ranges between 0.34 percent and 0.67 percent of GDP, for a band of 3 to 6 percent of inflation. Thus, it seems that the SARB's current inflation target band of 3-6 percent provides quite a good approximation in terms of welfare.

KEYWORDS: Cointegration; Money Demand; Welfare cost of Inflation

JEL Classification: E31; E41; E52

## 1 Introduction

Studies on welfare cost of inflation have been the focus of extensive theoretical and empirical analyses in both the recent and more distant past. Using Bailey's (1956) consumer surplus approach, as well as, the compensating variation approach, Lucas (2000) provided estimates of the welfare cost of inflation for the US economy based on annual data for the period of 1900 to 1994. Lucas' (2000) calculations, based on the log-log money demand specification, indicated that reducing the interest rate from 3% to zero would yield a benefit equivalent to an increase in real output of about 0.009 (or 0.9 percent).

Serletis *et al.* (2004), in their study dealing with the welfare cost of inflation for Canada and the United States, however, came up with much smaller figures, compared to Lucas (2000), when they used recent advances in the field of applied econometrics to estimate the interest elasticity of money demand. Unlike Lucas (2000), Ireland (2007), however, showed that a semi-log money demand specification fits the post 1980 US data better than a log-log econometric model. Based on the estimation of the semi-log money demand model, Ireland (2007) found that a 10 percent rate of inflation when compared to price stability would imply a welfare cost of 0.21 percent of income. This figure, though lower than that of Lucas (2000) and Serletis *et al.* (2004), was in line with Fisher's (1981) findings of 0.30 percent and a value of 0.45 percent obtained earlier by Lucas (1981).

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Given that welfare cost estimates differ remarkably based on alternative money demand functions, we in this study aim to first derive a money demand function that appropriately defines the South African money market, and then, in turn, use it to obtain welfare cost estimates of inflation. For this purpose, we look at quarterly data over the period of 1965:02 to 2007:01, and given the econometric problems of non-stationary data, use the Johansen (1991, 1995) cointegration technique to obtain a long-run money demand relationship. Note measures of welfare cost of inflation, are important for any economy, but more so in a country like South Africa, where the central bank targets inflation.<sup>1</sup> To put it differently, we try and investigate how substantial are the welfare costs of inflation under the current inflation target zone of 3 to 6 percent pursued by the South African Reserve Bank<sup>2</sup>, and if there is a need to rethink the band of the target in terms of the welfare cost of inflation. To the best of our knowledge, this is the first attempt to measure the welfare cost of inflation for the South African economy.

The remainder of the paper is organized as follows: Section 2 provides a brief summary of the theoretical issues regarding the estimation of the welfare cost of inflation, while, Section 3 and 4, respectively, discusses the data and presents the estimation of the log-log and the semi-log money demand specifications. Section 4 also calculates the welfare cost estimates for the South African economy. Finally, Section 5 concludes.

## 2 The Theoretical Foundations

As indicated by Lucas (2000), money demand specification is vital in determining the appropriate size of the welfare cost of inflation. Lucas (2000) contrasts between two competing specifications for money demand. One, inspired by Meltzer (1963), relates the natural logarithm of  $m$ , a ratio of money balances to nominal income, and the natural logarithm of a short-term nominal interest rate  $r$ . Formally, this can be expressed as follows:

$$\ln(m) = \ln(A) - \eta \ln(r) \quad (1)$$

where  $A > 0$  is a constant and  $\eta > 0$  measures the absolute value of the interest elasticity of money demand. Another specification, adapted from Cagan (1956), links the log of  $m$  to the level of  $r$  via the following equation:

$$\ln(m) = \ln(B) - \xi r \quad (2)$$

where  $B > 0$  is a constant and  $\xi > 0$  measures the absolute value of the semi-elasticity of money demand with respect to the interest rate.

By applying the methods outlined in Bailey (1956), Lucas (2000) transformed the evidence on money demand into a welfare cost estimate. Note Bailey (1956) described the welfare cost of inflation as the area under the inverse money demand function, or the “*consumers’s surplus*”, that could be gained by reducing the interest rate to zero from an existing (average or steady-state)

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<sup>1</sup>See Ludi and Ground (2006) for a great compilation on the history of monetary policy of South Africa.

<sup>2</sup>Though, the inflation target is for the CPIX, we use the CPI inflation for our calculations, mainly due to the fact that the CPIX series does not exist for the whole sample period used, given that South Africa’s decision to move to an inflation targeting regime only began in February of 2000. In addition, the correlation between the CPIX inflation and the CPI inflation over the period of 1999:02 to 2007:02 was found to be 0.81. So given this high correlation and the fact that the average rates of the CPI and the CPIX inflation were relatively close, specifically 5.12 and 6.23 percent respectively, studying the welfare cost of CPI inflation is very similar to studying the welfare cost for an inflation in the CPIX.

value. So if  $m(r)$  is the estimated function, and  $\psi(m)$  is the inverse function, then the welfare cost can be defined as:

$$w(r) = \int_{m(r)}^{m(0)} \psi(x) dx = \int_0^r m(x) dx - rm(r) \quad (3)$$

As seen from Equation 3, obtaining a measure for the welfare cost amounts to, integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue  $rm$  to deduce the deadweight loss.

Since the function  $m$  has the dimensions of a ratio to income, so does the function  $w$ . The value of  $w(r)$ , represents the fraction of income that people needs, as compensation, in order to be indifferent between living in a steady-state with an interest rate constant at  $r$  or an identical steady state with an interest of close or equal to zero. Given this, Lucas (2000) shows that when the money demand function is given by (E1) or is  $m(r) = Ar^{-\eta}$ , the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(r) = A \left( \frac{\eta}{1-\eta} \right) r^{1-\eta} \quad (4)$$

While, for a semi-log money demand specification i.e.,  $m(r) = Be^{-\xi r}$ ,  $w(r)$  is obtained by the following formula:

$$w(r) = \frac{B}{\xi} [1 - (1 + \xi r) e^{-\xi r}] \quad (5)$$

As can be seen from (Equations 4) and (Equations 5), an estimate of the interest elasticity of money demand is crucial in evaluating the welfare cost of inflation, and, hence, we first need to obtain the long-run relationship between the ratio of money balance to income and a measure of the opportunity cost of holding money, captured by a short-term nominal interest rate.

### 3 Data

In this study, we use quarterly time series data from the second quarter of 1965 (1965:02) to the first quarter of 2007 (2007:01) for the South African economy, which, in turn, are obtained from the South African Reserve Bank (SARB) Quarterly Bulletin and the International Financial Statistics of the IMF. The variables used in this study are the money balances ratio ( $rm3$ ), generated by dividing the broad measure of money supply (M3)<sup>3</sup> by the nominal income (nominal GDP), and short term interest rate, in our case proxied by the 91 days Treasury bill rate ( $tbr$ ).<sup>4</sup> All series, except for the Treasury bill rate are seasonally adjusted. Further, for the estimation of the log-log specification both the ratio of money balances and the Treasury bill rate are transformed into their logarithmic values, and are denoted by  $lrm3$  and  $ltbr$ , respectively.

<sup>3</sup>Though, in the literature, welfare costs of inflation has generally been obtained using a narrow definition of money, we chose M3, since we believe that a broad monetary aggregate captures the role of money better than a narrowly defined version of the same. In addition, the ratio of M3 to GDP was found to be least volatile. Finally, the choice was further motivated to take account of possible financial innovations that have taken place in the South African economy over the period of our concern.

<sup>4</sup>We also use the percentage change at seasonally adjusted annualized rates of the CPI to obtain the rate of inflation, and, hence, the real rate of interest. See below, for further details.

## 4 Empirical Results

As is standard in time series analysis, we start off by studying the univariate characteristics of the data. In this regard, we performed tests of stationarity on our variables ( $lrm3$ ,  $ltbr$  and  $tbr$ ) using the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips- Perron (PP) test. As can be seen from Table 1, the variables were found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests, while for the KPSS test, the null of stationarity was rejected. As the variables were found to be non-stationary, it paved the way for the Johansen test for cointegration between  $lrm3$  and  $ltbr$  in (1) and  $lrm3$  and  $tbr$  in (2).

At this juncture it is important to point out a possible concern in the analysis. These statistical tests which first analyzes the stationarity and then checks for cointegration between  $lrm3$  and  $ltbr$  in (1) and  $lrm3$  and  $tbr$  in (2), requires, as Ireland (2007) puts it, a “somewhat schizophrenic view of those data” since, in a linear framework, the analysis of the log-log model requires  $ltbr$  to follow an autoregressive process with a unit root, while the identical analysis of the semi-log model requires  $tbr$  to be  $I(1)$ . Bae (2005) actually provides a detailed discussion of the case in which both the models can be estimated under the common assumption that  $tbr$  follows an autoregressive unit root process, with the log-log specification being viewed as a non-linear relationship between  $lrm3$  and  $tbr$  and the semi-log model viewed as a linear framework for the same two variables. As in Ireland (2007), we follow Anderson and Rasche (2001), by treating both as linear functions linking  $lrm3$  and  $ltbr$  in (Equations 1) and  $lrm3$  and  $tbr$  in (Equations 2) and, thus, putting the two models on “equal footing *ex ante*”.

But, before we tested for cointegration, a test for the stability of the VAR model, including a constant as an exogenous variable was performed. Given that no roots were found to lie outside the unit circle for the estimated VAR based on 4 lags under both the log-log and the semi-log specifications, we conclude that the VARs are stable and suitable for further analysis.<sup>5</sup> Note the choice of 4 lags was based on the unanimity of two alternative lag-length criteria, namely the Schwarz information criterion and the Hannan-Quinn Information criterion for the log-log money demand specification, and the Sequential Modified LR test statistic for the semi-log money demand model. Before we proceed further, it is important to point out that though four criteria, namely the Final Prediction Error, the Akaike Information, the Schwarz Information and the Hannan-Quinn Information, overwhelmingly suggested the choice of two lags for the semi-log specification, no cointegration could be detected using the Johansen test with two lags. However, as has been reported below, the cointegration test based on 4 lags, suggested by the Sequential Modified LR test statistic, picked up one cointegrating relationship.

Once the issues of stability and the optimal lag length were settled, we tested for the cointegrating relationship based on the Johansen (1991, 1995) approach. For this purpose, we included four lags in the VAR, and allowed the level data to have linear trends, but the cointegrating equations to have only intercepts. Based on the Pantula Principle, both the Trace and the Maximum Eigen Value tests, showed that there is one stationary relationship in the data ( $r = 1$ ) at 5 percent level of significance for both the log-log and the semi-log specifications. The results have been reported in Tables 2 and 3.<sup>6</sup>

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<sup>5</sup>Tests indicating the stability of the VAR have been suppressed to save space. However, they are available from the authors upon request.

<sup>6</sup>As in Ireland (2007), we also used the Phillips-Ouliaris (1990) test for cointegration. However, unlike Ireland (2007), the test could not detect any cointegrating relationship between the chosen variables. Hence, the results of

Given one cointegrating relationship ( $r = 1$ ), the Johansen (1991, 1995) procedure gives the maximum likelihood estimates of the unrestricted cointegrating relation  $\beta' X_t$ . Even if the unrestricted  $\beta$  is uniquely determined, depending on the chosen normalization,  $\beta$  is not necessarily meaningful from an economic point of view. Therefore, an important part of long-run cointegration analysis is to impose (over-) identifying restrictions on  $\beta$  to achieve economic interpretability (Hendry *et al.* 2000).

As we are more interested in the relationship between the money balance ratio and interest rate, for both specifications,  $lrm3$  was restricted to unity. Given that we have only one cointegrating vector, the normalizing restriction on  $lrm3$  is enough to exactly identify the long-run relationship. However, we encountered two serious econometric problems with this restriction. First, the restriction was not binding. Secondly, the adjustment coefficient of  $lrm3$  was insignificant under both the specifications. Imposing an additional zero restriction on the adjustment coefficient of  $lrm3$  did ensure binding restrictions, but at the cost of suggesting that the ratio of real balance to income was in fact exogenous and we should not be normalizing on  $lrm3$ . Given this, we decided to normalize on the interest rate variable, i.e.,  $ltbr$  for the log-log specification and  $tbr$  for the semi-log specification. Further, with the adjustment coefficients on  $lrm3$  still being insignificant in both the models, we restricted them to zero, and obtained binding restrictions.<sup>7</sup> Note with  $lrm3$  now treated as the right-hand side variable, weak exogeneity of the same is what should be expected. The adjustment coefficients of  $ltbr$  and  $tbr$  were negative and significant, with them correcting for 6.9 percent and 8.08 percent of the disequilibrium in the next period, respectively. Based on the above restrictions, the obtained long-run relationship for the log- log specification is as follows:

$$ltbr = -5.275983 - 4.789793(lrm3) \quad (6)$$

[-3.87971]

While for the semi-log model, the relationship is given by:

$$tbr = -0.171261 - 0.454711(lrm3) \quad (7)$$

[-3.88877]

Figures 1 and 2 depict the cointegrating relationships under the log-log and the semi-log specifications respectively. As can be seen, the residuals of the two cointegrating equations are mean-reverting around zero and are stationary, which implies that the estimated cointegrating relations are appropriate.<sup>8</sup> Note what we have in equations (6) and (7) are actually the inverse of the money demand functions, with rate of interest as the dependant variable. Alternatively, we can view equations (6) and (7) as long-run rules for the treasury bill rate. Whatever we choose to call these equations is not important to our cause, but it is the values of the coefficients of these two estimated cointegrating relationships, that are more relevant. The obtained interest elasticity, in absolute term, equals to 0.2088 and the interest semi elasticity is equal to 2.1991, both of which

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the test have been suppressed to save space. They are, however, available upon request.

<sup>7</sup>Note the value of the LR test statistics for binding restrictions, both long- and short-run, for the log-log and the semi-log specifications respectively, were  $\chi^2(1)=0.0036$  (0.9522) and  $\chi^2(1)= 0.4041$  (0.5250), where the numbers in the parenthesis indicates the probability of committing a Type I error.

<sup>8</sup>Diagnostic tests on the residuals revealed that there is no autocorrelation in both the log-log and the semi-log specifications.

are obtained by taking the reciprocal of the coefficients corresponding to  $lrm3$  in equations (6) and (7) respectively. Importantly, the signs of the interest rate elasticity and semi-elasticity, in both the specifications, adhere to economic theory. Based on these two, elasticities, we are now ready to calculate the welfare cost of inflation as outlined in Lucas (2000), and described above in equations (E4) and (E5).

The estimates of the intercept and slope coefficient reported under the log-log specification imply values of  $A = 0.3323$  and that of  $\eta = 0.2088$ , while for the semi-log specification the values of  $B = 0.6862$  and that of  $\xi = 2.1991$ .<sup>9</sup> Plugging these values into the corresponding formula for the welfare cost measures, given by equations (E4) and (E5) respectively, and using the fact that the average real rate of interest<sup>10</sup> over this period was equal to 7.70 percent, so that a zero rate of inflation would also imply a nominal rate of interest equal to 7.70 percent, we obtain the baseline value of  $w$  under price stability. Naturally then, a value of  $r = 10.70$  corresponds to a three percent rate of inflation, while, when  $r = 13.70$ , the economy experiences a six percent inflation, and so on. So the welfare costs of inflation are evaluated by subtracting the value of  $w$  at an inflation equal to zero from the value of the same at a positive rate of inflation.

Given above in Table 4 are the measures of the welfare costs of inflation, under the log-log and the semi-log specifications for the inflation rates of 3, 6, 10 and 15 percent, respectively. For an inflation rate of 3 percent, the cost of inflation under both the log-log and the semi-log specifications are 0.34 percent of GDP. However, as the inflation rate increase from 6 percent to 15 percent, the cost of inflation in the log-log model ranges between 0.67 percent of GDP and 1.56 percent of GDP, while for the semi-log money demand function the welfare cost varies between 0.76 percent of GDP and 2.41 percent of GDP.<sup>11</sup> So, the two specifications provide clearly different measures of the cost of inflation with the semi-log function imposing greater welfare loss on the economy as the inflation rate increases beyond the 3 percent mark.

So the pertinent question now is: Which one of the two inverse money demand specifications represents the money market of South Africa better? To resolve this issue we look at couple aspects of the two alternative money demand models: First, we compare simple linear and exponential plots of the relationship between  $tbr$  and  $lrm3$  with the scatter plot of these two variables, and; second, we look at the  $R^2$  and *Adjusted R<sup>2</sup>* values of these two fits of the data.<sup>12</sup> In sum, we basically analyze the goodness of fit for the two specifications. As can be seen from Figure 3, it is impossible to choose between the two models based on the linear and the exponential plots of the data. However, with the  $R^2$  and the *Adjusted R<sup>2</sup>* values of the inverse money demand relationship captured by the log-log specification being higher than the corresponding values of the semi-log model,<sup>13</sup> we decided to rely more on the results from the former. In addition, recall that although there existed overwhelming evidence that suggested the choice of two lags for the semi-log

<sup>9</sup>Note the values for A and B are easily obtained by realizing that:  $A = [\exp(-5.27598)]$  raised to the power of 0.208819, and  $B = [\exp(-0.171261)]$  raised to the power of 2.199149.

<sup>10</sup>Note, as in Ireland (2007), we define the real rate of return to be equal to the difference between the nominal interest rate and the inflation rate, where the inflation rate is obtained as the percentage change in the seasonally adjusted series of the CPI. In addition, the real rate of interest was found to be stationary based on the ADF, the DF-GLS, the KPSS and the PP tests of unit roots.

<sup>11</sup>Note that these obtained values for the welfare cost of inflation are comparatively higher than those reported in Fischer (1981), Lucas (1981), Lucas (2000) and Ireland (2007) for the US economy.

<sup>12</sup>Note given that the plots are based on  $tbr$  and  $lbr$ , the linear trend fitted to the data gives us the semi-log inverse money demand relationship, while, the exponential trend when taken logs will yield the log-log inverse money demand model.

<sup>13</sup>The  $R^2$  and the *Adjusted R<sup>2</sup>* values of the log-log model are 0.1517 and 0.1466 respectively, while those of the semi-log function are 0.1443 and 0.1391 respectively.

specification, no cointegration could be detected using the Johansen test with two lags. We, thus, had to use 4 lags, based on the Sequential Modified LR test statistic, to obtain a stable long-run money demand relationship. Based on this, one can, perhaps, argue that the semi-log specification is relatively less reliable, in comparison to the log-log model, as to depicting a true picture of the South African inverse money demand, over the period in concern. Alternatively, the bottom line of all this discussion is that, we tend to believe, that the welfare cost measures obtained from the log-log inverse money demand relationship is more appropriate. This implies that the welfare cost of inflation for South Africa ranges between 0.34 percent and 0.67 percent of GDP, for a band of 3 to 6 percent of inflation.

## 5 Conclusion

This paper uses the Johansen (1991, 1995) cointegration technique to first obtain an appropriate long-run money demand relationship for the South African economy and then, in turn, deduce welfare cost estimates based on the inverse money demand function, as outlined in Lucas (2000). For this purpose, we look at quarterly data over the period of 1965:02 to 2007:01 and estimate a log-log function and a semi-log specification. Based on the fits of the specifications, we decided to rely more on the welfare cost measure obtained under the log-log inverse money demand model. Our estimates suggest that the welfare cost of inflation for South Africa ranges between 0.34 percent and 0.67 percent of GDP, for a band of 3 to 6 percent of inflation. Though, these measures are way higher when compared to the estimates observed in the literature, they are reasonably low. Based on our estimates, we can conclude that the SARB's current inflation target band of 3-6 percent provides quite a good approximation in terms of welfare, at least in comparison to a Friedman-type deflationary rule of zero nominal rate of interest.

However, it is important to point that our welfare cost estimates merely measures the distortion in the money demand due to positive nominal interest rates. But as argued by Dotsey and Ireland (1996), in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output. In addition, as pointed out by Feldstein (1997), interactions between inflation and a non-indexed tax code can add immensely to the welfare cost of inflation. Given these two additional sources of inflation costs, there is no denying the fact that one can achieve, possibly, larger gains by reducing the inflation target below 3 percent, the lower limit of the inflation target band.

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Table 1: Unit Root Tests

Series	Model	ADF		PP	KPSS	DF-GLS	Conclusion
		$\tau_\tau \tau_\mu \tau$	$\phi_3 \phi_1$	$\tau_\tau \tau_\mu \tau$	$\tau_t \tau_\mu$	$\tau_t \tau_\mu$	
LRM3	$\tau_t$	-1.39	10.03***	-2.10	0.31***	-1.31	Non- stationary
	$\tau_\mu$	-1.50	13.79**	-2.26	0.31	-1.32	
	$\tau$	-0.63		-0.63			
D(LRM3)	$\tau_t$	-18.92***	179.02***	-21.57***	0.07	-19.01***	Stationary
	$\tau_\mu$	-18.76***	351.99***	-19.97***	0.35*	-18.68***	
	$\tau$	-18.80***		-19.99***			
LTBR	$\tau_t$	-2.89	22.42***	-2.29	0.29***	-2.78*	Non- stationary
	$\tau_\mu$	-2.60*	32.53***	-2.26	0.91***	-1.42	
	$\tau$	-0.79		-0.84			
D(LTBR)	$\tau_t$	-7.35***	26.99***	-7.23***	0.03	-7.29***	Stationary
	$\tau_\mu$	-7.35**	53.97***	-7.30***	0.09	-6.76***	
	$\tau$	-7.36***		-7.32***			
TBR	$\tau_t$	-2.76	16.25***	-2.29	0.27***	-2.73*	Non- stationary
	$\tau_\mu$	-2.63	23.88***	-2.30	0.74***	-1.74*	
	$\tau$	-0.93		-0.77			
D(TBR)	$\tau_t$	-7.98***	31.86***	-7.89***	0.03	-8.02***	Stationary
	$\tau_\mu$	-7.98***	63.74***	-7.90***	0.08	-7.89***	
	$\tau$	-8.00***		-7.93***			

\*(\*\*) (\*\*\*) indicates statistical significance at 10(5)[1] percent level.

Table 2: ESTIMATION AND DETERMINATION OF RANK (*Log-Log*)

Null hypothesis	Alternative hypothesis	Test statistic	0.05 critical value	Prob. **
<i>Trace Statistic</i>				
$r=0$	$r=1$	18.86965*	15.49471	0.0149
$r=1$	$r=2$	0.111350	3.841466	0.7386
Trace test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
<i>Maximum Eigenvalue Statistic</i>				
$r=0$	$r=1$	18.75830*	14.26460	0.0091
$r=1$	$r=2$	0.111350	3.841466	0.7386
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

Table 3: ESTIMATION AND DETERMINATION OF RANK (Semi-Log)

Null hypothesis	Alternative hypothesis	Test statistic	0.05 critical value	Prob.**
<i>Trace Statistic</i>				
$r=0$	$r=1$	19.67238*	15.49471	0.0110
$r=1$	$r=2$	0.197347	3.841466	0.6569
Trace test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
<i>Maximum Eigenvalue Statistic</i>				
$r=0$	$r=1$	19.47503*	14.26460	0.0068
$r=1$	$r=2$	0.197347	3.841466	0.6569
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

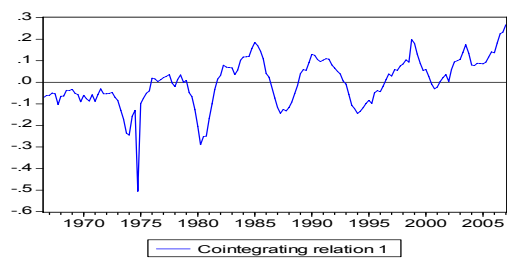


Figure 1. Cointegrating relationship of the Log-Log Specification

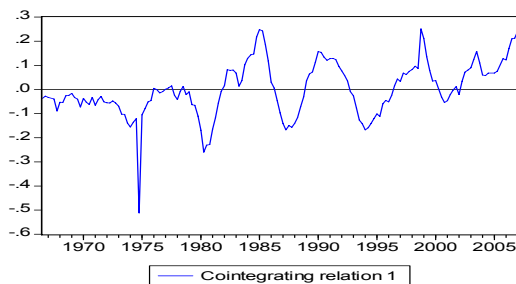


Figure 2. Cointegrating relationship of the Semi-Log Specification

Table 4: Welfare Costs of Inflation

Inflation Rate	Log-Log	Semi-Log
0.03	0.0034	0.0034
0.06	0.0067	0.0076
0.10	0.0108	0.0143
0.15	0.0156	0.0241

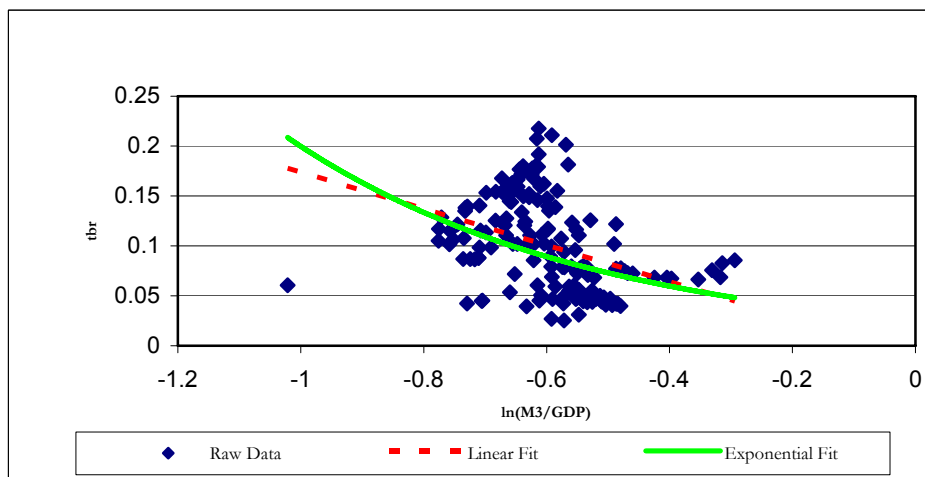


Figure 3. Inverse Money Demand Function of South Africa, 1965:02-2007:01.