UNIVERSITY OF CAPE TOWN

Exchange Rates and Product Variety

Witness Simbanegavi[1]

Working Paper Number 60

[1] School of Economics, University of Cape Town

Exchange Rates and Product Variety*

Witness Simbanegavi†
School of Economics, University of Cape Town, Private Bag
Rondebosch, 7701 RSA.

Abstract

We study the role of exchange rate variability in the firm's choice of whether to offer one or two varieties. We show that variability induces the firm to vertically segment markets (offer two varieties). This happens because variability in the exchange rate affects income dispersion and hence the firm's incentives to extract consumer surplus. To better extract surplus, the firm offers two price-quality menus, a high quality variant geared for top-end surplus extraction and a low quality variant to address market coverage concerns.

Keywords: exchange rate variability, income dispersion, surplus extraction, product variety

JEL Classification: F23, L12

1 Introduction

For firms selling across national borders, the exchange rate is an important factor in strategic planning and behaviour. Fluctuations in the exchange rate have a bearing on exporting firms' competitiveness and hence profitability. Baldwin and Krugman (1989) show that the level of the exchange rate matters for the firm's incentives to enter/ exit a foreign market.

Although there exists a large literature on firm behaviour under variable exchange rates (Dornbusch, 1987; Bodnar et al. 2002; Friberg, 1999; 2001) to name but a few, no study (to my knowledge) has considered the effect of exchange rate variability on firms' product variety. Yet, one of the important aspects of firm strategy concerns the number of varieties (product mix). According to the Economist (November 2001), the launch of the Euro has seen some firms in Europe cutting on

*I thank Richard Friberg, Thomas von Ungern-Sternberg and an anonymous referee for valuable comments. I also thank participants to the (2004) NOITS conference in Copenhagen, SNEE (2004) conference in Mölle, Sweden and participants to the Lunch Workshop at the Stockholm School of Economics for helpful comments. Financial support from Vetenskapsrådet, ERSA and the UCT's Block Grants is gratefully acknowledged. All remaining errors are mine.

†Witness.Simbanegavi@uct.ac.za

the number of varieties they produce. The launch of the Euro meant a permanent reduction in exchange rate variability to zero within the EMU. The question we address in this paper is the following: Does exchange rate variability matter for a firm's choice of the number of varieties to produce?

In many cases, firms offer multiple varieties of the same product. These varieties may be differentiated by quality, size or other subtle characteristics. A simple example is that of a bookseller, who can sell the book as a paperback, hardcover or both. A more elaborate example is that of a car manufacturer, for example, BMW. There is the 1 series, 3 series the 5 and 7 series respectively. Moreover, within each series, there are several different varieties – differentiated by such things as engine capacity, leather upholstery, entertainment etc. In such circumstances, firms may not necessarily exit a foreign market when the exchange rate dips too low as Baldwin and Krugman (1989) purports. Rather, the firm may just alter its product range.

The goal of this paper is to contribute to our understanding of the effects of exchange rate variability (or lack of) on product variety. We consider a monopoly firm selling in the Home market and the Foreign market. Consumers in both markets are uniformly distributed according to willingness to pay for quality. The firm cannot perfectly price discriminate. We assume that the Law of One Price (LOP) holds. The monopolist's problem is to choose whether to produce a single variety or two varieties.

We find that exchange rate variability engenders product variety. This is due to the fact that changes in the exchange rate not only affect the purchasing power of consumers, they also affect income dispersion and hence the firm's incentives to extract consumer surplus. Higher income dispersion makes it harder for the firm to extract surplus with a single variety. A single variety forces the firm to price out too many consumers but at the same time fails to extract much surplus from the top end of the market. Clearly, market coverage and surplus extraction are incompatible when the firm offers a single variety. Hence variability in the exchange rate induces the firm to choose the two varieties strategy (i.e., vertically segment markets). This is a classic case of second degree price discrimination – but here driven by exchange rate volatility.

Close in scope to the present study is Friberg (2001), who studies how variability in the exchange rate affects a monopoly firm's incentives to "horizontally" segment international markets. Other related literature include Gabszewicz et al. (1986) who study a monopoly firm's optimal product mix when the feasible range of qualities is bounded and Bonnisseau and Lahmandi-Ayed, (2001) who study a monopoly firm's incentives to vertically segment markets when consumers are distributed according to intensity of preference rather than willingness to pay.

The paper is organized as follows: Section 2 sets out and analyzes the model. Results are stated and discussed in section 3 and section 4 concludes the paper.

2 The Model

2.1 Model and Preliminaries

Our model is closely related to Lambertini (2000). Consider a monopoly firm selling in Home (H) and Foreign (F) markets. The firm can choose from two mutually exclusive technologies – a "monovariety" technology that allows it to produce a good of quality q only and a "multivariety" technology that allows it to produce two qualities; a high quality, q_2 and a low quality, q_1. If the firm chooses the multivariety technology, it has to sell both varieties in each market and moreover, it has to pay an additional fixed cost $K > 0$ over and above that associated with the monovariety technology. Thus, K can be interpreted as the cost of vertically segmenting markets. For simplicity, we normalize the fixed cost associated with the monovariety technology to zero. Quality is chosen from the interval $(0, \infty)$. Total production cost for any variety q_ℓ is $\Gamma_\ell = tq_\ell^2 x_\ell$; $t > 0$ is a constant and x_ℓ is the output of variety ℓ. Thus marginal cost of producing a unit of variety q_ℓ is $c_\ell(q_\ell) = tq_\ell^2$. Observe that, for a given quality level, the marginal cost of output is constant whereas the marginal cost of quality is increasing.

Consumers in each country are uniformly distributed on $[\eta\underline{\theta}, \eta\overline{\theta}]$; $\underline{\theta} = \overline{\theta} - 1 > 0$; where θ denotes the marginal willingness to pay for quality and η is a constant measuring the level of affluence. We assume (throughout the paper) that $\eta \equiv 1$ for Home consumers and $\eta \geq 1$ for Foreign consumers. That is, the Foreign consumers are at least as well off as the Home consumers. Consumers have unit demands. A generic consumer's net utility takes the form;

$$U(\theta) = \begin{cases} \eta\theta q_i - P_{ij}; i \in \{1, 2\}; j \in \{H, F\} \\ 0 \text{ otherwise} \end{cases}. \tag{1}$$

where P_{ij} is the price of quality i in country j.

Let S be the the exchange rate – the units of Home currency needed to buy one unit of Foreign currency. We make the following assumptions

Assumptions A1: (i). $\overline{\theta} \leq \frac{8}{5}$ and (ii). $S \in [0.72, 1.28]$.

The first assumption ensures that the market is only partially covered and the second assumption ensures that demands are nonnegative when the firm sells both varieties in each market[1]. We further assume that the exchange rate is symmetrically

[1] For the derivation of the upper bound to $\overline{\theta}$, see Appendix A.

distributed with a mean of unit and a finite variance, σ^2. We also assume that LOP holds, that is, $P_{iH} = SP_{iF}$.

In the monovariety case, a consumer with willingness to pay θ gets surplus $\eta\theta q - P_j$ if she buys a unit of the good with quality q and 0 otherwise. Let $\widehat{\theta}$ denote the consumer for whom the individual rationality constraint just binds. Then, $\widehat{\theta} = P_j/\eta q$. Thus, in each market, j, the firm faces the demand:

$$x_j = \overline{\theta} - P_j/\eta q; j = H, F. \tag{2}$$

In the multivariety case, a consumer with willingness to pay θ gets surplus $\eta\theta q_2 - P_{2j}$ when buying quality q_2 and surplus $\eta\theta q_1 - P_{1j}$ when buying quality q_1 and surplus zero when not buying. The consumer buys quality q_2 rather than quality q_1 only if $\eta\theta q_2 - P_{2j} \geq \eta\theta q_1 - P_{1j}$. Let $\widehat{\theta}_2$ denote the consumer indifferent between buying qualities q_2 and q_1. Then, $\widehat{\theta}_2 = (P_{2j} - P_{1j})/\eta(q_2 - q_1)$ and all consumers with $\theta > \widehat{\theta}_2$ buy quality q_2. Consumers with $\theta < \widehat{\theta}_2$ either buy quality q_1 or do not buy at all. They buy q_1 if and only if $\eta\theta q_1 - P_{1j} \geq 0$. Thus, in each country, j, the firm faces the following demands for the low and high quality varieties respectively;

$$\begin{pmatrix} x_{1j} \\ x_{2j} \end{pmatrix} = \begin{pmatrix} \frac{P_{2j} - P_{1j}}{\eta(q_2 - q_1)} - \frac{P_{1j}}{\eta q_1} \\ \overline{\theta} - \frac{P_{2j} - P_{1j}}{\eta(q_2 - q_1)} \end{pmatrix}. \tag{3}$$

2.2 Profits

The firm maximizes expected profits. The uncertainty arises from the fact that the level of the exchange rate will only be known after some irrevocable decisions have already been taken. We model the firm's decision as a two stage game. In the first stage and before the realization of the exchange rate, the firm decides on the quality level(s) q (q_1 and q_2). To simplify our analysis, we suppose that the choice of quality is made assuming that the exchange rate equal its expected value of unity[2]. Thereafter, the exchange rate is revealed and the firm makes a second move, the choice of price(s).[3] We solve the problem backwards, starting with the second stage decision.

Let $n \in \{1, 2\}$ be the number of varieties offered by the firm and π_k, $k = I, II$ be the firm's profit when it offers k varieties. The firm's profits are denominated

[2] In an appendix available from the author on request, we discard this assumption and instead let the firm choose the optimal quality (*ex ante*), given the distribution of the exchange rate. The conclusions of the paper are qualitatively unaffected but the expressions quickly get messy.

[3] This assumption implies that prices are perfectly flexible. An alternative and perhaps more realistic assumption would be that both prices and qualities are chosen before the exchange rate in known. This however, complicates the model greatly.

in the Home currency, that is, Foreign earned profits have to be converted into the Home currency equivalent. Given the qualities chosen in the first stage, the firm's second stage behaviour is described by,

$$\prod = \max_{P_{iH}, P_{iF}} \sum_{i=1}^{n} \left\{ \left(P_{iH} - tq_i^2\right) x_{iH} \ + \left(SP_{iF} - tq_i^2\right) x_{iF} \right\} \text{ s.t. } P_{iH} = SP_{iF}. \tag{4}$$

Substituting the demand functions in (2) and (3) into the objective function, (4), and differentiating with respect to prices gives the equilibrium prices as a function of the quality levels (we do not report these here). Substituting the equilibrium prices back into (4) and letting $S = 1$, the firm's first stage behaviour (choice of quality) is described by,

$$\mathbf{q}^* = \arg\max_{\mathbf{q}} \sum_{i=1}^{n} \left(P_i^* - tq_i^2\right) \left(x_{iH}\left(P_i^*\right) + x_{iF}\left(P_i^*\right)\right). \tag{5}$$

Differentiating (5) with respect to quality, q, and simplifying the resulting expressions we get[4]

$$q^* = \frac{2\eta\overline{\theta}}{3t\left(1+\eta\right)} \tag{6}$$

$$P_H^* = \frac{2\left(1+3S+4S\eta\right)\eta^2\overline{\theta}^2}{9t\left(1+S\eta\right)\left(1+\eta\right)^2} = SP_F^* \tag{7}$$

$$\pi_I = \frac{2\left(\left(2\eta+3\right)S-1\right)^2\eta^2\overline{\theta}^3}{27St\left(1+\eta S\right)\left(1+\eta\right)^3} \tag{8}$$

when the firm offers a single variety ($n = 1$) and

$$q_i^* = \frac{2i\eta\overline{\theta}}{5t\left(1+\eta\right)}; i = 1, 2 \tag{9}$$

$$P_{iH}^* = \frac{2i\left(\left(1+S\eta\right)i + 5S\left(1+\eta\right)\right)\eta^2\overline{\theta}^2}{25t\left(1+S\eta\right)\left(1+\eta\right)^2} = SP_{iF}^* \tag{10}$$

$$\pi_{II} = \frac{\left(1-\left(4+2\eta\right)S+\left(5+6\eta+2\eta^2\right)S^2\right)4\eta^2\overline{\theta}^3}{25St\left(1+\eta\right)^3\left(1+S\eta\right)} - K \tag{11}$$

when the firm offers two distinct varieties ($n = 2$).

We see from (6) – (11) that qualities, prices and profits are increasing in market affluence $(\overline{\theta}, \eta)$ but decreasing in the cost of quality, t. Notice also that a depreciation of the Home currency (increase in S) raises the Home price, P_H, but lowers the Foreign price, P_F and an appreciation (decrease in S) has the opposite effect.

[4] Since the calculations are not of any particular interest here, we relegate all calculations to Appendix B.

Thus, a depreciation is tantamount to an increase (decrease) in the wealth of Foreign (Home) consumers and the opposite is true for an appreciation.

From (6), $q^* = 2\eta\overline{\theta}/3t\,(1+\eta)$ and from (9), $q_1^* = 2\eta\overline{\theta}/5t\,(1+\eta)$ and $q_2^* = 4\eta\overline{\theta}/5t\,(1+\eta)$. Comparing q^* and q_1^* on the one hand and q^* and q_2^* on the other gives $q^* - q_1^* = 4\eta\overline{\theta}/15t\,(1+\eta) = 2\,[q_2^* - q^*] > 0$. Hence,

Lemma 1 *When consumers are uniformly distributed according willingness to pay for quality, and the marginal cost of quality is increasing, a single variety firm produces a good of intermediate quality compared to a multivariety firm. That is,* $q_1^* < q^* < q_2^*$.

This observation is new[5] and it contrasts sharply with the findings of Gabszewicz et al. (1986) and Bonnisseau and Lahmandi-Ayed (2001) that a single product monopolist pools all consumers on the top quality. The objective of the firm is to extract as much consumer surplus as possible. Observe that under our assumptions, price is convex in the quality level (see equation B2 in appendix B). Consequently, top end surplus extraction calls for a high quality whereas greater market coverage calls for a low quality (and hence low price). When the firm offers a single variety, this poses a dilemma. Greater market coverage can only be achieved at the expense of top-end surplus extraction and vice-versa. To minimize incongruence, the firm settles for an intermediate quality – a compromise that permits modest surplus extraction without pricing out too many consumers[6]. When the firm offers two qualities, the tension alluded to above falls away. The firm tailors the high quality variety for surplus extraction and the low quality variety for market coverage. Thus, q_1 is much lower and q_2 is much higher.

The tension alluded to above is absent in the models of Gabszewicz et al. (1986) and Bonnisseau and Lahmandi-Ayed (2001). In their models, quality is costless and moreover, markets are always fully covered. For these reasons, bunching occurs on the top quality. In the present model, however, the marginal cost of quality depends on the quality level. As the quality level increases, so does the price and as a result, lower willingness to pay consumers are driven out of the market. The need to balance surplus extraction and market coverage implies that bunching occurs on an intermediate quality.

[5] Although Lambertini (2000) employs a similar model to the one of this paper, he does not compare the qualities across the single and the multivariety strategies. Instead, he contrasts the monopoly outcome to the social planner outcome.

[6] The degree of market coverage matters here for two reasons. First, the difference in willingness to pay between the poorest and the richest individuals is not huge ($\underline{\theta} = \overline{\theta} - 1$) and second, the market is relatively poor ($\overline{\theta} \leq \frac{8}{5}$).

Below we consider the relationship between profits and the exchange rate. Figure 1 below plots π_I and π_{II} for $\eta = 1$, $K = 0$ and $\overline{\theta}^3/t = 1$.[7]

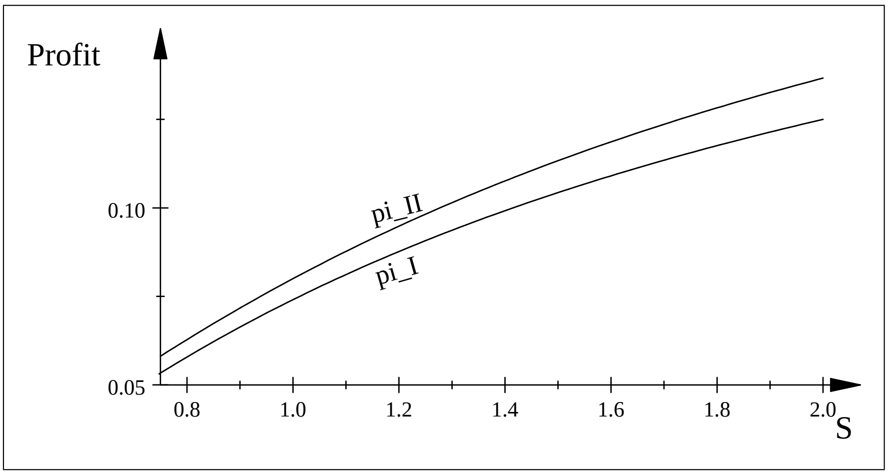

Figure 1: Profit as a Function of the Exchange Rate, S.

There are two interesting things to note about Figure 1. First, π_I and π_{II} are concave in the exchange rate and second, π_I appears more concave than π_{II} and they diverge as the Home currency depreciates.

Concavity of the profit function in the exchange rate, means that the firm loses more in bad times (appreciations) than it gains in good times (depreciations). Hence, greater variability in the exchange rate leads to lower expected profits.

A depreciation of the Home currency lowers Home market profits but raises profits from the Foreign market (denominated in foreign currency) at a decreasing rate[8]. Because now Foreign profits are converted at a more favourable exchange rate, the Home currency equivalent of the Foreign profits increases with the exchange rate and this more than makes up for the fall in Home market profits. This gives a concave shape to the profit function.

What is most striking about Figure 1 is the observation that π_I is more concave than π_{II}. This means that exchange rate variability hurts the firm more if it offers a single variety. Offering two varieties allows the firm to reduce the sensitivity of profits to exchange rate surprises (i.e., it makes the profit function less concave). Hence, one may conjecture that greater variability in the exchange rate may induce the firm to choose a multivariety strategy. In the next section, we show that this is

[7] Since $\overline{\theta}^3/t$ enters multiplicatively in the profit functions, it does not affect the curvature of the profit functions. Hence we can, without loss of generality, assume that $\overline{\theta}^3/t = 1$.

[8] Profit in the Foreign market increases at a decreasing rate because the LOP imposes a much tighter restriction as the Home currency depreciates.

indeed the case.

3 Variability and the Number of Varieties

Let $\triangle\pi \equiv \pi_{II} - \pi_I$ denote the profit difference – the difference between the profit with two varieties and the profit with a single variety (i.e., the benefit to the firm from vertically segmenting markets when LOP holds). We study the relationship between the profit difference and the exchange rate. In subsection 3.1, we consider the case where the Home and Foreign markets are equally affluent. In subsection 3.2 we relax the symmetry assumption and subsection 3.3 assesses the quantitative significance of our main result.

The profit difference, as a function of the exchange rate, S, is given by;

$$\triangle\pi\left(S;\eta\right) = \frac{\overline{\theta}^3}{t}\frac{\left(29 - \left(66 + 8\eta\right)S + \left(45 + 24\eta + 8\eta^2\right)S^2\right)2\eta^2}{675S\left(1+\eta\right)^3\left(1+S\eta\right)} - K. \tag{12}$$

3.1 Symmetric markets $(\eta = 1)$

When markets are equally affluent ($\eta = 1$), (12) reduces to:

$\triangle\pi\left(S\right) = \kappa\left(77S^2 - 74S + 29\right)/S\left(1+S\right), \kappa = \overline{\theta}^3/2700t$. Let $\overline{S} = 1.19$. Independent of $\overline{\theta}$ and t; $\triangle\pi\left(S\right)$ is convex for $S \leq \overline{S}$.[9] Since S is symmetrically distributed with mean of unit, we restrict ourselves (in this section) to the symmetric interval $[\underline{S}, \overline{S}] = [0.81, 1.19]$. Figure 2 below plots the profit difference as a function of the exchange rate.

[9] Formally, $\frac{\partial^2(\triangle\pi(S))}{\partial S^2} = \kappa\frac{174S+174S^2-302S^3+58}{S^3(1+S)^3} > 0 \forall S < 1.19364$.

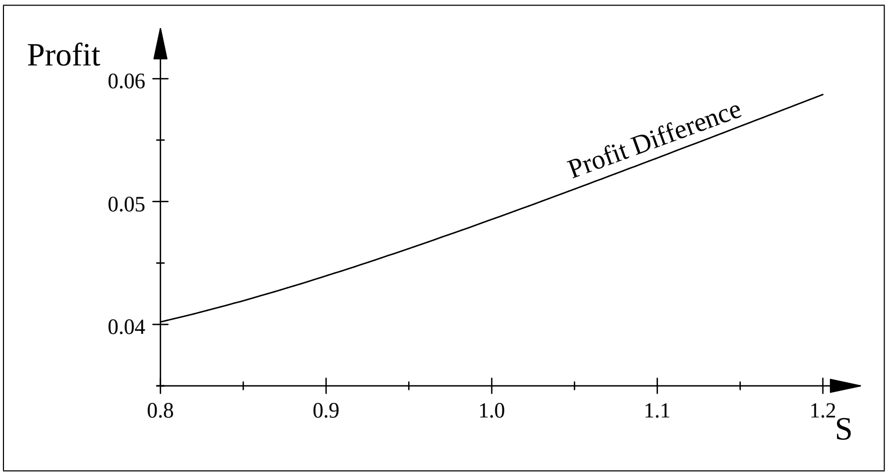

Figure 2: Profit difference as a function of exchange rate, S
for $\overline{\theta}{=}\frac{8}{5}, t = .5, K = 0.$

We see from Figure 2 that when variability is restricted to the interval $\left[\underline{S}, \overline{S}\right]$, the profit difference is convex in S. That is, the expected profit difference is increasing in exchange rate volatility. Suppose we choose K – the cost of vertically segmenting markets – so that the firm is indifferent between the single and the two variety strategies when the exchange rate is equal to unity. That is, K^* solves the equation: $\triangle\pi\left(1\right) \equiv \pi_{II}(1) - \pi_I(1) = 0$. Then we have;

Proposition 1 *A firm that is indifferent between offering a single variety and two varieties when the exchange rate is fixed and equal to its mean (of unity) strictly prefers to offer two varieties when the exchange rate is mildly stochastic.*

Proof. We have seen above that $\triangle\pi\left(S\right)$ is convex for $S \in \left[\underline{S}, \overline{S}\right]$. Because the exchange rate is symmetrically distributed, it follows that $E_S\left[\triangle\pi\left(S\right)\right] > \triangle\pi\left(1\right) = 0$. Hence the firm offers two varieties. ■

The intuition rests on the interaction between the following two observations; viz, that exchange rate variability has wealth effects and that a single variety makes surplus extraction more difficult (Lemma 1). Exchange rate variability affects the purchasing power of consumers, but more importantly here, it affects income dispersion[10]. It is this effect (effect on income dispersion) that matters for the firm's choice of product range (see, for example, Gabszewicz et al. 1986). For example, a

[10] In our model, consumers have unit demands, so an increase in income will not translate into a corresponding increase in demand by an individual consumer.

depreciation makes the Foreign market richer but also generates greater dispersion in willingness to pay. This confronts the firm with a dilemma: On the one hand, as the Foreign market gets richer, the firm finds it more costly to price out many consumers (by charging a higher price). On the other hand, the higher spread in willingness to pay creates incentives for the firm to want to charge a higher price so as to extract more surplus from the top end of the market. To resolve this dilemma, the firm needs to offer both high and low quality varieties. This raises profits by enhancing surplus extraction at the top and at the same time permitting greater market coverage[11].

With increasing marginal cost of quality, it is possible that the unit margin is lower for the high quality variety compared to the intermediate variety (Srinagesh and Bradburd, 1989; p. 103). However, this is not the case in the present model. In fact, it can be shown that for all S in $[0.72, 1.28]$, *the unit margin is higher for quality* q_2 *compared to quality* q *and higher for quality* q *compared to quality* q_1. This discrepancy in the ability to extract *net* surplus ensures that, when the exchange rate is stochastic, second degree price discrimination is more profitable than selling a single variety to all consumers.

The ability to extract surplus is however curtailed by arbitrage concerns. A "strong" depreciation of the Home currency implies a significant reduction in Foreign prices and this weakens surplus extraction in the Foreign market. However, the *conversion effect* – the effect of the level of the exchange rate on Foreign earned profits expressed in terms of the Home currency – mitigates this diminished ability to extract surplus by converting Foreign profits at a more favourable rate. It is the interplay between the conversion effect and the LOP that determines the curvature of the profit difference. The stronger the depreciation, the more LOP binds and the less convex the profit difference becomes.

The above Proposition parallels Friberg (2001) – exchange rate volatility affects consumers' purchasing power and hence the firm's incentives to price discriminate. In Friberg (2001), the firm responds to exchange rate variability by horizontally segmenting markets in order to third degree price discriminate. Here, we allow for spatial arbitrage and the firm responds to exchange rate variability by vertically segmenting markets in order to second degree price discriminate.

We have shown that exchange rate variability matters for firms' decisions on the number of varieties to produce. We find some support, albeit anecdotal, for our finding. The Economist (November, 2001) discusses how multiproduct firms selling in the EU are responding to the launch of the Euro. The introduction of the

[11] Gabszewicz (1983) show that there are instances where a larger heterogeneity in willingness to pay may lead to fewer (and not more) varieties being offered by a monopolist.

Euro meant a permanent reduction of exchange rate variability to zero within the EMU. According to the Economist, some firms, for example, Unilever and Procter & Gamble have started trimming the number of brands they offer so that they can concentrate on a few brands while others, for example, Nestle, Henkel and Danone are increasingly using the same brand name on their products across Euroland. These observed responses are in line with the predictions of our model. Elimination of exchange rate variability diminishes product variety because it reduces (expected) future dispersion in income and hence the incentive to crowd the product space[12].

3.2 Asymmetric markets[13]

In this subsection, we assume that the Foreign market is richer[14], that is, $\eta > 1$. Figure 3 plots $\triangle\pi\left(S;\eta\right)$ for different values of η.

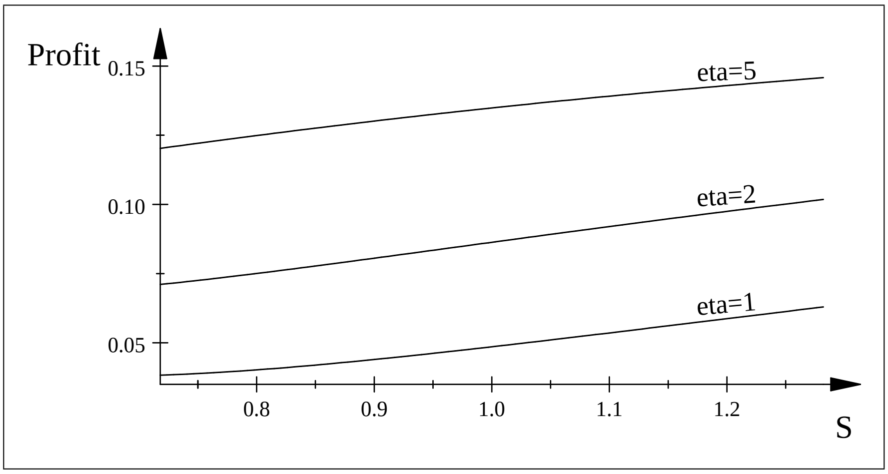

Figure 3: Profit difference as a function of the exchange rate, S. $\overline{\theta}=\frac{8}{5}$, $t = 0.5$, $K = 0$

Two things are immediate from Figure 3. First, the profit difference, $\triangle\pi\left(S;\eta\right)$, is increasing in η (i.e., an increase in affluence raises the profitability of the multivariety strategy) and (ii) the profit difference becomes less convex as the Foreign market gets richer.

When the Foreign market gets richer, the firm responds by raising the quality level and this allows the firm to charge higher prices. When the firm sells two

[12] Prior to the formation of the EMU, the now EMU countries were characterized by semi-fixed exchange rates (mild volatility). That is, exchange rate variability between the currencies of the now EMU member countries was rather low compared to variability with non EMU countries.

[13] The analysis here is only exploratory and therefore results are only suggestive.

[14] It makes no (qualitative) difference whether the Home or the Foreign market is richer.

varieties, both the quality gap and the price gap increase with η and this augurs well for surplus extraction. More importantly, the unit margin increases with η and the increase is larger the higher the quality level[15]. Clearly therefore, vertical segmentation permits greater surplus extraction at the top as well as greater market coverage[16]. Consequently, profits increase more with affluence under vertically segmented markets – hence the profit difference increases with η. As before however, LOP constrains top end surplus extraction more as the Foreign market gets richer and this makes the profit difference less convex.

3.3 Importance of the result, a quantitative assessment[17]

As we can see (figure 2), the *profit difference* function is relatively flat. This leads us to raise the question: How much does exchange rate variability raise expected profits relative to profits when the exchange rate is permanently fixed at unity? Is this increase quantitatively important?[18]

We calculate the percentage increase in the profit difference (gross of the fixed cost, K) for chosen realizations of the exchange rate.[19] We find a progressive effect of exchange rate variability on percentage increase in profits. For variability within 0.05, 0.15 and 0.18 units from the mean, expected profits increase by as much as 0.1%, 1% and 1.5% respectively.[20]

[15] That is, $\partial\left(P - tq_i^2\right)/\partial\eta = \mu q_i, \mu > 0$; which is increasing in the quality level (P is given by equation B2 in Appendix B).

[16] Although demand for the high quality variety falls when η increases (because prices increase with η), the displaced consumers turn to the low quality variety – rather than exit the market. With a single variety however, an increase in η significantly lowers market coverage as all displaced consumers have no option but to leave the market.

[17] We also examined the effects of relaxing the LOP assumption but since this is the focus of Friberg (2001), we mention the results in passing (and refer the interested reader to Friberg, 2001). Relaxing the LOP assumption confirms his result that: *The expected benefit from (horizontally) segmenting the Home and Foreign markets increases with exchange rate volatility.* A consequence of LOP is that the Home and Foreign prices are perfectly negatively correlated and this impacts negatively on the firm's ability to extract surplus. Relaxing LOP allows the firm to charge the optimal price in each market. Since exchange rate volatility implies a divergence between the Home and Foreign prices, relaxing LOP (segmenting the markets) is profitable.

[18] In Appendix C we provide an alternative way of assessing the quantitative significance of Proposition 1. In particular, we ask the question: If the firm is indifferent between the single and the two variety strategies in the absence of exchange rate volatility, what is the percentage increase in K that would make the firm to remain indifferent when we allow for exchange rate volatility? We reach a similar conclusion.

[19] Calculations are available from the author on request.

[20] The range of variability of the exchange rate suggested here is quite modest. For example, in the period between years 2000 and 2006, the Rand (South African currency) to USD exchange rate moved from over 12 Rands to the USD to below 6 Rands to the USD.

These numbers are not huge, but they are also non-trivial. For example, if the profit difference under a "fixed" exchange rate were ZAR 1 000 000, then a one percent increase in expected profits (due to exchange rate variability) would mean a ZAR 10 000 increase in expected profits.[21] This is not insignificant.

4 Conclusion

We extend the literature on monopoly product mix by considering how variability in the exchange rate affects the variety range offered by a monopoly firm selling at home and abroad. We find that exchange rate variability induces the firm to expand the number of varieties produced. The mechanism works through the effect of exchange rate volatility on the dispersion of income. A higher dispersion of income makes it harder for the firm to significantly extract surplus from the top end of the market under a single variety strategy. Hence, variability in the exchange rate leads to more varieties being offered. Our finding is of great interest in light of the adoption of the Euro. The result strengthens when we relax the "perfectly" integrated markets assumption.

Our model has limitations. In the paper, we assumed perfectly flexible prices. However, the literature on pricing in international markets shows that pass-through of exchange rate changes into import prices is generally incomplete. A notable extension therefore would be to allow for price rigidity in the model. This, however, is left for future research.

[21] ZAR is the acronym for the South African currency, the Rand.

Appendix

Appendix A: Partial Market Coverage

Partial market coverage obtains whenever $\underline{\theta} < \frac{P}{q}$, where $\frac{P}{q}$ is the marginal willingness to pay (MWTP) for quality of the individual indifferent between purchasing a unit of quality q at price P and purchasing nothing. Let $\overline{\theta}_{jk}$ be the highest WTP consistent with partial market coverage in market j when the firm offers k varieties.

Consider the single variety case. For the Home market, the condition $\underline{\theta} < \frac{P_H}{q}$ implies $\overline{\theta}_{H1} \equiv \frac{6(1+S)}{5-S}$ and in the Foreign market, the condition $\underline{\theta} < \frac{P_F}{q}$ implies $\overline{\theta}_{F1} \equiv \frac{6S(1+S)}{(3S+1)(2S-1)}$, where we make use of the fact that $\underline{\theta} = \overline{\theta} - 1$.

In the two varieties case, market coverage is determined by the low quality variety. In the Home market, the condition $\underline{\theta} < \frac{P_{1H}}{q_1}$ implies $\overline{\theta}_{H2} \equiv \frac{10(1+S)}{9-S}$ and in the Foreign market, the partial market coverage condition $\underline{\theta} < \frac{P_{1F}}{q_1}$ implies $\overline{\theta}_{F2} \equiv \frac{10S(1+S)}{10S^2-S-1}$, where P_{1j} is the price of quality 1 in market $j; j = H, F$.

In the Home market, $\overline{\theta}_{H1} - \overline{\theta}_{H2} = \frac{8S+4S^2+4}{S^2-14S+45} > 0$ for all S in the relevant range. That is, the binding constraint is $\overline{\theta}_H \leq \overline{\theta}_{H2}$. In the Foreign market, $\overline{\theta}_{F1} - \overline{\theta}_{F2} = \frac{4S+8S^2+4S^3}{2S-15S^2-16S^3+60S^4+1} > 0$ for all S in the relevant range. Hence, the binding constraint is $\overline{\theta}_F \leq \overline{\theta}_{F2}$. Thus partial market coverage in the two varieties case implies partial market coverage in the single variety case **but not the other way round**. We have partial market coverage in both markets with two varieties if $\overline{\theta} < \min\left\{\overline{\theta}_{H2}, \overline{\theta}_{F2}\right\}$. Evaluating $\overline{\theta}_{H2}$ and $\overline{\theta}_{F2}$ at $S = \{0.7, 2\}$ gives $\min\left\{\overline{\theta}_{H2}, \overline{\theta}_{F2}\right\} = \{1.6216\}$. In a sense, partial market coverage requires that markets be rather poor.

Appendix B: Derivation of Profits

Single variety strategy

Let π_I be the profit when the firm sells a single variety in both markets. In the second stage, the firm chooses prices P_H and P_F, given quality q chosen in the first stage, and her behaviour is described by

$$\pi_I = \max_{P_H, P_F} \left\{\left(P_H - tq^2\right) x_H + \left(SP_F - tq^2\right) x_F\right\} \text{ s.t. } P_H = SP_F. \tag{B1}$$

where the demands (x_H and x_F) are given by equation (2).

Simplifying the first order conditions of (B1) gives

$$P_H^* = \frac{1}{2S\eta + 2}\left(2Sq\overline{\theta}\eta + q^2 t + Sq^2 t\eta\right) = SP_F^*. \tag{B2}$$

Substituting (B2) into (B1) (assuming $S = 1$) gives π_I as a function of q only.

In the first stage, the firm chooses q to maximize $\pi_I(q)$ and her optimal behaviour is described by[22]

$$q^* = \arg\max_q \left(P^* - tq^2\right)\left(x_H\left(P^*\right) + x_F\left(P^*\right)\right). \tag{B3}$$

Differentiating (B3) with respect to q and simplifying yields

$$q^* = 2\eta\overline{\theta}/3t\left(1+\eta\right). \tag{B4}$$

Substituting (B4) into (B2) gives

$$P_H^* = 2\left(1+3S+4S\eta\right)\eta^2\overline{\theta}^2/\left[9t\left(1+\eta S\right)\left(1+\eta\right)^2\right] = SP_F^* \tag{B5}$$

and substituting (B5) and (B4) into (B1) gives

$$\pi_I^* = \left(4S\eta+6S-2\right)\left(2S\eta+3S-1\right)\eta^2\overline{\theta}^3/\left[27St\left(1+\eta S\right)\left(1+\eta\right)^3\right]. \tag{B6}$$

Two variety strategy

Let π_{II} be the profit when the firm sells two varieties in each market. As before, the firm chooses quality based on the expected exchange rate and then after the revelation of the exchange rate, choose prices. We solve the problem backwards. In the second stage, given the qualities q_1 and q_2 chosen in stage 1, the firm chooses prices, $P_{iH}, P_{iF}; i = 1, 2$ to solve,

$$\pi_{II} = \max_{P_{iH},P_{iF}} \sum_{i=1}^{2}\left(P_{iH} - tq_i^2\right)x_{iH} + \sum_{i=1}^{2}\left(SP_{iF} - tq_i^2\right)x_{iF} - K \text{ s.t. } P_{iH} - SP_{iF} = 0. \tag{B7}$$

where the demands (x_{iH} and x_{iF}) are given by equation (3). Differentiating (B7) with respect to P_{iH} and P_{iF} and simplifying the first order conditions yield

$$P_{iH}^* = \left(2S\overline{\theta}\eta q_i + tq_i^2 + St\eta q_i^2\right)/\left(2S\eta+2\right) = SP_{iF}^*. \tag{B8}$$

Substituting (B8) into (B7) (assuming $S = 1$) gives $\pi_{II}(q_1, q_2)$. In the first stage, the firm chooses q_1 and q_2 to maximize $\pi_{II}(q_1, q_2)$. Thus

$$\mathbf{q}^* = \arg\max_{q_1,q_2} \sum_{i=1}^{2}\left(P_i^* - tq_i^2\right)\left(x_{iH}\left(P_i^*\right) + x_{iF}\left(P_i^*\right)\right). \tag{B9}$$

[22] Notice that $\pi_I(q) \neq \pi_I$ since now we assume $S = 1$. Observe also that $P_H^* = P_F^* = P^*$.

Differentiating (B9) with respect to q_i and simplifying yields

$$q_i^* = 2i\eta\overline{\theta}/5t\,(1+\eta)\,; i = 1, 2. \tag{B10}$$

Substituting (B10) into (B8) gives

$$P_{iH}^* = 2i\left(\left(1+S\eta\right)i + 5S\left(1+\eta\right)\right)\eta^2\overline{\theta}^2 / \left[25t\left(1+S\eta\right)\left(1+\eta\right)^2\right] = SP_{iF}^* \tag{B11}$$

and substituting (B11) and (B10) into (B7) gives the profit function under the two variety strategy as

$$\pi_{II}^* = \frac{\left(1 - 4S - 2S\eta + 5S^2 + 6S^2\eta + 2S^2\eta^2\right)4\eta^2\overline{\theta}^3}{25St\left(1+\eta\right)^3\left(1+S\eta\right)} - K\ . \tag{B12}$$

Appendix C: Importance of the Result, a Quantitative Assessment

Although Proposition 1 is qualitatively quite intuitive, we still need to assess it's quantitative significance. The question we seek to answer is the following: Does the finding of Proposition 1 matter quantitatively? Our approach here – which is an alternative to the approach in the text (subsection 3.3) – is to calculate the percentage increase in K that is required in order for the firm to remain indifferent between the single and the two variety strategies if we allow for reasonable exchange rate volatility.[23] Assuming $\eta = 1$ as before, π_I and π_{II} are respectively given by: $(\pi_I, \pi_{II}) = \left(\frac{\overline{\theta}^3}{t}\frac{(5S-1)^2}{108S(S+1)}, \frac{\overline{\theta}^3}{t}\frac{1+13S^2-6S}{50S(S+1)} - K\right)$. In Proposition 1, we defined K such that, in the absence of exchange rate volatility (i.e., when the exchange rate is permanently fixed at its mean value of unity), the firm is indifferent between the single and the two variety strategies. Solving for K (assuming $\overline{\theta} = \frac{8}{5}, t = 0.5$.) then gives:

$$K^* = 0.048545$$

That is, when the cost of vertically segmenting the markets is K^*, the firm is indifferent between offering a single variety and offering two varieties.

Now let us allow for exchange rate variability. Suppose that for each draw of the exchange rate there are two potential (symmetric) outcomes, S' and S'', each with equal probability and satisfying $\frac{1}{2}S' + \frac{1}{2}S'' = 1$. Let $(S', S'') = (0.82, 1.18)$. This is a depreciation /appreciation of about 18%. This is fairly common in the currency markets.

[23] This alternative approach was suggested to me by a referee.

Now the expected profit with a single variety is

$$E_S(\pi_I) = \frac{1}{2}\pi_I(0.82) + \frac{1}{2}\pi_I(1.18)$$

and the expected profit with two varieties is

$$E_S(\pi_{II}) = \frac{1}{2}\pi_{II}(0.82) + \frac{1}{2}\pi_{II}(1.18).$$

Substituting in the expressions and simplifying we get;

$$E_S(\pi_I) = 0.598\,21$$

and

$$E_S(\pi_{II}) = 0.647\,46 - K.$$

The firm will be indifferent between the single and the two variety strategies with an 18% depreciation/appreciation if and only if $E_S(\pi_{II}) = E_S(\pi_I)$. Therefore the value of K that makes the firm indifferent is:

$$\widetilde{K} = 0.647\,46 - 0.598\,21 = 0.049\,25.$$

What is the percentage increase in K that would make the firm indifferent? This is straight forward. We take the difference between $\widetilde{K}$ and K^* and divide it by K^* and then multiply the resulting expression by 100. Thus:

$$\%\triangle K = \left(\frac{\widetilde{K} - K^*}{K^*}\right)100 = \left(\frac{0.049\,25 - 0.04\,854\,5}{0.04\,854\,5}\right)100 \approx 1.5\%$$

Thus for the firm to be indifferent between the single and two variety strategies in the face of mild exchange rate volatility (18%), K would have to increase by about 1.5%. This is not an insignificant increase. Thus we conclude that the result is quantitatively significant.

References

[1] Baldwin, Richard and Krugman, Paul (1989), "Persistent Trade Effects of Large Exchange Rate Shocks", *Quarterly Journal of Economics*, 104, 635-654.

[2] Baum, Christopher F., Caglayan, Mustafa and Barkoulas, John T. (2001), "Exchange Rate Uncertainty and Firm Profitability", *Journal of Macroeconomics*, 23, 565-576.

[3] Bodnar, Gordon M., Dumas, Bernard and Marston, Richard C. (2002), "Pass-Through and Exposure", *Journal of Finance*, 57, 199-231.

[4] Bonnisseau, Jean-Marc and Lahmandi-Ayed, Rim (2001), "Multiproduct Firms in Vertically Differentiated Markets", *Manuscript*.

[5] Dornbusch, Rudiger (1987), "Exchange Rates and Prices", *American Economic Review*, 77, 93-106.

[6] Friberg, Richard (1999), "Exchange Rates and the Firm", *MacMillan Press LTD*.

[7] Friberg, Richard (2001), "Two Monies, Two Markets? Variability and the Option to Segment", *Journal of International Economics*, 55, 317-327.

[8] Gabszewicz, Jean Jaskold (1983), "Blue and Red Cars or Blue Cars Only: A Note on Product Variety", *Economica*, New Series, 50, 203-206.

[9] Gabszewicz, Jean Jaskold; Shaked, Avner; Sutton, John and Thisse, Jacques-Francois (1986), "Segmenting the Market: The Monopolist's Optimal Product Mix", *Journal of Economic Theory*, 39, 273-289.

[10] Goldberg, Pinelopi K. and Knetter, Michael M. (1997), "Goods Prices and Exchange Rates: What Have we Learned?", *Journal of Economic Literature*, 35, 1243-1272.

[11] Lambertini, Luca (2000), "Does Monopoly Undersupply Product Quality?", *Manuscript,* Università degli Studi di Bologna.

[12] Salant, Stephen W. (1989), "When is Inducing Self-Selection Suboptimal For a Monopolist?", *Quarterly Journal of Economics*, 104, 391-397.

[13] Srinagesh, Padmanabhan and Bradburd, Ralph M. (1989), "Quality Distortion by a Discriminating Monopolist", *American Economic Review,* 79, 96-105.

[14] The Economist, November 2001.