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Equilibrium Pricing When Only Some Goods Are Advertised

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Equilibrium Pricing When Only Some Goods Are Advertised*

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Abstract

We study how price advertising of a subset of products affects equilibrium pricing and advertising under low and high product differentiation. We find that, when firms sell products with the same reservation price, loss-leader pricing obtains only when differentiation is low. However, when reservation prices differ, equilibrium may entail loss-leader pricing when differentiation is high. This enables us to shed some light on the seemingly paradoxical empirical findings in the marketing literature that loss-leader pricing fails to increase store traffic, loss-leader sales and hence to increase profits.

Keywords: Informative advertising; loss leader pricing; multiproduct firms

JEL Classification: L13; L15; M37

1 Introduction

The effect of advertising on prices has long been a subject of great interest to economists. Important contributions include Benham (1972) who finds, in an empirical study of the eyeglass market, that advertising is associated with lower prices; Butters (1977) who provides the first equilibrium analysis of price advertising and Grossman and Shapiro (1984) who study price advertising in a Hotelling model. Grossman and Shapiro assume full market coverage (all informed consumers make a purchase) and that firms each sell a single product. They show that, in oligopoly markets, higher advertising is associated with lower prices.

It is, however, well established that many firms advertise only a subset of the products they sell – a supermarket, for instance, advertises a handful of prices but sells hundreds of

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products.[1] What is the effect on prices of advertising only a subset of products? Milyo and Waldfogel (1999)'s empirical study of the effect of price advertising on prices suggests that prices of unadvertised products are higher when advertising is allowed. Theoretically, little is known. Indeed, in his extensive survey, Bagwell (2003; p. 51) writes, "Recent work, however, suggests that the distinction between the effect of advertising on the prices of advertised and unadvertised products warrants greater attention".

The goal of this paper is to deepen our understanding of the effects of price advertising when firms advertise only a subset of their products. We study price advertising under two product differentiation regimes, namely, "low" and "high" differentiation. We consider two firms, each selling two products but only advertising a single product. Advertising messages are randomly distributed over consumers, who are assumed to be uninformed about prices and firm locations unless they are reached by advertising.

We show that the existence of loss-leader pricing crucially depends on how strong competition between the firms is.[2] In particular, for sufficiently low distance (differentiation) between firms, the advertised good is priced below cost, otherwise the advertised good is priced above cost. We provide exact conditions (on the level of differentiation) where the pricing strategy changes from prices above cost to loss leader pricing. By emphasizing the role of the degree of product differentiation, we shed some light on the seemingly paradoxical findings of Walters and MacKenzie (1988) of a "weak" link between loss leader pricing and store traffic and profits.

Closely related are Lal and Matutes (1994) who study equilibrium pricing by multiproduct firms and Ellison (2005) who studies a vertically differentiated goods model in which firms only advertise the low quality good. However, neither Lal and Matutes nor Ellison explicitly model the advertising decision. In reality, advertising is an important strategic tool. Modelling the advertising decision allows us to study the interaction between advertising and prices. Moreover, both Lal and Matutes (1994) and Ellison (2005) only consider the case where the market is fully covered. However, as Soberman (2004) show, for some constellations of the differentiation parameter, some informed consumers find it profitable not to purchase.[3]

[1] We take it as a given that firms advertise only a subset of their products. Ellison (2005; pages 607-611) discusses some of the reasons why firms may want to advertise only a subset of their products.

[2] A good is termed a loss leader if it is priced below marginal cost.

[3] Also related is Bagwell and Ramey (1994) who show that "ostensibly uninformative" advertising may bring about coordination economies. We differ with them in several ways; First, in our model, advertising

The paper is organized as follows. Section 2 sets out the model. We analyze the model in section 3. Section 4 examines welfare implications and section 5 concludes.

2 Model and Preliminaries

Our model is an extension of the Grossman and Shapiro (1984) model to multiproduct firms. We consider a linear city of unit length served by two firms, 1 and 2, located at points 0 and 1 respectively. Each firm sells two independent products, product 1 and product 2. However, across firms, the products are substitutes.[4] Firms advertise only one (and the same) good and advertising is truthful. Firms randomly sent out advertisements to consumers. Let ϕ_i denote the advertising (ad) reach of firm $i; i = 1, 2$. The cost of reaching fraction ϕ_i of consumers is denoted $A(\phi_i)$, where $A' > 0$ and $A'' > 0$. For what follows, let $A(\phi_i) = a\phi_i^2/2; a > t/2$. Each good is produced at a constant marginal cost, c, and firms simultaneously and non-cooperatively choose prices and advertising intensities to maximize profits. There is no entry or exit and there are no fixed costs.

Consumers are uniformly distributed on $[0, 1]$. That is, each consumer is identified by a point on the unit interval that corresponds to her most preferred brand. Consumers are uninformed about prices and firm locations unless they are reached by advertising. Each informed consumer buys at most one unit of each product and uninformed consumers stay out of the market. A unit of good 1(2) generates gross surplus of $v_1(v_2)$ and consumers incur a shopping cost of t per unit distance. Whenever they find it profitable to purchase, partially informed consumers buy both goods from the same store. This, however, is not obvious with fully informed consumers.[5]

Given the advertising intensities, ϕ_1 and ϕ_2, the market is delineated as follows: Fraction $\phi_1\phi_2$ of consumers receive advertising messages from both firms and are thus fully informed. Fraction $\phi_i(1 - \phi_j); i, j = 1, 2; j \neq i$ receive ads from firm i but not firm j and hence are partially informed. Fraction $(1 - \phi_1)(1 - \phi_2)$ receive no ads from either firm and are thus uninformed. We assume that $\phi_1\phi_2$ is large enough so that firms find it worthwhile to

is (directly) informative whereas in theirs, it is not. Secondly, whereas in our model firms advertise only a subset of the products they sell, in theirs, they neither advertise the prices nor the products they sell. Rather, firms advertise for instance, their size. Also, in Bagwell and Ramey, product differentiation is unimportant.

[4] Firm 1's good k is an imperfect substitute for firm 2's good $k; k = 1, 2$.

[5] We get around this problem by assuming that consumers expect firms to charge the same price for the unadvertised product. This effectively rules out shopping at both stores.

compete for the fully informed consumers.[6]

Let p_{11} and p_{21} denote firm 1 and respectively, firm 2's advertised prices and let p_{12}^E and p_{22}^E denote the expected prices of the unadvertised products (The first subscript denotes the firm while the second denotes the product). There are three possible configurations of the expected prices; either $p_{12}^E = p_{22}^E$, $p_{12}^E < p_{22}^E$ or $p_{12}^E > p_{22}^E$. In the second and third configurations, the analysis gets complicated (see Appendix B.). For what follows, we suppose consumer expectations are such that $p_{12}^E = p_{22}^E$. Lemmas 1 and 2 below greatly simplify the construction and respectively, structure of the demand functions.

Lemma 1 *If* $p_{12}^E = p_{22}^E$, *fully informed consumers will buy both goods from a single firm.*

Proof. Let $p_{12}^E = p_{22}^E$. Suppose, without loss of generality, that $p_{11} \leq p_{21}$. In either case ($p_{11} = p_{21}$ or $p_{11} < p_{21}$), for any consumer $x \in (0,1)$, buying from both firms involves a cost t whereas buying from firm 1 only involves a cost $tx < t$. It follows therefore that the consumer at x will buy both goods from firm 1 rather than buying from both firms. Hence, no consumer will buy from both firms. ■

Lemma 1 is intuitive. If it is costly to visit a store, a necessary condition for fully informed consumers to shop around is that each firm quotes the lowest price in exactly one of the products – with no tie.

Fully informed consumers purchase from whichever firm gives them the greatest surplus. A consumer located at $x \in (0,1)$ gets surplus $v_1 + v_2 - p_{11} - p_{12}^E - tx$ buying from firm 1 and surplus $v_1 + v_2 - p_{21} - p_{22}^E - t(1-x)$ buying from firm 2. Let $\widehat{x}$ denote the location of the indifferent consumer. Then, firm i faces demand $\widehat{x} = (p_{j1} - p_{i1} + t)/2t$ from the fully informed consumers. For partially informed consumers, demand is determined (only) by individual rationality. Let y_i denote the location of a consumer who receives only firm i's ad(s). Buying yields surplus $v_1 + v_2 - p_{i1} - p_{i2}^E - ty_i$ while not buying yields surplus zero. Firm i thus faces demand $\widetilde{y}_i = \left(v_1 + v_2 - p_{i1} - p_{i2}^E\right)/t$ from the partially informed consumers (where $\widetilde{y}_i$ denotes the indifferent consumer).[7]

Hence, the firm faces the demand:

$$D_i = \phi_i \left[\left(1-\phi_j\right)\left(v_1 + v_2 - p_{i1} - p_{i2}^E\right)/t + \phi_j \left(p_{j1} - p_{i1} + t\right)/2t\right]; i \neq j.$$

[6] In an appendix available from the author on request, we provide a necessary condition for firms to compete for the fully informed consumers.

[7] However, if $v_1+v_2-p_{i1}-p_{i2}^E > t$, all consumers who receive at least one ad from firm i will make a purchase, that is, $\widetilde{y}_i = 1$.

Let $\overline{p} = p_{12} = p_{22}$ be the equilibrium price of the unadvertised good. We make the following "intuitively obvious" assumption.

Assumption A1 $\overline{p} \geq \max\{p_{11}, p_{21}\}$.

Assumption A1 simply says that the unadvertised price cannot be lower than the advertised price. If it were, the firm would be better off advertising that price instead, since consumers' visitation decisions are predicated on the advertised price. An immediate consequence of Assumption A1 is that;

Lemma 2 *If consumers expect firms to charge the same price for the unadvertised good, then, in equilibrium, firms will charge the reservation price. That is, if* $p_{12}^E = p_{22}^E$*, then* $\overline{p} = v_2$.

Proof. Let $\overline{p}$ be the equilibrium profit maximizing price of the unadvertised good. Suppose, contrary to the lemma, that $\overline{p} \neq v_2$. To start with, suppose $\overline{p} < v_2$ and that price is continuous. Then, $\exists \varepsilon > 0 : p = \overline{p} + \varepsilon < v_2$. Since consumers have unit demands, the demand that each firm faces is independent of the price of good 2 (the unadvertised price). Therefore, for any firm i with advertised price p_{i1} and for any $\varepsilon > 0$, $\pi_i(p_{i1}, \overline{p} + \varepsilon) > \pi_i(p_{i1}, \overline{p})$. In other words, $\overline{p}$ is not profit maximizing – a contradiction. Hence, we must have $\overline{p} > v_2$. However, for $\overline{p} > v_2$, visiting consumers only buy the advertised product. In fact, for any $\varepsilon > 0$, $\pi_i(p_{i1}, v_2 + \varepsilon) < \pi_i(p_{i1}, p)$, for any $p \in (\max\{p_{11}, p_{21}\}, v_2]$. Hence, $\overline{p} > v_2$ is not profit maximizing – a contradiction. Hence, it must be the case that $\overline{p} = v_2$. ■

What Lemma 2 says is the following; If consumers expect firms to charge the same price for the unadvertised good, then, the only equilibrium price that satisfies those expectations is $\overline{p} = v_2$. Given Lemma 2, firm i's demand reduces to

$$D_i = \phi_i \left[\left(1 - \phi_j\right)(v_1 - p_{i1})/t + \phi_j (p_{j1} - p_{i1} + t)/2t \right]; i \neq j. \tag{1}$$

We label the above model Standard (S) when firms each sell a single product and differentiation is low. When instead, firms each sell two products, we label the model Regime L (L) when differentiation is low and Regime H (H) when differentiation is high.

3 Analysis

In subsection 3.1, we jointly analyze the Standard model and regime L. To this effect, we let r denote the regime (S or L) and let I be an indicator variable such that $I = 0$ if $r = S$ and $I = 1$ if $r = L$. In subsection 3.2, we study the case of high differentiation and in subsection 3.3, we give a summary of our main findings.

3.1 Low Differentiation

When $v_1 - p_{i1} \geq t$, all consumers who receive at least one ad make a purchase. In particular, the partially informed consumer who travels the longest distance (the whole unit interval) gets non-negative surplus.[8] Thus, $\widetilde{y}_i \equiv \frac{v_1 - p_{i1}}{t} = 1$. We make the following assumption;

Assumption A2 $c + \sqrt{2at} < \min\{v_1, v_2\}$.

Assumption A2 is needed to ensure that, in equilibrium, consumers visit the stores in the single product case. If the equilibrium price is greater than or equal to $\min\{v_1, v_2\}$, visiting consumers get negative surplus since they incur some positive transportation costs and hence no consumer would visit a store.[9]

Let p_{21} and ϕ_2 be the advertised price and respectively, advertising level chosen by firm 2. Since $p_{12} = v_2$, firm $1'$s behaviour is described by

$$\pi_1 = \max_{p_{11},\phi_1} \left(p_{11} - c + I\left(v_2 - c\right)\right)\phi_1 \left[1 - \phi_2 + \phi_2\left(p_{21} - p_{11} + t\right)/2t\right] - a\phi_1^2/2. \tag{2}$$

The first order conditions, evaluated at the symmetric equilibrium: p^r, ϕ^r are;

$$(\pi_p = 0) \qquad p^r - c + I\left(v_2 - c\right) = \left(2t - t\phi^r\right)/\phi^r \tag{3}$$

$$(\pi_\phi = 0) \qquad p^r - c + I\left(v_2 - c\right) = 2a\phi^r/\left(2 - \phi^r\right). \tag{4}$$

[8] See Appendix A for the derivation of limits to the region of low differentiation.

[9] This assumption is implicit in the analyses of, for example, Bagwell (2003; Section. 5), Tirole (1988; Chap. 7) and Soberman (2004).

It is immediate from (3) that higher advertising is associated with lower advertised prices. Solving (3) and (4) yields;

$$p^r = c + \sqrt{2at} - I(v_2 - c) \quad (5)$$

$$\phi^r = 2/\left(1 + \sqrt{2a/t}\right) \quad (6)$$

and substituting (5) and (6) into (2) gives

$$\pi^r = 2a/\left(1 + \sqrt{2a/t}\right)^2. \quad (7)$$

The advertised price is given by $p^S = c + \sqrt{2at}$ in regime S ($I = 0$) and by $p_1^L = 2c + \sqrt{2at} - v_2 = p^S - (v_2 - c) < p^S$ in regime L ($I = 1$). That is, firms advertise lower prices when they sell multiple products but only advertise a subset. Although consumers pay the reservation price for the unadvertised good in regime L, the advertised price is lower as competition is more intense.

Observe that $\pi^S = \pi^L$ in (7), that is;

Proposition 1 *When firms advertise only a subset of their products, equilibrium profits are invariant with respect to an increase in the number of products.*

This is explained by the fact that when each firm carries multiple products, but only advertises a subset, the ability to extract the entire consumer surplus on the unadvertised good raises the incentives to increase store traffic. This leads firms to offer price reductions on the advertised good. Price cutting continues until the loss from price reductions equals the gain from the sale of the unadvertised good – hence profits are invariant.

Our invariance result resembles a standard finding in the switching costs literature. In fact, our model can be reinterpreted as a two period model in which consumers have switching costs and each firm sells a single product in each period. Firms offer bargains to entice consumers followed by rip-offs when consumers are locked in and the fierce competition for market share in the first period may totally dissipate potential profits from exploiting locked in consumers (Klemperer, 1987).[10]

[10] However, in equilibrium, consumers are not fooled. They know they are going to be ripped. Because firms cannot commit to not fleece consumers once they are locked in, they have to offer price discounts as a way to commit to leave consumers sufficient surplus to make the relationship (shopping trip) worthwhile.

3.2 High differentiation

We say that differentiation is high if, given the prices, at least one partially informed consumer does not make a purchase. Under high differentiation, the degree of product differentiation is such that $t \in \left(\frac{v_1+v_2}{2} - c, \frac{2(v_1+v_2-2c)}{3}\right)$ and the demand that firm i faces is given by equation (1) exactly.[11]

Firm $1'$s behaviour is described by

$$\pi_1^H = \max_{p_{11},\phi_1} (p_{11} + v_2 - 2c)\,\phi_1 \left[(1-\phi_2)(v_1 - p_{11})/t + \phi_2 (p_{21} - p_{11} + t)/2t\right] - a\phi_1^2/2.$$

The first order necessary conditions for an equilibrium are;

$$\pi_p = 4c + 2(v_1 - v_2) + (t - 2c - (2v_1 - v_2))\,\phi^H - \left(4 - 3\phi^H\right) p_1^H = 0 \tag{8}$$

$$\pi_\phi = \left(p_1^H + v_2 - 2c\right)\left(\phi^H/2 + \left(1 - \phi^H\right)\left(v_1 - p_1^H\right)/t\right) - a\phi^H = 0. \tag{9}$$

Solving (8) for p_1^H gives,

$$p_1^H = \left[4c + 2(v_1 - v_2) + (t - 2c - (2v_1 - v_2))\,\phi^H\right] / \left(4 - 3\phi^H\right). \tag{10}$$

Equation (10) gives the equilibrium price as a function of the advertising intensity. Due to complexity of the first order conditions, we cannot explicitly solve for p_1^H and ϕ^H. Substituting (10) into the objective function, profits are given by

$$\pi^H = R\left(\phi^H\right) - a\left(\phi^H\right)^2/2$$

where $R\left(\phi^H\right)$ is the firm's total revenue when its reach is ϕ^H.

[11] In regime H, we have that $t/2 < v_1 - p < t$. Intuitively, the condition says that for all possible equilibrium prices, the consumer located at $x = 1/2$ finds it profitable to purchase while the partially informed consumer who travels the entire unit distance finds it not profitable to purchase. The first inequality, evaluated at the full information (highest) price, gives the upper bound to regime H while the second inequality gives the lower bound. Evaluating this condition at $p_{1F}^H = t + 2c - v_2$ gives $t \in \left(\frac{v_1+v_2}{2} - c, \frac{2(v_1+v_2-2c)}{3}\right)$.

3.3 Main Results

To recapitulate, p_1^L, ϕ^L and π^L (p_1^H, ϕ^H and π^H) are respectively the equilibrium advertised price, advertising and profit under low (high) differentiation. We summarize equations (5) and (10) below;

Proposition 2 *Let $v_1 = v_2 = v$. When firms advertise only a subset of their products, the advertised good is priced below cost when differentiation is low but is priced above marginal cost when differentiation is high. The unadvertised good is priced at its reservation value.*

Proof. First, from (5), $p_1^L = 2c + \sqrt{2at} - v = c + \sqrt{2at} - (v - c)$. Clearly, $p_1^L < c$ if and only if $\sqrt{2at} - (v - c) < 0$ and $\sqrt{2at} - (v - c) < 0$ if and only if $c + \sqrt{2at} < v$. But this is nothing other than Assumption A2. Hence, we conclude that indeed $p_1^L < c$. Second, differentiation is high if the differentiation parameter, t, is such that $v - c < t < \frac{4}{3}(v - c)$. Let $\underline{t} = v - c$. Notice that (from (10)) $\partial p_1^H / \partial t > 0$. Therefore, let $\underline{p_1^H} \equiv p_1^H \left|_{t=\underline{t}}\right.$. Then, $\underline{p_1^H} = c$. Since $t > \underline{t} = v - c$, it follows that $p_1^H = \left(4c + (t - 2c - v)\phi^H\right) / \left(4 - 3\phi^H\right) > \underline{p_1^H} = c$. The proof of the second part of the result is given in Lemma 2. ■

This result is intuitive. Since consumers do not search, market shares are determined solely by the advertised prices. Holding the firm's advertising reach constant, the lower the advertised price the greater the likelihood that each ad received results in a sale. When differentiation is low, a firm that successfully undercuts a rival can substantially increase its market share. Moreover, since firms can rip-off visiting consumers on the unadvertised good, they compete more aggressively for market share and thus end up pricing below cost. When differentiation is high however, price advertising is primarily informative. Products are less similar and therefore price differences have to be large to induce consumers to switch to the distant supplier. Hence there is less rationale for pricing below cost.

An example of a market in which differentiation is generally low is the grocery retail market. Supermarkets, for instance, sell products that are almost (if not exactly) physically similar. Competition therefore is mainly on prices. With similar products, consumers do not have a strong inclination to buy from a particular store (consumers are "footloose") and as a result, price competition is intense. The presence of an unadvertised good only exacerbates the price competition. This leads to lower advertised prices.

Observe that our loss-leader pricing result differs from that of Lal and Matutes (1994). In Lal and Matutes, equilibrium advertised prices may well exceed marginal cost. In contrast, in our model, for all parameter values, the equilibrium necessarily entail advertised prices below marginal cost when differentiation is low. Proposition 2 thus allows us to pin down the sufficient conditions for loss-leader pricing. As a corollary, we have;

Corollary 1 *When firms each sell two products with the same reservation price, the following conditions are sufficient for loss-leader pricing; (i) Low differentiation and (ii) Firms advertise only a subset of their products.*

Next we consider the case when reservation prices differ, that is, $v_1 \neq v_2$. We summarize the results below;

Proposition 3 *Let* $v_1 \neq v_2$*. If* $t < \frac{v_1+v_2-2c}{2}$ *and firms advertise only a subset of their products, the advertised good is priced below cost irrespective of whether firms advertise the low or the high reservation price good.*

Proof. (The proof mimics that of Proposition 2 above). ■

When firms advertise the low reservation value good, competition for market share is tougher as the firm that succeeds in attracting more consumers will sell more units at the higher unadvertised (reservation) price. The larger the difference between the reservation prices the greater the incentive to undercut. Coupled with the fact that differentiation is low, this leads to a much lower equilibrium advertised price. When firms advertise the high reservation price good instead, there are two opposing effects. On the one hand, low differentiation induces firms to compete more aggressively for market share. On the other hand, firms realize that visiting consumers will pay a lower (reservation) price on the unadvertised good and this restrains the aggression. However, the low differentiation effect dominates and firms advertise prices below marginal cost in either case.

As a corollary to Proposition 3, we have the following;

Corollary 2 *When products have different reservation values, firms are indifferent as to which good to offer as a loss-leader.*

Proof. The profit in either case is $\pi = 2a/\left(1+\sqrt{2a/t}\right)^2$. ■

We showed earlier (Proposition 2) that when firms sell products with the same reservation value, loss-leader pricing obtains *only* when differentiation is low. Does this hold in general? It turns out that loss-leader pricing is possible under high differentiation. More precisely,

Proposition 4 *Let* $t \in \left(\frac{v_1+v_2}{2} - c, \frac{2(v_1+v_2-2c)}{3}\right)$ *and* $v_1 \neq v_2$. *When reservation values are sufficiently different, equilibrium* ***may*** *entail loss-leader pricing when firms advertise the low reservation price good. However, when firms advertise the high reservation value good, the advertised price exceeds marginal cost for all parameter constellations.*

Proof. Let firms advertise good i so that p_i^H is the advertised price. Notice that (from (10)) $\partial p_i^H/\partial t > 0$. Since $t > \underline{t} = \frac{v_1+v_2}{2} - c$, it follows that $p_i^H = \frac{4c+t\phi^H-2c\phi^H+2(v_i-v_j)-\phi^H(2v_i-v_j)}{4-3\phi^H} > p_i^H\left|_{t=\underline{t}} \equiv \underline{p_i^H} = c + \frac{(v_i-v_j)}{2}; i,j = 1,2; i \neq j\right.$. If $v_i < v_j$, advertising good i (the low reservation price good) gives $\underline{p_i^H} < c$. Thus, for $t \to \left(\frac{v_1+v_2}{2} - c\right)^+$ and for $|v_i - v_j|$ large, $p_i^H < c$. However, if $v_i > v_j$, then advertising good i (the high reservation price good) gives $\underline{p_i^H} > c$ and hence $p_i^H > c$. ■

For $t \to \left(\frac{v_1+v_2}{2} - c\right)^+$, most partially informed consumers make a purchase and thus competition for market share can be intense. Moreover, a higher unadvertised price adds to the incentives to compete vigorously. Thus, when firms advertise the low reservation price good, undercutting may result in prices below cost. However, when the differentiation parameter is sufficiently large, the incentive to undercut is reduced. Hence, for large t, firms advertise prices above marginal cost when they advertise the low valuation good. When firms advertise the high reservation price good however, it is never optimal to advertise prices below marginal cost. The fact that differentiation is high (products are less similar) and the fact that the unadvertised (reservation) price is lower when firms advertise the high reservation price good both induce firms to advertise higher prices.

The above results allow us to rationalize the "surprising" finding of Walters and MacKenzie (1988) that, in retail markets, loss-leader pricing fails to stimulate store traffic and hence is unprofitable. Walters and MacKenzie interpret their finding as "pointing to the fact that locational convenience and overall price perceptions are more important determinants of patronage than weekly specials" (p. 60). We turn their explanation on its head. Because differentiation is typically low in the retail sector, weekly specials are crucial determinants of visitation. As a result, a firm that offers such specials would substantially increase its market share if rivals would not follow suit. Realizing this, firms always try to match price cutting by rivals and this enables them to maintain their market shares.

This is a typical prisoner's dilemma. A firm that succeeds in undercutting its rival can greatly increase its profits since it then faces a large demand and sells the unadvertised good at its reservation price. On the other hand, if both firms undercut (symmetrically), then each firm maintains its market share. In this sense, undercutting is a (weakly) dominant strategy for each individual firm. However, this strategy does not maximize joint profits and firms settle for an equilibrium with low advertised prices but with the same level of demand.[12] This makes loss-leader pricing appear as if it were less important.[13] We find our argument more convincing, for if price specials were unimportant, why are retailers placing greater emphasis on hotter price specials? (Lal and Matutes, 1994; p. 345).

We next highlight the interaction between advertising and prices. One of the features that distinguish our model from the models of Lal and Matutes (1994) and Ellison (2005) is that with regard to information, consumers are ex-post heterogeneous – some receive ads and some don't. In Lal and Matutes and respectively, Ellison, all consumers are fully informed as regards to the advertised prices. Below we compare the "equilibrium advertised" prices to the "full information" prices.[14]

Proposition 5 *Let p_{1F}^L (p_{1F}^H) be the full information price in regime L (H). Then, $p_{1F}^L < p_1^L$ and $p_{1F}^H > p_1^H$.*

The full information price is $t + 2c - v_2$ in both regimes. Note however that prices under low and high product differentiation are not directly comparable since the ratio of t to v is different in the two regimes. Under full information and low differentiation, the equilibrium of our model collapses to that of Lal and Matutes (1994). However, because our model is an extension of the single product models of Grossman and Shapiro (1984) and Soberman (2004), we can (conclusively) show that the advertised good is priced below marginal cost whereas Lal and Matutes cannot.[15]

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[12] That loss leader pricing does not increase demand is a consequence of the unit demands assumption. With downward sloping demands, total demand can increase in equilibrium but market shares will not.

[13] Since competitors always match price cutting by rivals, the full effect of loss leader pricing on store traffic and profits is never realized in equilibrium. This gives a biased reading of the importance of loss leader pricing. This suggests a different empirical method to test for the effect of loss leader pricing – counterfactual analysis. What would be the effect on profits of firm i were competitors to not reciprocate when firm i lowers its price?

[14] By full information, we mean that every consumer in the market is aware of the advertised prices ($\phi \to 1$). We reserve the term "perfect information" for the case where all consumers are fully informed of all prices.

[15] The equilibrium in the single product case provides sufficient restrictions on the parameters to enable us to fully characterize the equilibrium in the multiproduct case. More precisely, in the single product case, the equilibrium price is given by: $p^S = c + \sqrt{2at}$. Since the reservation price for this good is v and firms advertise to inform consumers, $p^S = c + \sqrt{2at}$ is an equilibrium price only if $p^S < v$. For, if $p^S \geq v$, no consumer will visit a store when faced with positive transport costs. Letting marginal cost equal zero (as in Lal and Matutes), the equilibrium advertised price in our two good model is $p_1^L = t - v$ (same as in Lal and Matutes). But since $\sqrt{2at} < v$ (from the single product model) and $a > t/2$, it follows that $t < v$. Hence, $p_1^L = t - v < 0 =$ marginal cost.

When differentiation is low, the equilibrium advertised price exceeds the full information price. The reason is that when the market is covered, demand is less elastic in the presence of informational product differentiation (Bagwell, 2003; p. 75). This informational differentiation takes the form of some consumers being informed of the prices at only one of the stores. Because demand is less elastic, firms can afford to charge higher prices.

In contrast, the full information price exceeds the equilibrium advertised price when the market is not covered. In this case, demand is more elastic in the presence of informational product differentiation. Thus imperfect information increases firms' market power when differentiation is low but decreases firms' market power when differentiation is high. In other words, the effect of improvements in information depends on the differentiation regime – a point that appears to have eluded the literature (Soberman (2004) is an exception). Below we characterize the relationship between the advertised prices and the advertising intensity.

Corollary 3 *An increase in advertising is associated with lower (higher) advertised prices when differentiation is low (high).*

Proof. $\partial p_1^L/\partial\phi = -2t/\phi^2 < 0$ and $\partial p_1^H/\partial\phi = 4\left(c + t - \frac{v_1+v_2}{2}\right)/\left(3\phi - 4\right)^2 > 0$ ■

This result has equivalences in Soberman (2004; Propositions 1 and 2). When differentiation is low, demand by partially informed consumers is price inelastic while demand by fully informed consumers is price sensitive. An increase in the advertising intensity raises the share of fully informed consumers (reduces informational differentiation) in the market and this puts pressure on prices. When differentiation is high however, partially informed consumers are more price sensitive compared to fully informed consumers. Thus higher advertising, by increasing the share of fully informed consumers, reduces overall demand elasticity and thus enables firms to charge higher prices.

Although we cannot solve explicitly for the optimal advertising level under high differentiation, a numerical exercise shows that both the advertising intensity and profit decrease as the advertising cost, a, increases. There are at least three channels by which an increase in the advertising cost propagates itself into lower profit. Firstly, an increase in the advertising cost directly increases the advertising outlay and hence lowers profit. Secondly, an increase in the advertising cost reduces the advertising level and this directly lowers demand and, other things equal, lowers profit. Thirdly, an increase in the advertising cost, by reducing the advertising intensity, leads to lower advertised prices (Corollary 3) and hence lower profit.

Figure 1 plots profit as a function of the advertising intensity, for different values of a.

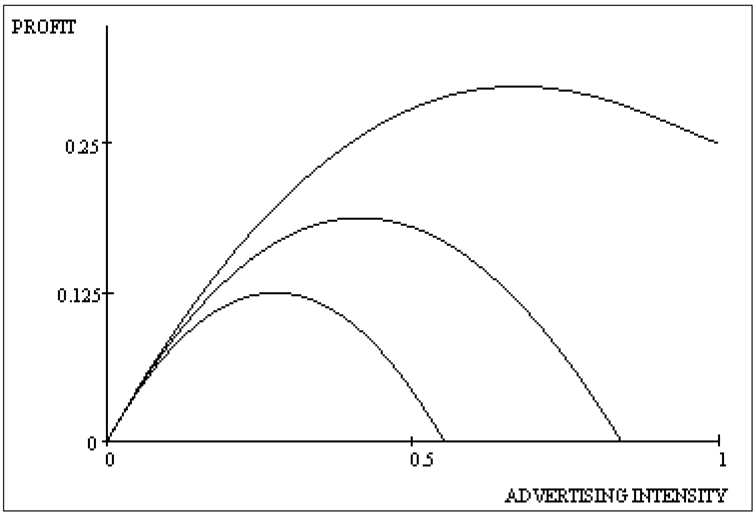

Figure 1. Profit as a function of the advertising intensity, for different values of a. ($v = 1, c = 0, t = 1.1$; $a = .6$ (highest curve), $a = 1.4$ (intermediate curve), $a = 2.5$ (lowest curve)).

We summarize the above observations in the following statement:

Remark 1 *First, for a given advertising intensity, profit increases as the advertising cost decreases. Second, as the advertising cost decreases, the optimal advertising level increases and moreover, profits become less sensitive to small perturbations in the advertising intensity. That is, the profit function becomes flatter in the neighborhood of the optimal advertising level.*

4 Welfare

We consider the welfare implications of informative advertising when firms advertise only a subset of their products. We restrict ourselves to the case of low differentiation.[16]

4.1 Market Equilibrium

To start with, we consider welfare in the market equilibrium. We define welfare as the sum of total profits and consumer surplus. As we saw in Proposition 1, profits do not change when firms move from the single to the multiproduct configuration. Hence,

[16] We cannot compare the optimal outcome to the equilibrium outcome under high differentiation because the regions of t consistent with partial market coverage do not intersect.

Proposition 6 *When firms increase the number of products they sell, but only advertise a subset, consumer surplus (and hence welfare) increases.*

Proof. $\phi^L = \phi^S$ and $\pi^L = \pi^S$ but $p^L < p^S$ ■

Intuitively, introducing the second good intensifies competition for market share and this generates spillovers for consumers (in the form of lower prices).

4.2 Social optimum

Suppose the firms studied above are instead run by a benevolent planner. The objective of the planner is to maximize welfare. We derive the socially optimal advertising levels for the single and multiproduct cases.

Let each firm offer n products; $n \in \{1, 2\}$. For consumers who receive ads from both firms (measure ϕ^2), the average transportation cost is $t/4$. Because each good is priced at marginal cost, c, the average net benefit per consumer is $n(v-c) - t/4$.[17] For consumers who only receive a single ad (measure $2\phi(1-\phi)$), the average net benefit per consumer is $n(v-c) - t/2$. The planner chooses the advertising level, ϕ, to maximize,

$$W_n = \phi^2 (n(v-c) - t/4) + 2\phi(1-\phi)(n(v-c) - t/2) - a\phi^2$$

where W_n is the welfare when each firm offers n products. This gives,

$$\phi_n^{Social} = (4n(v-c) - 2t) / (4n(v-c) + 4a - 3t) \tag{11}$$

$$W_n = (2n(v-c) - t)^2 / (4n(v-c) + 4a - 3t). \tag{12}$$

An important policy question concerns the welfare effects of changes in the cost of advertising and how that relates to the number of products offered. We have that:

Proposition 7 *An increase in the cost of advertising lowers welfare and the effect on welfare is larger when firms offer multiple products. That is,* $\partial W_n/\partial a < 0$ *and* $\left|\frac{\partial W_2}{\partial a}\right| > \left|\frac{\partial W_1}{\partial a}\right|$.

Proof. The first part of the proposition follows immediately from (16). As for the second

[17] Marginal cost pricing ensures that there is no consumption distortion and hence maximizes welfare.

part, differentiating (12) with respect to a and simplifying we get that $\left|\frac{\partial W_2}{\partial a}\right| > \left|\frac{\partial W_1}{\partial a}\right|$ if and only if $2s(4a-t) > 0$. Since $a > t/2$ (by assumption), the result follows immediately. ■

The first part of the proposition is due to Grossman and Shapiro (1984). Since price equal marginal cost, apart from raising the advertising outlay, an increase in the cost of advertising lowers the socially optimal level of advertising. These two (higher advertising outlay and reduced advertising intensity) both work to reduce welfare. For the second part notice that since transport costs are independent of the number of products purchased from any one supplier, surplus per visiting consumer increases with the number of products. Since an increase in the advertising cost lowers the advertising level – which means fewer consumers make a purchase, the result follows immediately.

That $\left|\frac{\partial W_2}{\partial a}\right| > \left|\frac{\partial W_1}{\partial a}\right|$ applies equally to the market outcome. Since welfare increases with the number of products in the market equilibrium (Proposition 6), it follows that reducing the share of informed consumers in the market lowers welfare more the larger the number of products.

4.3 Market versus Planner

Below we compare the market determined to the socially optimal advertising level. The market determined advertising level is given by $\phi^L = 2/\left(1+\sqrt{2a/t}\right)$ while the socially optimal level is given by $\phi_2^{Social} = (8s-2t)/(8s+4a-3t)\,;\ s = v-c$. A divergence may occur since the objectives of the planner and private firms generally differ. Solving the equation $\phi_2^{Social} - \phi^L = 0$ we get;

$$(a_1, a_2) = \left(\frac{(4s-3t)^2+(t-4s)\sqrt{16s^2-40st+17t^2}}{16t}, \frac{(4s-3t)^2-(t-4s)\sqrt{16s^2-40st+17t^2}}{16t}\right)$$

Observe that since $t-4s < 0$, $a_1 < a_2$. Since both ϕ_2^{Social} and ϕ^L are monotonically decreasing in a and $\lim_{a\to\frac{t}{2}} \phi^L = 1 > \lim_{a\to\frac{t}{2}} \phi_2^{Social}$, clearly, in the interval $(t/2, a_1)$, it must be the case that $\phi^L > \phi_2^{Social}$. In the interval (a_1, a_2), $\phi^L < \phi_2^{Social}$ and for $a > a_2$, $\phi^L > \phi_2^{Social}$. Thus, from a social welfare perspective, the market determined advertising level is excessive for a close to $t/2$, but too low for intermediate values of a.

However, if we fix a and s, so that both ϕ^L and ϕ_2^{Social} are only functions of the differentiation parameter, t, we get the result (due to Hamilton (2004; Proposition 2)) that: *Compared to the social optimum, the market undersupplies informative advertising for suf-*

ficiently homogeneous brands but oversupplies advertising for more differentiated brands.

Below we attempt an explanation for this apparent divergence. First, by increasing the number of products they sell, firms create benefits for all visiting consumers (Proposition 6). However, firms cannot appropriate the benefits so created (Proposition 1). This suggests that firms have lower incentives to inform consumers. Thus, compared to the social planner, nonappropriability of consumer surplus leads firms to undersupply informative advertising. Second, when a firm reaches a consumer who already has received advertising from the competitor, there is, on average, realignment of consumers among firms (the matching effect). Fully informed consumers buy from the nearest store, thereby saving on transportation costs. Firms do not care about this benefit and hence tend to underprovide informative advertising. However, firms care about business stealing. When a firm reaches a consumer who already has received advertising from a rival, the resulting realignment of consumers may create business for the advertising firm. If it does, the competitor loses $v + p^L - 2c > t$ while the consumer saves on average $t/2$ on transportation costs.[18] This ad thus generates a welfare loss from the social stand point. This suggests that the market determined advertising level may be excessive. However, whether the market over or underprovides advertising depends on which effect dominates.

We end this section with a *caveat.* As Ellison (2005; p. 619) notes: "Models ... with unit demands are poorly suited to welfare analysis". This is because there isn't much consumption distortion in these models. Consumption distortion only takes the form of changes in the number of consumers purchasing and changes in prices have no effect on the quantity demanded by any individual consumer as long as the price is less than the reservation price.[19] With downward sloping demand curves, one expects that consumers would buy more of the loss-leader (stocking) and less of the unadvertised good. Pesendorfer (2002) provide evidence supportive of stocking. He finds that demand for ketchup increases sharply during the sale period but also falls sharply after the sale.[20]

[18] The competitor loses $v - c$ on the unadvertised good and $p^L - c$ on the advertised good. $v + p^L - 2c = \sqrt{2at} > t$.

[19] I thank Jonas Häckner for this observation.

[20] This explains why many firms tend to offer limited quantities of the loss leader good, or to restrict the quantity purchased of the loss leader by any individual consumer.

5 Concluding Remarks

We study price advertising when firms advertise only a subset of their products. We find some support for the empirical findings that price advertising affects advertised and unadvertised prices differently. We also pin down the sufficient conditions for loss leader pricing. Based on this analysis, we provide a game theoretic (and coherent) explanation to the finding of Walters and Mackenzie (1988) that loss leader pricing fails to increase store traffic and hence profits.

Welfare effects are as expected – welfare decreases as the advertising cost increases. One drawback of our framework, especially for welfare, is the assumption of unit demands. A complete welfare analysis of loss leader pricing requires consumers to have downward sloping demands, so that the demand that each firm faces depends on both the advertised and the unadvertised prices.

Another worthy extension is to endogenize the choice of which products to advertise. This would allow firms to advertise different products. Other extensions include generalizing the model to more than two products.

Appendix

Appendix A: Derivations of Limits to the region of Low Differentiation

We want to derive restrictions on the differentiation parameter, t, that guarantee that the market will be fully covered.[21] That is, the consumer who travels the entire unit distance finds it profitable to purchase. We proceed by showing that if the advertised price, p_{i1}, exceeds $v_1 - t$, firm i's profits can be increased by reducing the price to below or about $v_1 - t$. In other words, the maximum price observed cannot exceed $v_1 - t$. Technically, if $v_1 - t$ is the maximum price, then, for $p_{i1} > v_1 - t$, we must have $\frac{\partial \pi_1}{\partial p_{i1}} < 0$. That is, any price higher than $v_1 - t$ yields lower profit. Consider firm 1.

Suppose then that $p_{11} > v_1 - t$. Then, $D_1 = \phi_1 \left((1 - \phi_2) \frac{v_1 - p_{11}}{t} + \phi_2 \frac{p_{21} - p_{11} + t}{2t} \right)$ and $\pi_1 = (p_{11} + v_2 - 2c) \phi_1 \left((1 - \phi_2) \frac{v_1 - p_{11}}{t} + \phi_2 \frac{p_{21} - p_{11} + t}{2t} \right) - \frac{a\phi_1^2}{2}$. Differentiating with respect to p_{11} gives, $\frac{\partial \pi_1}{\partial p_{11}} = \phi_1 \left((1 - \phi_2) \frac{v_1 - p_{11}}{t} + \phi_2 \frac{p_{21} - p_{11} + t}{2t} + (p_{11} + v_2 - 2c) \frac{\phi_2 - 2}{2t} \right)$ and $\frac{\partial^2 \pi_1}{\partial p_{11}^2} = \frac{\phi_1}{t} (\phi_2 - 2) < 0$. That is, firm i's profit is concave in own price. $v_1 - t$ being the maximum price consistent with full market coverage, it follows that $\frac{\partial \pi_1}{\partial p_{11}} |_{p = v_1 - t} \leq 0$. $\frac{\partial \pi_1}{\partial p_{11}} |_{p = v_1 - t} = (\phi - 2) (v_1 + v_2 - 2c - 2t) \leq 0$ only if $v_1 + v_2 - 2c - 2t \geq 0 \iff t \leq \frac{v_1 + v_2}{2} - c$. Thus, for $t < \frac{v_1 + v_2}{2} - c$, prices higher than $v_1 - t$ cannot be observed. That is, $p + t \leq v_1$ – which is the condition for full market coverage when firms advertise good 1.

Appendix B: Consumer Expectations and Purchase Decisions

We take up the issue of the purchase decisions of consumers given their expectations of the prices of the unadvertised good. To recapitulate, there are three cases to consider; In case i, $p_{12}^E = p_{22}^E$ and this is the case studied in the paper; In case ii, $p_{12}^E < p_{22}^E$ and in case iii, $p_{12}^E > p_{22}^E$. We consider only case ii since case iii is similar.

Let $p_{12}^E < p_{22}^E$. Depending on the advertised prices, each fully informed consumer has three options; (i) plan to buy both goods from firm 1, (ii) plan to buy both goods from firm 2 and (iii) plan to buy from both firms.

[21] We closely follow Soberman (2004).

Option i yields higher surplus than option ii if and only if $2v - p_{11} - p_{12}^E - tx \geq 2v - p_{21} - p_{22}^E - t(1-x) \iff x \leq \widehat{x} = \left(p_{21} + p_{22}^E - p_{11} - p_{12}^E + t\right)/2t$. Consumers with locations $x \in [0, \widehat{x})$ plan to buy both goods at firm 1 while those with locations $x \in (\widehat{x}, 1]$ plan to buy both goods at firm 2. Similarly, option i yields higher surplus than option iii if and only if $2v - p_{11} - p_{12}^E - tx \geq 2v - p_{21} - p_{12}^E - tx - t(1-x) \iff x \leq \widetilde{x}_1 = (p_{21} - p_{11} + t)/t$. Consumers with locations $x \in [0, \widetilde{x}_1)$ plan to buy both goods from firm 1 while those with locations $x \in (\widetilde{x}_1, 1]$ plan to buy good 1 from firm 2 and good 2 from firm 1. This alternative is viable only if $p_{21} < p_{11}$. For, if $p_{21} \geq p_{11}$, you incur unnecessary travel expenses by shopping from both stores. Similarly, option ii yields higher surplus than option iii if and only if $2v - p_{21} - p_{22}^E - t(1-x) \geq 2v - p_{21} - p_{12}^E - tx - t(1-x) \iff x \geq \widetilde{x}_2 = \left(p_{22}^E - p_{12}^E\right)/t$. Consumers with locations $x \in [0, \widetilde{x}_2)$ plan to buy from both firms while those with locations $x \in (\widetilde{x}_2, 1]$ plan to buy both goods from firm 2.

Suppose $p_{21} < p_{11}$. If $\widetilde{x}_1 < \widehat{x} < \widetilde{x}_2$, then all consumers with locations $x \in (\widetilde{x}_1, \widetilde{x}_2)$ plan to buy good 1 from firm 2 and good 2 from firm 1. Therefore, firm 1 faces demand $\widetilde{x}_1 = (p_{21} - p_{11} + t)/t$ for the advertised good and demand $\widetilde{x}_2 = \left(p_{22}^E - p_{12}^E\right)/t$ for the unadvertised good from the fully informed consumers. Hence,

$$\pi_1 = \left(p_{11} + p_{12}^E - 2c\right)\phi_1(1-\phi_2)\tfrac{2v - p_{11} - p_{12}^E}{t} + \phi_1\phi_2\left\{(p_{11} - c)\widetilde{x}_1 + \left(p_{12}^E - c\right)\widetilde{x}_2\right\}.$$

The profit function is a bit complicated. The first part constitute profit from the captive consumers while the second part constitute profit from the fully informed consumers.

However, if $\widetilde{x}_2 \leq \widetilde{x}_1$, then option iii is dominated and a generic consumer either buys both goods from firm 1 or from firm 2. Firm 1 then faces demand $\widehat{x} = \left(p_{21} + p_{22}^E - p_{11} - p_{12}^E + t\right)/2t$ from the fully informed consumers and profits are given by;[22]

$$\pi_1 = \left(p_{11} + p_{12}^E - 2c\right)\left(\phi_1(1-\phi_2)\frac{2v - p_{11} - p_{12}^E}{t} + \phi_1\phi_2\widehat{x}\right).$$

[22] Also notice that if $p_{11} < p_{12}$, then both goods are cheaper at firm 1 and obviously, no consumer finds it worthwhile to buy from both firms. That is, option iii is dominated.

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