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Abstract

We consider firms and multi-member households operating in a competitive market environment. Households are endowed with resources (commodity bundles) and shares of firm ownership. Household members are characterized by individual preferences, possibly with intra-household consumption externalities. Household decisions adhere to the collective rationality model. Existence of general equilibrium and validity of the first welfare theorem are investigated.

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1 Introduction

Traditional economic theory has treated "households" and "consumers" as synonyms. But would a formal distinction between a household as an economic entity and its constituents make a significant difference beyond a mere descriptive improvement? It does, if one is interested in household labor supply to market as well as household production, in the differential effect of taxes, subsidies and public goods on household members, to name just a few instances. Numerous theoretical and empirical studies have examined household related issues, usually relying on partial equilibrium analysis.

We are interested in the behavior and welfare of multi-member households in a general equilibrium context. This framework allows to investigate the feedback between decisions at the micro-level, by households and their members, and macro-variables, in particular market clearing prices. In special cases, one is able to perform comparative statics with respect to exogenous model parameters, like in Gersbach and Haller (2009). Incorporating multi-member households requires specifying the decision making of such households. Haller (2000) pioneered the analysis of general equilibrium models with multi-member households operating in a competitive market environment. His approach was motivated and influenced by the model of collective rationality of households forwarded by Chiappori (1988, 1992).[1] Haller considers a finite pure exchange economy and assumes collective rationality in its most general form: A household acts collectively in the market, with efficient bargaining within the household. In a competitive equilibrium, each household makes an efficient choice under its budget constraint, and markets clear. The crucial feature is that efficient choice by the household refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the household.

[1] Alternative models of household decision making have been introduced by Lundberg and Pollack (1993, 1994) and Apps and Rees (2009), among others.

First and foremost, two questions arise once a general equilibrium model with multi-member households is developed: Does the presence of multi-member households impair the efficiency of competitive equilibrium allocations where efficiency or Pareto optimality, to be precise, is defined in terms of individual preferences? Does the presence of multi-member households impede the existence of competitive equilibria? *Prima facie*, one might be inclined to think that the welfare properties of competitive equilibrium allocations depend on the details of intra-household bargaining. The key insight of Haller (2000) is that those details do not matter for the validity of the first welfare theorem. It suffices that in equilibrium, every household makes an efficient choice under its budget constraint and, by doing so, exhausts its budget. This neither requires nor rules out specific bargaining protocols or decision rules as long as efficient household decisions are reached — and the budget gets exhausted. Regarding the second question, it turns out that the aggregate excess demand of a multi-member household has similar properties as the excess demand of traditional consumers, which suggests that an equilibrium existence result should obtain via the excess demand approach. Gersbach and Haller (1999) and Sato (2009) take the excess demand approach to show equilibrium existence for economies à la Haller (2000). Gori (2010), using homotopy techniques, obtains equilibrium existence for economies with Nash-bargained household decisions and no intra-household consumption externalities.

The existing body of work on general equilibrium models with multi-member households has been confined to pure exchange economies.[2] Here we take a first pass at a general equilibrium model with multi-member households and production. We address the question whether equilibrium existence results and the first welfare theorem can be extended from a pure exchange

[2]See Gersbach and Haller (1999, 2001, 2005, 2009, 2010, 2011), Gori (2010), Gori and Villanacci (2011), Haller (2000).

context to a model with production. Extending results for pure exchange economies to economies with production is often, but not always straightforward. A more or less straightforward extension holds true for equilibrium existence and the first and second welfare theorem for finite Arrow-Debreu economies, though the proofs turn out to be more elaborate for economies with production than for pure exchange economies.

We find that extension of the first welfare theorem proves rather straightforward, indeed, though one of us — like perhaps the more sceptical or more cautious reader — had to be convinced by an explicit proof. Equilibrium existence with production is shown by following a strategy of proof different from Gersbach and Haller (1999) and Sato (2009) on the one hand and Gori (2010) on the other hand. We define induced household preferences for aggregate household consumption so that household choices and production plans reside in the same Euclidean space. Resorting to a theorem of Debreu (1982) based on the simultaneous optimization or social equilibrium approach of Arrow and Debreu (1954), we then show existence of a competitive equilibrium for this artificial economy — which translates into an equilibrium of the actual economy.

In the next section, we introduce the model and state and demonstrate our first main result, a first welfare theorem for finite economies with multi-member households and production. Section 3 is devoted to equilibrium existence. Section 4 offers concluding remarks.

2 Model and First Main Result

We consider an economy with a finite number of commodities, firms and households. The main departure from the traditional model is that a household can have several members, each with their own preferences. There are $\ell \geq 1$ continuous commodities, labeled $l \in \{1, \ldots, \ell\}$. Thus the commodity space is $\mathbb{R}^\ell$.

The population of consumers is divided into finitely many households $h = 1, \ldots, n$, with $n \geq 2$. Each household h consists of finitely many members $i = hm$ with $m = 1, \ldots, m_h$, $m_h \geq 1$. Put $I = \{hm : h = 1, \ldots, n; m = 1, \ldots, m_h\}$, the finite population of individuals to be considered. There are finitely many firms $j = 1, \ldots, f$, with $f \geq 1$. Let $H = \{1, \ldots, n\}$ denote the set of households and $J = \{1, \ldots, f\}$ denote the set of firms.

2.1 Technologies and Firm Decisions

Each firm j has a non-empty production set or technology $Y_j \subseteq \mathbb{R}^\ell$. The special case of $Y_j = \{0\}$ for all j amounts to a pure exchange economy. The objective of a firm is to maximize its profit, to the extent possible. For a price system $p \in \mathbb{R}^\ell_+$ and a firm j, let

$$Y_j(p) = \arg \max_{y_j \in Y_j} py_j$$

be the set of j's profit maximizers. $Y_j(p)$ may be empty. In case $Y_j(p) \neq \emptyset$, set

$$\pi_j(p) = \max_{y_j \in Y_j} py_j,$$

so that $\pi_j(p) = py_j$ for $y_j \in Y_j(p)$.

2.2 Allocations and Individual Preferences.

A generic individual $i = hm \in I$ has consumption set $X_i = \mathbb{R}^\ell_+$. Let $\mathcal{X} \equiv \prod_{i \in I} X_i$ be the set of consumption profiles and $\mathcal{Y} \equiv \prod_{j \in J} Y_j$ be the set of production profiles. Then the allocation space is $\mathcal{X} \times \mathcal{Y}$.

The consumption bundle of a generic individual i is denoted by x_i with $x_i \in X_i$. Let $\mathbf{x} = (x_i)$, $\mathbf{x}' = (x'_i)$ denote generic elements of $\mathcal{X}$. For $h = 1, \ldots, n$, define $\mathcal{X}_h = \prod_{m=1}^{m_h} X_{hm}$ with generic elements $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm_h})$.

If $\mathbf{x} \in \mathcal{X}$ is a consumption profile, then for $h = 1, \ldots, n$, household consumption is given by $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm_h}) \in \mathcal{X}_h$. We will allow for the possibility of consumption externalities. Following Haller (2000), we shall restrict attention to the case where such consumption externalities, if any, exist only between members of the same household. This is captured by the notion of **intra-household externalities**: For $i \in h$, the welfare of individual i depends only on household consumption $\mathbf{x_h}$. More specifically, we assume that the preferences of individual i have a **utility representation** $U_i : \mathcal{X}_h \longrightarrow \mathbb{R}$.

We adopt from Gersbach and Haller (2001) the concept of local non-satiation of multi-person households:

Definition 1 *A household h is* **locally non-satiated** *if for every $\mathbf{x_h} \in \mathcal{X}_h$ and every $\epsilon > 0$, there exists $\mathbf{x'_h} \in \mathcal{X}_h$ with*

$$\| \mathbf{x_h} - \mathbf{x'_h} \|_{m_h \ell} < \epsilon \text{ and } (U_i(\mathbf{x'_h}))_{i \in h} > (U_i(\mathbf{x_h}))_{i \in h}.$$ [3]

For local non-satiation of household h to hold it suffices that there exist a member hm and a commodity $c(h) \in \{1, \ldots, \ell\}$ such that (a) the welfare of hm is strictly increasing in $x_{hm}^{c(h)}$, hm's consumption of commodity $c(h)$ and (b) the welfare of all other household members is unaffected or positively affected by hm's consumption of commodity $c(h)$.

Local non-satiation for all households implies the budget exhaustion property (2) assumed in Proposition 1. Local non-satiation for all households is also one of the assumptions of Proposition 2.

[3] $\| \cdot \|_d$ denotes the Euclidean norm on a d-dimensional Euclidean space. We use the notation $\gg$, $>$ and $\geq$ for vector inequalities.

2.3 Property Rights and Household Decisions

Household h is endowed with a commodity bundle $\omega_h \in \mathbb{R}^\ell$, $\omega_h > 0$. The aggregate or social endowment is $\omega = \sum_h \omega_h$. Moreover, household h owns a share $\theta_{hj} \geq 0$ of firm $j \in J$. For each firm, $\sum_h \theta_{hj} = 1$.

Given a price system $p \in \mathbb{R}^\ell$ such that $Y_j(p) \neq \emptyset$ for all j, household h has wealth or income

$$\mathrm{w}_h(p) = p\,\omega_h + \sum_{j \in J} \theta_{hj}\pi_j(p).$$

Now consider a household h and a price system $p \in \mathbb{R}^\ell$. For $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm_h}) \in \mathcal{X}_h$,

$$p * \mathbf{x_h} = p \cdot \left(\sum_{m=1}^{m_h} x_{hm} \right)$$

denotes the total household expenditure on household consumption plan $\mathbf{x_h}$ at the price system p. As p and $\mathbf{x_h}$ are of different dimension for multi-member households, we use the $*$-product in lieu of the familiar inner product. If household wealth or income $\mathrm{w}_h(p)$ is well defined, then h's **budget set** is given as $B_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h \ : \ p * \mathbf{x_h} \leq \mathrm{w}_h(p)\}$. Next we define the **efficient budget set** $EB_h(p)$ by:

$\mathbf{x_h} = (x_{h1}, \ldots, x_{hm_h}) \in EB_h(p)$ if and only if $\mathbf{x_h} \in B_h(p)$ and there is no $\mathbf{x'_h} \in B_h(p)$ such that

$$\begin{aligned} U_{hm}(\mathbf{x'_h}) &\geq U_{hm}(\mathbf{x_h}) \quad \text{for all } m = 1, \ldots, m_h; \\ U_{hm}(\mathbf{x'_h}) &> U_{hm}(\mathbf{x_h}) \quad \text{for some } m = 1, \ldots, m_h. \end{aligned}$$

Thus efficient choice by the household refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the household.

2.4 Feasibility and Optimality

An allocation $(\mathbf{x}, \mathbf{y}) = ((x_i)_{i\in I}, (y_j)_{j\in J}) \in \mathcal{X} \times \mathcal{Y}$ is **feasible** if

$$\sum_{i\in I} x_i = \omega + \sum_{j\in J} y_j. \tag{1}$$

A feasible allocation $(\mathbf{x}, \mathbf{y}) = ((x_i)_{i\in I}, (y_j)_{j\in J})$ is **Pareto optimal** if there is no other feasible allocation $(\mathbf{x}', \mathbf{y}') = ((x_i')_{i\in I}, (y_j')_{j\in J})$ such that

$$\begin{aligned} U_{hm}(\mathbf{x_h'}) &\geq U_{hm}(\mathbf{x_h}) \quad \text{for all } h = 1, \ldots, n;\ m = 1, \ldots, m_h; \\ U_{hm}(\mathbf{x_h'}) &> U_{hm}(\mathbf{x_h}) \quad \text{for some } h = 1, \ldots, n;\ m = 1, \ldots, m_h. \end{aligned}$$

General Equilibrium: A **competitive equilibrium** is a triple $(p; (\mathbf{x}, \mathbf{y}))$ consisting of a price system p and an allocation $(\mathbf{x}, \mathbf{y}) = ((x_i)_{i\in I}, (y_j)_{j\in J})$ such that

1. $y_j \in Y_j(p)$ for all $j \in J$;
2. $\mathbf{x_h} \in EB_h(p)$ for all $h \in H$;
3. $(\mathbf{x}, \mathbf{y})$ is feasible, i.e., it satisfies (1).

In a general equilibrium, each firm maximizes profits, every household makes an efficient choice under its budget constraint and markets clear. We obtain a **first welfare theorem** for economies with multi-member households and production:

Proposition 1 *Let $(p; (\mathbf{x}^*, \mathbf{y}^*))$ be a competitive equilibrium such that*

$$p * \mathbf{x_h} = \mathrm{w}_h(p) \text{ for all } h = 1, \ldots, n;\ \mathbf{x_h} \in EB_h(p). \tag{2}$$

Then $(\mathbf{x}^*, \mathbf{y}^*)$ is a Pareto optimal allocation.

PROOF. Suppose $(\mathbf{x}^*, \mathbf{y}^*)$ is not Pareto optimal. Then there exists a feasible allocation $(\mathbf{x}', \mathbf{y}')$ such that

$$\begin{aligned} U_{hm}(\mathbf{x_h'}) &\geq U_{hm}(\mathbf{x_h^*}) \quad \text{for all } h = 1, \ldots, n;\ m = 1, \ldots, m_h; \\ U_{hm}(\mathbf{x_h'}) &> U_{hm}(\mathbf{x_h^*}) \quad \text{for some } h = 1, \ldots, n;\ m = 1, \ldots, m_h. \end{aligned}$$

Hence there exists at least one household h with

$$\begin{aligned} U_{hm}(\mathbf{x_h'}) &\geq U_{hm}(\mathbf{x_h^*}) \quad \text{for all } m = 1, \ldots, m_h; \\ U_{hm}(\mathbf{x_h'}) &> U_{hm}(\mathbf{x_h^*}) \quad \text{for some } m = 1, \ldots, m_h. \end{aligned}$$

Since $\mathbf{x_h^*} \in EB_h(p)$, we get $\mathbf{x_h'} \notin B_h(p)$ for such an h:

$$p * \mathbf{x_h'} > \mathrm{w}_h(p). \tag{3}$$

(3) holds for all such households. For the remaining households, $U_{hm}(\mathbf{x_h'}) = U_{hm}(\mathbf{x_h^*})$ for all $m = 1, \ldots, m_h$. If $\mathbf{x_h'} \in B_h(p)$, then $\mathbf{x_h'} \in EB_h(p)$ because of $\mathbf{x_h^*} \in EB_h(p)$ and, consequently, $p * \mathbf{x_h'} = \mathrm{w}_h(p)$ because of (2). If $\mathbf{x_h'} \notin B_h(p)$, then (3) holds. In any case,

$$p * \mathbf{x_h'} \geq \mathrm{w}_h(p). \tag{4}$$

(3) and (4) yield

$$p \sum_{i \in I} x_i' = \sum_{h \in H} p * \mathbf{x_h'} > \sum_{h \in H} \mathrm{w}_h(p).$$

Now $py_j^* \geq py_j'$ for each $j \in J$. Hence

$$
\begin{aligned}
& p \sum_{i \in I} x_i' > \sum_{h \in H} \mathrm{w}_h(p) \\
= \; & \sum_{h \in H} \left[p\omega_{h \in H} + \sum_{j \in J} \theta_{hj} p y_j^* \right] \\
= \; & p \sum_{h \in H} \omega_h + \sum_{h} \sum_{j \in J} \theta_{hj} p y_j^* \\
\geq \; & p\omega + \sum_{h \in H} \sum_{j \in J} \theta_{hj} p y_j' \\
= \; & p\omega + p \sum_{j \in J} y_j' \sum_{h \in H} \theta_{hj} \\
= \; & p\omega + p \sum_{j \in J} y_j' = p \left[\omega + \sum_{j \in J} y_j' \right],
\end{aligned}
$$

contradicting $\sum_{i \in I} x_i' = \omega + \sum_{j \in J} y_j'$, the feasibility of $(\mathbf{x}', \mathbf{y}')$. Hence to the contrary, $(\mathbf{x}^*, \mathbf{y}^*)$ has to be Pareto optimal. □

3 Existence

In contrast to the existence proofs for pure exchange economies in Gersbach and Haller (1999) and Sato (2010), who take the excess demand approach, we rely on the simultaneous optimization or social equilibrium approach of Arrow and Debreu (1954). In order to make the latter applicable, we replace each household's consumption set $\mathcal{X}_h$ by the aggregate consumption set $\mathcal{A}_h = \mathbb{R}_+^\ell$ so that (aggregate) consumption bundles and production plans have equal dimension. We also need preferences on $\mathcal{A}_h = \mathbb{R}_+^\ell$ that reflect household preferences. We are going to define a utility function $V_h : \mathcal{A}_h \to \mathbb{R}$ with the desired properties.

3.1 Preferences on Aggregate Household Consumption

Consider a household h with members $i = hm, m = 1, \dots, m_h$ and household consumption set $\mathcal{X}_h = \prod_{m=1}^{m_h} X_{hm}$. We introduce the notation $\mathcal{A}_h = \mathbb{R}_+^\ell$ for the household's aggregate consumption set. We further define a canonical mapping $A_h : \mathcal{X}_h \longrightarrow \mathcal{A}_h$ that assigns to each household consumption plan $\mathbf{x_h} = (x_{h1}, \dots, x_{hm_h})$ the aggregate consumption $A_h(\mathbf{x_h}) = \sum_m x_{hm}$. For each $a_h \in \mathcal{A}_h$, we are interested in the inverse image $A_h^{-1}(a_h)$, the household consumption plans that give rise to the aggregate consumption a_h for household h. For all $a_h \in \mathcal{A}_h$, $A_h^{-1}(a_h) \neq \emptyset$, since $(a_h, 0, \dots, 0) \in A_h^{-1}(a_h)$ and $(a_h/m_h, \dots, a_h/m_h) \in A_h^{-1}(a_h)$, for example. Two properties obviously hold:

(P1) The correspondence $A_h^{-1} : \mathcal{A}_h \twoheadrightarrow \mathcal{X}_h$ is convex and compact valued.

(P2) The correspondence $A_h^{-1} : \mathcal{A}_h \twoheadrightarrow \mathcal{X}_h$ is continuous.

Next fix for household h a utilitarian social welfare function $W_h : \mathcal{X}_h \to \mathbb{R}$ of the form

$$W_h(\mathbf{x_h}) = \sum_{m=1}^{m_h} c_{hm} \cdot U_{hm}(\mathbf{x_h}) \text{ for all } \mathbf{x_h} \in \mathcal{X}_h$$

where $c_h = (c_{h1}, \dots, c_{hm_h}) \in \mathbb{R}_{++}^{m_h}$. If each U_{hm} is continuous, then W_h is continuous and because of the compactness and non-emptiness of $A_h^{-1}(a_h)$,

$$V_h(a_h) = \max_{\mathbf{x_h} \in A_h^{-1}(a_h)} W_h(\mathbf{x_h}) \tag{5}$$

is well defined for all $a_h \in \mathcal{A}_h$. Moreover:

(P3) If each U_{hm} is continuous and concave, then $V_h : \mathcal{A}_h \to \mathbb{R}$ is continuous and concave.

Namely, W_h is continuous. In addition, (P1) and (P2) hold. Hence continuity of V_h follows from Berge's Maximum Theorem. If each U_{hm} is concave, then W_h is concave as well. Now let $a_h, a_h' \in \mathcal{A}_h$ and $\lambda \in (0,1)$. There exist $\mathbf{x_h} \in A_h^{-1}(a_h)$ and $\mathbf{x'_h} \in A_h^{-1}(a'_h)$ such that $V_h(a_h) = W_h(\mathbf{x_h})$ and $V_h(a'_h) = W_h(\mathbf{x'_h})$. Further, $\lambda \cdot \mathbf{x_h} + (1-\lambda) \cdot \mathbf{x'_h} \in A_h^{-1}(\lambda \cdot a_h + (1-\lambda) \cdot a'_h)$ and $W_h(\lambda \cdot \mathbf{x_h} + (1-\lambda) \cdot \mathbf{x'_h}) \geq \lambda \cdot W_h(\mathbf{x_h}) + (1-\lambda) \cdot W_h(\mathbf{x'_h})$. Therefore, $V_h(\lambda \cdot a_h + (1-\lambda) \cdot a'_h) \geq W_h(\lambda \cdot \mathbf{x_h} + (1-\lambda) \cdot \mathbf{x'_h}) \geq \lambda \cdot W_h(\mathbf{x_h}) + (1-\lambda) \cdot W_h(\mathbf{x'_h}) = \lambda \cdot V_h(a_h) + (1-\lambda) \cdot V_h(a'_h)$. This shows concavity of V_h.

3.2 Equilibrium Existence Result

We are now prepared to state an equilibrium existence result. Let $Y = \sum_j Y_j$ denote the aggregate production set.

Proposition 2 *A competitive equilibrium exists if*
for every consumer $i = hm$,
    (C) U_i *is continuous and concave;*
for every household h,
    (H) $\omega_h \gg 0$ *and local non-satiation holds;*
for every firm j,
    (F) $0 \in Y_j$;
for the aggregate production set Y,
    (Y) Y *is closed and convex;* $Y \cap (-Y) = \{0\}; \mathbb{R}^\ell_- \subseteq Y$.

PROOF. Suppose (C) for all consumers i, (H) for all households h, (F) for all firms j, and (Y) for the aggregate productions set. Fix a utilitarian social welfare function $W_h : \mathcal{X}_h \to \mathbb{R}$ for every household h. Consider the finite Arrow-Debreu economy $\mathfrak{E} = ((\mathcal{A}_h, V_h, \omega_h)_{h\in H}, (\theta_{hj})_{(h,j)\in H\times J}, (Y_j)_{j\in J})$ where $\mathcal{A}_h = \mathbb{R}^\ell_+$ and V_h is given by (5) for $h \in H$. Then:

(i) Each $\mathcal{A}_h$ is closed, convex, and bounded from below.

(ii) Each "consumer" h is locally non-satiated.

Namely, let $a_h \in \mathcal{A}_h$ and $\varepsilon > 0$. Let $\mathbf{x_h} \in A_h^{-1}(a_h)$ with $V_h(a_h) = W_h(\mathbf{x_h})$. Because of (H), local non-satiation holds for household h: There exists $\mathbf{x'_h} \in$

$\mathcal{X}_h$ with $\| \mathbf{x_h} - \mathbf{x'_h} \|_{m_h\ell} < \epsilon/m_h$ and $(U_i(\mathbf{x'_h}))_{i\in h} > (U_i(\mathbf{x_h}))_{i\in h}$. Then $W_h(\mathbf{x'_h}) > W_h(\mathbf{x_h})$. Let $a'_h \equiv A_h(\mathbf{x'_h}) \in \mathcal{A}_h$. It follows $\| a_h - a'_h \|_\ell =$ $\| A_h(\mathbf{x_h}) - A_h(\mathbf{x'_h}) \|_\ell = \| \sum_{m=1}^{m_h}(x_{hm} - x'_{hm}) \|_\ell \leq \sum_{m=1}^{m_h} \| x_{hm} - x'_{hm} \|_\ell \leq$ $m_h \| \mathbf{x_h} - \mathbf{x'_h} \|_{m_h\ell} < \epsilon$. Moreover, $V_h(a'_h) \geq W_h(\mathbf{x'_h}) > W_h(\mathbf{x_h}) = V_h(a_h)$. Hence there exists $a'_h \in \mathcal{A}_h$ such that $\| a_h - a'_h \|_\ell < \epsilon$ and $V_h(a'_h) > V_h(a_h)$. This shows that "consumer" h is locally non-satiated.

Further:

(iii) Each V_h is continuous and concave, by (P3).

(iv) $\omega_h \gg 0$ for all h.

(v) $0 \in Y_j$ for all j.

(vi) Y is closed and convex; $Y \cap (-Y) = \{0\}; \mathbb{R}^\ell_- \subseteq Y$.

(i)–(vi) imply that $\mathfrak{E}$ satisfies the hypothesis of Theorem 5 of Debreu (1982). Therefore, the economy $\mathfrak{E}$ has a competitive equilibrium $((a^*_h)_{h\in H}, (y^*_j)_{j\in J}, p^*)$ in the sense of Debreu. For each $h \in H$, choose $\mathbf{x_h}^* = (x^*_{h1}, \ldots, x^*_{hm_h}) \in A_h^{-1}(a^*_h)$ with $V_h(a^*_h) = W_h(\mathbf{x_h}^*)$. Let $\mathbf{x}^* = (x^*_i)_{i\in I}$ and $\mathbf{y}^* = (y^*_j)_{j\in J}$. We claim that $(p^*; (\mathbf{x}^*, \mathbf{y}^*))$ is a competitive equilibrium of the economy with multi-member households $h \in H$.

1. $y^*_j \in Y_j(p^*)$ for all $j \in J$, by the definition of a competitive equilibrium of $\mathfrak{E}$.

2. $\mathbf{x}^*_\mathbf{h} \in EB_h(p^*)$ for all $h \in H$. Namely, $V_h(a^*_h) = \max\{V_h(a_h) : a_h \in \mathcal{A}_h,\, p^*a_h \leq \mathrm{w}_h(p^*)\}$. Since $a^*_h = A_h(\mathbf{x}^*_\mathbf{h})$, we get $p^* * \mathbf{x}^*_\mathbf{h} = p^*a^*_h \leq \mathrm{w}_h(p^*)$ and, therefore, $\mathbf{x}^*_\mathbf{h} \in B_h(p^*)$. If $\mathbf{x}^*_\mathbf{h} \notin EB_h(p^*)$, then there exists $\mathbf{x'_h} \in B_h(p^*)$ such that $U_{hm}(\mathbf{x'_h}) \geq U_{hm}(\mathbf{x}^*_\mathbf{h})$ for all $m = 1, \ldots, m_h$ and $U_{hm}(\mathbf{x'_h}) > U_{hm}(\mathbf{x}^*_\mathbf{h})$ for some $m = 1, \ldots, m_h$. Choose such an $\mathbf{x'_h}$. Then $W_h(\mathbf{x'_h}) > W_h(\mathbf{x}^*_\mathbf{h})$. Hence for $a'_h = A_h(\mathbf{x'_h})$: $a'_h \in \mathcal{A}_h$, $p^*a'_h =$ $p^* * \mathbf{x'_h} \leq \mathrm{w}_h(p^*), V_h(a'_h) \geq W_h(\mathbf{x'_h}) > W_h(\mathbf{x}^*_\mathbf{h}) = V_h(a^*_h)$, contradicting the fact that $V_h(a^*_h) = \max\{V_h(a_h) : a_h \in \mathcal{A}_h,\, p^*a_h \leq \mathrm{w}_h(p^*)\}$. Therefore, $\mathbf{x}^*_\mathbf{h} \in EB_h(p^*)$ has to hold.

3. $(\mathbf{x}^*, \mathbf{y}^*)$ is feasible, i.e., it satisfies (1). Namely, $\sum_{h\in H} a_h^* = \omega + \sum_{j\in J} y_j^*$ implies $\sum_{i\in I} x_i^* = \sum_{h\in H} \sum_{m=1}^{m_h} x_{hm}^* = \sum_{h\in H} A_h(\mathbf{x_h^*}) = \sum_{h\in H} a_h^* = \omega + \sum_{j\in J} y_j^*$ and, thus, (1).

We have shown that $(p^*; (\mathbf{x}^*, \mathbf{y}^*))$ satisfies conditions 1.–3. of a competitive equilibrium for the economy with multi-member households $h \in H$. This demonstrates the claim and completes the proof. □

4 Concluding Remarks

Proposition 1 means that, by and large, competitive exchange among profit maximizing firms and multi-member households satisfying the collective rationality model yields Pareto optimal allocations. Obviously, local non-satiation of households prevails and a fortiori the budget exhaustion property (2) holds if all individuals exhibit strict monotonicity in own consumption and all intra-household consumption externalities are nonnegative. Example 3.1 in Haller (2000) illustrates that local non-satiation of households can still hold if all individuals in multi-member households experience specific negative consumption externalities. But Example 1 in Sato (2009) demonstrates that certain negative consumption externalities can lead to violation of (2) (and of local non-satiation) and yield suboptimal equilibrium allocations. Still, equilibrium allocations are always weakly Pareto optimal without any further assumptions.

Proposition 2 states existence of a competitive equilibrium for a finite economy with profit maximizing firms and multi-member households satisfying the collective rationality model, under almost standard assumptions. The only exception is the assumption of local non-satiation of households which possibly can be replaced by weaker but less transparent assumptions. Without any assumption of this kind, one can expect an equilibrium with free disposal at best.

General equilibrium analysis often includes a second welfare theorem, core inclusion, core equivalence, and related issues. We are going to briefly discuss these topics as well as household production.

4.1 Second Welfare Theorem

Proposition 6 of Gersbach and Haller (2001) asserts validity of a second welfare theorem for a pure exchange economy with fixed household structure. The proof of the proposition applies the separating hyperplane theorem. Like similar proofs in the literature, it can easily incorporate production.

4.2 Core Theory

Haller (2000) presents a H-core inclusion result where in the definition of the H-core or household core only unions of households in H qualify as coalitions. Again, the budget exhaustion property (2) proves instrumental. There is a sizeable literature on coalition-production economies, e.g. Böhm (1974) and Hildenbrand (1974, Ch. 4), where each coalition is endowed with its own technology. This approach provides an elegant way to extend the methods developed for pure exchange economies, but tends to ignore individual ownership of means of production. In other cases, individual private property in firms does not fully apply. Debreu and Scarf (1963) assume that production technologies are publicly available and exhibit constant returns to scale so that issues related to corporate control are absent. Allingham (1975) and Aliprantis, Brown and Burkinshaw (1987) assume divisibility of technologies and that each shareholder controls a fraction of the firm's technology which avoids conflicts among shareholders.

In general economies with production and private property, the question arises when and how a coalition of consumers or households can alter the production plan of a firm that is not entirely owned by the coalition. First attempts to deal with this intricate question and to take fully into account the ramifications of private property have been made by Haller (1991) and Xiong and Zheng (2007). This complex issue is left to future research.

4.3 Household Production

By most accounts, household production creates substantial value in most economies. It was the article of Becker (1965) that positioned the use of time and household production firmly within economic theory. At the micro level, Becker's work and a rich subsequent literature (see e.g. Apps and Rees (2009) for a discussion) demonstrate that household production constitutes a major determinant of household welfare. For those reasons, a comprehensive account and description of household activities and intra-household allocation ought to include household production.[4]

Delineating household production in general equilibrium frameworks, however, proves difficult. The most stringent definition would require that the household uses its own factors of production to produce goods for its own consumption only. But households which are autarkic with respect to all factors of production barely exist. For example, to bake a cake, most of the basic ingredients are typically purchased in the market. A less stringent definition requires that labor and capital are owned by the household whereas intermediate products can be obtained in the market. Yet even then, households living in rental housing, for example, would not qualify for household production.

Within a general equilibrium framework, Gilles and Diamantaras (2003) assume that each consumer is endowed with his own home production set. They distinguish between tradeable and non-tradeable commodities. A consumer's productive activity is only considered household or home production if the output consists of non-tradeable commodities. Individuals can own

[4] At the macro level, the value of household production is significant and could be around 35 percent of GDP in developed countries (see Apps and Rees (2009), p. 32).

and consume non-tradeable commodities, but they are restricted to consuming their home produced quantities of these non-tradeables. Now almost any commodity is tradeable at some time in some place. But that is exactly Gilles's and Diamantaras's point: tradeability is an endogenous, temporal and local feature. Gilles and Diamantaras (2003) demonstrate that welfare analysis involving transfers — adopting a valuation equilibrium concept to be precise — can be performed with their formalization of home production and its connection with tradeability. Yet showing existence of a competitive equilibrium that *ceteris paribus* respects private property rights seems beyond reach.

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