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Applying Hubbert Curves and Linearization to Rock Phosphate

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Abstract

Rock phosphate is a critical, nonrenewable resource for which there is no known substitute in agriculture. Cordell, Drangert, and White (2009) use Hubbert methodology (1956) to estimate the peak—the year after which production will monotonically decline—of world rock phosphate production at 2033–34. This note assesses the applicability of Hubbert's (1949) peak methodology to world rock phosphate production, based on (a) the ability of the model to produce accurate in-sample and out-of-sample forecasts and stable estimates of ultimately recoverable reserves, and (b) the degree to which the rock phosphate market approximates the theoretical conditions underpinning the Hubbert model. In both respects, the application of Hubbert methodology to rock phosphate is found to be problematic.

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During the past four years, rising world food prices and the global economic downturn increased the ranks of the world's food insecure from 848 million to 925 million by September 2010, reversing decades of slow yet steady progress in reducing hunger (WFP and FAO 2010). While the human costs have been considerable, the political consequences have been significant as well. Food prices sparked demonstrations and riots in 48 countries during 2007–08. While prices receded in 2009, they reached historic highs in February 2011—and were once again implicated in political turmoil. High food and fuel prices were among the grievances motivating the demonstrations that led to the ouster of Tunisian President Zine El Abidine Ben Ali and Egyptian President Hosni Mubarak.

For some, the crises were further evidence that food insecurity is the inevitable consequence of overpopulation and outstripping the world's finite resources, expressed most vocally by proponents of peak arguments. Peak commodity arguments, most notably peak oil (Deffeyes 2008) and peak phosphate (Cordell, Drangert, and White 2009), contend that modern agricultural production is based on the use of non-renewable inputs, which by definition will eventually be depleted. Problems will arise not when the resources are depleted, but once peak production—the maximum rate of extraction—is reached and the resource enters a terminal decline in productivity.

Modern agriculture requires the application of phosphorous-, nitrogen-, and potassium-based fertilizers. Both phosphorous and potassium are sourced primarily from non-renewable mined deposits of rock phosphate and potash. Cordell, Drangert, and White (2009) placed the timing of peak at roughly 2030, and argued that world reserves of rock phosphate, at the time estimated to total 16 billion metric tons (mt), would be depleted in the next 50 to 100 years. Subsequently, the US Geological Survey (USGS) released new estimates of phosphate reserves that placed the total at 65 billion (2011); assuming Hubbert methodology is in fact applicable to rock phosphate production, based on this revised estimate of reserves, the peak would occur around 2090.

The question is whether Hubbert methodology at present can be fruitfully applied to world rock phosphate production. This note concerns both the empirics and theory on which peak arguments are premised: the Hubbert curve, which forecasts the timing of peak production of a resource, and Hubbert linearization, a technique for estimating ultimately recoverable reserves (URR) of a resource. It briefly introduces the methodology and then applies it to data on world phosphate production from 1900–2010. The Hubbert curve is assessed according to its ability to accurately backcast—perform in-sample prediction—and its ability to subsequently forecast out-of-sample production. Hubbert linearization is assessed in terms of its accuracy in forecasting current world reserves of rock phosphate. In both cases, the methods are found to rest on tenuous empirical grounds. The Hubbert curve fails to accurately forecast out-of-sample production, while Hubbert linearization-derived estimates of URR are highly model dependent and therefore unreliable. From a theoretical perspective, Hubbert methodology hinges on several core assumptions about rates of resource exploration and extraction that fail to take into account the behavior of firms in response to variable prices, and that are inconsistent with the dynamics of rock phosphate production.

At base, peak arguments as applied to non-renewable resources have to be correct: There is only so much rock phosphate on Earth. However, the data suggest that Hubbert methodology for estimating peak production and URR of rock phosphate rests on tenuous empirical and theoretical foundations.

THE HUBBERT MODEL: BASICS AND FORECAST ABILITY

Originally developed for addressing production in oil provinces, the Hubbert model contends that production of a non-renewable resource follows a bell-shaped symmetrical curve, with the peak level of production occurring when 50 percent of the resource has been recovered.

The model rests on three assumptions:

1. Production of a finite resource fits a logistic growth curve, $(t) = \frac{1}{1+e^{-t}}$, where P is equal to cumulative discoveries at time (t).
2. Exploration effort is constant over time.
3. The time lag between discovery and production is constant over time (Giraud et al. 2010, Giraud 2011).

The factors that account for differing rates of discovery as a function of time are geological, rather than market driven. From these basic assumptions about the nature of resource discovery and production, two related techniques for estimating peak production and URR have been developed: Hubbert curves and Hubbert linearization.

Hubbert Curves

The Hubbert curve, developed by Hubbert (1956), fits observed production data of a resource over time to a symmetric distribution in order to forecast the timing of peak production. In doing so, the model assumes that production of a finite resource fits a logistic growth curve: Production rises gradually at first before increasing rapidly, peaking, and subsequently decreasing rapidly. In the Hubbert model, production increases monotonically up to the peak year, at which point production decreases monotonically. The data are then fit to a logistic or Gaussian distribution based on three parameters: URR, the estimated peak year, and a shape parameter that determines the rate of growth and decline. Parameters are chosen to maximize the in-sample predictive power of the model, i.e., goodness of fit, usually holding URR constant at a value determined by survey estimates or Hubbert linearization.

Hubbert Linearization

Hubbert linearization (HL) consists of plotting the cumulative production of a resource at year t (C(t)) versus the yearly production divided by the cumulative production (P(t)/C(t)), and fitting a regression line to a subsample of the data to solve for C(t), the URR, when P(t)/C(t) = 0. This technique is supposed to generate estimates of URR based simply on production data to date.

However, the fitting of the regression line can only be done once the graph has settled into a stable linear regime (Déry and Anderson 2007). Thus, Hubbert linearization requires the researcher to make untestable decisions about which data to include in the calculation of the regression line, which suggests a caveat emptor approach to model interpretation.

Model Validation: Hubbert Curve Forecasting and Backcasting Performance

If one is to use models for forecasting, the model must be validated. There are two basic types of validation. The first is in-sample forecasting, or backcasting, in which the model is assessed on its ability to replicate observed variability in the data from which the parameters were derived. The second is out-of-sample forecasting, where portions of the observed data are used to fit the model, and the model is subsequently assessed according to its ability to replicate observed variability in data withheld from the fitting process.

A comparison of Hubbert curves using world rock phosphate production data from 1900–1988[1] and 1900–2008, depicted in figure 1, illustrates issues related to forecast ability. The Hubbert-derived model does not accurately backcast rock phosphate production for the entire period. The model fit for the period 1900–1988 is very good ($R^2 = 0.97$), indicating that through 1988, the market for rock phosphate approximated the conditions that underpin the Hubbert-derived model. However, the period 1989–2008 has been marked by a different dynamic, as evidenced by the reduced explanatory power of the full sample model ($R^2 = 0.91$).[2]

Put somewhat differently, the restricted sample (1900–1988) model does a very poor job of forecasting actual levels of rock phosphate production for the following 20-year period (1989–2008), even though its backcast ability is high. If the Hubbert curve from 1900–1988 were the appropriate model for future production levels, actual annual production would have been, on average, 45.1 percent higher for the post-1989 period.

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1. 1988 was selected as the cut point because that was the year actual production "peaked" in the sample, at 166 million mt.

2. This model is the closest to that reported by Cordell, Drangert and White (2009).

	Full sample, 2010 estimate[a] (1900–2008)	Restricted sample, 2010 estimate (1900–1988)
Peak year	2031	2027–28
Production at peak (mt)	205,822,483	330,518,447
URR (mt)	22,700,500,000	22,700,500,000
Shape parameter	44	35
Model R^2	0.914	0.974

a. Reanalysis conducted using the Gaussian distribution, which performs equivalently to the logistic for the purposes of estimating peak production (Bartlett 2000).

Model validation: Hubbert linearization

Recall that Hubbert linearization (HL) requires the researcher to make important choices about which subset of the data is used to fit the regression model necessary to estimate URR. Several examples should suffice to demonstrate the sensitivity of HL-derived estimates to sample selection. Figure 2 shows the HL technique applied to data from the period 1968–2008,[3] which returns an estimated URR of 8.26 billion mt—only a third of 2010 USGS estimates, and roughly one-ninth of 2011 estimates (USGS 2010, 2011). Note that the R^2 of the regression line is relatively high (R^2 = 0.936)—according to the internal logic of the model, the estimate is on firm footing. Figure 3 shows the same analysis, but using only the period 1993–2008 to fit the regression line. While the R^2 of the regression line is much lower (R^2 = 0.583), the resulting estimate of URR is 18.5 billion mt—much closer to the 22.7 billion calculated from 2010 USGS reserve estimates, and over ten billion mt greater than the first estimate.

Which estimate is the "correct" estimate? Unfortunately, the HL technique does not provide an obvious benchmark for assessing the accuracy of the forecasts. The model that better fits the linear dynamic in the P(t)/C(t) data returns a less accurate estimate of URR than the model based on a less linear subsample of the data. The only way to compare the two is if the URR quantity is known, in which case the need for the technique is obviated.

Both the Hubbert curve and HL methods are on tenuous empirical and theoretical ground. Empirically, the Hubbert curve fails to accurately forecast out-of-sample production, which should be trivial if the dynamics of phosphate production are captured by the Hubbert curve. HL-derived estimates are highly model dependent, in the sense that estimates are driven by analyst's decisions over which data to use in fitting the model, and there are no clear, uniformly applied standard criteria governing selection. Of the two, Hubbert curve techniques have the virtue of having clear metrics for assessing model performance: goodness of fit for backcasts and forecasts.

3. Data for 1920–2008 are plotted in order to provide context.

THEORETICAL ISSUES: MARKET DYNAMICS AND ROCK PHOSPHATE RESERVE ESTIMATES

Recall that Hubbert methodology is based on three core assumptions:

1. Production of a finite resource fits a logistic growth curve, $(t) = \frac{1}{1+e^{-t}}$, where P is equal to cumulative discoveries at time (t).
2. Exploration effort is constant over time.
3. The time lag between discovery and production is constant over time.

How reasonable are these assumptions? The first would is an assumption about the rate at which the marginal time unit of exploration effort translates into reserves. This is dependent on the nature of the geological resources.

Assumption 2 requires that exploration effort not be price elastic: that rational economic actors do not factor prevailing market prices into present decisions about allocating effort and resources to exploration. Assumption 3 requires that new discoveries be translated into production at a constant rate over time; leaving a larger proportion of reserves in the ground and keeping production constant (or even allowing production to increase at a slower rate than the rate of discovery) is not possible. Thus assumption 3 also requires that rational economic actors do not factor expectations about pricing in to their decisions.

The market conditions that would need to obtain for the model to hold, thus, would be that of non-collusive firms, each without market power, offering a homogeneous product and facing constant or declining real prices (otherwise, incentives to stockpile could exist). Under these conditions, rational economic actors would constantly maximize production, as the marginal quantity of product would have no impact on the prevailing market price.

If firms have market power or face price variation, these assumptions become unrealistic. Hubbert's forecasts of peak world oil production (1956) predated the emergence of OPEC—a cartel designed to artificially create market scarcity by consciously suppressing production levels below the technological maximum capacity. While no such cartel exists in phosphate production, the market is characterized by a small number of large producers. The USGS reports that Morocco and Western Sahara, which account for 15.1 percent of global production and over 75 percent of listed world reserves, currently operate their mines at constant rates less than maximum production—behavior that would be considered irrational unless there were market-based incentives to do so. Even the most vocal proponents of peak phosphate arguments acknowledge that political and economic factors ranging from the fall of the Soviet Union to the 2007–08 speculation-driven spikes in commodity markets affect global demand and production levels.

Assumption 2 further requires that exploration effort be constant. Yet survey effort, and thus reserve estimates, are price elastic. Until 2007–08, when food prices drove up demand for agricultural inputs, prices for rock phosphate had been falling for thirty years (see figure 4). Higher prices spurred both survey effort (i.e., the search for new concentrations of valuable minerals) and the conversion of known geologic resources—mineral concentrations that have been sampled and surveyed—to reserves, the extraction of which is economically feasible given prevailing technology and market prices. By comparative standards, survey effort is relatively low. Barclays Capital estimates that the energy industry will spend $490 billion on oil and natural gas exploration globally in 2011 against world oil production valued at $2.5 trillion in 2010;[4] industry-wide spending on rock phosphate exploration, in contrast, was $2.6 million against $21.6 billion in FY 2010–11.

When survey effort is relatively low, URR estimates are likely to be inaccurate. This is problematic, as URR defines the area under the curve and is thus the single most important parameter for applying Hubbert curves to estimating the timing of peak production. Furthermore, reserve estimates over the past forty years have varied widely, from over 50 billion mt in 1970 to just 11 billion mt in 1995 (figure 4). Year-to-year revisions of reserve estimates for important producers can vary widely: China's estimated reserve was reduced from 6.6 billion mt tons in 2008 to 4.1 billion mt in 2009. This reduction at the time amounted to one-seventh of total global reserves. Between 2010 and 2011, Morocco and Western Sahara's estimated reserves increased from 5.7 billion mt to 50 billion mt. These issues point to the need for more comprehensive surveys of existing resources (see IFDC 2010), and a better understanding of the relationship between geological resources and reserves.

CONCLUSIONS

Cordell, Drangert, and White's (2009) application of Hubbert methodology to world phosphate production has spurred significant interest in the long-term sustainability of reliance on rock phosphate mining for critical inputs to modern agriculture. This note assesses the applicability of Hubbert methodology to rock phosphate on both empirical and theoretical grounds. Empirically, Hubbert curves produce inaccurate out-of-sample forecasts of production, while Hubbert linearization results in URR estimates that are highly model dependent. Theoretically, the kind of market conditions that would have to obtain for Hubbert methodology to apply to rock phosphate are unrealistic. Finally, as a practical point, reserve estimates for rock phosphate have been highly variable and subject to frequent revision. Thus, the underlying data warrant closer inspection and more effort is needed to systematically survey world phosphate reserves and resources.

4. "Oil Industry Set for Record Exploration Spending in 2011," *Voice of America*, December 29, 2010.

Hubbert methodology is inappropriate for forecasting world phosphate peak production and URR at present. That does not mean that rock phosphate reserves are infinite, nor does it diminish the very real problems of fertilizer runoff and eutrophication or the laudability of efforts to reclaim phosphorous from human and animal waste (Cordell, Drangert, and White 2009, Carpenter and Bennett 2011, Childers et al. 2011). It simply means that peak phosphate is a questionable argument on which to predicate these efforts at conservation.

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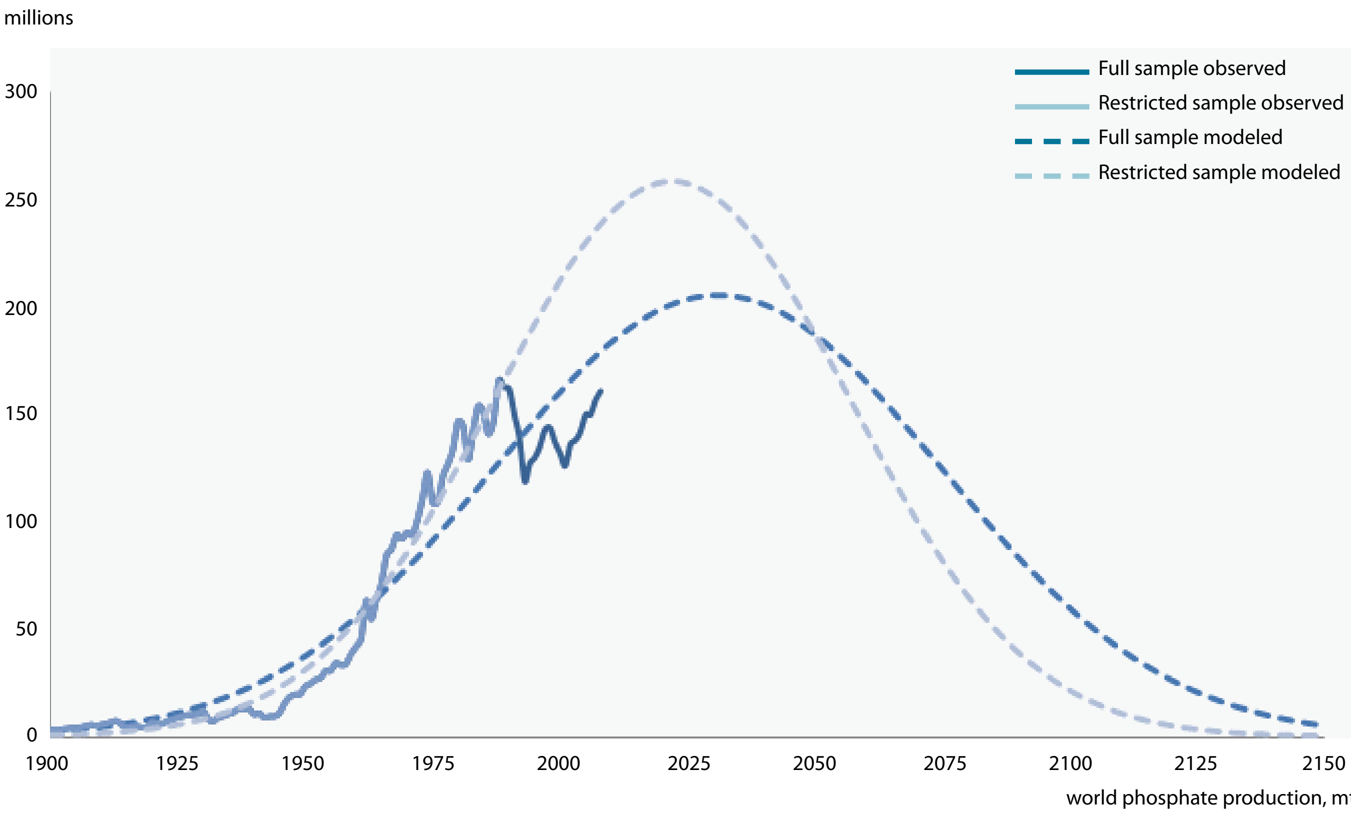
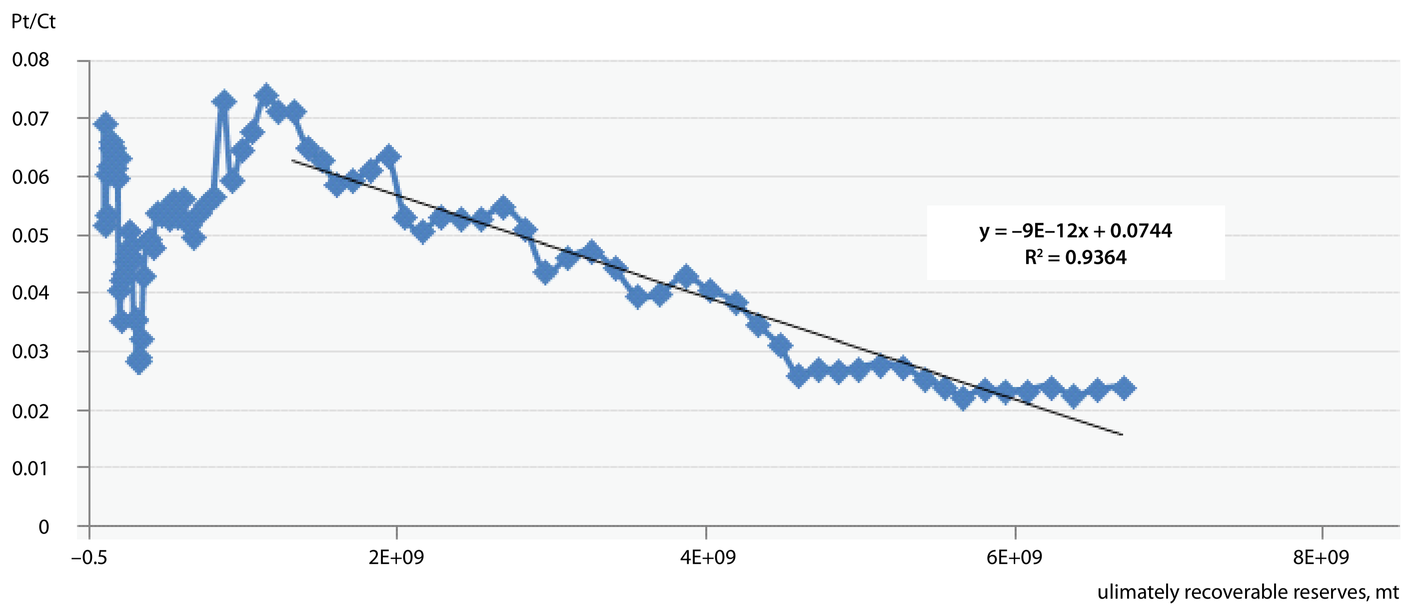
Figure 1 Hubbert curves for world rock phosphate production, 1990–2008



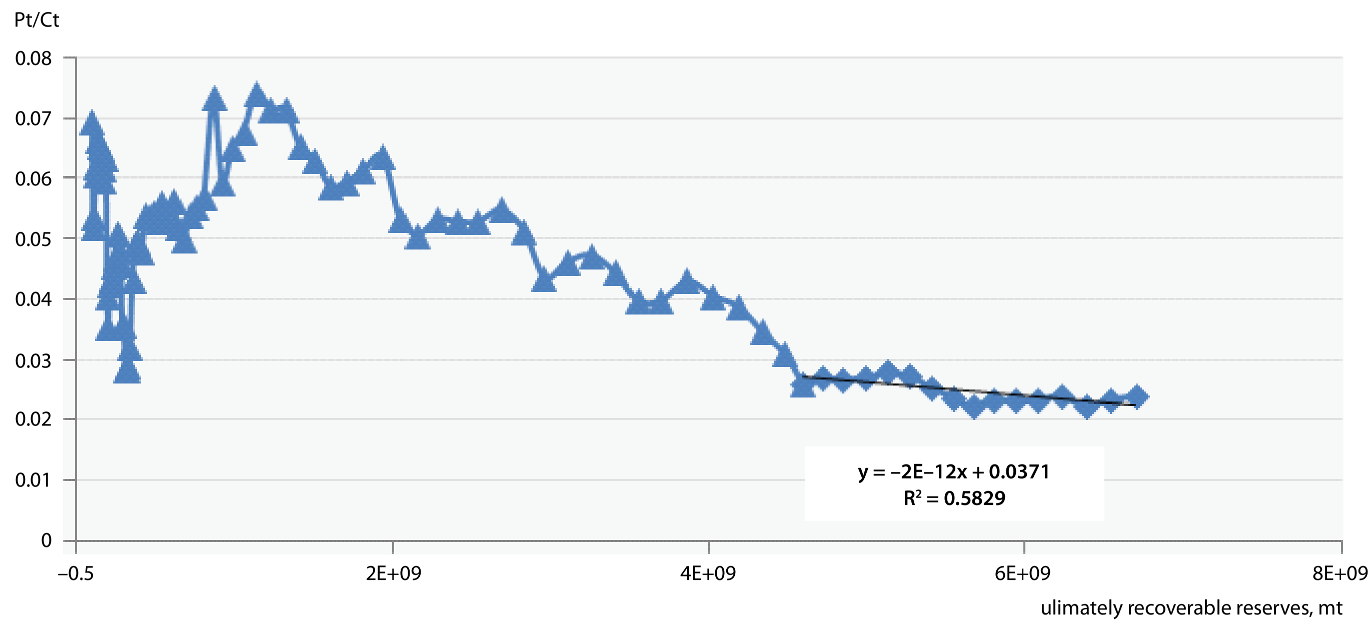
Source: Author's calculations based on USGS estimates (2010).

Figure 2 Hubbert linearization, rock phosphate, 1968–2008



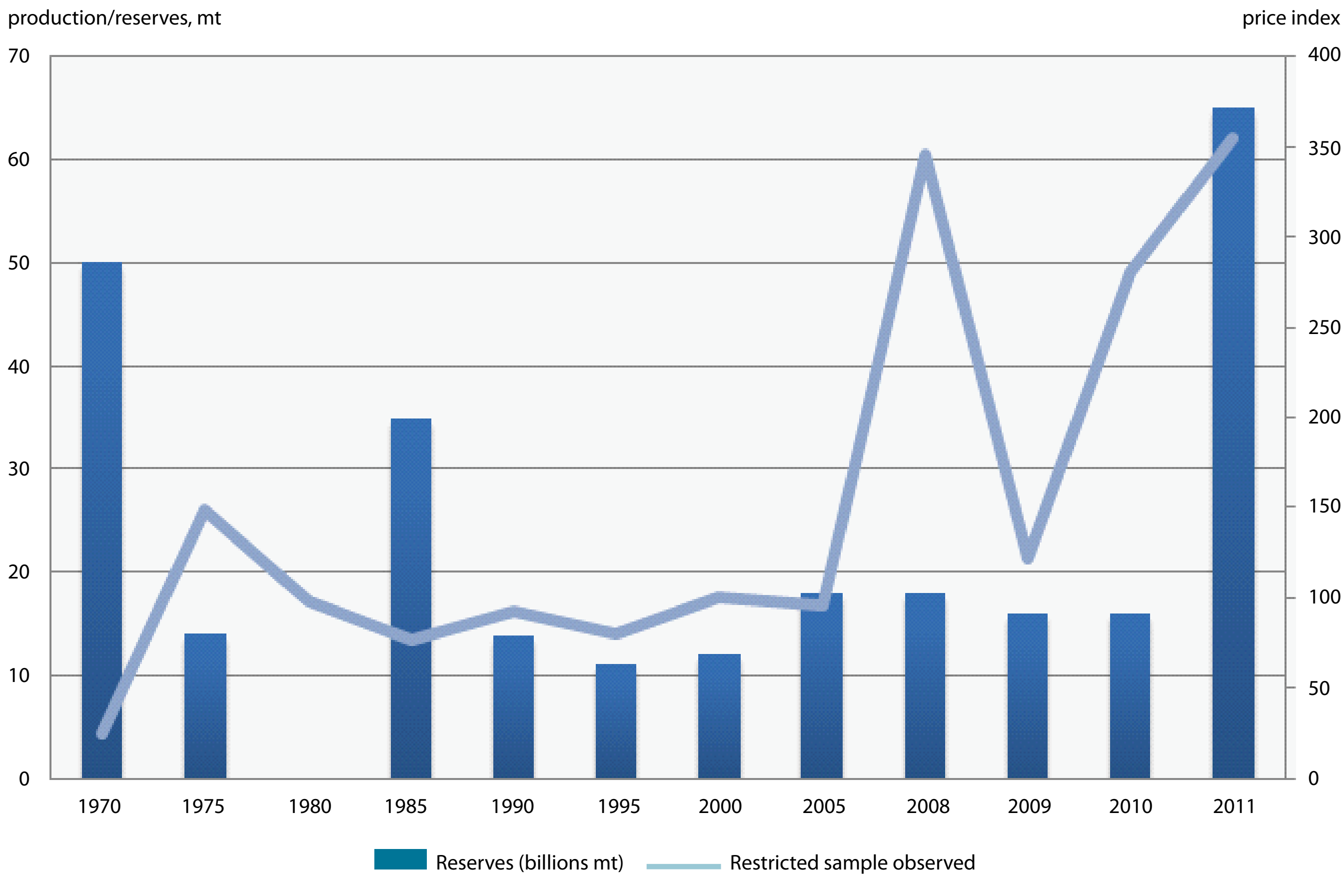
Source: Author's calculations, based on USGS estimates (2010).

Figure 3 Hubbert Linearization, Rock Phosphate, 1993–2008



Source: Author's calculations, based on USGS estimates (2010).

Figure 4 Reserve estimates, price (100=2000), and production, 1990–2011



Source: United Nations Conference on Trade and Development, annual US Geological Survey Mineral Commodities Summaries.