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Insider Trading, Informational Efficiency and Allocative Efficiency

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ABSTRACT

A dominant, net buyer of a certain asset receives a private signal that is correlated with its mean value. We call this insider a Boesky Insider when the quality of the received signal is such that the future value of the asset can be predicted with certainty. We show that even an infinitesimal probability of a Boesky Insider results in informational inefficiency of prices. Insisting that the equilibrium be continuous in the signal accentuates the inefficiency to the extent that no information is conveyed. The informational inefficiency notwithstanding, the regime that allows insider trading can result in greater liquidity and is, in an ex-ante sense, Pareto superior when compared to a regime in which insider trading is banned.

JEL CLASSIFICATION: C72, G12, G14

KEYWORDS: Efficient Markets, Insider Trading, Perfect Bayesian Equilibrium, Pooling, Public Confidence, Zero Probability Event.

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1 Introduction

“The public’s out there throwing darts at a board, sport. I don’t throw darts at a board - I bet on sure things” - Gordon Gecko in Wall Street.

The notion that agents have heterogeneous beliefs about the value of assets is neither new, nor surprising. In some sense, all participants in securities markets believe that they have superior information to those with whom they trade. In this sense we all consider ourselves insiders to a certain degree. The fact that other traders have different views, based on different information does not lead to a fear of trading, nor a precipitous decline in the liquidity of markets. Yet what if the person with whom we were trading might know that value of the security for sure? What if they were the CEO of a company about which there was takeover speculation; of a company which was about to announce a large earnings downgrade, or upgrade; of a company which was about to file for bankruptcy? Market participants, regulators and commentators fear the possibility of just such an insider and it is a major motivation for bans on insider trading.

In this article, we study a model of a securities market involving a dominant, price setting insider. The insider receives private information about a parameter that is correlated with the future value of a certain asset, and sets the unit price at which she is willing to acquire the asset. When this estimate is perfect so that the insider *knows* the future value, then we call that type of insider a Boesky Insider¹. Our main finding is that even if the probability of there being such an insider is *infinitesimal*, prices are necessarily informationally inefficient in that all equilibria are at least partially pooling. What is more, the extent of the informational inefficiency is extreme if the equilibrium is also required to be continuous in this parameter. For in this case every equilibrium is fully pooling.

The above result calls into question the traditional law and economics view² that allowing insiders to trade on the basis of their information will improve informational efficiency. Yet informational inefficiency in itself is hardly the criterion for evaluating the desirability of a policy. Therefore, taking the informational inefficiency of

¹The colorful term “Boesky” insider is suggested by the activities of the infamous insider trader Ivan Boesky.

²See Ausubel [1990] for an exposition of this view.

prices as a given, we consider its welfare effects. Specifically, we are interested in whether regulating insider trading will necessarily lead to greater welfare, and whether it will result in greater liquidity, as measured by the expected quantity that is sold.

We study a very simple legal restriction on insider trading which may be viewed as a stylization of a commonly observed method aimed at providing all the market participants with a level playing field so to speak. According to this rule, after receiving the private signal, the insider has two options. She can either abstain from trading in the market, or she can make her private information public and then trade. We shall assume that all such public announcements are perfectly verifiable. While perfect verifiability of private information is a strong assumption, it clearly biases the case for regulation.

Given the structure of our model, it turns out that it is always advantageous for an insider to make her information public and trade on that basis. As the reports are perfectly verifiable, the regime is by design, informationally efficient. We shall refer to this regulated regime as the NI regime.

We compare the welfare of the market participants under the regulated and informationally efficient NI regime with their corresponding welfare under a particular informationally inefficient fully pooling equilibrium in the unregulated market. This pooling equilibrium, which we shall frequently refer to as the PE regime has the property that the quantity traded is identical to that under the NI regime. The existence of such a pooling equilibrium is guaranteed under the assumption that the outsiders display constant absolute risk aversion and certain simple relationships between the parameters that determine the riskiness in the risky asset and the spread in the distribution of the signals.

A comparison of the PE regime and the NI regime offers several interesting insights. First, by construction, the liquidity in the PE regime is the same as that of the NI regime. Therefore, informational inefficiency does not imply lower liquidity. Second, it turns out that depending on the parameter values, the PE regime can Pareto Dominate the NI regime while the converse can never be the case. The reason for this is as follows. Under the NI regime, it turns out that the quantity acquired by the *Insider* is a constant. The price however varies. By construction, the supply under the PE regime is the same as that under the NI regime but the price, being a pooling equilibrium, is also constant. Consequently, the PE regime offers the

outsider additional insurance. This makes it possible for an outsider to supply the same quantity as in the NI regime at a lower average price and still be as well off. As the *Insider* is risk-neutral, the lower average price at which she acquires the same quantity improves her payoff also. In fact, we show that when the signals are uniformly distributed over a large range, the payoffs under the unregulated PE regime Pareto dominate those under the regulated NI regime.

The early theoretical literature on the efficiency effects of insider trading argued that insiders would reveal their private information (see Grossman [1976], Grossman [1989], Allen [1981], Glosten and Milgrom [1985] and Verrecchia [1982], among others.). These papers assumed a competitive market in the sense that the large trader could not affect the price of the asset through their strategic choice. They find the equilibria that are separating and hence that prices are informationally efficient. (See Ausubel (1990) and Ausubel (1989) however.) The informational efficiency of prices and other actions has also been considered in imperfectly competitive environments, including the works of Gould and Verrecchia [1985], Grinblatt and Ross [1985], Kyle [1985], Kyle [1989], Altug [1991], Laffont and Maskin [1990], Bhattacharya and Spiegel [1991], Fishman and Hagerty [1992] and Benabou and Laroque [1992] among others. Most of these papers restriction attention to strategies that are linear in the underlying variable.

Ausubel [1990] and Laffont and Maskin [1990] are among the comparatively few contributions that, to our knowledge, compare the informational and allocative efficiency of insider trading. The latter is in an important reconsideration of most of the earlier literature. Laffont and Maskin criticize the preoccupation with linear equilibria in most other works and focus instead on the equilibrium that is most favorable to the insider. Indeed, in the binary signal model they consider, any strategy is trivially linear. They show that the most favorable equilibrium for the insider (whenever such an equilibrium exists) is the one that is fully pooling. The reason that a pooling equilibrium improves the payoff relative to the separating equilibrium is that the expected equilibrium price is more or less the same in a separating and a pooling equilibrium. To ensure incentive compatibility however, the quantity traded must be constrained and lower in a separating equilibrium which in turn implies that the insider prefers the pooling equilibrium. Depending on the parameter values, this may even benefit the outsiders. Adopting the Pareto

principle for ranking the two equilibria, they then argue that an insider may be able to influence beliefs at an ex-ante stage so that the pooling equilibrium emerges and thus prices are informationally inefficient.

Our model is related to Laffont and Maskin [1990] but is different in two key respects. First, we allow the type of the insider to be a continuous variable. Second, we introduce the notion that there may be Boesky insiders, albeit with an infinitesimal probability. If one insists on the equilibrium being continuous, which as a requirement is clearly weaker than linearity, and is a condition that is not relevant in Laffont and Maskin [1990], our finding is that every equilibrium is fully pooling. Therefore the issue of ranking equilibria that differ in informational content does not arise.

The rationale for why the ban on insider trading does not benefit the insider is also very different to the role played by quantity constraints in the Laffont and Maskin's explanation of a pooling equilibrium being favorable for the insider. In fact, in our model the ban on insider trading acts as a pre-commitment device enabling the insider, being a Stackelberg leader, to fully exploit her dominant position by allowing her to credibly reveal her private information. In particular, she will not face the quantity constraints that are implied by a separating equilibrium. Therefore one might expect that the insider will necessarily prefer the NI regime to the PE regime. This, however, is not the case, the reason being that in the PE regime the constant pooling price faced by the outsiders offers them additional insurance. This allows them to supply the same expected quantity as they would under the NI regime but for a lower (expected) price.

Ausubel [1990] offers a public confidence argument³ for regulating insider trading by introducing an ex-ante investment stage involving potential insiders and outsiders. Potential insiders may prefer regulation at the later trading stage so that it allows for greater investment at the ex-ante stage. Regulation can Pareto-dominate the unregulated regime. Our work omits the ex-ante investment stage although in our model too there is a break down of public confidence resulting from the presence of perfectly informed insiders. The loss in public confidence notwithstanding, regulation may be Pareto Dominated, in contrast to the findings of Ausubel [1990]. The difference can be ex-

plained in part by the fact that there is imperfect competition in our model whereas the markets are perfectly competitive in Ausubel's model. We discuss this further in Section 5.1.

1.1 Brief Intuition For Informational Inefficiency

In the model we study, the insider privately observes a parameter \tilde{S} that is correlated with the mean of a risky asset. She then acts as a Stackelberg leader and sets a price $p(\tilde{S})$ at which she is willing to acquire the asset. The insider is risk neutral while the small traders are risk averse. When the signal \tilde{S} is a noisy estimate of the mean value of the asset, for risk sharing reasons, there are gains from trade even if it becomes common-knowledge. On the other hand if a particular realization of the signal, say $\tilde{S} = s^*$, is fully informative of the future value of the asset so that the dominant trader is a Boesky insider, there is no further risk. Consequently if s^* were common-knowledge among all traders in the market, the otherwise risky asset and the risk-free bond are perfect substitutes.

Given the above observation, if there were to be a fully revealing equilibrium in which a positive quantity is traded, then the dominant trader is forced to set the price of the asset given s^* to be such that the return on the asset equals the return on the risk-free bond. Otherwise there would be an arbitrage opportunity. If one now assumes that the equilibrium price is continuous in the signal, for other values of the signal close to s^* , the differential on the mean return between the bond and the risky asset is negligible. Therefore at a noisy signal sufficiently close s^* , the risk averse traders would strictly prefer to sell almost their entire endowment of the risky asset and invest the proceeds in the bond. Incentive compatibility conditions, which require the quantity traded to be non-decreasing, then force the outsiders to trade their entire endowment for all signals higher than s^* , thus contradicting the hypothesis that the price is fully revealing.

In a nutshell, the fact that there is a Boesky insider constrains the return on the asset at s^* to equal that of the risk-free bond. Even though the probability of this event is infinitesimal, due to incentive compatibility, it will have repercussions for all other values of s^* .

The rest of the paper is organized as follows. In Section 2, we present the basic model and discuss the assumption of Boesky Insiders. Section 3 considers the case of insider trading and presents three results: Proposition 1, the result on informational inefficiency,

³Benabou and Laroque [1992] also offers a related rationale for regulating Insider trading.

Proposition 2, a sufficient condition for checking when an arbitrary price-quantity combination can be supported as a pooling equilibrium outcome and Proposition 3 which identifies the equilibrium described as the PE regime earlier. In Section 4 we introduce the legal restriction on insider trading and compare the welfare in PE and NI regimes. Proposition 4 and (its) Corollary 1 contain the welfare comparisons between the regulated and unregulated regimes.

We emphasize that Proposition 1 is the only result that relies on the presence of Boesky insiders. As such, the rest of the paper, particularly the comparisons of the welfare effects of the informational inefficient pooling equilibrium in the PE regime vis-a-vis the regulated NI regime, can be read independently of this result.

2 The Model

Two assets are traded in period 1, the values of which are realized in period 2. One of these assets is a risk free bond that offers a sure return of $R > 1$, with its current price being normalized to one. The period 2 value of the second asset, on the other hand, is the realization of a random variable, say \tilde{V} which follows a normal distribution whose mean and variance are not a priori known.

In period one however, a large dominant trader receives a private signal \tilde{S} by which she becomes informed that conditional on $\tilde{S} = s$, the distribution of \tilde{V} is normal with the mean s and variance σ_s^2 . The signal itself is continuously distributed according to a probability distribution $F(\cdot)$ with $[a, b]$ as its support. We shall refer to the large trader as the *Insider* or Player D of type s when she observes $\tilde{S} = s$.

Player D has an initial wealth of m units of money and a zero unit endowment of the risky asset. After finding out her type, she acts as a Stackelberg leader by setting the price at which she is willing to acquire the asset. The *large trader's strategy* is therefore a function $p : [a, b] \rightarrow \mathbb{R}_+$. For each $s \in [a, b]$, the strategy $p(\cdot)$ specifies a price $p(s)$ at which Player D is willing to trade the asset. We shall routinely refer to a strategy of Player D as a "pricing strategy". The expected profit of Player D of type s if she acquires q units of the risky asset at a price ρ is:

$$V(q, \rho, s) = Rm + q(s - R\rho) \quad (1)$$

We assume that Player D is risk neutral and maximizes expected

profits. It is only natural to allow the large trader the option of refusing to trade. The utility of no-trade is mR .

The market also contains a unit mass of identical, risk-averse price taking traders. As we assume that no such trader directly observes the signal \tilde{S} , we will often refer to them as outsiders. Each outsider has an endowment of w_0 units of money one unit of the risky asset. If an outsider were to supply q units of the risky asset at a unit price of ρ and invest the proceeds in the risk-free bond, her future (stochastic) wealth is $\tilde{w} = w_0R + q\rho R + (1 - q)\tilde{V}$.

Assume that an outsider satisfies the expected utility hypothesis and let $u(\cdot)$ denote the appropriate von Neumann-Morgenstern expected utility functional. If the outsider were fully informed that the signal is $\tilde{S} = s$, then her expected utility is $E[u(\tilde{w})|\tilde{S} = s]$. The actual information that an outsider has will depend on the regime that we are studying. In Section 4 where insider trading is regulated, she would know the true signal. In Section 3 where the insider is allowed to trade on the basis of her information, the optimal allocation, what she knows is endogenously determined by how much information the equilibrium strategy of the insider will reveal. In general, we represent her information about the insider's type as a distribution function $G(\cdot)$ with some measurable subset of $[a, b]$ as its support. Given her belief, her payoff can now be written as

$$U(q, G, \rho) = \int_a^b E[u(\tilde{w})|\tilde{S} = s]dG(s) \quad (2)$$

Let $x(\rho, s)$ denote the utility maximizing supply of risky asset by an outsider who believes that \tilde{V} is distributed normally with mean s and variance $\sigma_s^2 \equiv \sigma^2$. Therefore

$$x(\rho, s) = \arg \max_q \frac{1}{\sigma_s \sqrt{2\pi}} \int_{-\infty}^{\infty} u(w_0 + q\rho R + (1 - q)(s + \epsilon)) e^{-\epsilon^2/2\sigma} \partial \epsilon \quad (3)$$

Throughout the paper we assume $u(\cdot)$ is strictly concave and twice continuously differentiable. We also assume that $u(\cdot)$ satisfies non-decreasing absolute risk aversion. An important implication of these restrictions is that $x(\rho, s)$ is non-decreasing⁴ in s for a given ρ .

When the conditional variance is independent of the signals, i.e. $\sigma_s^2 = \sigma^2 > 0$, the model is similar to most of finance literature

⁴See Laffont and Maskin [1990] for a proof.

involving rational expectations⁵. In fact, if we were to make this assumption, apart from the fact that there is a continuum of types, the model is identical to the one studied by Laffont and Maskin [1990]. Such an assumption does not however allow one to say that one type of insider is better informed than another. One extreme form to introduce this possibility is to say, as we do here, that the signal is either “noisy” or “accurate”. When the s is a noisy signal, $\sigma_s^2 = \sigma^2 > 0$ whereas $\sigma_s^2 = 0$ if the signal is considered to be accurate. In the latter case, we shall refer the corresponding type as a Boesky Insider. Let

$$B = \{s \in [a, b] : \sigma_s^2 = 0\}$$

denote the set of all Boesky Insiders. A significant innovation of this work is the study of the implications of the “Boesky Insider” for informational efficiency.

2.1 The Boesky Insider

There are numerous situations where an insider could be perfectly informed about the future value of a security. The situation where a tender offer is being considered is a classic case. In many instances the recommendation of the board will be sufficient to ensure that the offer will be accepted. An insider who knew that the board had decided to recommend the offer would know that the share price would become equal to the offer price. A company which is about to sign a merger agreement will have a definitive value known to those who know that the agreement will be signed. A company which is going to perform a share buyback will have a share price equal to the buyback price. If a company is about to file for bankruptcy (in the sense that its liabilities exceed its assets) the value of its equity is zero. These situations are widespread. They also arise where the information does not imply a specific price of an underlying asset but derivative securities, side contracts or other financial instruments are involved⁶. For instance: when the holder of barrier options, so called

⁵See Grossman [1989] for example.

⁶An infamous example of such a case involved Ivan Boesky himself. As the *Wall Street Journal* reported “Mr. Boesky testified that Mr. Mulheren responded ‘I understand’ after the arbitrageur told him ‘it would be great if Gulf & Western [Industries Inc. stock] traded at 45.’ Mr. Mulheren then bought 75,000 shares of the stock just before the market closed, causing the price to climb to \$45 – which triggered an agreement by G&W to buy back Mr. Boesky’s huge stake” -

”knock-ins” or ”knock-outs”, receives information that the price of the underlying asset will more above or below a certain price, or where a ”collar” is in place⁷ which means that the holder is certain of their payoff outside a defined band of prices of the underlying asset. This highlights that there are many possible instances where certainty of information translates directly into certain about the value of a security.

It is important to distinguish here between certain knowledge of an event taking place which will which will affect the price of a security, and certain knowledge of the price of a security. It is the latter which we mean by a Boesky insider. Clearly there are many more instances of the former than the latter. Yet, as we shall see that for the purposes of our analysis there need only be an infinitesimally small probability that such a Boesky insider may exist.

3 Insider Trading, The Boesky Insider & Informational Efficiency

We model the situation where an insider can trade freely based on her information as a signaling game. In this game, the *Insider* chooses a pricing strategy as described in the previous section. An *outsider’s strategy* is a mapping $Q : \mathfrak{R}_+ \rightarrow [0, 1]$ that represents the share of the risky asset that she will supply for each price. In order to describe an equilibrium one must also specify *conditional beliefs* of an outsider, $\{G_\rho\}_{\rho \geq 0}$ where $G_\rho(\cdot)$ is a distribution function whose support is a measurable subset of $[a, b]$. Upon seeing a price ρ , an outsider believes that the *Insider’s* type is distributed according to G_ρ . We will refer to $\{G_\rho\}_{\rho \geq 0}$ as a *system of beliefs*.

Definition 1 A Perfect Bayesian Equilibrium consists of a pair of strategies $\{p, Q\}$ and a system of beliefs $\{G_\rho\}_{\rho \geq 0}$ such that:

- (i) For all ρ in the range of p , G_ρ is the conditional probability distribution of \tilde{S} obtained in a Bayesian fashion.

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⁷This consists of buying a ”cap” (say through the purchase of a call option) and selling a ”floor” (say through the sale of a put option). A ”zero cost collar” is a special case where the cost of the cap is equal to the proceeds generated by the sale of the floor.

(ii) $p(s) \in \arg \max_{\rho} (s - \rho)Q(\rho)$ and $p(s) \leq s/R$.

(iii) $Q(\rho) \in \arg \max_{q \in [0,1]} U(q, \rho, G_{\rho})$ for all ρ .

Conditions (ii)-(iii) in the above definition of an equilibrium require that traders' choices are optimal given their beliefs. Condition (ii) also takes into account that a dominant trader can refuse to trade. Condition (i) forces the small traders to have rational expectations.

The following further terminology is used to describe equilibria. A PBE is said to be *fully revealing* if the equilibrium pricing strategy p is such that $p(s) \neq p(t)$ whenever $s \neq t$. It is said to be *completely pooling* if $p(s) = p(t)$ for all s, t . A strategy that is neither fully revealing nor *completely pooling* is said to be *partially revealing*.

A PBE is said to be *continuous*, if the equilibrium pricing strategy is continuous. It is worthwhile noting that in a continuous equilibrium one does not require that Q is continuous. As we show in Lemma (1), continuity of $Q(\cdot)$ in the range of $p(\cdot)$ is in fact implied by the continuity of the latter through the incentive compatibility constraints.

Proposition 1 *Suppose that the set of Boesky Insiders is not empty, i.e. $B \neq \emptyset$. In an equilibrium either there is no trade or, if there is trade the following hold:*

1. *Every equilibrium is partially pooling.*
2. *If the equilibrium is continuous, then it is fully pooling.*

The formal proof which is somewhat involved is in the Appendix. Here we shall briefly sketch an argument that will convey the main intuition by considering why an equilibrium cannot involve a positive amount of trade by every type of *Insider* and be fully revealing. Also, assume that there is exactly one type s^* who is a Boesky Insider and that $u(w) = -\exp\{-\gamma w\}$ so that the outsiders display constant absolute risk aversion of $\gamma > 0$. Then⁸,

$$x(\rho, s) = \begin{cases} 1 & \text{if } s \leq \rho R, \\ 1 - \frac{s - \rho R}{2\nu} & \text{if } s - 2\nu \leq \rho R \leq s, \\ 0 & \rho R \leq s - 2\nu \end{cases} \quad (4)$$

⁸See Chapter 4, Huang and Litzenberger [1990] for the algebra leading to Formula (4).

where $\nu = \gamma\sigma^2/2$.

Given an equilibrium pricing strategy $p(\cdot)$ let $q(s) = Q(p(s)) > 0$ denote the associated equilibrium supply at s . As the dominant trader can always choose not to trade, (Condition (ii) for an equilibrium) we have $p(s) \leq s/R$ for all s . Using routine arguments from the theory of incentives it can be shown that $p(\cdot)$ and $q(\cdot)$ are both non-decreasing. (See Lemma 1 in the Appendix.) The monotonicity must hold in *every* equilibrium – whether it is separating or otherwise. If the equilibrium is fully revealing then $p(\cdot)$ and $q(\cdot)$ must be increasing. From this it is clear that $0 < q(s) < 1$ for all $s \in (a, b)$. For if $q(s) = 0$ for some s , then $q(t) = 0$ for all $t \leq s$. Likewise if $q(s) = 1$ for some s then $q(t) = 1$ for all $t \geq s$. In either case $q(\cdot)$ would not be increasing.

By way of contradiction now suppose that $p(\cdot)$ is a continuous and fully revealing equilibrium pricing strategy. When a small trader observes the price $p(s^*)$ it becomes common-knowledge that the large trader has observed $\tilde{S} = s^*$ and that the future price of the otherwise risky asset is now s^* for sure. Consequently, the bond and the asset are now perfect substitutes. If $p(s^*) < s^*/R$, then the return on the bond is lower than the return on the risky asset. As there is no risk in either asset, a small trader's unique best response is to retain her entire endowment, i.e. $q(s^*) = 0$, in contradiction of the hypothesis that every type trades a positive amount.

Therefore we are left with the possibility that $p(s^*) = s^*/R$. We will now argue that this too leads to a contradiction by showing that $q(s^*) = 1$. The requirement that $q(s)$ is a best response to Player D 's price $p(s)$ tells us that $q(s) = x(p(s), s)$ for all $s \neq s^*$. We know that $q(s)$ cannot be either zero or one in a fully revealing equilibrium. Therefore from Eq. (4)

$$q(s) = 1 - \frac{s - p(s)R}{2\nu} \quad \forall s \neq s^*. \quad (5)$$

Since $p(s^*) = s^*/R$ and $p(\cdot)$ is assumed to be continuous $\lim_{s \rightarrow s^*} q(s) = 1$. In the Appendix (Lemma 1) we show that $q(\cdot)$ must be continuous whenever $p(\cdot)$ is continuous. Therefore $q(s^*) = \lim_{s \rightarrow s^*} q(s) = 1$ which violates the monotonicity of $q(\cdot)$. Therefore $p(\cdot)$ cannot be fully revealing.

Note that even though the foregoing arguments were presented as though the equilibrium is fully revealing, we have not used the full

force of this assumption. Indeed, we could have virtually repeated the arguments to conclude that any continuous equilibrium must be pooling in a *neighborhood* of s^* . It remains to rule out then the possibility that a continuous equilibrium pricing strategy $p(\cdot)$ can be partially revealing.

Again, suppose by way of contradiction, that there is an equilibrium such that in an interval $S = [\alpha, \beta]$, the equilibrium price is a constant \bar{p} but is strictly increasing in a left neighborhood of α . When a small trader sees the price \bar{p} , through Bayesian updating, she believes that Player D 's type is distributed according to the posterior distribution $G(s) = (F(s) - F(\alpha))/(F(\beta) - F(\alpha))$. Therefore the equilibrium supply of the asset is, say, $\bar{q} \in \arg \max_q U(q, G, \bar{p})$. Through an application of the mean value theorem of integral calculus, it follows that $\bar{q} = x(\bar{p}, \bar{s})$ for some $\bar{s} \in (\alpha, \beta)$. On the other hand $\lim_{s \uparrow \alpha} q(s) = \lim_{s \uparrow \alpha} x(p(s), s) = x(\bar{p}, \alpha)$. As $x(\bar{p}, \alpha) > x(\bar{p}, \bar{s})$ we arrive at the contradiction that $q(\cdot)$ is continuous at α . A symmetric argument applies that the strategy cannot be increasing in any right neighbourhood of β .

When we prove the general case, we are first required to prove that in *any* equilibrium either every type trades a positive quantity or there is no trade. This leads to the conclusion that $0 < q(s) < 1$ for all s . Then a no-arbitrage argument as above establishes that $p(s^*) = s^*/R$. Using various assumptions that an analogue of $x(\rho, s)$ must satisfy, we are able to show that $p(\cdot)$ must be pooling in a neighborhood of s^* . Finally, an argument similar to the one in the previous paragraph shows that $p(\cdot)$ is fully pooling if it is also continuous.

3.1 Existence of Equilibrium

In case of binary signals, the setup considered by Laffont and Maskin [1990], a pooling equilibrium may not exist⁹ unless the signals are sufficiently close, although a separating equilibrium always exists. Here the presence of Boesky Insiders rule out the possibility of a separating equilibrium. Therefore the question of existence of an equilibrium requires a reconsideration. In this section, we provide two sets of results. The first is a sufficient condition for checking whether a candidate price-quantity pair (ρ^*, q^*) can be supported as an outcome of a pooling equilibrium. This is Proposition 2 which

applicable for an arbitrary $u(\cdot)$ satisfying non-decreasing risk aversion. After this, we shall consider in more detail the case when outsiders display constant absolute risk-aversion.

Define then:

$$\pi(s) = mR + \max_p (s - \rho R)x(\rho, b) \quad (6)$$

$\pi(s)$ is the maximum profit of a type s *Insider* when an outsider believes that the former is the highest type b .

Proposition 2 (ρ^*, q^*) can be supported as the outcome of a pooling equilibrium if:

$$0 \leq R\rho^* \leq a \quad (7a)$$

$$q^* = \arg \max_q U(q, F, \rho^*) \quad (7b)$$

$$v(a) \geq \pi(a) \text{ and } v(b) \geq \pi(b) \quad (7c)$$

$$\text{where } v(s) = mR + (s - \rho^* R)q^*.$$

The formal proof is contained in the Appendix. The conditions in Eq. (7a) and Eq. (7b) are easy to understand. Eq. (7a) ensures that it is individually rational for the *Insider* to trade regardless of her the signal she receives. This also ensures that the posterior belief conditional on observing ρ^* is the same as the prior belief. Eq. (7b) ensures that the supply of q^* is a best response of the outsider given her beliefs, and the offered price. The strength of the Proposition is that just the two inequalities in Eq. (7c) are sufficient to ensure that (ρ^*, q^*) is an equilibrium outcome.

Indeed, in order to support the above as an equilibrium outcome, we need to specify what happens at all the nodes out of the equilibrium. We fix the beliefs of the outsiders as follows: Whenever they see a price other than ρ^* , they believe that the *Insider* has received the highest signal. In other words, they believe that the *Insider*'s type is b . Therefore if the *Insider* were to choose a price that differs from ρ^* and the outsiders were to best respond with the above beliefs, the payoff of type s *Insider* is bounded above by $\pi(s)$. To ensure that the *Insider* does not have an incentive to deviate, her equilibrium payoff of $v(s)$ must be at least $\pi(s)$ for all s . The Proposition simplifies verification of this condition by asserting that

⁹See Proposition 9 of their paper.

it suffices to ensure that the extreme types a and b do not have an incentive to deviate. The proof of this fact involves showing that $\pi(\cdot)$ is convex. As $v(\cdot)$ is linear, if the inequalities in Eq. (7c) are satisfied, then $v(s) \geq \pi(s)$ for all s .

We now consider in greater detail the existence issue when an outsider's absolute risk-aversion is a constant γ . Let

$$\delta = \frac{E[\tilde{S}e^{-\gamma\tilde{S}/2}]}{E[e^{-\gamma\tilde{S}/2}]} \quad (8)$$

and recall $\nu = \gamma\sigma^2/2$. The parameter δ depends only the conditional prior distribution of \tilde{S} , whereas ν depends on the degree of risk-aversion and the variance of the idiosyncratic risk. The existence of a pooling equilibrium depends on the relationship between these two parameters. Consider the following inequalities:

$$\nu \leq \delta \leq \nu + a \quad (9a)$$

$$\nu \geq \frac{(b-a)^2}{4(b-\delta)}. \quad (9b)$$

Proposition 3 *Suppose that $u(w) = -e^{-\gamma w}$. Suppose the resulting parameters (ν, δ) satisfy Inequality (9a) when $\nu \leq (b-a)/2$ and also Inequality (9b) when $\nu \geq (b-a)/2$. Then a fully pooling equilibrium exists in which the Insider acquires half the endowment of risky asset of the outsider at the price $(\delta - \nu)/R$.*

Hereafter, we shall refer to a regime with the equilibrium outcomes described in the above Proposition as the PE regime and let $(\rho^{pe}, q^{pe}) = ((\delta - \nu)/R, 1/2)$ to denote the equilibrium outcome.

The reader may find Figure 1 drawn in the (δ, ν) space helpful to visualize the regions in which an the equilibrium outcomes described in Proposition 3 are feasible.

Proof We will verify that (ρ^{pe}, q^{pe}) satisfies Eq. (7b-7c) and apply Proposition 2. Equality (9a) is a simple rearrangement of Eq. (7a).

To see that (ρ^{pe}, q^{pe}) satisfies Eq. (7b), first write the payoff of an outsider from choosing q when facing a constant price ρ , given that the posterior belief is the same as the prior F :

$$U_I = U(q, F, \rho) = -e^{-\gamma w_0 R - \gamma q \rho R + \gamma \nu (1-q)^2} \\ \times E[e^{-\gamma(1-q)\tilde{S}}]$$

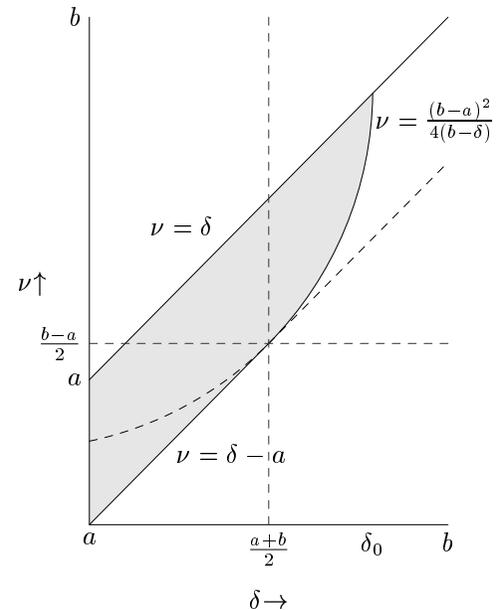


Figure 1: Equilibrium exists if (δ, ν) lies in the shaded region.

The picture is constructed as follows. Clearly $a < \delta < b$ while $\nu \geq 0$. Inequality (9a) requires that (δ, ν) lie between the two parallel lines $\nu = \delta$ and $\nu = \delta - a$. The RHS of Inequality (9b), regarded as a function of δ , is increasing, convex and is supported by the line $\nu = \delta - a$, with $\delta = (a+b)/2$ being the point of tangency. It also has a fixed point at $\delta_0 = b + \sqrt{a(2b-a)}$ which, it may be verified lies to the left of b . The picture is drawn schematically with these points in mind. The shaded region is the set described by the Inequalities (9b-9b).

U_I is concave in q . By differentiating $\log(-U_I)$ with respect to q yields the following first order condition for an optimum:

$$\rho = \frac{E[\tilde{S}e^{-\gamma(1-q)\tilde{S}}]}{E[e^{-\gamma(1-q)\tilde{S}}]} - \gamma(1-q)\sigma^2.$$

which clearly holds at $(\rho, q) = (\rho^{pe}, q^{pe})$.

It remains to verify the two inequalities in Eq. (7c). Checking that $v(b) \geq \pi(b)$ is relatively straightforward. For, $\pi(b)$ is the monopoly profit of the type b *Insider* under full information. This is achieved if she acquires half the endowment of the risky asset at a unit price $\rho = (b - \nu)/R$. On the other hand, in the proposed equilibrium, she acquires an identical quantity at the price ρ^{pe} . Since $\delta < b$, it follows that $\pi(b) \leq v(b)$.

Seeing $\pi(a) \leq v(a)$ is little more involved. When the supply is being determined as per $x(\rho, b)$, it is clear from Eq. (4) that unless a price of at least $b - 2\nu$ is offered, an outsider will refuse to supply a positive quantity. When ν is sufficiently low, so that there is not much idiosyncratic risk in the asset, the desired price $(b - 2\nu)$ will be greater than a . In this case, $\pi(a) = mR$ as no positive quantity will be acquired by the type a *Insider*. For such values of ν , $\pi(a) \leq v(a)$ holds trivially as long as Inequality (9a) holds. In contrast, when $\nu \geq (b - a)/2$, there interior solution to type a *Insider's* maximization problem which yields

$$\pi(a) = mR + \frac{\nu}{2} \left(1 - \frac{b - a}{2\nu}\right)^2 \quad (10)$$

Comparing the above with $v(a) = mR + (a - \delta + \nu)/2$ shows that $\pi(a) \leq v(a)$ is equivalent to Inequality (9b). ■

For subsequent analysis, it is useful to record the respective equilibrium utility of the *Insider* and the outsiders in under the PE regime:

$$V_{PE} = mR + (\bar{S} - \delta + \nu)/2 \quad (11)$$

$$U_{PE} = -e^{\gamma w_0 + 3\gamma\nu/4} e^{-\gamma\delta/2} E[e^{-\tilde{S}/2}]. \quad (12)$$

4 Regulated vs. Unregulated Insider Trading

In this section we analyze the welfare implications of regulating insider trading. Under the regulation we consider, after the *Insider*

receives the signal, she has one of two options. She can publicly declare the signal she has received and then trade. If she chooses not to disclose, then she must abstain from trading.

The above restriction is a fairly accurate representation of the most common restriction on insider trading. However, we stylize the environment by assuming that any disclosure of the *Insider* is fully verifiable. This is, of course, an idealization. In reality, convincing a court of exactly how much a trader knew so that she is in breach of the Securities and Exchange Act is an uphill task for the prosecution. Yet it is the assumption of perfect verifiability that provides the most hospitable environment for making a case against insider trading. We will refer to this regime with the legal restriction on insider trade as the NI regime. Define

$$\nabla = \frac{E[e^{-\gamma\tilde{S}}]}{E[e^{-\gamma\tilde{S}/2}]} - e^{-\gamma\delta/2}. \quad (13)$$

The following result provides a comparison of the welfare of the traders under the PE and the NI regimes depending on how ∇ , δ and \bar{S} , the different parameters of the prior distribution F , relate to one another.

Proposition 4 *Suppose that the hypotheses of Proposition 3 hold and in addition $\nu \leq a$.*

1. *If $\delta > \bar{S}$, regulation makes the Insider better off and the outsiders worse off.*
2. *If $\delta < \bar{S}$, regulation makes the Insider worse off. The outsiders are also worse off if (and only if) $\nabla > 0$.*

Proof For almost all s , by revealing her private information, a type s *Insider* faces the supply curve $x(\cdot, s)$, which is given by Eq. (4). By hypothesis $\nu \leq a$. Therefore, reaction of the outsiders as a given, the *Insider* of type s achieves a positive maximum profit by quantity $q^{ni} = 1/2$ and setting a price

$$p^{ni}(s) = (s - \nu)/R. \quad (14)$$

and the expected price is therefore

$$\rho^{ni} = \bar{S} - \nu. \quad (15)$$

First substitute $p^{ni}(s)$ to get $\tilde{w} = w_0R + s - \nu + \tilde{c}/2$ to be the random wealth in state s under NI for all¹⁰ s . The ex-ante utility of the *Insider* and a typical outsider respectively obtained by substituting appropriately in Eq. (1) and Eq. (2) as being:

$$V_{NI} = mR + \nu/2 \quad (16)$$

$$U_{NI} = -e^{-\gamma w_0 R + 3\gamma\nu/4} E[e^{-\gamma\tilde{S}}] \quad (17)$$

Direct algebraic manipulation shows that

$$V_{PE} > V_{NI} \quad \Leftrightarrow \quad \delta < \bar{S} \quad (18)$$

and that

$$U_{PE} > U_{NI} \quad \Leftrightarrow \quad \nabla > 0. \quad (19)$$

To complete the proof, it suffices to show that ∇ is necessarily positive whenever $\delta > \bar{S}$. This can be seen as follows:

$$\begin{aligned} & E[e^{-\gamma\tilde{S}}] - e^{-\gamma\delta/2} E[e^{-\gamma\tilde{S}/2}] \\ & > E[e^{-\gamma\tilde{S}}] - e^{-\gamma\bar{S}/2} E[e^{-\gamma\tilde{S}/2}] \\ & > E[e^{-\gamma\tilde{S}}] - E[e^{-\gamma\bar{S}/2}] E[e^{-\gamma\tilde{S}/2}] \quad (\text{Since } e^{-x} \text{ is convex}) \\ & \geq 0 \end{aligned}$$

where the last inequality follows from the fact that $E[X^2] \geq E[X]^2$ for any random variable X . ■

Though its proof is somewhat mechanical, fortunately it is possible to offer some intuition for the welfare comparisons presented in Proposition 4. Note that under both the regimes, the *Insider* acquires an amount $q^{pe} = q^{ni} = 1/2$, independent of the signal she receives. The important difference between the regimes is that under the NI regime, the price is a random variable $p^{ni}(\tilde{S})$ whereas under the PE regime, the price is a constant. It is this variation in prices that causes the various welfare effects.

After all the *Insider*, being risk-neutral, does not care about the idiosyncratic risk. The question of whether she prefers the PE or the NI regime is simply a question of comparing the (expected) prices,

which are $\rho^{ni} = \bar{S} - \nu$ and $\rho^{pe} = \delta - \nu$. The equivalence in Eq. (18) is then immediate. An outsider on the other hand cares about the idiosyncratic risk. When $\delta > \bar{S}$ the expected price she receives for her asset under NI regime, namely ρ^{ni} is lower than the expected price under the PE regime, namely ρ^{pe} . What is more the former has a greater variance. Naturally, regulation makes the risk averse outsider worse off in this case. This leads to Part 1 of the Proposition.

In contrast, when $\delta < \bar{S}$, it does not necessarily follow that an outsider prefers regulation. Even though the expected price she receives for her endowment is now higher under regulation, the price is random. Therefore if the difference between δ and \bar{S} is not too large, then she too, like the *Insider*, may well prefer the pooling equilibrium to regulated trade – the higher expected price being the risk premium. If this were true, then the pooling equilibrium is Pareto superior to the regulated outcome as stated by Part 2 of the Proposition.

The fact that the *Insider* may prefer the informationally inefficient PE regime must seem somewhat unintuitive. After all the regulation acts as a pre-commitment device allowing her to credibly reveal her information. One might have thought this possibility for the *Insider* to fully exploit her monopoly position without being constrained quantity constrained (as in Laffont and Maskin [1990]) to to credibly reveal her information cannot be improved upon. Yet, the Proposition shows this need not be the case. The resolution to this puzzle comes from supply side.

With full information, it is the quantity that is traded is a constant. The *Insider* is forced to charge a price that depends on her signal to induce the outsiders to supply the quantity q^{ni} in each state. To escape the variability in the price fluctuations, the outsiders may prefer to offer the same quantity for a lower price. It is evident then the the likelihood of this being the case depends on the particulars of the prior distribution F .

To the extent that the legal restriction works as a pre-commitment device is similar to that Ausubel [1990]. It is a worthwhile remark that the restriction leads to the opposite conclusion from ours.

Some indication of when the parameters are such that PE Pareto dominates the NI regime can be had from considering the case where $(b-a)/2 > a$. In this case, the region in the parameter space to which Proposition 4 applies to is shown as the shaded triangle in Figure 2, as it is only in this region that $\nu \leq a$. Observe that the triangle does

¹⁰This is true for all s as the assumption $\nu \leq a$ ensures that acquiring $q^{ni} = 1/2$ at the price $p^{ni}(s)$ is individually rational.

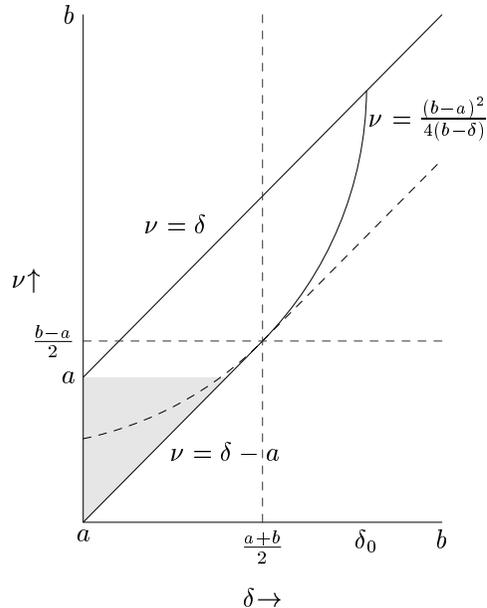


Figure 2: Proposition 4 applies only in the shaded region.

not change as b increases. However, δ is a function of b , and is likely to be increasing in b . Therefore for b sufficiently large, Part 1 of the Proposition may be inapplicable. This for instance is the case when F is uniformly distributed on $[a, b]$. Direct computation reveals that

$$\delta = 2 + \frac{ae^{-a/2} - be^{-b/2}}{e^{-a/2} - e^{-b/2}}$$

in which case $\lim_{b \rightarrow \infty} \delta = (a + 2)$ and $\bar{S} = (a + b)/2 > \delta$. In this case, if the outsiders are better off, Part 2 of Proposition 4 must be true, that is the PE regime is Pareto Superior to NI.

Corollary 1 *Suppose that $u(w) = -e^{-w}$. Suppose F is uniformly distributed on $[a, b]$ such that $(b - a) > 4$. For all ν such that $\delta - a \leq \nu \leq a$, the informationally inefficient PE regime Pareto dominates the regulated and informationally efficient NI regime.*

Proof Direct computation shows that

$$A = \frac{E(e^{-\bar{S}})}{E(e^{-\bar{S}/2})} = \frac{e^{-a/2} + e^{-b/2}}{2}.$$

The log derivative A is $-b/2$, while the log derivative of $\exp\{-\delta/2\}$ is $-\partial\delta/\partial b$ which is exponential in b . Therefore $e^{-\delta}$ decreases at a much faster rate than does A with respect to b . Moreover $\lim_{b \rightarrow a} Ae^{-a/2}$ and $\lim_{b \rightarrow \infty} e^{-a/2}/2$. Therefore $\lim_{b \rightarrow a} \nabla = 0$ (while $\lim_{b \rightarrow \infty} \nabla = e^{-a/2}(0.5 - e^{-1}) > 0$). Therefore, $\nabla > 0$ for all b . Since $\delta < a + 2$ and $\bar{S} = (a + b)/2$, we have $\delta < \bar{S}$. Apply Part 2 of Proposition 4 to complete the proof. ■

5 Discussion and Conclusion

5.1 Public Confidence, Liquidity and Insider Trading

Insider trading was outlawed in most countries long ago¹¹. These bans do not stop so-called corporate "insiders" (such as managers, officers and directors) from trading *per se*. The bans outlaw trading, by a person¹², on the basis of material, price-sensitive, non-public information. There are well documented fairness as well as efficiency arguments in favor of such legislation. Though the academic

¹¹For instance the Securities and Exchange Commission ("SEC") has prohibited insider trading since the Securities and Exchanges Act of 1934 (Section 10(b) and SEC Rule 10b-5 generally and Section 14(e) and Rule 14e-3 with respect to takeovers) and strengthened such position with the Insider Trading Sanctions Act of 1984 and the Insider Trading and Securities Fraud Enforcement Act of 1988. Many other countries' bans are also long held.

¹²Who qualifies as a "person" varies by jurisdiction and is a matter of some debate. In many countries outside the US it is taken to mean any person, regardless of their relationship with the company in question. In the United States attention has generally been restricted to those who owe a fiduciary duty to the company. However, the boundary of who owes such a duty is somewhat uncertain. This was partially clarified in *Chiarella v United States* where an employee of a financial printer (Vincent Chiarella) was charged by the SEC with illegal insider trading but later cleared by the Supreme Court. The SEC has also pursued cases under the "misappropriation theory" which holds that even those without a fiduciary duty may be charged with deceitful acquisition and misuse of information belonging to a party to whom they owe a fiduciary duty. The landmark case was *James H. O'Hagan v United States* (1996). The Supreme Court found in favor of the SEC. It had previously been uncertain whether they supported the misappropriation doctrine, since being split 4-4 in *Carpenter v United States* (1987).

Recently the SEC has sought (under section 14(e)-3 of the Securities and Exchange Act) to broaden illegal insider trading to any person (at least in the case of a takeover bid), even if they do not owe any fiduciary duty, much like in other jurisdictions. This has been rejected (as exceeding the SEC's authority) by the Court of Appeals in *Chesterman v United States* (1990).

literature has focused almost exclusively on direct efficiency arguments, (Ausubel 1990 is a notable exception), the public rationale for regulation of insider trading is heavily focused on notions of public confidence. It is frequently claimed by policymakers, commentators and market participants that insider trading "undermines public confidence" in securities markets. The following quotations are illustrative:

If the investor thinks he's not getting a fair shake, he's not going to invest, and that is going to hurt capital investment in the long run." - Arthur Levitt (former Chairman SEC and American Stock Exchange)¹³

"Powerful Wall Street tycoons convinced former President Bill Clinton not to pardon junk-bond king Michael Milken in a furious last-minute battle, The Post has learned. Personal phone calls to Clinton and White House aides - arguing that the widely expected Milken pardon would undermine public confidence in financial markets - tipped the balance...The opponents of a pardon included Citigroup's influential Robert Rubin, a former treasury secretary in the Clinton administration."¹⁴

"It should give impetus to those dealing in securities and thereby bring back public confidence." - Franklin D. Roosevelt proposing to Congress the legislation which became the Securities Act of 1933 (quoted in Ausubel 1990)¹⁵.

The Public Confidence argument is directly concerned with the quality of information which insiders possess. Indeed, it is largely concerned with extreme cases. The rationale for prohibition of trading on "material, price sensitive, non-public information" is not to prevent market professionals doing their job. In the words of the

¹³"The Epidemic of Insider Trading - The SEC is Fighting a Losing Battle to Halt Stock-Market Abuses" *Business Week* 4/29/85

¹⁴*New York Post* 2/1/01

¹⁵As Ausubel (1990) notes the Securities Act of 1933 was a forerunner to the Securities and Exchange Act of 1934, and was concerned with information disclosure in Initial Public Offerings ("IPOs").

Wall Street Journal, "Market professionals earn their keep by analyzing companies, crunching numbers, interviewing company executives. Their job, in other words, is to come up with material information not yet discovered by the market."¹⁶

The rationale for insider trading laws is not to prevent people trading when they *sense* something, rather when they *know* something. Such laws are designed to stop Boesky Insiders from trading. There is significant evidence that corporate insiders earn above market returns. Seyhun (1992) documents that in the United States "corporate insiders earned an average of 5.1 percent abnormal profits over a one-year holding period between 1980 and 1984, increasing further to 7.0 percent after 1984". This kind of insider trading, though it may be technically illegal, has no apparent negative effect on liquidity and public confidence. Indeed, the huge rise in the liquidity of US equities markets over the 1980s and '90s has been at least correlated with a more than fourfold rise in trading of corporate insiders (Seyhun, 1992). Such trading, illegal or not, is rarely, if ever, pursued by the SEC. It is the Boesky and Milken who raise the spectre of diminished public confidence in securities markets. Not because they are insiders, but because they are Boesky Insiders.

The quantity traded in the equilibrium identified in Proposition 3 is as high as the quantity traded when insider trade is regulated. Therefore, trading on the basis of private information does not necessarily lead to lower liquidity. This observation is somewhat different from the findings of Glosten (1988) and Bajeux and Rochet (1989), who argue that insider trading decreases liquidity and Leland (1992) who as part of a general exploration of the efficiency of insider trading, finds similarly. A somewhat different argument is presented in Ausubel (1990) which articulates an efficiency argument for public confidence in securities markets. Utilizing a two stage model in which there is first investment in a productive asset, and then merely exchange, he argues that insider trading leads to reduced confidence by outsiders and that this may in fact damage insiders through a reduction in investment. Our work does not directly address the important issue of public confidence. A reconsideration of Ausubel's work when the insiders differ in the quality of their information would be a fruitful exercise.

¹⁶*Wall Street Journal* 11/18/86.

5.2 Equilibrium Selection

To draw a direct comparison with the work of Laffont and Maskin (1990), one could consider our model when $B = \emptyset$, i.e. there are no Boesky Insiders. In this case one can construct a separating equilibrium by solving an appropriate differential equation¹⁷. In this equilibrium, the insider's expected profit would be lower than they would have been under NI. This is because under NI the large trader does not face the quantity constraints. For the parameters identified by Proposition 4 or Corollary 1, the pooling equilibrium of Proposition 3 which dominates regulated trade will also dominate the revenue under the separating equilibrium.

If one agrees with Laffont and Maskin that the large trader can influence beliefs in the market to select her favorite equilibrium, then it may be in the interest of the large trader sow the "grain of doubt" about being a Boesky Insider, even if she is not. If they are able to sow this "grain of doubt", *and trade occurs*, they will get their preferred (pooling) equilibrium.

5.3 Robustness of Equilibria

One way to overcome the sensitivity of continuous PBE to zero probability events to assume that incentive compatibility conditions are satisfied except perhaps on a set of measure zero. This would rule out the impact of a zero probability event. Even if one assumes this position, Proposition 1 could be reinterpreted as a failure of equilibria to be robust to small perturbations. For one could just as well assume that $B = (-\epsilon, \epsilon)$ so that the set of Boesky Insiders is a set of measure zero. Any continuous equilibrium is necessarily fully pooling for all $\epsilon > 0$, even though the case $\epsilon = 0$ may allow for a fully separating equilibrium.

5.4 Conclusion

The existing literature on insider trading has tended to over emphasize the role of informational efficiency. To achieve a more complete understanding of the allocative inefficiency, or otherwise, of insider trading, and possible regulatory remedies, one must consider public

¹⁷There may not be an explicit solution. With constant absolute risk aversion, the separating equilibrium pricing strategy, if it exists, is implicitly given by $p(s) = \nu \log(s - p(s) - 2\nu) - s/2 + C$, where C is a constant of integration.

confidence issues in addition to those considered here. This may involve combining features of the type of model considered in Ausubel (1990) with those contained in this paper.

6 Appendix

Given an equilibrium $\{p, Q, G_\rho\}$ recall that $q(s) = Q(p(s))$ is the equilibrium supply of the risky asset in state s . Let

$$V(t, s) = mR + (s - p(t))q(t) \quad (20)$$

$$v(s) = V(s, s). \quad (21)$$

$v(s)$ is the equilibrium payoff of the dominant trader and $V(t, s)$ is the payoff of a type s trader if she pretends to be of type t and offers the price $p(t)$ instead $p(s)$ as required in the equilibrium.

The proof of Proposition 1 Lemma 1-3. Of these only Lemma 3 relies on the fact that $B \neq \emptyset$.

Lemma 1 *If $p(\cdot)$ is an equilibrium pricing strategy and $q(\cdot)$ the corresponding equilibrium supply, the following hold:*

1. $q(\cdot)$ is non-decreasing.
2. Let $\alpha = \sup\{s | q(s) = 0\}$ if $q(s) = 0$ for some s and $\alpha = a$ otherwise. In the region $(\alpha, b]$, $p(\cdot)$ is non-decreasing and is increasing if and only if $q(\cdot)$ is also increasing.
3. $v(\cdot)$ is continuous.
4. If p is continuous, then q is also continuous.

Proof In an equilibrium, a type s trader must have no incentive to choose the price $p(t)$, where $s \neq t$. Therefore

$$\begin{aligned} v(s) &\geq V(t, s) \\ &= v(t) + (s - t)q(t). \end{aligned} \quad (22)$$

As the above must hold for all s, t we have

$$(s - t)q(t) \leq v(s) - v(t) \leq (s - t)q(s) \quad (23)$$

If $s > t$ it is clear from above that $q(s) \geq q(t)$ and therefore q is non-decreasing. This proves part 1.

To see that p must also be non-decreasing, assume by way of contradiction that there exist $\alpha < s < t$ such that $p(s) > p(t)$. Then

$$\begin{aligned} v(s) &= mR + (s - Rp(s))q(s) \\ &< mR + (s - Rp(t))q(t) \\ &\leq V(t, s) \end{aligned}$$

contradicting that $p(\cdot)$ is an equilibrium. We leave it to the reader to follow the above inequalities to conclude that $p(\cdot)$ is increasing if and only if $q(\cdot)$ is increasing. This completes Part 2.

Part 3 is immediate from Equation (23) while Part 4 is immediate from the definition of $v(s)$.

Lemma 2 *Suppose $p(\cdot)$ is a fully revealing equilibrium pricing strategy. The associated equilibrium supply $q(\cdot)$ is such that either $q(s) = 0$ for all $s \in [a, b]$ or $q(s) > 0$ for all $s \in [a, b]$.*

Proof Let $p(\cdot)$ be a fully revealing equilibrium. Define $\alpha = \sup\{s : q(s) = 0\}$ and suppose, by way of contradiction that $a \leq \alpha < \beta$.

Since $q(\cdot)$ is monotonic, it follows that $q(s)$ is zero for all $s < \alpha$ and is positive for all $s > \alpha$. By pretending to be of type $s < \alpha$, the dominant trader can ensure a payoff of mR . Therefore $mR + q(s)(s - Rp(s)) \geq mR$ for all s which in particular implies $p(s) \leq s/R$ for all $s > \alpha$. Taking the limit on both sides of this inequality as s decreases to α , we note that $\rho^+ \leq \alpha/R$ where ρ^+ is the right-hand limit of $p(\cdot)$ at α .

Now suppose $\rho^+ < \alpha/R$. Then we can find $t < \alpha < s$ such that $p(s) < t/R$. The equilibrium payoff of type t is mR . On the other hand if she pretends to be of type s , then her payoff is $mR + q(s)(t - Rp(s))$ which is strictly greater than her equilibrium payoff. This contradicts the fact that $p(\cdot)$ is an equilibrium pricing strategy.

This leaves us with the possibility that $\rho^+ = \alpha/R$. Now for all $s > q(s)$ (except perhaps for $s = s^*$), as $p(\cdot)$ is fully revealing, we must have $q(s) = x(p(s), s)$ for all $s > \alpha$. By taking the limits on both sides of the above we note that the right hand limit of $q(\cdot)$ at α is $\lim_{s \downarrow \alpha} x(\alpha/R, \alpha) = 1$. As $q(\cdot)$ is monotonic, this implies $q(s) = 1$ for all $s > \alpha$.

Lemma 3 *Suppose $B \neq \emptyset$ and $p(\cdot)$ is an equilibrium pricing strategy such that $q(s) > 0$ for all $s \in [a, b]$. Then $p(\cdot)$ cannot be fully revealing.*

Proof The proof is again by contradiction. Suppose $p(\cdot)$ is fully revealing and if possible $q(s) > 0$ for all s .

At s^* , once a small trader observes the price $p(s^*)$, it becomes common-knowledge that the actual return on the risky asset is $s^*/p(s^*)$. If this is higher than R , then $q(s^*) = 0$ and we have a contradiction. On the other hand if $s^*/p(s^*) < R$, then the return on the bond is higher than the risky asset and hence $q(s^*) = 1$. Monotonicity of $q(\cdot)$ would then imply that $q(s) = 1$ for all $s \geq s^*$ which in turn contradicts the fact that $p(\cdot)$ is fully revealing.

It remains to consider the case when $p(s^*) \geq s^*/R$. Let ρ^+ denote the right hand limit of $p(\cdot)$ at s^* . Since $p(\cdot)$ is increasing this limit is well defined. Moreover, $\rho^+ \geq p(s^*) = s^*/R$. Since for all $q(s) = x(p(s), s)$ for all $s \geq s^*$, $\lim_{s \downarrow} q(s) = x(\rho^+, s^*)$. Since $\rho^+ \geq p(s^*) = s^*/R$, it we have $x(\rho^+, s^*) = 1$. Again, by monotonicity of $q(\cdot)$, $q(s) \geq x(\rho^+, s^*) = 1$ for all $s \geq s^*$ and we have a contradiction as before.

Proof (Proposition 1). Suppose $q(t) > 0$ for some $s \in [a, b]$. If $p(\cdot)$ were fully revealing, then by Lemma 2, $q(s) > 0$ for all $s \in [a, b]$ violating Lemma 3. Therefore $p(\cdot)$ cannot be fully revealing.

To complete the proof, we must show that if $p(\cdot)$ is continuous, then it is fully pooling. Let α be as we defined in the proof of Lemma 2 and by hypothesis $a < \alpha$. As $p(\cdot)$ is continuous, there exists $\epsilon > 0$ such that $p(\cdot)$ is strictly increasing in the region $(\alpha - \epsilon, \alpha)$. Therefore $q(s) = x(s, p(s))$ for every $s \in (\alpha - \epsilon, \alpha)$ and hence $\lim_{s \uparrow \alpha} q(s) = x(p(\alpha), \alpha)$. On the other hand, $q(\alpha) = x(p(\alpha), \alpha)$ for some $s \in (\alpha, \beta)$. Therefore, $\lim_{s \uparrow \alpha} q(s) \neq q(\alpha)$ violating the fact that $q(\cdot)$ must also be continuous when $p(\cdot)$ is continuous. A similar argument shows that $\beta = b$.

Proof (Proposition 2) Let

$$\rho^m(s) = \arg \max_{\rho} (s - \rho)(x(\rho, b))$$

$$q^m(s) = x(\rho^m(s), b).$$

By definition, $\pi(s) = mR + (s - \rho^m(s))q^m(s)$. By the Envelope Theorem, $\pi'(s) = q^m(s)$. Arguments similar to those in the proof of Part I of Lemma 1 can now be used to show that $\rho^m(s)$ and $q^m(s)$ are both (weakly) increasing in s and hence $\pi(\cdot)$ is convex.

Take the system of beliefs to be as in the proof of Proposition : a small trader's posterior on \tilde{S} is the same as her prior if $\rho = \rho^*$ and when $\rho \neq \rho^*$ she believes the highest type b .

q^* by construction is a best response to the realized price ρ^* . Moreover, the beliefs of the small trader are in accordance with Bayesian updating.

Given $\rho^* < a/R$, it is clear that Player D is better off trading the equilibrium $q^* > 0$ than not trading. If a type s trader were to deviate and choose a price $\rho \neq \rho^*$, given the beliefs of the small trader, she faces the $x(\cdot, b)$ as the supply of risky asset. Therefore, her maximum payoff from such a deviation is $\pi(s)$. To prevent a deviation, one must have $v(s) \geq \pi(s)$ for all s . Since $s = \lambda a + (1 - \lambda)b$ for some $\lambda \in [0, 1]$

$$\begin{aligned} v(s) &= \lambda v(a) + (1 - \lambda)v(b) \\ &\geq \lambda \pi(a) + (1 - \lambda)\pi(b) \\ &\geq \pi(s) \end{aligned} \tag{24}$$

where the first of the above inequalities is by hypothesis and the second is due to the convexity of π .

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