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under Stochastic Emissions

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Spatial Economic Theory of Pollution Control under Stochastic Emissions

Abstract. This paper examines the effectiveness of environmental policies in curtailing pollution of a firm which is operating in a space economy under stochastic emissions. We consider a general n-input planar space production-location model, in which the output is produced jointly with the byproduct pollution. Although production is non-stochastic, the resulting pollution emission is assumed to have a random component, and the polluting firm must make its production and location decisions before the uncertainty is resolved. We provide some propositions concerning the comparative statics of the polluting firm's location choices, urban pollution concentration, and the relative impact of regulation via emission taxes to that of emission standards.

Key words: location theory; pollution control; emission uncertainty

JEL classification: D80; H25; Q28; R38

1. Introduction

In recent years, much effort has been channeled into studying the effects of environmental policy on plant location. There are significant contributions to the literature on how a firm's location *within a region* is influenced by environmental policies. Studies along this line include Mathur (1976), Gokturk (1979), Forster (1987) and Hwang and Mai (2003). The results they obtain indicate that a tax and/or regulation on urban pollution concentration may not succeed in pushing the polluting firm away from the urban center. The other strand of the literature addresses the issue of plant endogeneity in a *two-region* model that involves two polluting firms whose location decisions are influenced by the environmental policies. In the analyses of Markusen, et al. (1993), Motta and Thisse (1994) and Ulph (1994), it is assumed that the environmental policy is given or that the environmental targets are exogenous, environmental policies are shown to have a very strong impact on a firm's choice of location. In Markusen, et al. (1995), Rauscher (1995), Hoel (1997), Markusen (1997), Ulph and Valentini (1997) and Bárcena-Ruiz and Garzón (2003), however, the environmental policy is considered to be endogenous and there are strategic interactions between governments. These studies show that the Nash equilibria of the game are generally not Pareto optimal.

The literature is cast entirely in a deterministic world. This is somewhat surprising, since pollution is random by nature. For given production levels, there may exist considerable uncertainty regarding the magnitude of pollution consequences. Fuel and other inputs may be random in quality, while weather and other stochastic environmental factors may also contribute to the level of pollution generated (Hennessy and Roosen, 1999).[1] As it is well known that uncertainty affects the efficiency of the firm's performance, the purpose of this paper is to extend the analysis by introducing emission uncertainty.

Specifically, to investigate the effectiveness of environmental policies in curtailing

[1]Even if pollution is nonrandom, measurement errors may give rise to stochastic penalities.

the pollution of a firm, we consider a general n-input planar space production-location model, which subsumes the linear market stipulated by Mathur (1976), Gokturk (1979) and Forster (1987) and the Weberian triangle adopted by Hwang and Mai (2003) as special cases. The output is produced jointly with the byproduct pollution. Although production is nonstochastic, the resulting pollution emission is assumed to have a random component, and the polluting firm must make its production and location decisions before the uncertainty is resolved.

In addition to the Introduction, this paper has four other sections. Section 2 poses the expected utility maximization problem for the polluting firm, and also characterizes the optimal production and location conditions. Section 3 develops the comparative statics concerning the effects of emission control on the polluting firm's choice of location and the pollution concentration in the urban center, and in Section 4 we compare the impact of regulation via emission taxes to that of emission standards on the polluting firm's expected utility and urban pollution concentration. Section 5 concludes.

2. The Model

It is assumed that that there are $n+1$ relevant sites for the polluting firm: one is the market site for the output, which can be thought of as a high density urban center (point C in Figure 1), while the others are the locations from where the inputs come (points $L_1, \ldots, L_n$ in Figure 1).

[Place Figure 1 about here.]

According to Figure 1, h is the distance between the firm's plant location R and urban center C; and s_i is the distance between the firm's plant location R and L_i, the site of input M_i. Based on the law of cosine, the distances $s_1, \ldots, s_n$ can be expressed as

$$s_1 = \sqrt{d_1^2 + h^2 - 2d_1 h \cos\theta},$$

$$s_i = \sqrt{d_i^2 + h^2 - 2d_i h \cos|\theta_i - \theta|}, \qquad i = 2, \ldots, n,$$

where d_i denotes the distance between the urban center C and L_i; and $\theta, \theta_2, \ldots, \theta_n$ are, respectively, the angles $L_1CR, L_1CL_2, \ldots, L_1CL_n$. Given the sites of C and $L_1, \ldots, L_n$, it is evident that each firm's plant location is determined once θ and h are chosen.

Output q and pollution emissions ω are joint products of the inputs. The production function of the firm is specified as

$$Q = F(M_1, \ldots, M_n),$$

where F is a twice continuously-differentiable function with the properties that $F_i \equiv \partial F/\partial M_i > 0$ and $F_{ii} \equiv \partial^2 F/\partial M_i^2 \leq 0, \quad i = 1, \ldots, n$.

Although production is assumed to be nonstochastic, the resulting pollution emission is held to be proportional to production and to have a random component:

$$\omega = \gamma Q \varepsilon. \tag{1}$$

Here, γ is a constant of proportionality that can be called the pollution coefficient. The multiplicative source of randomness, ε, is assumed to have unit expected value and standard deviation σ, namely

$$\varepsilon = 1 + \sigma z, \tag{2}$$

with $E(z) = 0, \sigma_z^2 = 1$ and $\Pr(z > -1/\sigma)$=1. The randomness may arise from variations in input quality (e.g., more sulfurous coal) or in weather variables such as rain and wind (Hennessy and Roosen, 1999).

The government can use one of two instruments for controlling pollution, an *emission tax*, where emissions are taxed at a rate t per unit of pollution emission, or an *emission standard*, where the government announces an upper limit $\bar{\omega}$ on emission quantity, with a fine τ being imposed per unit of excess emission. We will not analyze how the government makes the choice of the tax rate or emission standard, but will simply assume they are given. It is also assumed that the government can perfectly monitor the firm's emissions. Thus, there is no room for moral hazard, and the firm will pay a pollution bill exactly according to the amount of pollutants they emit.

The profit of the firm is

$$\pi = (p - rh)Q - \sum_{i=1}^{n}(p_i + r_i s_i)M_i - \chi.$$

It is assumed that the firm is a price taker in all markets, $(p, p_1, \ldots, p_n)$ are the prices of output and inputs, $(r, r_1, \ldots, r_n)$ denote the constant transport rates per unit quantity and distance of output and inputs, and $p_i + r_i s_i$ is the delivered price of input M_i; χ is the pollution payment of the firm, which equals $t\omega$ if the environmental policy is an emission tax, whereas $\chi = \tau(\omega - \bar{\omega})$ if an emission standard is imposed.[2]

Without knowing the exact pollution emission, the firm has to make decisions concerning production and plant location. It is assumed that the objective of the firm is to maximize the expected utility of profits. The firm's attitude towards risk is summarized by an increasing, concave von Neumann-Morgenstern utility function U, with $U' \equiv dU/d\pi > 0$ and $U'' \equiv d^2U/d\pi^2 < 0$. That is, the firm seeks to

$$\max_{M_1,\ldots,M_n,\theta,h} E[U(\pi)].$$

To explore the firm's optimal decisions, a two-stage optimization algorithm is used. Moreover, at each stage of the maximization problem, it is supposed that an interior solution exists, and that the second-order conditions are satisfied.

The first stage of the maximization problem involves finding the optimal input combination which maximizes the expected utility at a given location choice (h, θ). The first-order conditions for the first-stage maximization are

$$\frac{\partial E[U(\pi)]}{\partial M_i} = E[U' \cdot \pi_i] = E[U' \cdot ((p - rh - \upsilon)F_i - P_i)] = 0, \qquad i = 1, \ldots, n, \quad (3)$$

[2]We assume $\omega > \bar{\omega}$ throughout the analysis for the sake of simplicity.

where $P_i \equiv p_i + r_i s_i$, and $\upsilon \equiv \partial\chi/\partial M_i = t\gamma\varepsilon \quad (\tau\gamma\varepsilon)$ if the pollution control policy is an emission tax (standard).

In view of (3), we see that the following relationship always holds:

$$\frac{F_i}{F_j} = \frac{P_i}{P_j}, \qquad i, j = 1, \ldots, n, \quad i \neq j. \tag{4}$$

We have from (4) that, *ceteris paribus*, the firm's expansion paths are straight lines through the origin under emission uncertainty if the production function is homothetic. This equation is important in the comparative statics that follows.

Let $M_i(\theta, h)$ be the value of M_i satisfying (3), and let $\pi(\theta, h)$ be the corresponding maximum value of profits. The second stage of the optimization problem is to find the $(\theta,\, h)$ that maximizes $E[U(\pi(\theta, h))]$, which yields the following first-order conditions

$$\frac{\partial E[U(\pi(\theta, h))]}{\partial \theta} = -\sum_{i=1}^{n} r_i s_{i\theta} M_i(\theta, h) E[U'] = 0, \tag{5}$$

$$\frac{\partial E[U(\pi(\theta, h))]}{\partial h} = -[rQ + \sum_{i=1}^{n} r_i s_{ih} M_i(\theta, h)] E[U'] = 0. \tag{6}$$

Since $E[U'] > 0$, we know from (5) and (6) that

$$\sum_{i=1}^{n} r_i s_{i\theta} M_i(\theta, h) = 0, \tag{7}$$

$$rQ + \sum_{i=1}^{n} r_i s_{ih} M_i(\theta, h) = 0. \tag{8}$$

We are now in a position to examine the effect of emission control on the polluting firm's choice of plant location and on the pollution concentration in the urban center.

3. Emission Control, Production Location and Urban Pollution Concentration

3.1 When Will the Location Invariance Hold?

Hwang and Mai (2003) have demonstrated within a deterministic Weberian space model that the plant location of a polluting firm is invariant with respect to a change in emission standard if the production function is linear homogeneous. The following proposition extends their result to the most general n-input planar space case with stochastic emissions, and to the imposition of an emission tax:

Proposition 1. *Assume a world with emission uncertainty. The polluting firm's choice of location is invariant with respect to the imposition of an emission tax/standard if the production function is linear homogeneous.*

Proof: It is easy to see from (7) and (8) that the optimal (θ, h) will not change (and hence location invariance will occur) as long as $(M_1, ..., M_n)$ move proportionately and Q/M_i remains unaltered for any i after the imposition of an emission tax/standard. If the production function is linear homogeneous, $F_i(M_1, ...M_n)$ is homogeneous of degree zero for any i and, thus, (4) remains satisfied for any input combination proportional to $(M_1, \ldots, M_n)$. Since any proportionate change in $(M_1, ..., M_n)$ will also keep $F(M_1, ..., M_n)/M_i$ unaltered for any i, the same location choice must be chosen from (7) and (8). $\diamondsuit$

3.2 Predicting the Location Shift

It is also of interest to know the shift in location if the polluting firm's plant location is not invariant with respect to the imposition of an emission tax/standard. In their attempts, Mathur (1976) and Gorturk (1979) showed in a linear space model that an emission tax may not succeed in pushing the polluting firm away from the urban center. Hwang and Mai (2003) demonstrated unambiguously the impact of a change in emission standard on plant location when the production function is homogeneous of degree $k \neq 1$. In the following, we present results that generalize previous findings. The proof is given in Appendix 1.

Proposition 2. *Assume a world with emission uncertainty. The polluting firm's absolute risk aversion index is decreasing and its relative risk aversion index is increasing. The firm's choice of plant location will be farther away from (nearer to) the urban cen-*

ter in response to the imposition of an emission tax/standard if the production function is homogeneous of degree $k > (<) 1$.

Proposition 2 generalizes previous findings in the sense that the result is valid for any planar space, not just for the locational triangle or the linear market. It also holds for any number of inputs, not just for the case of one or two inputs, and holds for any form of emission uncertainty, not just for the case of a deterministic world.

To interpret the result of Proposition 2, note that from (7) and (8), the plant location is determined entirely by the trade-off between the transportation costs of the output and the inputs under emission uncertainty. Following the imposition of an emission tax/standard, the distribution of π shifts to the left. The expected profit is lower, and the variation in the profit distribution either increases proportionally (in the case of an emission tax) or remains constant (in the case of an emission standard). Given that the absolute risk aversion index is decreasing and the relative risk aversion index is increasing, the polluting firm considers itself poorer and tends to be *more* risk-averse. Therefore, the level of output *decreases*. The optimum input ratios, however, remain unchanged due to the homogeneity of the production function, and the input/output ratios rise/decline if the production function exhibits increasing/decreasing returns to scale. The pulling force of output, therefore, decreases/increases in relation to that of inputs and, as a result, the polluting firm's choice of plant location will be farther away from/nearer to the urban center.

3.3 Effect of Emission Control on Urban Pollution Concentration

Following Mathur (1976), it is assumed that the pollution concentration in the urban center, $\omega(0)$, depends upon the amount of emissions generated at the production site and upon the distance between the urban center and the production site, namely

$$\omega(0) = \omega \cdot \phi(h), \tag{9}$$

where $\phi(.)$ is the pollution diffusion function over distance with $\phi' < 0$ and $\phi'' \geq 0$.

Since $\omega(0)$ is random, we examine the effect of a stricter emission control policy on expected urban pollution concentration, which gives

$$\frac{\partial E[\omega(0)]}{\partial t} = \gamma \cdot [\frac{\partial Q}{\partial t}\phi + \frac{\partial h}{\partial t}\phi' Q], \quad (10)$$

$$\frac{\partial E[\omega(0)]}{\partial \bar{\omega}} = \gamma \cdot [\frac{\partial Q}{\partial \bar{\omega}}\phi + \frac{\partial h}{\partial \bar{\omega}}\phi' Q]. \quad (11)$$

The first terms on the right hand side of (10) and (11) represent the *output* effect of emission control on expected pollution concentration in the urban center, whereas the second terms represent the *locational* effect. Given decreasing absolute risk aversion and increasing relative risk aversion, the level of output decreases unambiguously in response to a stricter emission control policy.[3] However, it may not succeed in pushing the polluting firm away from the urban center if the firm's production function exhibits decreasing returns to scale (based on Proposition 2). In that case, if the locational effect outweighs the output effect, a stricter emission control policy may lead to a higher expected urban pollution concentration. This leads to

Proposition 3. *Assume a world with emission uncertainty. The polluting firm's absolute risk aversion index is decreasing and its relative risk aversion index is increasing. A stricter emission control policy will reduce expected pollution concentration in the urban center if the firm's production function exhibits non-decreasing returns to scale. However, it may result in an increase in expected urban pollution concentration if the production function exhibits decreasing returns to scale.*

4. Emission Taxes vs Standards

Policy-makers are regularly confronted with the task of choosing policy instruments to achieve environmental goals. In an effort to shed some light on the policy implications of the results, we address the following question below: Is some form of pollution

[3] Note that a *decrease* in $\bar{\omega}$ indicates that the government adopts a stricter emission control policy.

control policy more preferable to firms or governments than the other type?

In making comparisons of the impacts of emission taxes/standards, the concept of differential incidence is used,[4] which requires comparing the two instruments with equal expected emission payments, namely

$$t\gamma Q = \tau(\gamma Q - \bar{\omega}).$$

It follows that

$$\gamma Q dt = -\tau d\bar{\omega}. \tag{12}$$

Through the use of (12) and the envelope theorem, we have[5]

$$\frac{\partial E[U(\pi)]}{\partial t} + \frac{\gamma Q}{\tau} \cdot \frac{\partial E[U(\pi)]}{\partial \bar{\omega}} = -\gamma Q E[U'z], \tag{13}$$

$$\frac{\partial E[\omega(0)]}{\partial t} + \frac{\gamma Q}{\tau} \cdot \frac{\partial E[\omega(0)]}{\partial \bar{\omega}} = \gamma \cdot \{\phi \cdot (\frac{\partial Q}{\partial t} + \frac{\gamma Q}{\tau} \cdot \frac{\partial Q}{\partial \bar{\omega}}) + \phi' Q \cdot (\frac{\partial h}{\partial t} + \frac{\gamma Q}{\tau} \cdot \frac{\partial h}{\partial \bar{\omega}})\}. \tag{14}$$

Given that $U'' < 0$, the sign of (13) is unambiguously negative since $E[U'z] > 0$,[6] whereas that of (14) depends on the firm's attitudes toward risk and the characteristics of the production function. The following proposition summarizes the results, and the proof is given in Appendix 2.

Proposition 4. *Assume a world with emission uncertainty, and in which the polluting firm's expected emission payment is fixed. (i) The polluting firm's expected utility associated with the imposition of an emission tax will be lower than that associated with the imposition of an emission standard. (ii) Given that the polluting firm's absolute risk aversion index is decreasing and its relative risk aversion index is increasing, the imposition of an emission tax will result in lower expected pollution concentration in the urban center than will the emission standard if the production function exhibits non-decreasing returns to scale.*

[4]See Musgrave (1959).
[5]See footnote 2.
[6]The proof is similar to that previously demonstrated by Sandmo (1971, p. 67).

The result of Proposition 4(i) is intuitively appealing: recall that the imposition of an emission tax makes the variation in profit increase, whereas it remains unchanged in the case where an emission standard is imposed. Given that the polluting firm is risk averse, with a fixed expected emission payment, it would certainly prefer emission standards to taxes. To interpret part (ii) of Proposition 4, note that a greater variation in the profit distribution caused by the imposition of an emission tax will make the firm *more* risk averse than will the emission standard. Therefore, relative to the imposition of an emission standard, the polluting firm will decrease output more and the choice of plant location will be farther away from (the same distance from, nearer to) the urban center in response to the imposition of an emission tax if the production function is homogeneous of degree $k > (=, <) 1$. The expected pollution concentration in the urban center will thus be lower when the governments use emission taxes than when they use emission standards if the polluting firm's production function exhibits non-decreasing returns to scale.

5. Conclusion

This paper examines the effectiveness of environmental policies in curtailing the pollution of a firm which is operating in a space economy under stochastic emissions. Specifically, we consider a general n-input planar space production-location model. The output is produced jointly with the byproduct pollution. Although production is nonstochastic, the resulting pollution emission is assumed to have a random component, and the polluting firm must make its production and location decisions before the uncertainty is resolved. It is demonstrated that

(i) location invariance occurs for emission taxes/standards if the polluting firm's production function exhibits constant returns to scale (Proposition 1).

(ii) Given decreasing absolute risk aversion and increasing relative risk aversion, the polluting firm's choice of plant location will be farther away from (nearer to) the urban center in response to the imposition of an emission tax/standard if the production function exhibits increasing (decreasing) returns to scale (Proposition 2).

(iii) A stricter emission control policy may result in an increase in expected urban pollution concentration if the polluting firm's production function exhibits decreasing returns to scale (Proposition 3).

This paper also compares the impact of regulation via emission taxes to that in relation to emission standards on the polluting firm's expected utility and the pollution concentration in the urban center (Proposition 4). It shows that, for a given amount of expected emission payment, a risk-averse polluting firm would prefer emission standards to emission taxes. However, given that the polluting firm's absolute risk aversion index is decreasing and its relative risk aversion index is increasing, to reduce the expected pollution concentration in the urban center, the choice of an emission tax Pareto dominates an emission standard if the production function exhibits non-decreasing returns to scale.

The production-location framework employed in our analysis, while useful for providing insights into the output/spatial effects of environmental policy, has little to say regarding questions as to the choice of envionmental policies by governments. Extending the model to address optimal environmental taxation/standards is a promising area for future research. Alternatively, more extensive policy analysis, such as studying the effects of subsidies to a residual abating input or issuing a number of tradable permits, also appears to be an interesting direction for future research.

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Appendix 1. Proof of Proposition 2.

Totally differentiating (7) and (8) with respect to θ, h, t and $\bar{\omega}$ yields the following comparative static matrix:

$$\begin{pmatrix} \pi_{\theta\theta} & \pi_{\theta h} \\ \pi_{h\theta} & \pi_{hh} \end{pmatrix} \begin{pmatrix} d\theta \\ dh \end{pmatrix} = - \begin{pmatrix} \pi_{\theta t} \\ \pi_{ht} \end{pmatrix} dt - \begin{pmatrix} \pi_{\theta\bar{\omega}} \\ \pi_{h\bar{\omega}} \end{pmatrix} d\bar{\omega} \qquad (A.1)$$

where

$\pi_{\theta\theta} \equiv -\sum_{i=1}^{n}[r_i s_{i\theta\theta} M_i + r_i s_{i\theta} \cdot (\partial M_i/\partial\theta)] < 0$

$\pi_{\theta h} = \pi_{h\theta} \equiv -\sum_{i=1}^{n}[r_i s_{i\theta h} M_i + r_i s_{i\theta} \cdot (\partial M_i/\partial h)]$

$\pi_{hh} \equiv -\sum_{i=1}^{n}[r_i s_{ihh} M_i + (rF_i + r_i s_{ih}) \cdot (\partial M_i/\partial h)] < 0$

$\pi_{\theta t} \equiv -\sum_{i=1}^{n} r_i s_{i\theta} \cdot (\partial M_i/\partial t)$

$\pi_{ht} \equiv -\sum_{i=1}^{n}(rF_i + r_i s_{ih}) \cdot (\partial M_i/\partial t)$

$\pi_{\theta\bar{\omega}} \equiv -\sum_{i=1}^{n} r_i s_{i\theta} \cdot (\partial M_i/\partial\bar{\omega})$

$\pi_{h\bar{\omega}} \equiv -\sum_{i=1}^{n}(rF_i + r_i s_{ih}) \cdot (\partial M_i/\partial\bar{\omega})$

Via (A.1), we obtain the following comparative statics:

$$\frac{\partial h}{\partial t} = \frac{1}{D_2} \cdot (\pi_{h\theta}\pi_{\theta t} - \pi_{\theta\theta}\pi_{ht}) \qquad (A.2)$$

$$\frac{\partial h}{\partial\bar{\omega}} = \frac{1}{D_2} \cdot (\pi_{h\theta}\pi_{\theta\bar{\omega}} - \pi_{\theta\theta}\pi_{h\bar{\omega}}) \qquad (A.3)$$

where $D_2 \equiv \pi_{\theta\theta}\pi_{hh} - \pi_{\theta h}^2 > 0$ by the second-order condition of the second stage of the maximization problem.

Note that, from (7), $r_i s_{i\theta} M_i = \sum_{j\neq i} r_j s_{j\theta} M_j$. Therefore, we can rewrite $\pi_{\theta t}$ and $\pi_{\theta\bar{\omega}}$ as

$$\pi_{\theta t} = -\sum_{j\neq i} r_j s_{j\theta} \cdot [M_i \cdot (\partial M_j/\partial t) - M_j \cdot (\partial M_i/\partial t)]/M_i \qquad (A.4)$$

$$\pi_{\theta\bar{\omega}} = -\sum_{j\neq i} r_j s_{j\theta} \cdot [M_i \cdot (\partial M_j/\partial\bar{\omega}) - M_j \cdot (\partial M_i/\partial\bar{\omega})]/M_i \qquad (A.5)$$

If the production function is homogeneous of degree k, then

(i) Equation (4) remains satisfied when $M_1, \ldots, M_n$ change proportionally in response to a perturbation in t or $\bar{\omega}$, namely

$$\frac{(\partial M_i/\partial t)}{(\partial M_j/\partial t)} = \frac{(\partial M_i/\partial \bar{\omega})}{(\partial M_j/\partial \bar{\omega})} = \frac{M_i}{M_j}. \tag{A.6}$$

Substituting (A.6) into (A.4) and (A.5) gives $\pi_{\theta t} = \pi_{\theta \bar{\omega}} = 0$.

(ii) Since $Q = \sum_{i=1}^{n} F_i M_i - (k-1)Q$, (8) can be rewritten as $\sum_{i=1}^{n}(rF_i + r_i s_{ih})M_i - (k-1)rQ = 0$ and, therefore, we have $(rF_i + r_i s_{ih}) = -\sum_{j\neq i}(rF_j + r_j s_{jh})M_j/M_i + (k-1)rQ/M_i$, it follows that

$$\pi_{ht} = -(k-1)rQ \cdot (\partial M_i/\partial t)/M_i, \tag{A.7}$$

$$\pi_{h\bar{\omega}} = -(k-1)rQ \cdot (\partial M_i/\partial \bar{\omega})/M_i. \tag{A.8}$$

When t increases or $\bar{\omega}$ decreases, the firm's profit level shifts downward. Since the absolute risk aversion index is decreasing, the firm will decrease its optimal production, and hence $\partial Q/\partial t < 0$ and $\partial Q/\partial \bar{\omega} > 0$. Through the use of the chain rule, $\partial M_i/\partial t = (\partial M_i/\partial Q)\cdot(\partial Q/\partial t)$ and $\partial M_i/\partial \bar{\omega} = (\partial M_i/\partial Q)\cdot(\partial Q/\partial \bar{\omega})$. Since all inputs are assumed to be normal, $\partial M_i/\partial Q > 0 \ \forall i$, and thus $\partial M_i/\partial t < 0$ and $\partial M_i/\partial \bar{\omega} > 0$.

Substituting (A.7), (A.8) and $\pi_{\theta t} = \pi_{\theta \bar{\omega}} = 0$ into (A.2) and (A.3), we arrive at Proposition 2.

Appendix 2. Proof of Part (ii) of Proposition 4.

Note that the total variable cost function can be derived by minimizing the total variable cost subject to a given output level, namely

$$C(P_1, ..., P_n, Q) = \min_{M_1,...,M_n} \sum_{i=1}^{n} P_i M_i \quad \text{s.t.} \quad Q = F(M_1, ..., M_n)$$

If the production function is homothetic, as is well known, $C(.)$ can be expressed as the product of two functions:

$$C(P_1, ..., P_n, Q) = c(P_1, ..., P_n)H(Q).$$

Moreover, the following relationship always holds

$$\frac{H}{Q} \begin{matrix} > \\ = \\ < \end{matrix} H_Q \text{ if the production function exhibits } \begin{matrix} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{matrix} \text{ returns to scale.} \qquad (A.9)$$

The polluting firm's profit function can thus be rewritten as

$$\pi = [(p - rh)Q - c(P_1, ..., P_n)H(Q)] - \chi.$$

The objective of the firm is to find out the optimal level of output and the plant location in order to maximize $E[U(\pi)]$. The first-order conditions for the optimization problem are

$$\partial E[U(\pi)]/\partial Q = E[U'\pi_Q] = 0 \qquad (A.10)$$

$$\partial E[U(\pi)]/\partial \theta = -c_\theta E[U'] = 0 \qquad (A.11)$$

$$\partial E[U(\pi)]/\partial h = -(rQ + c_h H)E[U'] = 0 \qquad (A.12)$$

where $\pi_Q = p - rh - c(.)H_Q - t\gamma\varepsilon \;\; (= p - rh - c(.)H_Q - \tau\gamma\varepsilon)$ if an emission tax (standard) is imposed.

Totally differentiating (A.10)–(A.12) with respect to t and $\bar{\omega}$ yields

$$\frac{\partial Q}{\partial t} = \frac{\gamma D_1}{D}\{E[U'\varepsilon] + QE[U''\pi_Q\varepsilon]\} \qquad (A.13)$$

$$\frac{\partial Q}{\partial \bar{\omega}} = -\frac{\tau D_1}{D}E[U''\pi_Q] \qquad (A.14)$$

$$\frac{\partial h}{\partial t} = \frac{\gamma c_h c_{\theta\theta}}{D} \cdot (\frac{H}{Q} - H_Q)\{E[U'\varepsilon] + QE[U''\pi_Q\varepsilon]\}\{E[U']\}^2 \cdot H \qquad (A.15)$$

$$\frac{\partial h}{\partial \bar{\omega}} = -\frac{\tau c_h c_{\theta\theta}}{D} \cdot (\frac{H}{Q} - H_Q)E[U''\pi_Q]\{E[U']\}^2 \cdot H \qquad (A.16)$$

where D is the relevant Hessian determinant, and $D_1 \equiv (c_{\theta\theta} c_{hh} - c_{\theta h}^2)\{E[U'] \cdot H\}^2$. It follows that

$$\frac{\partial Q}{\partial t} + \frac{\gamma Q}{\tau} \cdot \frac{\partial Q}{\partial \bar{\omega}} = \frac{\gamma D_1}{D}\{E[U'\varepsilon] + QE[U''\pi_Q z]\} \tag{A.17}$$

$$\frac{\partial h}{\partial t} + \frac{\gamma Q}{\tau} \cdot \frac{\partial h}{\partial \bar{\omega}} = \frac{\gamma c_h c_{\theta\theta}}{D} \cdot (H/Q - H_Q)\{E[U'\varepsilon] + QE[U''\pi_Q z]\}\{E[U']\}^2 \cdot H \tag{A.18}$$

We have $D < 0$, $D_1 > 0$, and $c_{\theta\theta} > 0$ by the second-order conditions, and $c_h < 0$ from (A.12). Moreover, following similar procedures as those previously adopted by Sandmo (1971, pp. 68-70), it is easy to show that $E[U''\pi_Q] > 0$ if the absolute risk aversion index is decreasing, and $E[U''\pi_Q \pi] < 0$ if the relative risk aversion index is increasing. Accordingly, $E[U''\pi_Q z] > 0$ given that the absolute risk aversion index is decreasing and the relative risk aversion index is increasing. It follows that the sign of (A.17) is strictly negative, while that of (A.18) is positive (zero, negative) if the production function exhibits increasing (constant, decreasing) returns to scale based on (A.9). Put together, we see from (14) that part (ii) of Proposition 4 holds.

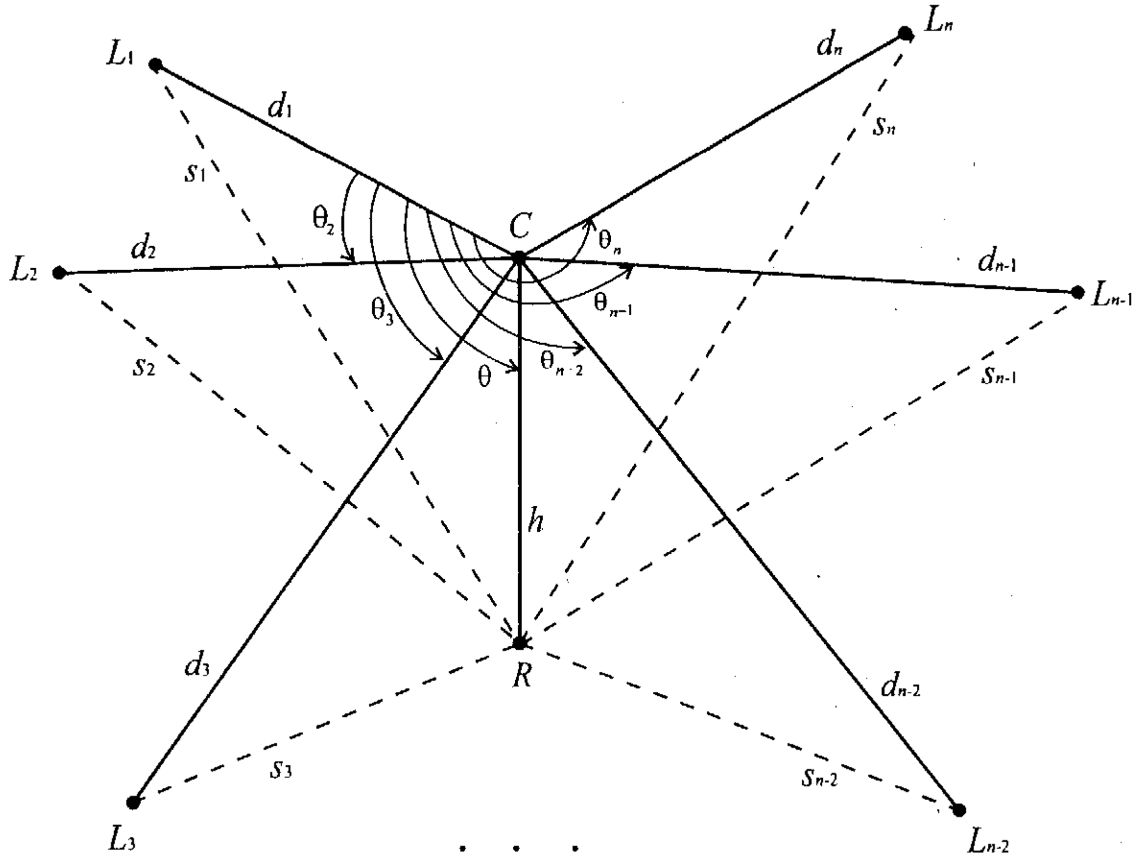

FIGURE 1: The n-input Planar Space.

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