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Abstract

This paper extends Graetz, Reinganum and Wilde's (1986) seminal work on tax compliance to the real-world scenario where the IRS (Internal Revenue Service) faces a budget constraint imposed upon her by the Congress. The paper consists of two parts. The first part is positive – we characterize the equilibria resulting from the interaction between taxpayers and the budget-constrained IRS. The second part is normative – we examine the efficiency implication of varying the size of the budget allocated to the IRS. It is shown that, to mitigate or eliminate the so-called "congestion effect," the IRS should be sufficiently budgeted and, in particular, we provide a case for the policy prescription that the size of the budget allocated to the IRS should be expanded as

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long as an additional dollar allocated could return more than an additional dollar of tax revenue.

1 Introduction

> "Unlike other government agencies, there is a positive return on money invested in the IRS. ... In its FY2007 budget recommendation, the Board calls for a modest increase in enforcement that would result in a real return on investment, ranging from three to six dollars on every dollar spent, resulting in $730 million revenue increase by FY2009 on a $242 million investment."
>
> IRS Oversight Board (2006, pp. 12-13)[1]

On the basis of a 3-1 to 6-1 return for an additional dollar invested, does it make sense for the Board to recommend an expanded IRS budget on enforcement? This paper provides a case for the positive answer.

We consider a model of tax compliance, which extends the seminal work of Graetz, Reinganum and Wilde (1986, hereafter GRW) to the real-world scenario where the IRS faces a budget constraint imposed upon her by the Congress. The paper consists of two parts. The first part is positive. We characterize the equilibria resulting from the interaction between taxpayers and the budget-constrained IRS, and study the impact of imposing budget constraints on the IRS. The second part is normative. We examine the efficiency implication of varying the size of the budget allocated to the IRS and, in particular, we ask: how much should we fund the IRS?

Unlike the classical work of Allingham and Sandmo (1972) and Yitzhaki (1974) on tax evasion, which treats the IRS actions as exogenous, the GRW model views the IRS as a

[1] "The IRS Oversight Board was created by the IRS Restructuring and Reform Act of 1998 (RRA 98), which was enacted to improve the IRS so that it may better serve the public and meet the needs of taxpayers." (see the web-site of the Board)

strategic player that interacts with taxpayers. The GRW model also differs from the principal-agent tax evasion model first introduced by Reinganum and Wilde (1985). As pointed out by GRW, the principal-agent model suffers from the time inconsistency problem since it requires that the IRS announce and commit to an audit policy, even though the precommitted audit policy will typically prove suboptimal once taxpayers submit their reported income. GRW emphasize that their interactive model follows the natural temporal sequence of decisions: first, taxpayers report their income, and only then does the IRS decide whether to perform tax audits.[2]

Graetz, Reinganum and Wilde (1984, hereafter GRW′) also extend the GRW model to account for the effect of imposing budget constraints on the IRS. However, their way of deriving equilibria is somewhat complicated. We believe our approach greatly simplifies the analysis. More importantly, we further address the efficiency issue across equilibria whereas they do not. We will compare our approach with the GRW′ approach after we derive the equilibria of our model.[3]

Slemrod and Yitzhaki (1987) investigate the same normative question as our paper. The main differences in modeling include: (i) while they study tax auditing *with commitment*, we study tax auditing *without commitment*; and (ii) while they subsume the IRS and the Congress under the rubric of a single player called "government," we treat the IRS and the Congress as two different players. Perhaps more interestingly, the policy prescription derived from our model starkly contrasts that derived from their model. Slemrod and Yitzhaki prescribe

[2]For recent surveys of the tax evasion literature, see Andreoni et al (1998), Slemrod and Yitzhaki (2002), Cowell (2004) and Sandmo (2005). Auditing is typically a negative-sum game between taxpayers and the IRS. However, using auditing as a threat, it may be feasible for the IRS to create a Pareto-improving outcome for both taxpayers and the IRS under some circumstances; see Chu (1990) and Ueng and Yang (2000, 2001) for the details.

[3]Macho-Stadler and Perez-Castrillo (1997) consider the effect of imposing budget constraints on the IRS in a principal-agent framework. They do not address the efficiency issue either.

that the size of the budget allocated to the IRS *should not* be expanded to the level where an additional dollar allocated would return just an additional dollar of tax revenue.[4] By contrast, we prescribe that the size of the budget allocated to the IRS *should* be expanded until an additional dollar allocated would return just an additional dollar of tax revenue. It is shown within our model that our policy prescription always dominates other policy prescriptions that fall short of equating the marginal revenue to the marginal cost of tax collection. The dominating criterion is based on the lower efficiency cost per dollar of tax collection. We will make more comparisons between our result and theirs when we address the efficiency issue.

The remainder of this article is organized as follows. Section 2 describes the basic model. Section 3 provides the full characterization of the equilibrium outcomes. Section 4 addresses the efficiency issue and provides an answer to the optimal funding of the IRS. Section 5 considers an extension of the basic model and Section 6 concludes.

2 Basic Model

Our basic model is essentially the same as the GRW model. For ease of exposition, however, we transform the GRW model into an equivalent, but simpler, model. GRW assume that taxpayers earn either high or low income. High-income taxpayers need to pay a high tax, while low-income taxpayers need to pay a low tax. We normalize the low income of the GRW model to zero so that either taxpayers earn an income or they do not. Only the taxpayers who earn the income need to pay tax in our model. GRW defend their simple setup by arguing that "the model might also be viewed as addressing issues of noncompliance across a

[4] This conclusion is also drawn by Usher (1986), Kaplow (1990), Mayshar (1991), and Sanchez and Sobel (1993). A feature of the Slemrod-Yitzhaki model that these papers have in common is the auditing with commitment. Except for the principal-agent model of Sanchez and Sobel (1993), these papers, similar to Slemrod and Yitzhaki (1987), subsume the IRS and the Congress under the rubric of a single player called "government."

relatively small range of income–for example, within a given audit class" (GRW, p. 17). We consider an extended model with multiple audit classes in Section 5.

Suppose that there is a unit mass of continuum taxpayers who may earn an income $y > 0$. This income need not be the total income earned. It may simply represent a particular type of income, say, income from vehicle sales or tip income. Those taxpayers who have income y are obliged to pay a positive tax T $(< y)$, while those who do not have y are obliged to pay nothing. The IRS knows that there is a $q \in (0, 1)$ portion of taxpayers who have y, and a $1 - q$ portion of taxpayers who do not have y. However, the IRS cannot identify a priori which taxpayer has y and which taxpayer does not have y. As emphasized by GRW, one of the distinct features of modern systems of income taxation is their self-reporting nature: the tax law requires taxpayers to file tax returns and report their own income to the IRS. The taxpayers who do not have y will always report nothing to the IRS truthfully. However, some taxpayers who have y may cheat and also report nothing. A cheater is subject to a fine $F > 0$ if he is discovered cheating by the IRS. This fine is imposed in addition to the tax T due with $T + F \leq y$ (the limited liability constraint). The taxpayers who have y are assumed to possess a common von Neumann-Morgenstern utility function over income, namely, $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ with $u' > 0$ and $u'' < 0$. They maximize the expected utility by choosing to report y or report nothing.[5]

After receiving a taxpayer's report, the IRS can decide whether to perform an investigative tax audit. Auditing is costly and the IRS has to bear a cost of $c > 0$ to verify each taxpayer. We assume as in GRW that the truth will be discovered once a tax audit is performed and

[5] There are the so-called "ghost" taxpayers, who fail to file their tax returns to the IRS as required by the tax law (Cowell and Gordon, 1995; Erard and Ho, 2001). In our simple model, those taxpayers who have y but report nothing to the IRS and those taxpayers who have y but do not report it to the IRS can be treated as the same. Thus, there are two possible interpretations for non-compliance in our model: (i) *underreporting* if taxpayers have y but report nothing to the IRS, and (ii) *non-filing* if taxpayers have y but fail to file tax returns with the IRS.

that it always pays off for the IRS to audit an evader (i.e. $T + F > c$). Under a given budget constraint I, the IRS's objective is to maximize the tax revenue collected (including taxes and fines), net of audit costs through auditing. Because auditing is costly, it is clear that the "profit-maximizing" IRS will only audit those taxpayers who report nothing. We assume that none of the taxpayers bear any additional cost during the auditing process. We also assume that the IRS cannot use the taxes or fines collected to finance her own auditing expenses.[6] The IRS takes the tax T , the fine F and the budget I as given in her auditing since these variables are predetermined by the Congress. Our focus is on the impact of I.[7]

The timing of this game is as follows. Given the realization of y, c, T, F and I, those taxpayers who do not have y always report nothing, and those taxpayers who have y simultaneously choose whether to report y or not. After observing the taxpayers' reports, the IRS randomly chooses to audit a fraction of taxpayers who do not report y. We solve the equilibrium of this game under the condition that the IRS's strategies are restricted to depend on the distribution of the taxpayers' reports only.[8]

3 Equilibrium

3.1 Equilibrium outcomes

In this subsection we will characterize all the equilibrium outcomes of the game.

[6] Wertz (1979, p. 144) describes the rule of the game: "An agency [the IRS] may not spend more on enforcement activities in a budget period than its legislature has appropriated for them. Deficiencies collected throughout the period are transmitted to the general government; they may not be used by the agency to expand its activities."

[7] Both the tax T and the fine F are also under the control of the Congress. However, their determination is not on the yearly basis as the budget I.

[8] This restriction implies that a unilateral deviation by a single taxpayer cannot influence the course of our game. It is a natural regularity requirement when there are many players; see, for example, Gul et al (1986).

3.1.1 Characterization

We use the notation α to denote the portion of cheaters among all taxpayers,[9] and the notation $\beta(\alpha)$ to denote the IRS's best audit response to α. It is clear that $\alpha \in [0, q]$ and $\beta(\alpha) \in [0, 1]$. Note that the IRS can observe α in our model. This is because the IRS knows by assumption that there is a $q \in (0, 1)$ portion of taxpayers who have y, but only a $q - \alpha$ portion of taxpayers report having y to the IRS.

For any $\alpha \in [0, q]$, let $R(\alpha)$ denote the IRS's (gross) expected revenue from a single audit. Thus,

$$R(\alpha) = \frac{\alpha}{\alpha + (1 - q)} (T + F)$$

where $\alpha + (1 - q)$ is the portion of taxpayers who do not report y. $R(\alpha)$ represents the marginal revenue of tax collection to the IRS when there is the α amount of cheaters. Observe that $R(0) = 0$, $R(q) = q(T + F)$, and $\frac{\partial R}{\partial \alpha} > 0$.

To characterize the equilibrium, we need to define several other notations. First, define $\bar{\beta}$ as the probability of audit such that a taxpayer who has income y is merely indifferent between reporting y and not reporting y. That is,

$$\bar{\beta} u(y - T - F) + \left(1 - \bar{\beta}\right) u(y) = u(y - T).$$

Secondly, define $\bar{\alpha}$ as the amount of α such that the IRS is merely indifferent between auditing and not auditing. That is,

$$R(\bar{\alpha}) = c. \tag{1}$$

Finally, define $\hat{\alpha}$ as the amount of α such that the IRS uses $\bar{\beta}$ as the audit probability and

[9] Thus, the portion of honest taxpayers (including those who have y and also report y to the IRS, and those who do not have y) equals $(q - \alpha) + (1 - q) = 1 - \alpha$. Note that the notation α in the GRW model denotes the portion of actual cheaters among those who are *potential cheaters*, while it denotes the portion of cheaters among *all taxpayers* in our model. That is, our α equals GRW's $q\alpha$.

just exhausts all her budget. That is,

$$\bar{\beta}(\hat{\alpha}+1-q)c = I.$$

For convenience, we shall call "the taxpayer who has y" simply "the taxpayer" from now on. We use (α^*, β^*) to denote the outcome of an equilibrium. Let Γ be the set of all equilibrium outcomes. The following proposition characterizes the set of equilibrium outcomes of the game.[10]

Proposition 1 *(i) If $R(q) \in [0, c)$, then $\Gamma = \{(q, 0)\}$.*

(ii) If $R(q) = c$, then $\Gamma = \{(q, \beta) : \beta \in [0, \min\{\frac{I}{c}, \bar{\beta}\}]\,\}$.

(iii) If $R(q) > c$, then

a) $\Gamma = \{(q, \frac{I}{c})\}$ for $I \in [0,\, \bar{\beta}\,(\bar{\alpha}+1-q)\,c)$;

b) $\Gamma = \{(\bar{\alpha}, \bar{\beta}), \left(\hat{\alpha}, \bar{\beta}\right), \left(q, \frac{I}{c}\right)\}$ for $I \in [\,\bar{\beta}\,(\bar{\alpha}+1-q)\,c, \bar{\beta}c]$;

c) $\Gamma = \{(\bar{\alpha}, \bar{\beta})\}$ for $I \in (\bar{\beta}c,\, \infty)$.

Remark 1 *In b) of Proposition 1 (iii), $\left(\hat{\alpha}, \bar{\beta}\right)$ will coincide with $(\bar{\alpha}, \bar{\beta})$ if $I = \bar{\beta}\,(\bar{\alpha}+1-q)\,c$, while it will coincide with $\left(q, \frac{I}{c}\right)$ if $I = \bar{\beta}c$. This can be seen directly from the definition of $\hat{\alpha}$.*

Remark 2 *Taxpayers report their income to the IRS before auditing and, therefore, they are first movers in the auditing game. However, unlike the leader in a Stackelberg game, knowing the IRS's response $\beta(\alpha)$ does not help the taxpayers much because no taxpayer can alter α by his single deviation and hence no taxpayer can affect β by his income report (remember our regularity assumption that the IRS's strategies only depend on the distribution of the taxpayers' reports). As a result of this "impotent" feature, the taxpayers de facto take the IRS's audit probability β as given. This explains why the subgame perfect equilibrium coincides with the Nash equilibrium in our model. This coincidence is consistent with Kalai's (2004) observation that the equilibria of simultaneous-move games are robust to a large variety of extensive or sequential modifications when there are many players.*

[10] The proofs of our propositions are all relegated to the Appendix.

3.1.2 Hard-to-tax

There are the so-called "hard-to-tax" taxpayers, which is a common referrence to small firms, smaller farmers, self-employed persons, and informal suppliers.[11] These taxpayers could be defined as those whose "tax amounts are quite low compared with the administration costs that would have to be incurred by the tax administration to assess the proper amount of tax." (Thuronyi, 2004, p. 102). This definition corresponds to the case where $R(q) \in [0, c]$ in our model. "Hard-to-tax" is not the same as "impossible-to-tax" after all. It is simply not profitable for the IRS to audit these taxpayers. As a result of lacking the motivation to audit, a very high level of evasion results in equilibrium ($\alpha^* = q$ in Proposition 1 (i) and (ii)).

The possibility that the corner solution $\alpha^* = q$ will occur when audit costs are very high has been noted by GRW. Our minor addition is to point out that if the equilibrium outcome $\alpha^* = q$ is associated with $R(q) \in [0, c]$, then pouring more resources into tax administration will be of little help in resolving the problem and $\alpha^* = q$ will still result. Bird and Casanegra de Jantscher (1992) emphasize that a common constraint usually faced by tax reforms in developing countries is the scarcity of resources for tax administration. This emphasis may need to be qualified in light of our finding here.[12]

3.1.3 Graphic illustration

The intuition underlying Proposition 1 (iii) is best understood in terms of the best-response graphs of the taxpayers and of the IRS. Figures 1a, 1b and 1c illustrate a), b) and c) of

[11] Informal suppliers are defined as: "individuals who provide products or services through informal arrangements which frequently involve cash-related transactions or 'off the books' accounting practice." (Internal Revenue Service, 1996, p. 43)

[12] Former IRS Commissioner Lawrence B. Gibbs was reported to have stated that the IRS will not collect "small amounts owed by such a huge number of taxpayers that collection efforts would not be cost-effective" (Los Angeles Times, 1987). Inspired by Gibbs' statement, Reinganum and Wilde (1988) consider a model in which the IRS will tolerate evasion in cases where collection efforts would not be cost-effective for the IRS.

Proposition 1 (iii), respectively. In each figure, the dotted curve represents the taxpayers' best response while the solid curve represents the IRS's. Note that taxpayers adopt pure rather than mixed strategies in our model. In drawing the dotted curve, it is assumed that there exists a representative taxpayer who will choose evasion with a probability of $\frac{\alpha}{q}$ and compliance with a probability of $\frac{q-\alpha}{q}$. Our pure-strategy outcome is realized through the representative taxpayer's infinite trials so that, due to the law of large numbers, the α amount of taxpayers will evade while the $q - \alpha$ amount of taxpayers will comply.

[Insert Figure 1 about here]

Note that the shape of the taxpayers' best-response curve remains the same as that in GRW. Note also that the shape of the IRS's best-response curve remains the same as that in GRW if $\alpha < \bar{\alpha}$. However, the shape of the IRS's best-response curve changes if $\alpha \geq \bar{\alpha}$. Two salient features stand out. First, the result $\beta(\alpha) \in [0, 1]$ would be true at $\alpha = \bar{\alpha}$ if there were no budget constraint. This is because $\bar{\alpha}$ is by definition the amount of α such that the IRS is indifferent between auditing ($\beta = 1$) and not auditing ($\beta = 0$). However, when the budget constraint is imposed, it may no longer be feasible for the IRS to support any $\beta \in [0, 1]$ as she would wish. Instead, we have $\beta(\alpha) \in [0, \min\{\frac{I}{c(\alpha+1-q)}, 1\}]$ at $\alpha = \bar{\alpha}$. In terms of the graph, the height of the IRS's best response curve at $\alpha = \bar{\alpha}$ may fall short of 1 as shown in Figures 1a and 1b. Secondly, when $\alpha > \bar{\alpha}$, $R(\alpha) > c$ will hold so that an incremental dollar of audit input could return more than an incremental dollar of tax revenue. As a result, the "profit-maximizing" IRS would audit for sure if there were no constraint on her budget. That is, $\beta(\alpha) = 1$ would hold for all $\alpha > \bar{\alpha}$. This may no longer be true when the budget constraint is imposed. Specifically, a binding budget constraint will bring down the feasible probability of audit that the IRS can support and, moreover, the larger the amount of evaders (i.e. a higher α), the lower the probability of audit that these evaders will face (i.e. a lower β). This "congestion" effect, which is emphasized by GRW$'$, is captured in our model by the downward-sloping part of the IRS's best response curve as $\alpha > \bar{\alpha}$.

Note that $\beta(\alpha) = \frac{I}{c(\alpha+1-q)}$ for $\alpha \geq \bar{\alpha}$ if $\beta(\alpha) \leq 1$. This explains the shape of the IRS's best-response curve as $\alpha \geq \bar{\alpha}$ in Figure 1 (i.e. $\frac{\partial\beta(\alpha)}{\partial\alpha} < 0$ and $\frac{\partial^2\beta(\alpha)}{\partial\alpha^2} > 0$). The intersection of the dotted and the solid curve in Figure 1 pins down the equilibrium of the game. There are three intersections in Figure 1b, while there is a single intersection in both Figures 1a and 1c. The former intersections represent the three possible equilibria characterized in b) of Proposition 1 (iii), while the latter intersection represents the unique equilibrium characterized in a) and c) of Proposition 1 (iii), respectively. We shall have more to say on these equilibrium outcomes later.

3.2 Discussion

This subsection discusses several issues related to Proposition 1.

3.2.1 Habitual complier

The original GRW model incorporates taxpayers who are inherently honest, in the sense that they report their incomes truthfully regardless of the incentive to cheat. GRW call these taxpayers "habitual compliers." We briefly discuss the effect of incorporating these taxpayers.

Suppose that ρ ($< q$) taxpayers are inherently honest and always report y to the IRS. Introducing these so-called "habitual compliers" to the model reduces the portion of strategic taxpayers who may cheat from q to $q - \rho$. In terms of Figure 1, it merely replaces the origin $(0, 0)$ with $(\rho, 0)$ and $\bar{\alpha}$ with $\bar{\alpha} + \rho$. As long as $\bar{\alpha} + \rho \leq q$, everything else remains the same and nothing essential changes with the incorporation.[13]

[13] If $\bar{\alpha} + \rho > q$, we would have $\bar{\alpha} > q - \rho$, which is infeasible since $q - \rho$ is the maximal portion of cheaters in the presence of ρ habitual compliers. See footnote 21 for more comments on habitual compliers.

3.2.2 Budget surplus

The profit-maxmizing objective function of the IRS follows GRW and is standard in the tax evasion literature. GRW consider other possible IRS objective functions, but conclude that the profit-maximizing objective "adequately captures both the general and the specific deterrence objectives often attributed to IRS enforcement policy." (GRW, p. 29)

Sticking to the assumption that the IRS maximizes her "profit," a budget surplus becomes possible as long as the IRS is not budget constrained in equilibrium. For example, when the equilibrium outcome $(\alpha^*, \beta^*) = (\bar{\alpha}, \bar{\beta})$ occurs, it is likely that the audit cost expended by the IRS will be less than the budget size appropriated by the Congress (i.e. $\bar{\beta}(\bar{\alpha} + 1 - q)c < I$). This possiblity raises a subtle question which seemed to go unnoticed in the past, namely, how the profit-maximizing IRS would "deal with" the budget surplus. Indeed, the situation of a budget surplus will always result in GRW's budget-unconstrained model. This is because the budget size in the GRW model is large enough for the IRS to support $\beta = 1$ for all $\alpha \geq \bar{\alpha}$, but $\beta^* = \bar{\beta} < 1$ in equilibrium. We do not address the "budget surplus" question directly in this paper. Instead, we keep the basic framework of the GRW model and devise a simple scheme to achieve two ends: (i) preserving the IRS objective of profit maximization, and (ii) forcing the IRS to conserve the use of the allocated budget and return the unused money back to the Congress.

Melumad and Mookherjee (1989) argue that it is difficult for a government to commit to the *allocation* of aggregate audit costs or aggregate revenues collected, but it is reasonable to assume that the government can make commitments based on these *aggregate* variables since they are publicly available as part of the process of budgetary appropriations and reviews of tax-collection agencies. In line with this argument, our scheme consists of two aggregate variables: the total tax revenue collected ($G \equiv (q - \alpha)T + \alpha\beta(T + F)$) and the budget surplus generated ($B \equiv I - \beta\,(\alpha + 1 - q)\,c$). The Congress uses the sum $G + B$ to evaluate the IRS's performance (the larger the sum, the higher the score), or even offers a fixed fraction of the sum $G + B$ to the IRS as her bonus.

If the IRS were to maximize G alone, she would exhaust all the budget with $B = 0$ even though an additional dollar of audit input could not return an additonal dollar of tax revenue (i.e. $R(\alpha) < c$). On the other hand, if the IRS were to be motivated to maximize B alone, she would simply do nothing and generate the budget surplus $B = I$, even though an additional dollar of audit input could return more than an additional dollar of tax revenue (i.e. $R(\alpha) > c$). Since the IRS is motivated to maximize the sum of G and B rather than either of them alone, she needs to trade off the loss of B against the gain in G when carrying out a tax audit. If $R(\alpha) > c$, the loss of B through expended audit cost will be more than compensated by the gain in G through collected tax revenue and, as a result, it will be worthwhile for the IRS to carry out the tax audit. On the other hand, if $R(\alpha) < c$, the loss of B will not be compensated by the gain in G and, therefore, it will be not worthwhile for the IRS to carry out the tax audit. This trade-off between G and B at the margin will drive the IRS to equate $R(\alpha)$ (the marginal revenue of tax collection) with c (the marginal cost of tax collection) as far as possible and, at the same time, conserve the use of the allocated budget as much as possible. In other words, the proposed scheme has achieved the two ends stated within our framework.[14]

3.2.3 Comparison with GRW′

The defining feature that distinguishes our model (with budget constraints) from the GRW model (without budget constraints) is the presence of the congestion effect: holding the IRS's budget constant, the higher the incidence of evasion, the lower the audit probability that an evader will face. This congestion effect is exhibited in our model through the modification of

[14] It must be admitted that many practical complications may arise if our scheme is put into effect in the real world. Nevertheless, the scheme may still serve as a useful start for taking into consideration these complications.

the IRS's best response in the GRW model. As to the taxpayers' best response, it remains the same as that in the GRW model (see our Figure 1).

GRW$'$ adopt a different approach: the congestion effect is incorporated into their model through the modification of the taxpayers' rather than the IRS's best response in the GRW model. They explain their approach as follows (pp. 6-7):

> "In that model [the GRW model] no budget constraint is imposed on the IRS so there is no direct interaction between the reporting strategies of the taxpayers – a Nash Equilibrium for the game can be characterized simply by considering the interaction between the IRS and a representative taxpayer. However, once a budget constraint is imposed on the IRS, the likelihood of audit facing one taxpayer depends on the reporting strategy of the other taxpayers. Thus it becomes useful to consider first the interaction between the taxpayers in selecting their reporting strategies, taking the IRS audit strategy as given."

Note that the so-called "IRS audit strategy" in the last sentence of the above quotation cannot be the probability of audit that the taxpayers actually face (i.e. β in our model). Faced with a given β, a taxpayer will comply if $\beta > \bar{\beta}$ and will evade if $\beta < \bar{\beta}$. This is true regardless of other taxpayers' decisions. In other words, there would simply be no interaction among the taxpayers, contrary to GRW$'$'s description.

The "IRS audit strategy" in the GRW$'$ model means something else, which is called "the probability of audit given exposure" by GRW$'$. As to "exposure," it has to do with the size of the IRS budget – the larger the budget, all else equal, the higher the probability of exposure to an audit. The probability of audit that the taxpayers actually face in the GRW$'$ model is the product of two probabilities: the probability of exposure, and the probability of audit given exposure. Given the IRS's probability of audit given exposure, GRW$'$ derive the so-called "taxpayer equilibrium" (all the taxpayers make mutually best responses to each other). The full equilibrium of the game is then determined by putting together (i) the taxpayer equilibrium, given the IRS's probability of audit given exposure; and (ii) the IRS's best choice over the probability of audit given exposure, given the taxpayer equilibrium. It turns out that the IRS's best choice over the probability of audit given exposure coincides with the IRS's best response in the GRW model (see their Figure 2).

As might be glimpsed from what we have just described, the GRW$'$ way of deriving equilibria is somewhat complicated. We believe our approach greatly simplifies the analysis. In particular, note that while the taxpayers' best response in the GRW model is replaced with the "taxpayer equilibrium" in GRW$'$, we avoid the step of deriving the "taxpayer equilibrium." As in the GRW model, our equilibria are characterized simply by considering the interaction between the IRS and a representative taxpayer.

4 Efficiency

In this section we turn our attention to evaluate and compare the degrees of efficiency in tax collection across different equilibria as the budget size I varies, making an attempt to answer the normative question: how much should we fund the IRS? Since variations in the budget I exert no impact on $\alpha^* = q$ if $R(q) \in [0, c]$ (Proposition 1 (i)-(ii)), our analysis is confined to the case where $R(q) > c$ (Proposition 1 (iii)).

The efficiency criterion we use for comparison is in terms of per dollar of tax collection. Specifically, we first add up (i) the full cost imposed on the taxpayers and (ii) the audit cost expended by the IRS to arrive at a sum, and then divide the resulting sum by the total tax revenue (including fines) collected. The full cost to the taxpayers includes the tax and fine paid plus the excess burden imposed. Slemrod and Yitzhaki (1987) emphasize that the tax revenue collected from the taxpayers merely represents a transfer between the private and the public sector and, therefore, it should not be counted as a cost to society. They employ the excess burden imposed on taxpayers plus the audit cost expended by the IRS as the cost to society. Our efficiency measure, which is defined in terms of per dollar of tax collection, is consistent with theirs. This is because both the numerator and the denominator of our efficiency measure have taken into account the tax revenue collected so that

(full cost imposed on taxpayers + audit cost expended by IRS}/total tax revenue collected =1 + (excess burden imposed on taxpayers + audit cost expended by IRS)/total tax revenue collected.

We will exploit some useful properties of the full cost imposed on the taxpayers when we make comparison of efficiency across different equilibria.

Slemrod and Yitzhaki (1987) subsume the IRS and the Congress under the rubric of a single player called "government." Through choosing *both* the probability of audit and the tax rate, the society is constrained to raise a given amount of tax revenue in their model. By contrast, the IRS and the Congress are two different players in our model. Because the tax structure is predetermined by the Congress and so is beyond the IRS's control, the tax revenue collected will as a rule be variable and not fixed. To facilitate the comparison across different equilibria, it is more convenient for us to define the efficiency criterion in terms of per dollar of tax collection.

Note that our per-tax-dollar criterion is consistent with the so-called "marginal cost of public funds," which is defined as the full cost to society of transferring a dollar to the government; see Slemrod and Yitzhaki (2002). Since our comparison is across different equilibria, we employ the average- rather than marginal-cost criterion.

4.1 Cost of public funds

Following Cowell (1990), we define the "cost of evasion" as the monetary amount that a taxpayer would just be prepared to pay in order to be guaranteed that he will get away with the tax evasion. It is an amount C such that

$$u(y-C)=\beta u(y-T-F)+(1-\beta)u(y). \tag{2}$$

The amount C can be decomposed into two components: the tax (including the fine) that a taxpayer expects to pay (r) and the risk premium that the taxpayer would be ready to pay in order to eliminate the exposure to audit risk (θ). That is, $C=r+\theta$, where $r=\beta(T+F)$. Note that $\theta>0$ as long as $u''<0$. Yitzhaki (1987) calls the risk premium θ the "excess burden of tax evasion" since it represents a deadweight loss beyond what would be imposed on the taxpayer if the expected tax revenue r were somehow collected by a lump-sum tax. The equality $C=T$ will obviously hold if the taxpayer is indifferent between evasion and compliance. This then implies that $T>\bar{\beta}(T+F)$, since (i) $C=\bar{\beta}(T+F)+\theta$ with $\theta>0$, and (ii) $\bar{\beta}$ is by definition the probability of audit that makes the taxpayers indifferent between evasion and compliance. We call this finding "Result 1" and will exploit it later.

By the implicit function theorem, we have C as a function of β. Taking the total derivative of equation (2) with respect to β gives

$$u'(y-C)(-C_{\beta})=u(y-T-F)-u(y).$$

Hence,

$$C_{\beta}=\frac{u(y)-u(y-T-F)}{u'(y-C(\beta))}>0 \tag{3}$$

and

$$C_{\beta\beta}=\frac{[u(y)-u(y-T-F)]u''(y-C(\beta))C_{\beta}}{[u'(y-C)]^2}<0. \tag{4}$$

That is, $C(\cdot)$ is a strictly increasing and concave function. We call this finding "Result 2" and will also exploit it later.

Let I^* denote the amount of audit cost expended by the IRS in an equilibrium. By our assumption that the IRS is not allowed to use the taxes or fines collected to finance her own audit cost, we have $I^*\leq I$. Under an equilibrium outcome (α^*,β^*), the total cost paid by the taxpayers and the Congress equals $S(\alpha^*, \beta^*, I^*) \equiv qC(\beta^*) + I^*$, while the corresponding total tax revenue collected by the IRS equals $G(\alpha^*, \beta^*, I^*) \equiv (q - \alpha^*) T + \alpha^* \beta^* (T + F)$. Thus, our efficiency measure is $\psi(\alpha^*, \beta^*, I^*) \equiv \frac{S(\alpha^*, \beta^*, I^*)}{G(\alpha^*, \beta^*, I^*)}$. Note that if $\alpha^* = \bar{\alpha}$ or $\alpha^* = \hat{\alpha}$, we can replace T with C in S. The reason for this is that when $\alpha^* = \bar{\alpha}$ or $\alpha^* = \hat{\alpha}$, some taxpayers will evade while others will not. All these taxpayers must be indifferent between evasion and compliance and, therefore, we have $C = T$.

Let $\bar{I} \equiv \bar{\beta}(\bar{\alpha} + 1 - q) c$, which is the minimal size of the budget that is capable of supporting $(\bar{\alpha}, \bar{\beta})$ (see Proposition 1 (iii)). Table 1 summarizes the total cost S and the tax revenue collected G as the equilibrium outcome (α^*, β^*) varies with I. Note that both S and G remain the same for all $I \geq \bar{I}$ if $(\alpha^*, \beta^*) = (\bar{\alpha}, \bar{\beta})$, and thus $\psi\left(\bar{\alpha}, \bar{\beta}, I\right) = \psi\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right)$ if $I \geq \bar{I}$. This result is due to our scheme of forcing the IRS to conserve the use of the allocated budget and return the unused money back to the Congress.

Table 1.

IRS's budget I	(α^*, β^*)	Total cost S	Tax collected G
$I < \bar{I}$	$(q, \frac{I}{c})$	$qC\left(\frac{I}{c}\right) + I$	$\frac{I}{c} q(T + F)$
$I \in [\bar{I}, \bar{\beta}c]$	$(q, \frac{I}{c})$	$qC\left(\frac{I}{c}\right) + I$	$\frac{I}{c} q(T + F)$
	$\left(\hat{\alpha}, \bar{\beta}\right)$	$qC\left(\bar{\beta}\right) + I$	$(q - \hat{\alpha}) T + \hat{\alpha}\bar{\beta}(T + F)$
	$(\bar{\alpha}, \bar{\beta})$	$qC\left(\bar{\beta}\right) + \bar{I}$	$(q - \bar{\alpha}) T + \bar{\alpha}\bar{\beta}(T + F)$
$I > \bar{\beta}c$	$(\bar{\alpha}, \bar{\beta})$	$qC\left(\bar{\beta}\right) + \bar{I}$	$(q - \bar{\alpha}) T + \bar{\alpha}\bar{\beta}(T + F)$

The following proposition shows that $(\bar{\alpha}, \bar{\beta})$ yields the lowest ψ among all possible equilibrium outcomes.

Proposition 2 *Of all possible equilibrium outcomes, $(\bar{\alpha}, \bar{\beta})$ yields the least cost per dollar of tax collection.*

Figure 2 depicts our efficiency measure ψ against the allocated budget I.[15] The value of

[15] Except for $\psi = \psi(\bar{\alpha}, \bar{\beta}, \bar{I})$, the line segment associated with $\psi = \psi(q, \frac{I}{c}, I)$ or $\psi = \psi(\hat{\alpha}, \bar{\beta}, I)$ need not be straight as shown in Figure 2.

$\psi\left(q, \frac{I}{c}, I\right)$ is strictly decreasing in I until $I = \bar{\beta}c$ (see the proof of Proposition 2). Beyond $\bar{\beta}c$, $(\alpha^*, \beta^*) = (q, \frac{I}{c})$ will no longer exist (see Table 1). The value of $\psi\left(\hat{\alpha}, \bar{\beta}\right)$ will appraoch $\psi\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right)$ if I approaches $\bar{I}$, while it will approach $\psi\left(q, \frac{I}{c}, I\right)$ if I approaches $\bar{\beta}c$. Indeed, $\left(\hat{\alpha}, \bar{\beta}\right)$ will coincide with $(\bar{\alpha}, \bar{\beta})$ if $I = \bar{I}$, but it will coincide with $\left(q, \frac{I}{c}\right)$ if $I = \bar{\beta}c$ (see Remark 1). As to $\psi\left(\bar{\alpha}, \bar{\beta}, I\right)$, it remains constant with respect to I. It is clear from Figure 2 that $(\bar{\alpha}, \bar{\beta})$ yields the least cost per dollar of tax collection among all possible equilibrium outcomes. Note that $\bar{\alpha}$ is the smallest equilibrium evasion possible within our model.

[**Insert Figures 2 and 3 about here**]

Suppose that $(\bar{\alpha}, \bar{\beta})$ is not the equilibrium outcome at the status quo. If we pour more resources into the IRS, the IRS's best-response curve will be shifted upward as shown in Figure 3. If we keep on pouring, it is clear that $(\bar{\alpha}, \bar{\beta})$, similar to that shown in Figure 1c, will eventually result as the unique equilibrium outcome. In contrast to outcomes $\left(\hat{\alpha}, \bar{\beta}\right)$ and $(q, \frac{I}{c})$ where the marginal tax revenue is greater than the marginal cost of collection (i.e. $R(\hat{\alpha}) > c$ and $R(q) > c$), the IRS equates the marginal tax revenue to the marginal cost of collection under the outcome $(\bar{\alpha}, \bar{\beta})$ (i.e. $R(\bar{\alpha}) = c$). Since outcome $(\bar{\alpha}, \bar{\beta})$ yields the least cost per dollar of tax collection, we obtain

Corollary 1 *The size of the budget allocated to the IRS should be expanded as long as an additional dollar allocated could return more than an additional dollar of tax revenue.*

A tax farmer, who is interested only in profit maximization, will expand the size of her audit resources if an additional dollar of audit input could return more than an additional dollar of tax revenue. Corollary 1 requires that the Congress support the "IRS as tax farmer."[16] This policy prescription contrasts with Slemrod and Yitzhaki's (1987) finding that the marginal tax revenue exceeds the marginal cost of collection at the optimum and,

[16] Should the IRS be simply privatized? Answering this question would take us beyond the scope of the present paper. Some would argue that collection costs tend to be lower for private than public agents

consequently, the "IRS as tax farmer" would lead to a socially excessive amount of resources devoted to tax collection. Put differently, the Congress should provide a smaller budget than the IRS would wish according to Slemrod and Yitzhaki, whereas the Congress should provide the budget that the IRS would wish according to our model.

To ensure that $(\bar{\alpha}, \bar{\beta})$ is the unique equilibrium outcome, $I > \bar{\beta}c$ must hold (see Table 1). Since the size of the population who report nothing equals $(\alpha^* + 1 - q)$ in equilibrium and since the IRS incurs a cost c for each person it verifies, the meaning of the inequality $I > \bar{\beta}c$ is clear: even if $\alpha^* = q$ (i.e. all taxpayers evade) so that $\alpha^* + 1 - q = 1$, the size of the budget allocated should still enable the IRS to support an audit probability higher than $\bar{\beta}$ (i.e. $\frac{I}{c(\alpha^*+1-q)} = \frac{I}{c} > \bar{\beta}$). The intuition behind this result is simple. Note that the taxpayers will comply if they expect $\beta > \bar{\beta}$. When $I > \bar{\beta}c$, it is feasible for the IRS to support an audit probability higher than $\bar{\beta}$ at all possible realized α's. This feasibility completely eliminates the taxpayers' self-fulfilling expectations that a widespread and rampant evasion may "congest" the IRS's tax administration to such an extent that it becomes impossible for the IRS to maintain $\beta > \bar{\beta}$ at some high α's. By contrast, the taxpayers' self-fulfilling expectations could support the realization of $\alpha^* = \hat{\alpha}$ or $\alpha^* = q$ if $I \leq \bar{\beta}c$.[17]

(Toma and Toma, 1992). However, one might worry about whether taxpayers' private information should be possessed by private agents. Through H.R. 4520, American Jobs Creation Act of 2004, the Congress gives the IRS the authority to use private collection agencies to collect IRS debt and pay them a bounty of up to 25 percent of the money they collect. This statute is strongly opposed by National Treasury Employees Union. One reason raised for the opposition is: "the IRS does not have the technology in place to ensure that taxpayer information is kept secure and confidential when it is handed over to the private collection agencies." (Kelley, 2005)

[17] At $\alpha^* = q$, the maximal probability of audit that the IRS can support equals $\frac{I}{c}$. If $\frac{I}{c}$ is greater than $\bar{\beta}$, the equilibrium $(\bar{\alpha}, \bar{\beta})$ can be ensured. If $\frac{I}{c}$ is not greater than $\bar{\beta}$, the equilibrium $(\bar{\alpha}, \bar{\beta})$ cannot be ensured. We focus on lifting $\frac{I}{c}$ above $\bar{\beta}$ by increasing I. There is another side of the same coin: lifting $\frac{I}{c}$ above $\bar{\beta}$ by reducing c. We comment on this alternative possibility at the end of the paper.

4.2 Intuition

As noted before, Slemrod and Yitzhaki (1987) and others, including Usher (1986), Kaplow (1990), Mayshar (1991) and Sanchez and Sobel (1993), all conclude that the size of the budget allocated to the IRS should fall short of equating the marginal revenue with the marginal cost of tax collection, whereas we conclude that it should equate the marginal revenue with the marginal cost of tax collection. Is there any intuition behind the difference? In this subsection we provide one.

Mayshar (1991) views the maximal tax revenue collected as a function of the IRS's enforcement budget and other variables such as the tax base and tax structure. He calls this function a "tax technology." Like the standard production function of the firm, the tax technology is a "black box" and its details are left unspecified. Because of the "black box" nature, Mayshar's tax technology can be interpreted to accommodate a variety of models, including Slemrod and and Yitzhaki's commitment model and our non-commitment model. Specifically, in terms of our notation, we can simply write $G = G(I)$, where $G(.)$ represents the tax technology. The net tax revenue or "profit" is then represented by $G(I) - I$.

What does $G(I) - I$ look like? Mayshar (1991) argues that it takes the shape of a Laffer curve as shown in Figure 4-1.[18] This shape seems to be typical for a profit function. Let us impose the set of indifference curves of a social welfare function $W(I, G-I)$ with $\frac{\partial W}{\partial I} < 0$ and $\frac{\partial W}{\partial (G-I)} > 0$ on the "Laffer curve." It is easy to see from Figure 4-1 that the optimal level of I is always lower than the level selected by the "profit-maximizing" IRS.

[Insert Figure 4 about here]

Figure 4-2 shows the shape of $G(I) - I$ in our model.[19] The "profit" $G(I) - I$ increases with I continuously until the level of I reaches $\bar{I}$. From $\bar{I}$ on, $G(I) - I$ takes a zigzag shape

[18] See Figure 1 in Mayshar (1991).

[19] The shape of $G(I) - I$ in Figure 4-2 can be derived directly from Table 1.

rather than the standard shape of a Laffer curve and, in particular, the value of $G(I) - I$ may jump discontinuously as I varies. The reason for the discontinuous jump in "profit" is obviously attributable to the existence of multiple equilibria, which are in turn attributable to the mitigation (if $\alpha^* = \hat{\alpha}$ rather than $\alpha^* = q$) or even elimination (if $\alpha^* = \bar{\alpha}$ rather than $\alpha^* = q$) of the congestion effect.[20] This mitigation or elimination of the congestion effect becomes feasible only if the size of the budget allocated to the IRS is large enough to satisfy $I \geq \bar{I}$. Let us impose the same set of indiffernce curves of the social welfare function $W(I, G - I)$ on the "zigzag curve." It is easy to see from Figure 4-2 that the equilibrium outcome $(\alpha^*, \beta^*) = (\bar{\alpha}, \bar{\beta})$ with $I = \bar{I}$ is the optimal solution. As we have explained in Figure 3, this optimal solution can be implemented by supporting the "IRS as tax farmer" and expanding the size of the budget allocated to the IRS as long as the marginal revenue is greater than the marginal cost of tax collection.

5 Extension

The model presented so far may well represent a particular audit class only, where the audit class is sorted on the basis of some observable taxpayer characteristics such as zip code, reported income level/source, occupation or age. GRW and Erard and Feinstein (1994), among others, interpret their audit rules within, not across, audit classes. The same kind of interpretation is equally applicable to our previous setting. In this section we consider an economy-wide model in which there are two or more audit classes.

Consider an economy in which there are $n \geq 2$ audit classes. Except for the audit cost, each audit class has identical structures and parameters as before. Without loss of generality, let $0 < c_1 < c_2 < \cdots < c_n$. We could consider a more general setting in which incomes earned

[20] Remember that the congestion effect is captured in our model by the downward-sloping part of the IRS's best response curve as $\alpha > \bar{\alpha}$.

vary across different audit classes. Specifically, let i and j denote two different audit classes, and $T_i < T_j$ and $F_i < F_j$ if $y_i < y_j$ (i.e. the higher the income earned in an audit class, the more the tax that needs to be paid and the larger the fine that needs to be imposed if evasion is detected). However, Proposition 1 reveals that what matters to our model is the relative rather than absolute relation between $T + F$ and c (i.e. $R(q) = q(T + F)$ relative to c). In view of this, we let $y_i = y_j$ and focus on the case where $c_i \neq c_j$ for simplicity.[21]

If $R(q) \leq c_i$, it is not profitable for the IRS to audit class i. We then have the same result as Proposition 1 (i)-(ii), that is, $\alpha_i^* = q$ and variations in I exert no impact on α_i^*. Similar to Proposition 2, we confine our analysis to the cases where $R(q) > c_i$.

Now suppose that $R(q) > c_n$. Define $\bar{\alpha}_i$ as the amount of α_i such that the IRS is merely indifferent between auditing and not auditing class i (i.e. $R(\bar{\alpha}_i) = c_i$). Let $I_i \geq 0$ be the IRS's budget allocation intended for auditing class i with $\sum_{i=1}^{n} I_i = I$. We have the following extension of Proposition 2.[22]

Proposition 3 *Of all possible equilibrium outcomes,* $\{(\bar{\alpha}_i, \bar{\beta})\}_{i=1}^{n}$ *yields the least cost per*

[21] Reinganum and Wilde (1986) extend the GRW model (without habitual compliers) to the situation where a taxpayer's income can take any value in some range. They focus on the fully separating equilibrium. As Erard and Feinstein (1994) point out, an unrealistic feature of this separating solution is that the IRS knows the true income of each taxpayer prior to performing any tax audit, even though in actual practice this is often not the case. Erard and Feinstein (1994) incorporate habitual compliers into the Reinganum-Wilde model, showing that the incorporation has important impacts on the equilibrium solution and, in particular, that it resolves the unrealistic feature mentioned above. The cost paid is that the resulting equilibrium is characterized by a highly nonlinear second-order differential equation, and Erard and Feinstein have to rely extensively on simulations to study the solution of their model. It should be noted that Erard and Feinstein (1994) do take into account the IRS's budget constraint. However, there are no multiple equilibria associated with the congestion effect in their model. Like GRW′, Erard and Feinstein (1994) do not address the efficiency issue.

[22] The budget allocation $\{I_1, I_2, ..., I_n\}$ that supports the outcome $(\bar{\alpha}_i, \bar{\beta})$ $(i = 1, 2, ..., n)$ is not unique. Therefore, we state $\{(\bar{\alpha}_i, \bar{\beta})\}_{i=1}^{n}$ instead of $\{(\bar{\alpha}_i, \bar{\beta}, I_i)\}_{i=1}^{n}$ in Proposition 3.

dollar of tax collection.

When there is only one audit class, $I > \bar{\beta}c$ must hold in order to ensure the least cost equilibrium outcome $(\bar{\alpha}, \bar{\beta})$ (see Table 1). What is the corresponding condition when there are two or more audit classes? The following proposition provides the answer.

Proposition 4 *To support $\{(\bar{\alpha}_i, \bar{\beta})\}_{i=1}^{n}$ as the unique equilibrium outcome of the game, $I > [c_n + \sum_{i=1}^{n-1}(\tilde{\alpha}_i + 1 - q)c_i]\bar{\beta}$ must hold,*

where $\{\tilde{\alpha}_i\}_{i=1}^{n-1}$ satisfy

$$\frac{R(\tilde{\alpha}_1)}{c_1} = \frac{R(\tilde{\alpha}_2)}{c_2} = \cdots = \frac{R(\tilde{\alpha}_{n-1})}{c_{n-1}} = \frac{R(q)}{c_n}. \tag{5}$$

Remark 3 *Melumad and Mookherjee (1989) argue that it is difficult for a government to commit to the allocation of aggregate audit costs or aggregate revenues collected, but it is reasonable to assume that the government can make commitments based on these aggregate variables since they are publicly available as part of the process of budgetary appropriations and reviews of tax-collection agencies. Our setting is consistent with this argument since the Congress can control I, but not I_i.*

Remark 4 *Erard and Ho (2004) use information from two data files of the IRS's Taxpayer Compliance Measurement Program for the tax year 1988 (one for those who file tax returns and the other for those who do not file tax returns) plus supplementary information on tip earners and informal suppliers to study the tax compliance behavior of 34 distinct occupational groups in the U.S. economy. They find that the share of tax liability that goes unpaid is 14.9% on average for all occupations as a whole, but that it varies substantially across different occupations, ranging from 51.1% (vehicle sales), 49.8% (tip earners), 44.1% (informal suppliers), 16.0% (mechanics and repairers), and 7.0% (doctors and dentists), to 5.4% (accountants, auditors and tax preparers). Since $\frac{R(\alpha_i^*)}{c_i} = \frac{R(\alpha_j^*)}{c_j}$ for $i \neq j$, $\frac{\partial R}{\partial \alpha} > 0$ and $0 < c_1 < c_2 < \cdots < c_n$, it is clear that $\alpha_1^* < \alpha_2^* < ... < \alpha_n^* \leq q$ must hold in our extended model. Supposing that* *audit classes are sorted on the basis of occupations, this result is consistent with Erard and Ho's finding with regard to the U.S. tax compliance continuum across occupations.*

The intuition underlying Proposition 4 is as follows. First, it is not possible to have $\alpha_i^* = \alpha_j^* = q$ for $i \neq j$, since this would lead to $\frac{R(q)}{c_i} \neq \frac{R(q)}{c_j}$ (the IRS can then improve her profit by re-allocating the budget between audit classes i and j). Thus, in any equilibrium there is at most one audit class such that all taxpayers cheat. Furthermore, this audit class must be the nth class because $\alpha_1^* < \alpha_2^* < ... < \alpha_n^* \leq q$ and the corresponding equilibrium must satisfy equation (5). The inequality condition stated in Proposition 4 essentially requires that the total budget allocated to the IRS be large enough for her to kill off this equilibrium. In Table 1, $(\bar{\alpha}, \bar{\beta})$ is the only equilibrium outcome that will survive once the budget size is large enough to eradicate the most "congestive" equilibrium evasion (i.e. $\alpha^* = q$). Analogously, in our extended model, $\{(\bar{\alpha}_i, \bar{\beta})\}_{i=1}^n$ is the only equilibrium outcome that will survive once the budget size is large enough to eradicate the most "congestive" equilibrium evasion (i.e. $\alpha_i^* = \tilde{\alpha}_i$ for $i = 1, 2, ..., n-1$ and $\alpha_n^* = q$).

6 Conclusion

We conclude our paper with four remarks. First, suppose that the audit classes in Section 5 are sorted on the basis of occupations. We can then extend our model to incorporate the taxpayers' occupational choice before the Congress's budget allocation.[23] As long as the size of the budget allocated is large enough to support the least cost equilibrium as the unique outcome, the IRS's auditing activities will not distort the taxpayers' occupational choice. This is because the equality $\bar{\beta}_i u(y - T - F) + (1 - \bar{\beta}_i) u(y) = u(y - T)$ holds for

[23] One can reverse the order of the choices: adding the taxpayers' occupational choice *after* the Congress's budget allocation. However, it seems that the occupational choice is a longer decision than the yearly budget allocation.

all $i = 1, 2, ..., n$ at the least cost equilibrium. That is, all taxpayers in all occupations are indifferent between evasion and compliance. This equality also indicates that horizontal equity in the ex ante sense will be obeyed. Other equilibria need not possess such desirable properties. For example, consider the equilibrium that satisfies equation (5). The outcome resulting from this equilibrium not only distorts the taxpayers' occupational choice but also violates horizontal equity. This is because the taxpayers in the nth audit class enjoy a higher expected utility than the taxpayers in other audit classes.

Second, Kau and Rubin (1981, p. 262) hypothesize that "there have been changes in production technologies which have directly led to an increase in the proportion of income which is subject to taxation." These changes are attributable to factors such as fewer self-employed individuals, improved record keeping due to increased incorporation, and the substitution of market production for home production. All of these changes presumably lower the IRS's cost of tax audits. North (1985, p. 392) puts forth a similar hypothesis: "The supply of government was made possible by new technology which, coupled with the consequences of growing market specialization, lowered the costs of government monitoring of income and wealth and increased the efficiency of government taxation." Kau and Rubin (1981) find empirical support for their hypothesis, and Ferris and West (1996) provide additional empirical support. In terms of our model, a lower cost of tax audit has three main effects: (i) it turns some taxpayers from being "hard-to-tax" into "not-so-hard-to-tax" (i.e. from $R(q) \leq c$ to $R(q) > c$ in Proposition 1), (ii) it lowers the threshold evasion that makes the IRS indifferent between auditing and not auditing (i.e. a lower $\bar{\alpha}$ defined in equation (1)), and (iii) it raises the probability of audit that the IRS can support under a budget constraint (i.e. a higher $\frac{I}{c(\alpha+1-q)}$). These effects are obviously important and should not be ignored. Nevertheless, as far as the yearly budget appropriation is concerned, it does not seem unreasonable to view the audit cost c as a parameter, which is beyond the control of both the IRS and the Congress.[24]

[24] The IRS is modernizing its forty-year-old information system through the Business Systems Modernization

Third, according to our model, the IRS will let go of "hard-to-tax" taxpayers. Increasing the size of the budget allocated to the IRS can do little about it. This result is attributed to the fact that there is a negative return on money invested in "hard-to-tax" taxpayers for the IRS. Wertz (1979) observes that the IRS is often expected by the Congress to "show a profit" on her enforcement activities. This could aggravate the "hard-to-tax" problem. Fixing the "hard-to-tax" is a thorny task and alternative strategies such as exempting these taxpayers or simply ignoring them have been proposed. We refer those who are interested in the issue to Alm et al (2004).

Fourth, we provide a case for the policy prescription that the size of the budget allocated to the IRS should be expanded as long as an additional dollar allocated could return more than an additional dollar of tax revenue (Corollary 1). Of course, like findings in other theoretical models, this result is built upon several assumptions which abstract a parsimonious model from the complicated real world. An assumption of the GRW model, on which our model is based, is that individual incomes take one of only two values (either high or low). This assumption may be restrictive in that it reduces the taxpayer problem to a simple comply/do not comply decision.[25] Other assumptions such as that true income will be discovered once a tax audit is performed, and that taxpayers suffer no additional cost during the auditing process may be problematic as well. It is arguable that the tax code itself is imperfect and that tax auditors are not uniform in interpreting the tax code. As a result, the so-called "true income" may never be known. "Mention the IRS, most people think of the dreaded tax audit." This vivid description of the IRS's tax audit by Slemrod and Bakija

program. The implementation of this program is expected to reduce the IRS's audit cost in the future; see IRS Oversight Board (2006).

[25] Two points are worth mentioning, however. First, as noted in footnote 5, there are two possible interpretations for non-compliance in our model: underreporting and non-filing. Whether or not to file tax returns is by nature a binary comply/do not comply decision. Second, we have extended the GRW model to multiple audit classes in Section 5.

(2004, p. 180) suggests that the auditing process itself may be highly costly to taxpayers. Note also that filing tax returns per se is assumed costless for individuals in our model. This seems inconsistent with the substantial efforts exerted by the IRS to provide the so-called "taxpayer service." Indeed, according to Professor Slemrod's (2005) testimony to the President's Advisory Panel on Federal Tax Reform, complying with the tax code per se costs individual taxpayers approximately \$85 billion a year. Despite these and other possible limitations of our model, we believe we have brought a fresh perspective to the important issue of how much to fund the IRS. Kaplow (1996, p. 144) wrote:

> "In the academic literature, it is well understood (although not always remembered or emphasized) that the proper cost-benefit analysis does not simply compare the enforcement cost to the revenue raised."

This claim may need to be qualified based on the thrust of this paper.

7 Appendix

Proposition 1.

Proof. (i) If $R(q) \in [0, c)$, the IRS's incremental expected revenue from a tax audit will always be less than her audit cost spent, regardless of what α is. Hence, the IRS never audits, that is $\beta^* = 0$. Since the IRS never audits, the taxpayer has no incentive to report y and, as a result, $\alpha^* = q$.

(ii) Suppose that $R(q) = c$. If $\alpha^* < q$, the IRS has no incentive to audit since $R(\alpha) < c$ for all $\alpha < q$. This implies that $\beta(\alpha^*) = 0$. However, with $\beta(\alpha^*) = 0 < \bar{\beta}$, every taxpayer would strictly prefer cheating, that is, $\alpha^* = q$, which yields a contradiction. This leaves us only the case of $\alpha^* = q$. Given $R(q) = c$, the IRS is indifferent between auditing and not auditing, that is, $\beta(q) \in [0, \min\{\frac{I}{c}, 1\}]$. However, for all taxpayers to choose cheating, we require that $\beta(q) \leq \bar{\beta}$. Hence, we obtain $\beta^* \in [0, \min\{\frac{I}{c}, \bar{\beta}\}]$.

(iii) Suppose that $R(q) > c$. Since $R(0) = 0$ and $\frac{\partial R}{\partial \alpha} > 0$, we have a unique $\alpha \in (0, q)$ such that $R(\alpha) = c$. This unique α is the $\bar{\alpha}$ defined in (1). Note that the sign of $R(\alpha) - c$ is the same as the sign of $\alpha - \bar{\alpha}$. The IRS's best audit response to α with the budget constraint is thus given by

$$\beta(\alpha) = \begin{cases} \min\{\frac{I}{c(\alpha+1-q)}, 1\} & \text{if} \quad \alpha > \bar{\alpha} \\ \in [0, \min\{\frac{I}{c(\bar{\alpha}+1-q)}, 1\}] & \text{if} \quad \alpha = \bar{\alpha} \\ 0 & \text{if} \quad \alpha < \bar{\alpha} \end{cases}$$

where $\alpha > \bar{\alpha}$ implies $R(\alpha) > c$ so that the IRS will either exhaust all her budget with $\beta(\alpha) = \frac{I}{c(\alpha+1-q)}$ or reach $\beta(\alpha) = 1$; $\alpha < \bar{\alpha}$ implies $R(\alpha) < c$ so that it is not profitable for the IRS to carry out any tax audit with $\beta(\alpha) = 0$; and $\alpha = \bar{\alpha}$ implies $R(\alpha) = c$ so that the IRS is indifferent between auditing and not auditing.

A taxpayer's best response will depend on his expectation concerning β. If he expects $\beta > \bar{\beta}$, he will report y. If $\beta < \bar{\beta}$, he will report nothing. If $\beta = \bar{\beta}$, he is indifferent.

Suppose $\alpha^* < \bar{\alpha}$, then $\beta(\alpha^*) = 0$, which implies that every taxpayer strictly prefers cheating, that is, $\alpha^* = q > \bar{\alpha}$, a contradiction.

Suppose $\alpha^* = \bar{\alpha}$, then $\beta(\alpha^* = \bar{\alpha}) \in [0, \min\{\frac{I}{c(\bar{\alpha}+1-q)}, 1\}]$. Since $\bar{\alpha} \in (0, q)$, it is required that a taxpayer be indifferent between reporting y and reporting nothing. Because $\bar{\beta}$ is the audit probability that makes the taxpayer indifferent between reporting and not reporting, the only equilibrium in this case is $\beta(\alpha^* = \bar{\alpha}) = \bar{\beta}$. Note that $\beta(\alpha^* = \bar{\alpha}) \in [0, \min\{\frac{I}{c(\bar{\alpha}+1-q)}, 1\}]$. Therefore, $\frac{I}{c(\bar{\alpha}+1-q)} \geq \bar{\beta}$ or, equivalently, $I \geq \bar{\beta}\, c(\bar{\alpha} + 1 - q)$.

Suppose $\alpha^* \in (\bar{\alpha}, q)$, then $\beta(\alpha^*) = \min\{\frac{I}{c(\alpha^*+1-q)}, 1\}$. To support $\alpha^* \in (\bar{\alpha}, q)$, which implies that a taxpayer is indifferent between reporting y and not reporting, we need $\beta(\alpha^*) = \frac{I}{c(\alpha^*+1-q)} = \bar{\beta}$, that is, $\alpha^* = \hat{\alpha}$ and $\beta^* = \bar{\beta}$. Since $\hat{\alpha} \in (\bar{\alpha}, q)$, we have $I = \bar{\beta} c(\hat{\alpha} + 1 - q) \in (\bar{\beta}(\bar{\alpha} + 1 - q) c, \bar{\beta} c)$.

Suppose $\alpha^* = q$, then $\beta(\alpha^*) = \min\{\frac{I}{c(q+1-q)}, 1\} = \min\{\frac{I}{c}, 1\}$. To support $\alpha^* = q$, which implies that a taxpayer prefers cheating, it is required that $\beta(\alpha^* = q) = \frac{I}{c} \leq \bar{\beta}$. Hence, we obtain $I \leq c\bar{\beta}$.

To sum up, $(\alpha^*, \beta^*) = (\bar{\alpha}, \bar{\beta})$ could result if $I \geq \bar{\beta}c(\bar{\alpha} + 1 - q)$; $(\alpha^*, \beta^*) = \left(\hat{\alpha}, \bar{\beta}\right)$ could result if $\bar{\beta}\left(\bar{\alpha} + 1 - q\right)c < I < \bar{\beta}c$; and $(\alpha^*, \beta^*) = (q, \frac{I}{c})$ could result if $I \leq \bar{\beta}c$. ∎

Proposition 2.

Proof. First, consider the comparison between $\left(\bar{\alpha}, \bar{\beta}\right)$ and $(\hat{\alpha}, \bar{\beta})$ when $\hat{\alpha} \neq \bar{\alpha}$. We know that $\hat{\alpha} > \bar{\alpha}$ and that $T > \bar{\beta}(T + F)$ (Result 1). Invoking these two results, it is straightforward to see from Table 1 that $G\left(\hat{\alpha}, \bar{\beta}, I\right) < G\left(\bar{\alpha}, \bar{\beta}, I\right)$. Since $\bar{I} < I$ if $\hat{\alpha} \neq \bar{\alpha}$, we also see from Table 1 that $S\left(\hat{\alpha}, \bar{\beta}, I\right) > S\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right)$. Putting these together yields $\psi\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right) < \psi\left(\hat{\alpha}, \bar{\beta}, I\right)$. That is, outcome $(\hat{\alpha}, \bar{\beta})$ always yields a higher cost per dollar of tax collection than outcome $(\bar{\alpha}, \bar{\beta})$ if $\hat{\alpha} \neq \bar{\alpha}$.

Next, consider the comparison between $\left(\bar{\alpha}, \bar{\beta}\right)$ and $\left(q, \frac{I}{c}\right)$. We will first show that $\psi(q, \frac{I}{c}, I)$ is strictly decreasing in I. Since $\psi\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right)$ is a constant, all we are left to show is that $\psi(q, \frac{I}{c}, I)$ remains higher than $\psi(\bar{\alpha}, \bar{\beta}, \bar{I})$ at $I = \bar{\beta}c$, the maximal size of the budget beyond which the equilibrium with outcome $(q, \frac{I}{c})$ will no longer exist (see Proposition 1 (iii)).

From (3) and (4), we see that C is strictly increasing and concave in β (Result 2). Therefore, for any $\lambda \in (0, 1)$ and $x > 0$, we have $\lambda C\left(x\right) + \left(1 - \lambda\right) C\left(0\right) < C\left(\lambda \cdot x + \left(1 - \lambda\right) \cdot 0\right)$. This leads to $\lambda C\left(x\right) < C\left(\lambda x\right)$, since $C\left(0\right) = 0$ by the definition of C in equation (2). Now, for any $I < I'$, choosing $\lambda = \frac{I}{I'}$ and $x = \frac{I'}{c}$ gives $\frac{I}{I'}C\left(\frac{I'}{c}\right) < C\left(\frac{I}{c}\right)$. Utilizing this result, we have

$$\psi(q, \frac{I}{c}, I) = \frac{qC\left(\frac{I}{c}\right) + I}{\frac{I}{c}q(T+F)} > \frac{q\frac{I}{I'}C\left(\frac{I'}{c}\right) + I}{\frac{I}{c}q(T+F)} = \frac{qC\left(\frac{I'}{c}\right) + I'}{\frac{I'}{c}q(T+F)} = \psi(q, \frac{I'}{c}, I').$$

This proves that $\psi(q, \frac{I}{c}, I)$ is strictly decreasing in I.

At $I = \bar{\beta}c$, we have $(q, \frac{I}{c}) = \left(\hat{\alpha}, \bar{\beta}\right)$ (Remark 1). Thus,

$$\begin{aligned} G\left(q, \frac{I}{c}, I\right) &= G(\hat{\alpha}, \bar{\beta}, I) = \left(q - \hat{\alpha}\right) T + \hat{\alpha}\bar{\beta}(T+F) \\ &< \left(q - \bar{\alpha}\right) T + \bar{\alpha}\bar{\beta}(T+F) = G\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right). \end{aligned}$$

At $I = \bar{\beta}c$, we also have

$$S\left(q, \frac{I}{c}, I\right) = qC(\frac{I}{c}) + I = qC\left(\bar{\beta}\right) + \bar{\beta}c > qC\left(\bar{\beta}\right) + \bar{\beta}\left(\bar{\alpha} + 1 - q\right)c$$
$$= S\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right).$$

Since $G\left(q, \frac{I}{c}, I\right) < G\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right)$ and $S\left(q, \frac{I}{c}, I\right) > S\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right)$ at $I = \bar{\beta}c$, we obtain $\psi(\bar{\alpha}, \bar{\beta}, \bar{I}) < \psi\left(q, \frac{I}{c}, I\right)$ at $I = \bar{\beta}c$.

Thus, for all $I \in [\bar{I}, \infty)$ (where the equilibrium $(\bar{\alpha}, \bar{\beta})$ may result) and $I' \in [0, \bar{\beta}c]$ (where the equilibrium $\left(q, \frac{I}{c}\right)$ may result),

$$\psi\left(\bar{\alpha}, \bar{\beta}, I\right) = \psi\left(\bar{\alpha}, \bar{\beta}, \bar{I}\right) < \psi\left(q, \frac{I}{c}, I = \bar{\beta}c\right) \leq \psi\left(q, \frac{I'}{c}, I'\right)$$

where the last inequality has utilized the property that ψ is decreasing in I. This means that $(\bar{\alpha}, \bar{\beta})$ gives a lower cost per dollar of tax collection than $\left(q, \frac{I}{c}\right)$. ■

Proposition 3.

Proof. First, if there exists an i such that $\alpha_i^* < \bar{\alpha}_i$, then from the definition of $\bar{\alpha}_i$ and because $\frac{\partial R}{\partial \alpha} > 0$, we know that $R\left(\alpha_i^*\right) < c_i$ and hence the IRS will choose $\beta_i^* = 0$. This implies that the taxpayers will all cheat, i.e., $\alpha_i^* = q$, a contradiction. Hence, we must have $\alpha_i^* \geq \bar{\alpha}_i$ for all i. Next, we can check that if $\{(\alpha_i^*, \beta_i^*)\}_{i=1}^n$ is an equilibrium outcome, then either $\alpha_i^* \in [\bar{\alpha}_i, q)$ and $\beta_i^* = \bar{\beta}$, or $\alpha_i^* = q$ and $\beta_i^* = \frac{I_i}{c_i} \leq \bar{\beta}$. Putting these together, we conclude that the resulting equilibrium outcomes in an audit class qualitatively follow that of Proposition 1 (iii). Finally, viewing the role of I_i intended for audit class i as that of I in the case of one audit class, the rest of the proof then follows the same logic as the proof of Proposition 2. ■

Proposition 4.

Proof. First, note that $R\left(\alpha_i\right) = \frac{\alpha_i}{\alpha_i - 1 - q}(T + F)$ is strictly increasing and continuous in α_i. Since $\frac{R(\bar{\alpha}_i)}{c_i} = 1$ and $\frac{R(q)}{c_i} > \frac{R(q)}{c_n} > 1$ $(i = 1, 2, ..., n-1)$, from the intermediate value theorem we know that there exist $\tilde{\alpha}_i \in (\bar{\alpha}_i, q)$ $(i = 1, 2, ..., n-1)$ that satisfy (5).

Step 1. Suppose that $I \leq [c_n + \sum_{i=1}^{n-1}(\tilde{\alpha}_i + 1 - q)c_i]\bar{\beta}$. It is straightforward to check that there exists an equilibrium outcome $\{(\alpha_i^*, \beta_i^*)\}_{i=1}^n$ such that $(\alpha_i^*, \beta_i^*) = (\tilde{\alpha}_i, \bar{\beta})$ for $i = 1, \ldots, n-1$ and $(\alpha_n^*, \beta_n^*) = \left(q, \frac{I_n}{c}\right)$, where $I_n = I - \sum_{i=1}^{n-1}(\tilde{\alpha}_i + 1 - q)\, c_i \bar{\beta}$. First, given $\alpha_i^* = \tilde{\alpha}_i$ $(i = 1, 2, ..., n-1)$ and $\alpha_n^* = q$, we see from equation (5) that the incremental revenue per dollar of tax audit is the same across all classes. This implies that the IRS cannot improve her profit by re-allocating the budget between different audit classes and changing β_i. Next, given $\beta_i^* = \bar{\beta}_i$ $(1, 2, ..., n-1)$ and $\beta_n^* = \frac{I_n}{c_n}$, we see that $\alpha_i^* = \tilde{\alpha}_i$ $(i = 1, 2, ..., n-1)$ is consistent with $\beta_i^* = \bar{\beta}_i = \bar{\beta}$ $(i = 1, 2, ..., n-1)$ and that $\alpha_n^* = q$ is consistent with $\beta_n^* = \frac{I_n}{c_n} \leq \bar{\beta}$ (Since $I \leq [c_n + \sum_{i=1}^{n-1}(\tilde{\alpha}_i + 1 - q)c_i]\bar{\beta}$ by assumption, we obtain $I_n \leq c_n\bar{\beta}$ from $I_n = I - \sum_{i=1}^{n-1}(\tilde{\alpha}_i + 1 - q)\, c_i\bar{\beta}$).

Step 2. Suppose that $I > [c_n + \sum_{i=1}^{n-1}(\tilde{\alpha}_i + 1 - q)c_i]\bar{\beta}$. We show below that $\{(\alpha_i^*, \beta_i^*)\}_{i=1}^n = \{\left(\bar{\alpha}_i, \bar{\beta}\right)\}_{i=1}^n$, that is, $\{\left(\bar{\alpha}_i, \bar{\beta}\right)\}_{i=1}^n$ is the unique equilibrium outcome of the game.

First, $\frac{R\left(\alpha_i^*\right)}{c_i}$ must equal a constant for all $i = 1, 2, \ldots, n$. Suppose not, and let i_0 be the audit class which has the highest incremental revenue per dollar of tax audit (i.e. $\frac{R(\alpha_{i_0}^*)}{c_{i_0}} > \frac{R\left(\alpha_i^*\right)}{c_i}$ for $i \neq i_0$). This implies that the IRS will audit the class i_0 with the first priority. Since $\frac{I}{\left(\alpha_{i_0}^*+1-q\right)c_{i_0}} > \frac{I}{c_{i_0}} > \bar{\beta}$ $(\frac{I}{c_{i_0}} \geq \frac{I}{c_n} > \bar{\beta} + \sum_{i=1}^{n-1}(\tilde{\alpha}_i + 1 - q)\frac{c_i}{c_n}\bar{\beta})$, we have $\beta_{i_0}^* > \bar{\beta}$, which leads to $\alpha_{i_0}^* = 0$. However, note that $R\left(\alpha_{i_0}^* = 0\right) = 0$ and $\frac{R(\alpha_i)}{c_i} \geq 0$ for all $i = 1, \ldots, n$, which yields a contradiction with $\frac{R(\alpha_{i_0}^*)}{c_{i_0}} > \frac{R\left(\alpha_i^*\right)}{c_i}$ for $i \neq i_0$.

Secondly, $\frac{R\left(\alpha_i^*\right)}{c_i} = 1$ must hold for all $i = 1, \ldots, n$. Suppose not. If $\frac{R\left(\alpha_i^*\right)}{c_i} < 1$, the IRS would have no incentive to audit and so $\beta_i^* = 0$. This leads to $\alpha_i^* = q$ for all i and $\frac{R(q)}{c_i} > \frac{R(q)}{c_n} > 1$, a contradiction. If $\frac{R\left(\alpha_i^*\right)}{c_i} > 1$, the profit-maxmizing IRS will exhaust the budget intended for each audit class. Since c_i is strictly increasing in i, α_i^* must be strictly increasing in i in order to have a constant $\frac{R\left(\alpha_i^*\right)}{c_i}$ for all i. Because of $\alpha_1^* < \alpha_2^* < ... < \alpha_n^* \leq q$, the highest $\{\alpha_i^*\}_{i=1}^n$ that generates the same incremental revenue per dollar of tax audit is that $\alpha_i^* = \tilde{\alpha}_i$ for $i = 1, \ldots, n-1$ and $\alpha_n^* = q$. This implies that $\alpha_i^* \leq \tilde{\alpha}_i < q$ for $i = 1, \ldots, n-1$, which in turn implies that $\beta_i^* \leq \bar{\beta}$ $(i = 1, 2, ..., n-1)$. Putting $\alpha_i^* \leq \tilde{\alpha}_i$ and $\beta_i^* \leq \bar{\beta}$

$(i = 1, 2, ..., n-1)$ together yields

$$I_n = I - \sum_{i=1}^{n-1} (\alpha_i^* + 1 - q)\, \beta_i^* c_i > I - \sum_{i=1}^{n-1} (\tilde{\alpha}_i + 1 - q)\, \bar{\beta} c_i > \bar{\beta} c_n$$

where the first equality has utilized the result that the IRS will exhaust the budget intended for each class and the last inequality comes from our premise. Therefore, we have $\beta_n^* = \frac{I_n}{c_n} > \bar{\beta}$, which leads to $\alpha_n^* = 0$, a contradiction.

Since $\frac{R(\alpha_i^*)}{c_i} = 1$ for all $i = 1, \ldots, n$, we have $\alpha_i^* = \bar{\alpha}_i$ $(i = 1, \ldots, n)$. To support $\alpha_i^* \in (0, q)$ such that the taxpayers are indifferent between auditing and not auditing, $\beta_i^* = \bar{\beta}$ must be true. ■

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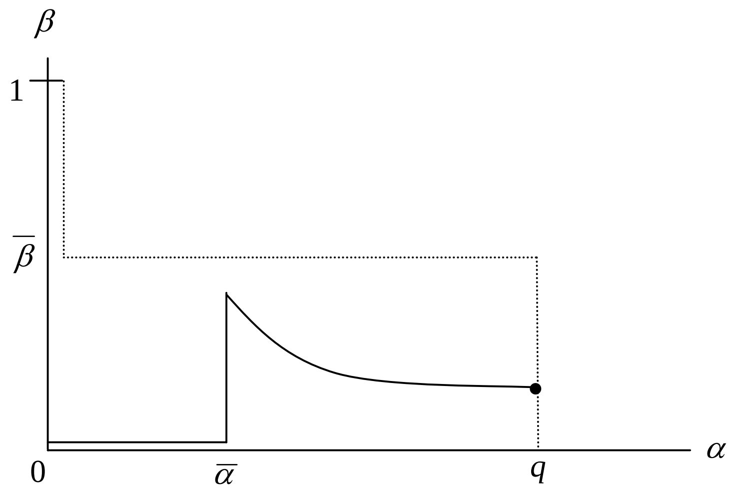

Figure 1a $\alpha^* = q$

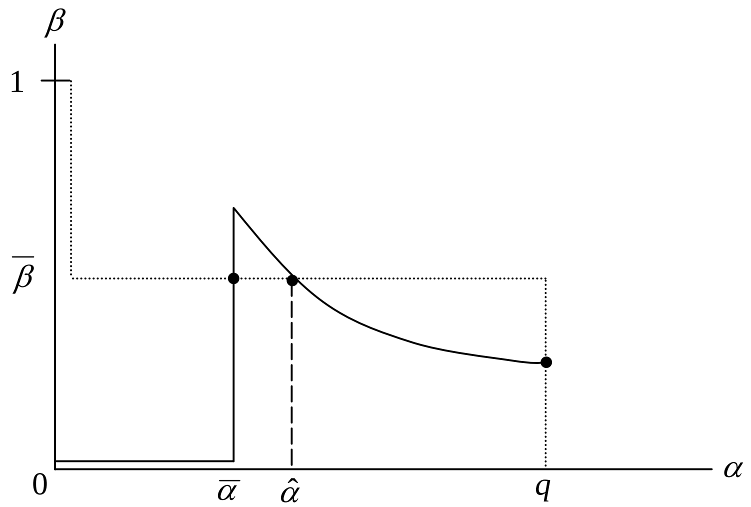

Figure 1b $\alpha^* = \overline{\alpha}, \hat{\alpha},$ or q

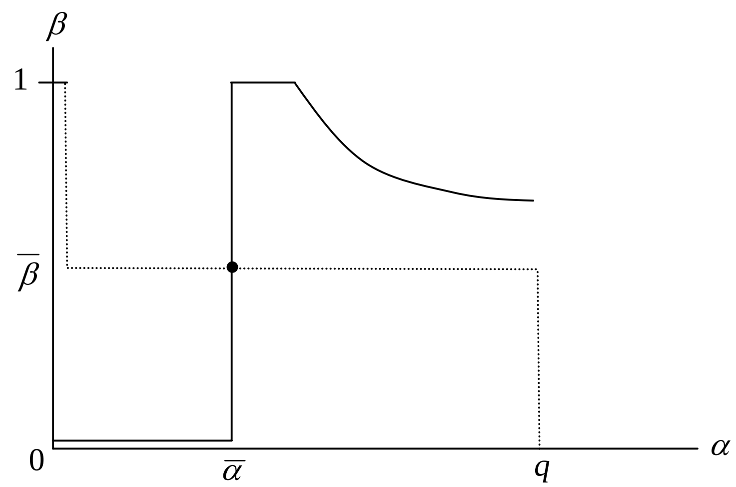

Figure 1c $\alpha^* = \overline{\alpha}$

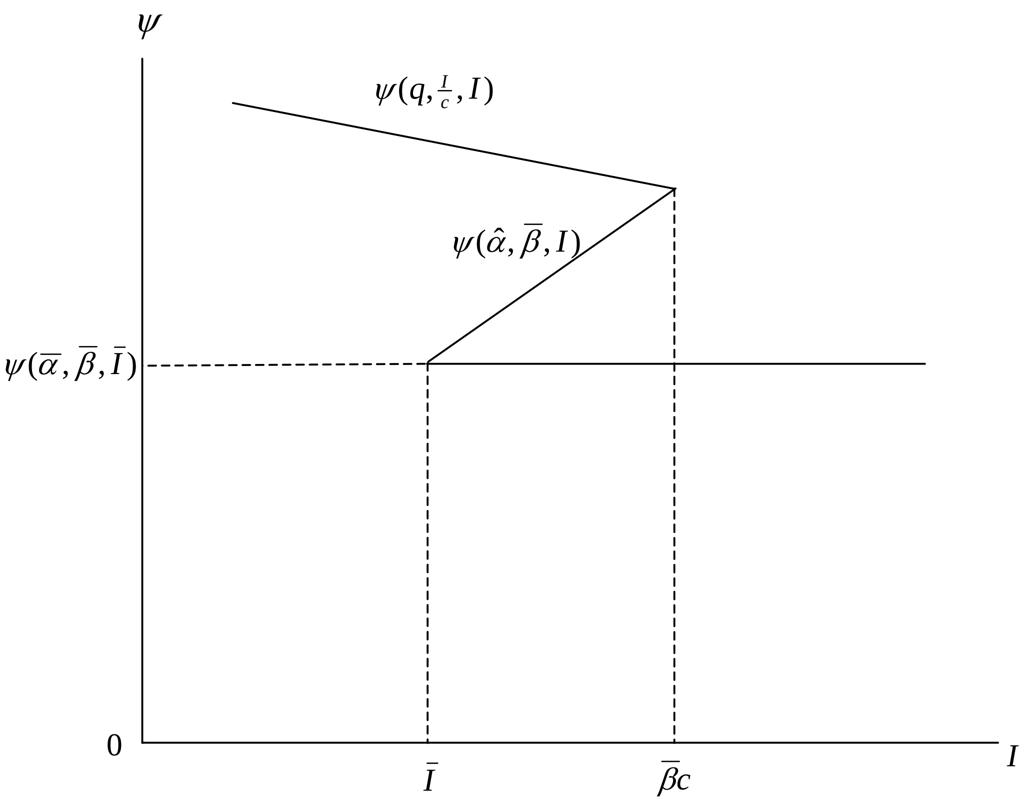

Figure 2 ψ against I

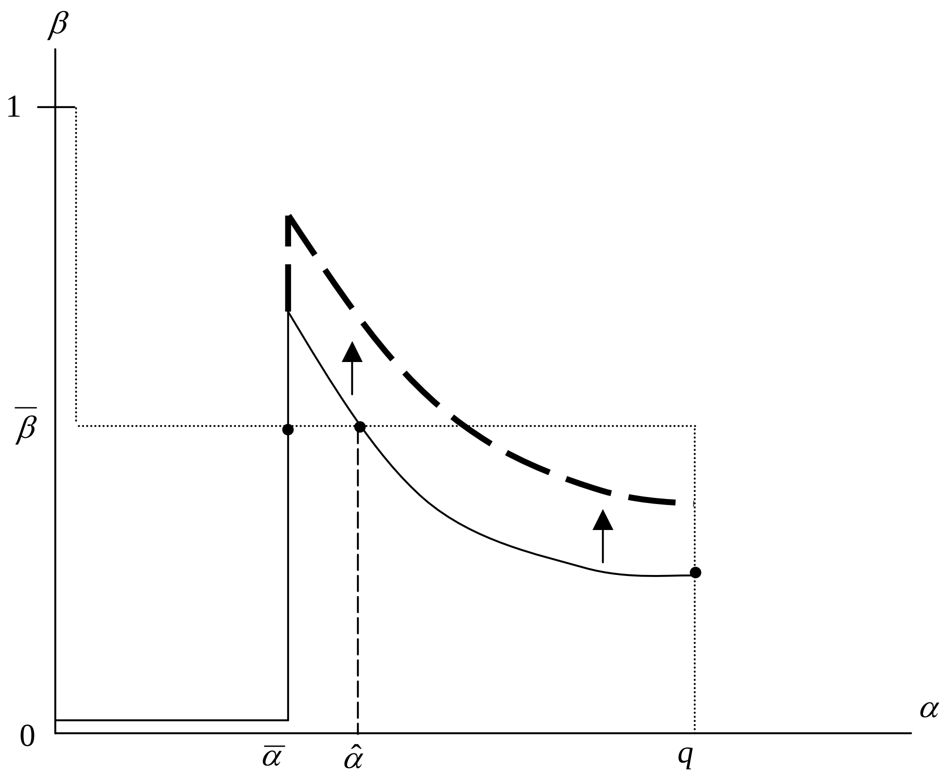

Figure 3 Shift of $\beta(\alpha)$ due to an increase in I

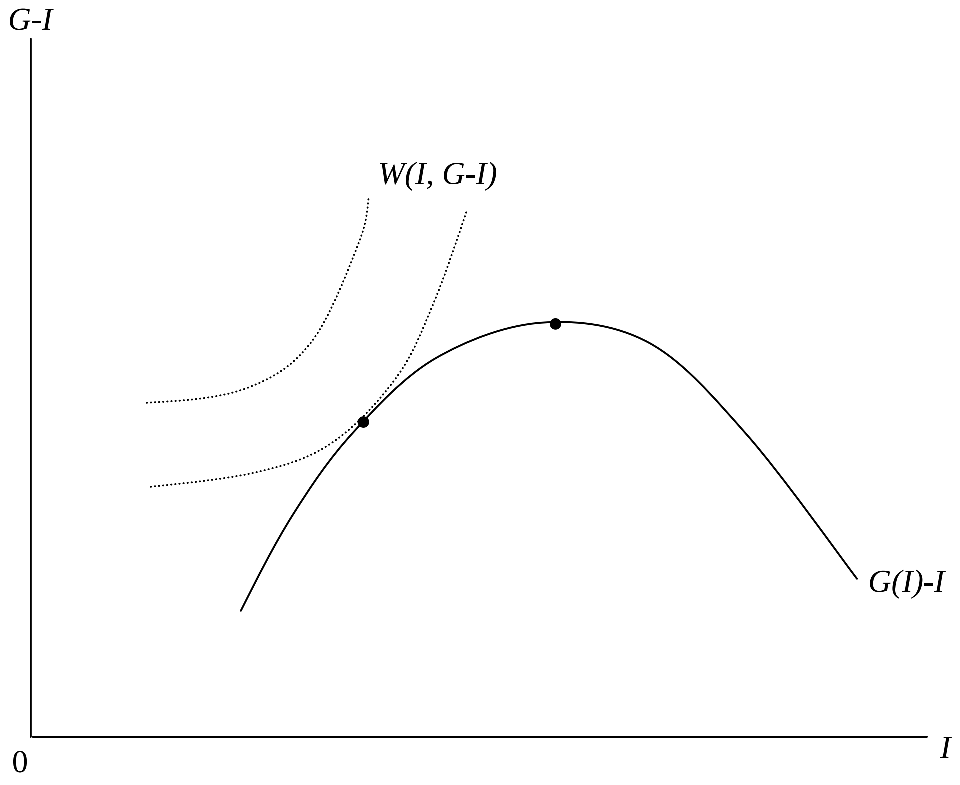

Figure 4-1 A typical profit function

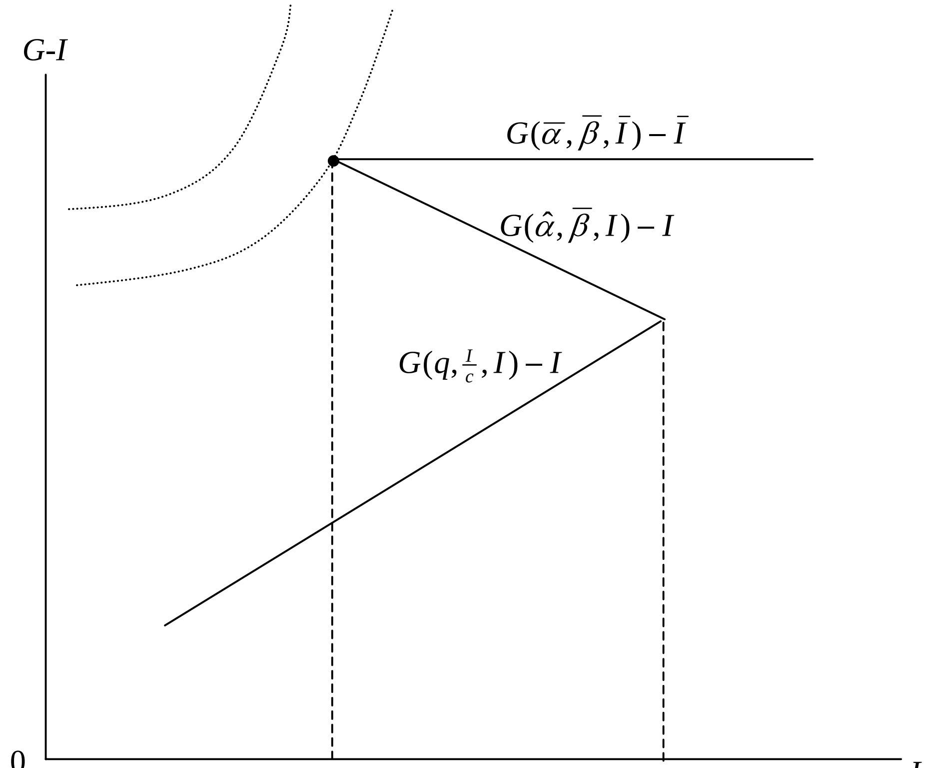

Figure 4-2 A profit function with congestion effects

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