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**WORKING PAPER SERIES**

**NO. 514 / AUGUST 2005**

**GAINS FROM  
INTERNATIONAL  
MONETARY POLICY  
COORDINATION**

**DOES IT PAY TO  
BE DIFFERENT?**

by Zheng Liu  
and Evi Pappa

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by Zheng Liu <sup>2</sup>  
and Evi Pappa <sup>3</sup>

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## Abstract

This paper presents a new argument for international monetary policy coordination based on considerations of structural asymmetries across countries. In a two-country world with a traded and a non-traded sector in each country, optimal independent monetary policy cannot replicate the natural-rate allocations. There are potential welfare gains from coordination since the planner under a cooperating regime internalizes a *terms-of-trade externality* that independent central banks tend to overlook. Yet, with symmetric structures across countries, the gains are quantitatively small. If the size of the traded sector differs across countries, the gains can be sizable and increase with the degree of asymmetry. The planner's optimal policy not only internalizes the terms-of-trade externality, it also creates a *terms-of-trade bias* in favor the country with a larger traded sector. Further, the planner tries to balance the terms-of-trade bias against the need to stabilize fluctuations in the terms-of-trade gap.

*JEL classification:* E52, F41, F42

*Keywords:* International Policy Coordination; Optimal Monetary Policy; Asymmetric Structures; Terms-of-Trade Bias.

## Non-Technical Summary

In an increasingly integrated world economy, should countries coordinate monetary policy or should they conduct monetary policy independently? Recent theoretical work within the new open-economy macro (NOEM) framework has reached a surprising conclusion: inward-looking policy is approximately optimal and there is not much to gain from policy coordination.

This paper provides a new argument for international monetary policy coordination based on considerations of structural asymmetries across countries. Our analysis is based on a framework that generalizes the standard two-country, one-sector model in the NOEM literature by considering multiple sectors of production in each country. In particular, we consider a traded sector, the products of which enter the consumption baskets in the households in both countries; and a non-traded sector, the products of which enter domestic baskets only. An innovation of this paper is that we allow the size of the traded sector to differ across countries to capture some of the structural asymmetries between countries at different stages of development. This feature is motivated by the observation that a more developed country has typically a larger service sector than does a less developed one, and the traded component in services is typically small.

In our welfare analysis, we obtain welfare measures through second-order approximations to the households' utility functions and to the private sector's optimizing conditions. We show that, with multiple sectors and thus multiple sources of nominal rigidities, optimal independent monetary policy cannot replicate the natural-rate allocations, creating a scope for coordination. The gains from coordination arise from two channels. The first channel is rather standard in the NOEM literature and is independent of the structural asymmetry in the model: if acting independently, a country's central bank tends to overlook the effect of terms-of-trade on the other country's well-being; whereas when the countries cooperate, this terms-of-trade externality would be properly recognized and efforts would be made to internalize it. The second channel is unique to our model and works only through structural asymmetries across countries: the planner's optimal policy under the cooperating regime creates a *terms-of-trade bias* that favors the country with a larger traded sector; and this bias has to be balanced against the need to stabilize fluctuations in the terms-of-trade gap, among other variables in the policy objective. Absent structural asymmetry, the welfare gains from coordination are quantitatively small under calibrated parameters. As the degree of asymmetry enlarges, so do the welfare gains in general. With plausible structural asymmetries, there are sizable gains. Thus, a stronger case for policy coordination can be made when the countries involved have asymmetric production and trading structures.

# 1 Introduction

As countries become more interdependent through international trade, should they conduct monetary policies independently or should they coordinate their policies? In other words, are there gains from international monetary policy coordination? This question lies at the heart of the intellectual discussions about optimal monetary policy in open economies. The literature has produced a strong conclusion in favor of inward-looking policies and flexible exchange-rate regimes. This conclusion has been drawn not only in the traditional literature within the Mundell-Fleming framework that features ad hoc stabilizing policy goals, but also in the more recent New Open-Economy Macro (NOEM) literature that features optimizing individuals, monopolistic competition and nominal rigidities, with the representative household's utility function serving as a natural welfare metric for optimal policy. In the traditional literature, many have argued that the gains from coordination are likely to be small because a flexible exchange-rate system would effectively insulate impacts of foreign disturbances on domestic employment and output [e.g., Mundell (1961) and the survey by McKibbin (1997)]. In the NOEM literature pioneered by Obstfeld and Rogoff (1995), it has been shown that, although gains from coordination are theoretically possible, they are quantitatively small [e.g., Obstfeld and Rogoff (2000a, 2002), Corsetti and Pesenti (2005)].

The remarkably strong conclusion about the lack of gains from coordination has stimulated a lively debate and a growing strand of literature in search of sources of coordination gains by enriching the simple framework built by Obstfeld and Rogoff (2000a, 2002). Several potential sources have been identified. For instance, the gains from coordination can be related to the degree of exchange-rate pass-through [e.g., Devereux and Engel (2003), Duarte (2003), and Corsetti and Pesenti (2005)].<sup>1</sup> Even with perfect exchange-rate pass-through, inward-looking monetary policy can be suboptimal and be improved upon by coordination, depending on the values of the intertemporal elasticity and the elasticity of substitution between goods produced in different countries [e.g., Clarida, et al. (2002), Benigno and Benigno (2003), Pappa (2004), Sutherland (2002a), and Tscharov (2004)]. Policy coordination may also produce welfare gains if the international financial markets are incomplete [e.g., Benigno (2001) and Sutherland (2002b)], policy makers have imperfect information [e.g., Dellas (2004)], or domestic shocks are imperfectly correlated across sectors [e.g., Canzoneri, et al. (2004)].

The present paper emphasizes the role of asymmetries in the production structure across countries in generating gains from policy coordination. To this end we build a two-country model in the spirit of the NOEM literature, with two production sectors within each country. One

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<sup>1</sup>Corsetti and Dedola (2002) show that, if the distribution of traded goods requires local inputs, then international markets would be endogenously segmented, rendering exchange-rate pass-through incomplete. This feature also provides a scope for international monetary policy cooperation.



sector produces traded goods that enter the consumption baskets in both countries, and the other sector produces non-traded goods that enter the domestic consumption basket only. To allow for real effects of monetary policy, we assume staggered price setting in both sectors.<sup>2</sup> A key point of departure from the NOEM literature is that we allow the share of traded goods in the consumption basket to be different across countries to capture an important cross-country difference in production and trading structures. As Figure 1 shows, a developed country has typically a much larger share of service value-added in GDP than does a developing country, and the traded component of services is small. In this sense, the asymmetric production and trading structure in our model can be interpreted broadly as characterizing countries at different stages of development. In the context of this model, we examine how the presence of multiple sectors and sectoral asymmetries across countries would affect macroeconomic stability and welfare under independent or cooperating central banks.

To help exposition, we assume log-utility in aggregate consumption, a unitary elasticity of substitution between domestically-produced traded goods and imported goods, and a unitary elasticity of substitution between traded and non-traded goods in the consumption baskets. In the absence of non-traded sectors, many authors have demonstrated that, under a comparable set of assumptions (i.e., log-utility and unitary elasticity of substitution for goods produced in different countries), optimal monetary policy for an independent central bank is inward-looking and there are no gains from coordination.<sup>3</sup> Introducing a non-traded sector and sticky prices in both sectors renders exchange-rate pass-through incomplete even under producer-currency pricing; meanwhile, it creates a policy trade-off facing independent central banks when sectoral shocks are imperfectly correlated, which gives rise to a scope for gains from policy coordination. Our simplifying assumptions also make it possible to derive second-order approximations to the households' utility functions even in the presence of multiple sources of nominal rigidities and sectoral asymmetry, and help us to obtain an analytical expression for the welfare criterion that can be used to compare outcomes of different policies. Despite their apparent restrictiveness, these assumptions do not prevent the model from generating significant coordination gains, nor

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<sup>2</sup>The NOEM literature on optimal monetary policy typically employs a simpler model with one-period predetermined prices, an advantage of which is its analytical tractability [e.g., Obstfeld and Rogoff (1995, 2000a, 2002), Corsetti and Pesenti (2005), Canzoneri, et al. (2004)]. However, assuming staggered price-setting as we do here instead of predetermined prices helps generate richer and arguably more realistic equilibrium dynamics [e.g., Clarida, et al. (2002), Kollmann (2002)] and is thus more appropriate for *quantitative* welfare analysis. In addition, as is well known in the closed-economy literature, staggered price-setting leads to inefficient price dispersion, giving rise to an additional source of inefficiency that optimal monetary policy needs to deal with.

<sup>3</sup>See, for example, Clarida et al. (2002), Benigno and Benigno (2003), and Pappa (2004), among others.



do they prevent us from studying the sensitivity of the results to some key parameters in the model.

The literature has long emphasized the importance of the non-traded sector in understanding international business cycle fluctuations [e.g., Stockman and Tesar (1995), Baxter, et al (1998), Corsetti, et al. (2003), and Ghironi and Melitz (2003)] and real exchange rate movements [e.g., Rogoff (1996) and Burnstein et al (2003)]. Empirical studies suggest that, at least for the OECD countries, a substantial part of aggregate fluctuations originates from sectoral shocks rather than national disturbances [e.g., Stockman (1988), and Marimon and Zilibotti (1998)]; and within each country, the time series processes generating productivity shocks in traded and non-traded sectors are quite different [e.g., Canzoneri, et al. (1999)]. These studies cast doubts on the ability of models with a single traded sector in explaining the transmission of shocks across countries. Yet, it is remarkable that studies of optimal monetary policy in open economies typically abstract from the non-traded sector or other multi-sector features of the actual economy by assuming that each country is completely specialized in a single traded sector, with no distinctions between sector-specific and country-specific shocks.<sup>4</sup>

Our paper contributes to the literature in three aspects. First, we study optimal monetary policy in an open-economy model that explicitly incorporates a non-traded sector, so that a monetary authority needs to confront a policy trade-off stemming from multiple sources of nominal rigidities and imperfectly correlated sector-specific shocks; whereas in the standard one-sector model with traded goods only, policy makers are not concerned about such trade-offs. Second, we make a methodological contribution to the literature by deriving an explicit expression for welfare under both independent central banks and a common planner. To our knowledge, we are the first to derive such a welfare criterion in an open economy with multiple sectors based on quadratic approximations of households' utility function.<sup>5</sup> Finally, the main value-added of the current paper in relation to the existing literature is that, by introducing non-traded goods and asymmetric production structures, without any further modifications, we are able to go beyond

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<sup>4</sup>A few notable exceptions include Obstfeld and Rogoff (2002) and Hau (2000), whose models feature a traded and a non-traded sector, with perfectly correlated shocks; Canzoneri, et al. (2004), who examine a version of the model presented in Obstfeld and Rogoff (2002), but allow imperfect correlations between sectoral shocks; Tille (2002), who presents a two-country model that features incomplete specialization of the countries in two types of traded goods (but with no non-traded goods), so that a distinction between sectoral shocks and national shocks arises; and Huang and Liu (2004a), who study a model with multiple stages of production and trade in intermediate goods. Unlike our work here, all of these studies maintain symmetric production and trading structures across countries.

<sup>5</sup>For a comprehensive description of the general approach to deriving the welfare criterion for optimal policy based on quadratic approximations to households' utility functions, see Michael Woodford (2003).

the special results obtained by Obstfeld and Rogoff (2002) concerning the welfare consequences of international monetary policy cooperation.

Our results can be easily summarized. With multiple sectors and thus multiple sources of nominal rigidities, optimal independent monetary policy cannot replicate the natural-rate allocations, creating a scope for coordination. Such gains materialize as the planner under the cooperating regime tries to internalize a terms-of-trade externality that independent central banks tend to overlook. Yet, in the absence of structural asymmetry, the gains obtained through this channel are quantitatively small under calibrated parameters. With asymmetric structures, we show that the planner's optimal policy creates a *terms-of-trade bias* in favor of the country that has a larger traded sector. This terms-of-trade bias needs to be balanced against the planner's desire to stabilize fluctuations in the terms-of-trade gap, among other variables in the policy objective derived from the first principle. The gains from coordination increase with the degree of structural asymmetry and can be sizable under calibrated parameters. Further, the gains increase with the share of imported goods in the traded consumption baskets and with the durations of pricing contracts; but decrease with the correlations of domestic shocks. To the extent that the production and trading structure in our model captures a difference between developing countries and developed ones, our results shed some light on the welfare consequences of international monetary policy coordination between countries at different stages of development.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 examines equilibrium dynamics. Section 4 discusses optimal monetary policy under independent central banks and under cooperation. Section 5 assesses the quantitative gains from policy coordination and studies their sensitivity to changes in a few key parameters in the model. Finally, Section 6 concludes. We focus on presenting the main results and intuitions in the text, and relegate detailed derivations to the Appendix.

## 2 The Model

Consider a world economy with two countries, home and foreign, each populated by a continuum of identical, infinitely-lived households. The representative household in each country is endowed with one unit of time, and derives utility from consuming a basket of final goods. The consumption basket consists of traded goods, either domestically produced or imported (e.g., manufacturing goods), and of non-traded goods (e.g., services). Final consumption goods are composites of differentiated intermediate goods produced in two sectors, a traded good sector, and a non-traded good sector. Production of intermediate goods requires domestic labor as the only input, which is supplied by domestic households. Labor is mobile across sectors, but not across countries. The

production and preference structures in the two countries are symmetric except that the share of traded goods in the final consumption basket may differ.

Time is discrete. In each period of time  $t = 0, 1, \dots$ , a productivity shock is realized in each intermediate-good sector. Firms and households make their optimizing decisions after observing the shocks. All agents have access to an international financial market, where they can trade a state-contingent nominal bond. The government in each country conducts monetary policy and uses lump-sum transfers to finance production subsidies.

## 2.1 Representative Households

The preferences of households are symmetric across countries, so we focus on the representative household in the home country. The utility function is given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t [\ln C_t - \Psi L_t], \quad (2.1)$$

where  $0 < \beta < 1$  is a subjective discount factor,  $C_t > 0$  denotes consumption,  $L_t \in (0, 1)$  denotes hours worked, and  $\mathbb{E}$  is an expectation operator.

The purchase of consumption goods is financed by labor income, profit income, and a lump-sum transfer from the government. In addition, the household has access to an international financial market, where state-contingent nominal bonds (denominated in home currency) can be traded. The period-budget constraint facing the household is given by

$$P_t C_t + E_t D_{t,t+1} B_{t+1} \leq W_t L_t + B_t + \Pi_t + T_t, \quad t = 0, 1, \dots, \quad (2.2)$$

where  $P_t$  is the price level,  $B_{t+1}$  is the holdings of the state-contingent nominal bond that pays one unit of home currency in period  $t + 1$  if a specified state is realized,  $D_{t,t+1}$  is the period- $t$  price of such bonds,  $W_t$  is the nominal wage rate,  $\Pi_t$  is the profit income, and  $T_t$  is the lump-sum transfer from the government.

The household maximizes (2.1) subject to (2.2). The optimal labor supply decision implies

$$\Psi C_t = W_t / P_t, \quad (2.3)$$

which states that the marginal rate of substitution between leisure and consumption equals the real consumption wage. The optimal consumption-saving decision is described by

$$D_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}}, \quad (2.4)$$

so that the intertemporal marginal rate of substitution equals the price of the state contingent bond. Define the nominal interest rate on a risk-free bond as  $R_t = [E_t D_{t,t+1}]^{-1}$ . Then (2.4) implies that

$$\frac{1}{C_t} = \beta E_t \left[ \frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}} R_t \right], \quad (2.5)$$

which is the familiar intertemporal Euler equation.

The final consumption basket consists of traded goods (domestically produced and imported) and non-traded goods. Denote  $C_{Nt}$  the composite good that is non-traded, and  $C_{Tt}$  the composite of goods that are traded. Then we have

$$C_t = \bar{\alpha} C_{Tt}^\alpha C_{Nt}^{1-\alpha}, \quad \bar{\alpha} = \alpha^{-\alpha} (1 - \alpha)^{\alpha-1}. \quad (2.6)$$

The traded component  $C_{Tt}$  is itself an aggregate of domestically produced good  $C_{Ht}$  and imported good  $C_{Ft}$ , that is,

$$C_{Tt} = \bar{\omega} C_{Ht}^\omega C_{Ft}^{1-\omega}, \quad \bar{\omega} = \omega^{-\omega} (1 - \omega)^{\omega-1}. \quad (2.7)$$

Solving the household's expenditure-minimizing problem yields the following demand functions for non-traded and traded goods:

$$C_{Nt} = (1 - \alpha) P_t C_t / \bar{P}_{Nt}, \quad C_{Tt} = \alpha P_t C_t / \bar{P}_{Tt}, \quad (2.8)$$

where  $\bar{P}_{Nt}$  is the price of final non-traded goods, and  $\bar{P}_{Tt}$  is the price of final traded goods, which are related to the price level  $P_t$  by

$$P_t = \bar{P}_{Tt}^\alpha \bar{P}_{Nt}^{1-\alpha}. \quad (2.9)$$

The induced demand functions for domestically produced traded goods and for imported goods are respectively given by

$$C_{Ht} = \omega \bar{P}_{Tt} C_{Tt} / \bar{P}_{Ht}, \quad C_{Ft} = (1 - \omega) \bar{P}_{Tt} C_{Tt} / [\mathcal{E}_t \bar{P}_{Ft}^*], \quad (2.10)$$

where  $\bar{P}_{Ht}$  is the price index of home-produced traded goods,  $\bar{P}_{Ft}^*$  is the price index of foreign-produced traded goods, and  $\mathcal{E}_t$  is the nominal exchange rate. These prices are related to  $\bar{P}_{Tt}$  by

$$\bar{P}_{Tt} = \bar{P}_{Ht}^\omega [\mathcal{E}_t \bar{P}_{Ft}^*]^{1-\omega} \quad (2.11)$$

Throughout our analysis, we assume that firms set prices in the sellers' local currency and the law-of-one-price holds, so that the cost of imported goods in the home consumption basket is simply the price of traded goods charged by foreign exporting firms, adjusted by the nominal exchange rate, as in (2.11).<sup>6</sup>

<sup>6</sup>Several recent studies discuss the effects of buyers' local currency pricing [e.g., Devereux and Engel (2003)], "dollar pricing" [e.g., Devereux, et al. (2003)], or some intermediate pricing behavior [e.g., Corsetti and Pesenti (2005)] on the international dimension of monetary policy. We focus here on sellers' local currency pricing, although it should not be too difficult to extend our analysis to a model with different pricing behaviors. The case of "dollar pricing" (or Euro pricing) can be particularly relevant in studying the gains from coordination between a more advanced economy and a less developed one, since there is evidence that the currency of the developed economy is typically used as the "vehicle currency" [see, for example, Goldberg and Tille (2005) and ECB (2005)]. In our view, this issue is of interest in itself and is important enough to deserve investigation in a separate paper.



## 2.2 Production Technologies and Optimal Pricing Rules

There are two sectors producing intermediate goods: a non-traded sector and a traded sector. In each sector, there is a continuum of firms producing differentiated products indexed in the interval  $[0, 1]$ . To produce intermediate goods in each sector requires labor input, with constant-returns-to-scale (CRS) technologies

$$Y_{Nt}(i) = A_{Nt}L_{Nt}(i), \quad i \in [0, 1], \quad (2.12)$$

and

$$Y_{Ht}(j) + Y_{Ht}^*(j) = A_{Tt}L_{Tt}(j), \quad j \in [0, 1], \quad (2.13)$$

where  $Y_{Nt}(i)$  is the output of type- $i$  non-traded intermediate goods;  $Y_{Ht}(j)$  is the output of type- $j$  traded intermediate goods sold in the domestic market, and  $Y_{Ht}^*(j)$  that exported to the foreign market;  $A_{Nt}$  and  $A_{Tt}$  are productivity shocks in the two sectors; and  $L_N$  and  $L_T$  are labor inputs in the non-traded and in the traded sector respectively. The logarithms of the productivity shocks in each sector follows a random-walk process, that is,

$$\ln(A_{k,t+1}) = \ln(A_{k,t}) + \varepsilon_{k,t+1}, \quad k \in \{N, T\}, \quad (2.14)$$

where  $\varepsilon_{Nt}$  and  $\varepsilon_{Tt}$  are mean-zero, iid normal processes with finite variances given by  $\sigma_N^2$  and  $\sigma_T^2$ , respectively. We allow the shocks to be correlated across sectors, with a correlation coefficient given by  $\rho_{TN}$  (they need not be perfectly correlated). The productivity shocks in the foreign country follow similar processes, and potentially correlated with the shocks in the home country, with correlation coefficients denoted by  $\rho_{TT}$  for traded sectors and  $\rho_{NN}$  for non-traded sectors.

There is a CES aggregation technology that transforms intermediate goods produced in each sector into final consumption goods according to

$$C_{Nt} = \left[ \int_0^1 Y_{Nt}(i)^{\frac{\theta_N-1}{\theta_N}} di \right]^{\frac{\theta_N}{\theta_N-1}}, \quad C_{Ht} = \left[ \int_0^1 Y_{Ht}(j)^{\frac{\theta_T-1}{\theta_T}} dj \right]^{\frac{\theta_T}{\theta_T-1}}, \quad (2.15)$$

where  $\theta_N$  and  $\theta_T$  denote elasticities of substitution between differentiated products in the two sectors. To ensure equilibrium existence, we assume that the  $\theta$ 's both exceed unity (see, for example, Blanchard and Kiyotaki (1987)).

By solving the cost-minimizing problem of the aggregation sector, we obtain the demand functions for each type of intermediate goods:

$$Y_{Nt}^d(i) = \left[ \frac{P_{Nt}(i)}{\bar{P}_{Nt}} \right]^{-\theta_N} C_{Nt}, \quad Y_{Ht}^d(j) = \left[ \frac{P_{Ht}(j)}{\bar{P}_{Ht}} \right]^{-\theta_T} C_{Ht}, \quad (2.16)$$

where  $P_{Nt}(i)$  is the price of type- $i$  non-traded intermediate goods,  $P_{Ht}(j)$  is the price of type- $j$  traded intermediate goods, and  $\bar{P}_{Nt} = \left[ \int_0^1 P_{Nt}(i)^{1-\theta_N} di \right]^{\frac{1}{1-\theta_N}}$  and  $\bar{P}_{Ht} = \left[ \int_0^1 P_{Ht}(j)^{1-\theta_T} dj \right]^{\frac{1}{1-\theta_T}}$  are the corresponding price indices.

Firms are price takers in the input market and monopolistic competitors in the product markets. In each sector, firms stagger their pricing decisions in the spirit of Calvo (1983). Specifically, in each period of time, each firm receives an i.i.d. random signal that determines whether or not it can set a new price. The probability that a firm can adjust its price is  $1 - \gamma_k$  in sector  $k \in \{N, T\}$ . By the law of large numbers, a fraction  $1 - \gamma_k$  of all firms in sector  $k$  can adjust prices, while the rest of the firms cannot.

If a firm who produces type- $i$  non-traded goods can set a new price, it chooses  $P_{Nt}(i)$  to maximize its expected present value of profits

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \gamma_N^{\tau-t} D_{t,\tau} [P_{Nt}(i)(1 + \tau_N) - V_{N\tau}] Y_{N\tau}^d(i), \quad (2.17)$$

where  $\tau_N$  is a production subsidy,  $V_{Nt}$  is the unit cost, which is identical across firms since all firms face the same input market, and  $Y_{Nt}^d(i)$  is the demand schedule for type  $i$  non-traded good described in (2.16). Regardless of whether a firm can adjust its price, it has to solve a cost-minimizing problem, the solution of which yields the unit cost function

$$V_{Nt} = W_t/A_{Nt}, \quad (2.18)$$

and a conditional factor demand function

$$L_{Nt} = \frac{1}{A_{Nt}} \int_0^1 Y_{Nt}^d(i) di. \quad (2.19)$$

The solution to the profit-maximizing problem gives the optimal pricing rule

$$P_{Nt}(i) = \frac{\mu_N}{(1 + \tau_N)} \frac{\mathbb{E}_t \sum_{\tau=t}^{\infty} \gamma_N^{\tau-t} D_{t,\tau} V_{N\tau} Y_{N\tau}^d(i)}{\mathbb{E}_t \sum_{\tau=t}^{\infty} \gamma_N^{\tau-t} D_{t,\tau} Y_{N\tau}^d(i)}, \quad (2.20)$$

where  $\mu_N = \theta_N/(\theta_N - 1)$  measures the steady-state markup in sector  $N$ . Similarly, the optimal pricing rule for a firm that produces type- $j$  traded good is given by

$$P_{Ht}(j) = \frac{\mu_T}{(1 + \tau_T)} \frac{\mathbb{E}_t \sum_{\tau=t}^{\infty} \gamma_T^{\tau-t} D_{t,\tau} V_{T\tau} [Y_{H\tau}^d(j) + Y_{H\tau}^{*d}(j)]}{\mathbb{E}_t \sum_{\tau=t}^{\infty} \gamma_T^{\tau-t} D_{t,\tau} [Y_{H\tau}^d(j) + Y_{H\tau}^{*d}(j)]}, \quad (2.21)$$

where  $\mu_T = \theta_T/(\theta_T - 1)$  measures the steady state markup in sector  $T$ . From solving the firm's cost-minimizing problem, we obtain the unit cost function

$$V_{Tt} = W_t/A_{Tt}, \quad (2.22)$$

and a conditional factor demand function

$$L_{Tt} = \frac{1}{A_{Tt}} \int_0^1 [Y_{Ht}^d(j) + Y_{Ht}^{*d}(j)] dj. \quad (2.23)$$

The economic structure of the foreign country is similar, except that the share of traded good in the consumption basket may differ from that in the home country. In particular, the foreign consumption basket is given by

$$C_t^* = \bar{\alpha}^* C_{Tt}^* \alpha^* C_{Nt}^{*1-\alpha^*}, \quad \bar{\alpha}^* = \alpha^{*-\alpha} (1 - \alpha^*)^{\alpha^*-1}, \quad (2.24)$$

where  $\alpha^*$  may not equal to  $\alpha$ . The foreign country's structure is otherwise symmetric to that of the home country's. In what follows, we denote all foreign variables with an asterisk and assume that all other parameters are identical to their counterparts in the home country.

### 2.3 Risk Sharing, Market Clearing, and Equilibrium

Since the state-contingent nominal bond is traded in the international financial market, the foreign household's optimal consumption-saving decision leads to

$$D_{t,t+1} = \beta \frac{C_t^*}{C_{t+1}^*} \frac{P_t^*}{P_{t+1}^*} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}. \quad (2.25)$$

By combining this equation with its home counterpart (2.4) and iterating with respect to  $t$ , we obtain a risk-sharing condition

$$Q_t = \phi_0 \frac{C_t}{C_t^*}, \quad (2.26)$$

where  $Q_t = \mathcal{E}_t P_t^* / P_t$  is the real exchange rate, and  $\phi_0 = Q_0 C_0^* / C_0$ . The risk-sharing condition links the real exchange rate to the marginal rate of substitution between consumption in the two countries, so that all households face identical relative price of consumption goods in the world market.

In equilibrium, each country's labor market as well as the world bond market clear. Since labor is mobile within each country (but not across countries), labor market clearing implies that

$$L_{Nt} + L_{Tt} = L_t, \quad L_{Nt}^* + L_{Tt}^* = L_t^*. \quad (2.27)$$

Also, in equilibrium, nominal bonds are in zero net supply in the world market, so that  $B_t + B_t^* = 0$ .

Our goal is to analyze optimal monetary policy under two alternative monetary regimes. One in which each country tries to maximize its own households' welfare, taking the other country's policy actions as given; and the other in which a world planner tries to coordinate the two countries' monetary policy so as to maximize their collective welfare. For this purpose, we do not specify a particular monetary policy rule. Instead, we solve for the optimal policy that maximizes the welfare objective under each regime, subject to the private sector's optimizing conditions. For any given monetary policy, we can define an equilibrium for this world economy.

An *equilibrium* consists of allocations  $C_t, C_{Nt}, C_{Tt}, L_t, B_{t+1}$  for the home household and  $C_t^*, C_{Nt}^*, C_{Tt}^*, L_t^*, B_{t+1}^*$  for the foreign household; allocations  $Y_{Nt}(i)$ , and  $L_{Nt}(i)$ , and price  $P_{Nt}(i)$  for



non-traded intermediate good producer  $i \in [0, 1]$  in the home country and  $Y_{Nt}^*(i)$ , and  $L_{Nt}^*(i)$ , and price  $P_{Nt}^*(i)$  for non-traded intermediate good producer  $i \in [0, 1]$  in the foreign country; allocations  $Y_{Ht}(j)$ ,  $Y_{Ht}^*(j)$ , and  $L_{Tt}(j)$ , and price  $P_{Ht}(j)$  for traded intermediate good producer  $j \in [0, 1]$  in the home country and  $Y_{Ft}^*(j)$ ,  $Y_{Ft}(j)$ , and  $L_{Tt}^*(j)$ , and price  $P_{Ft}^*(j)$  for traded intermediate good producer  $j \in [0, 1]$  in the foreign country; together with prices  $D_{t,t+1}$ ,  $\mathcal{E}_t$ ,  $Q_t$ ,  $P_t$ ,  $\bar{P}_{Nt}$ ,  $\bar{P}_{Tt}$ ,  $\bar{P}_{Ht}$ ,  $P_t^*$ ,  $\bar{P}_{Nt}^*$ ,  $\bar{P}_{Tt}^*$ ,  $\bar{P}_{Ft}^*$ , and wages  $W_t$  and  $W_t^*$ , that satisfy the following conditions: (i) taking the prices and the wage as given, the household's allocations in each country solve its utility maximizing problem; (ii) taking the wage and all prices but its own as given, the allocations and the price of each non-traded intermediate good producer in each country solve its profit maximizing problem; (iii) taking the wage and all prices but its own as given, the allocations and the price of each traded intermediate good producer in each country solve its profit maximizing problem; and (iv) the world market for bonds and the domestic markets for labor clear.

### 3 Equilibrium Dynamics

To facilitate analysis of optimal monetary policy, we first examine a useful benchmark in which price adjustments are flexible, and then describe the equilibrium dynamics under sticky prices. We call the allocations in the flexible-price equilibrium the “natural rate” allocations, and deviations of the allocations in the sticky-price equilibrium from their natural rate levels the “gaps.” In analyzing the equilibrium dynamics, we focus on log-deviations of equilibrium variables from their steady-state values (denoted by hatted variables).

#### 3.1 The Balanced-Trade Steady State and the Current Account

We begin by describing a balanced-trade steady state, in which all shocks are shut off (i.e.,  $A_k = A_k^* = 1$  for  $k \in \{N, T\}$ ) and the net export is zero. The net export in the home country is given by

$$\begin{aligned}
 NX_t &= \bar{P}_{Ht} C_{Ht}^* - \mathcal{E}_t \bar{P}_{Ft}^* C_{Ft} \\
 &= (1 - \omega) \alpha^* \mathcal{E}_t P_t^* C_t^* - (1 - \omega) \alpha P_t C_t \\
 &= (1 - \omega) \alpha^* \mathcal{E}_t P_t^* C_t^* \left[ 1 - \frac{\alpha}{\alpha^*} Q_t^{-1} \frac{C_t}{C_t^*} \right] \\
 &= (1 - \omega) \alpha^* \mathcal{E}_t P_t^* C_t^* \left[ 1 - \frac{\alpha}{\alpha^*} \phi_0^{-1} \right], \tag{3.1}
 \end{aligned}$$

where the second equality follows from the demand functions for final traded consumption goods as in (2.10) and its foreign counterpart, the third from the definition of the real exchange rate, and the last from the international risk-sharing condition (2.26). In the balanced-trade steady

state,  $NX = 0$  so that  $\phi_0$  is given by

$$\phi_0 = \frac{\alpha}{\alpha^*}. \quad (3.2)$$

Clearly, if the countries have symmetric structures, that is, if  $\alpha = \alpha^*$ , then we have  $\phi_0 = 1$  and, from the risk-sharing condition (2.26),  $C_t = Q_t C_t^*$ . Since the real exchange rate  $Q_t$  represents the relative price of foreign consumption basket in terms of home consumption, it follows that, under symmetric structures, international risk-sharing leads to equalized aggregation consumption (measured in identical units) across countries for each period  $t$ . Yet, in the presence of structural asymmetry, that is, in the more general case with  $\alpha \neq \alpha^*$ , we have  $\phi_0 \neq 1$  so that consumptions in the two countries are not necessarily equal (in conformable units) even with households trading the state-contingent assets in the international financial market. In this case, the  $\phi_0$  term represents a “risk-sharing wedge” that arises only in the presence of structural asymmetry in the global economy. It turns out, as we show below, the condition under which the risk-sharing wedge arises also leads to sizable welfare gains from international monetary policy coordination.<sup>7</sup>

Given that  $\phi_0 = \alpha/\alpha^*$ , equation (3.1) implies that the net export is zero not only in the steady state, but for all  $t \geq 0$ . With zero net export, along with the assumption that neither country has an initial outstanding debts, the equilibrium current account would be zero for all  $t$ . This result greatly simplifies our analytical derivations of the welfare criteria.

### 3.2 The Terms of Trade and Some Aggregation Results

The balanced-trade steady-state conditions described above imply that the risk-sharing condition can be rewritten as

$$Q_t = \frac{\alpha}{\alpha^*} \frac{C_t}{C_t^*}. \quad (3.3)$$

This condition is quite useful in obtaining the aggregate results and the approximated welfare objectives below.

We now derive some aggregate results to be used in obtaining the welfare objectives under optimal policies. Let  $Y_{Tt} = C_{Ht} + C_{Ht}^*$  denote aggregate demand for home-produced traded intermediate goods. We call  $Y_{Tt}$  the aggregate traded output. Similarly, let  $Y_{Nt} = C_{Nt}$  denote the aggregate non-traded output. Then we have

$$\bar{P}_{Ht} Y_{Tt} = P_t C_t [\omega \alpha + (1 - \omega) \alpha^* Q_t \frac{C_t^*}{C_t}] = \alpha P_t C_t, \quad (3.4)$$

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<sup>7</sup>Pesenti and Tille (2004) emphasize the importance of the risk-sharing wedge in analyzing gains from international monetary policy coordination in a one-sector open economy model with preset prices. The risk-sharing wedge in our model is somewhat different from theirs in that it is determined here by the balanced-trade steady-state conditions, so that it is independent of monetary policy; whereas in the Pesenti-Tille world, the wedge is given by the ratio of the expected marginal utility of consumption in the two countries, and is endogenous to policy. Such difference stems mainly from the different assumptions about the timing of portfolio choice decisions.

where the first equality follows from the demand functions for traded consumption as described in (2.8) and (2.10) and their foreign counterparts, and the second equality follows from the risk-sharing condition (3.3). Similarly, we can show that

$$\bar{P}_{Ft}^* Y_{Tt}^* = \alpha^* P_t^* C_t^*. \quad (3.5)$$

Let  $S_t = \mathcal{E}_t \bar{P}_{Ft}^* / \bar{P}_{Ht}$  denote the home country's terms of trade. It follows from (3.4) and (3.5), along with (3.3), that the terms of trade is given by the relative traded outputs. That is,

$$S_t = \frac{Y_{Tt}}{Y_{Tt}^*}. \quad (3.6)$$

Under Cobb-Douglas aggregation technologies, expenditure on non-traded goods is a constant fraction of total consumption expenditures in each country. In particular, we have

$$\bar{P}_{Nt} Y_{Nt} = (1 - \alpha) P_t C_t, \quad \bar{P}_{Nt}^* Y_{Nt}^* = (1 - \alpha^*) P_t^* C_t^*. \quad (3.7)$$

Equation (3.4) and the price index relations  $P_t = \bar{P}_{Tt}^\alpha \bar{P}_{Nt}^{1-\alpha}$  and  $\bar{P}_{Tt} = \bar{P}_{Ht}^\omega [\mathcal{E}_t \bar{P}_{Ft}^*]^{1-\omega}$  imply that the real demand for home-traded goods is given by

$$Y_{Tt} = \alpha C_t Q_{Nt}^{1-\alpha} S_t^{1-\omega}, \quad (3.8)$$

where  $Q_{Nt} = \bar{P}_{Nt} / \bar{P}_{Tt}$  denotes the relative price of non-trade goods. Similarly, (3.7) implies that the real demand for home non-traded goods is given by

$$Y_{Nt} = (1 - \alpha) C_t Q_{Nt}^{-\alpha}. \quad (3.9)$$

Use (3.9) to eliminate  $Q_{Nt}$  and (3.6) to eliminate  $S_t$  from (3.8), and go through the same procedure for the foreign country, we obtain

$$C_t = \bar{\alpha} Y_{Nt}^{1-\alpha} [Y_{Tt}^\omega Y_{Tt}^{*1-\omega}]^\alpha, \quad C_t^* = \bar{\alpha}^* Y_{Nt}^{*1-\alpha^*} [Y_{Tt}^{*\omega} Y_{Tt}^{1-\omega}]^{\alpha^*}. \quad (3.10)$$

Thus, aggregate consumption in each country is a weighted average of the non-traded output and a composite of traded outputs produced in the two countries.

From (2.16) and (2.19), the aggregate demand for labor in the non-traded sector is given by

$$L_{Nt} = \int_0^1 L_{Nt}(j) dj = \frac{1}{A_{Nt}} \int_0^1 Y_{Nt}^d(j) dj = \frac{G_{Nt}}{A_{Nt}} Y_{Nt}, \quad (3.11)$$

where  $G_N = \int_0^1 (P_N(j) / \bar{P}_N)^{-\theta_N} dj$  measures the price-dispersion within the sector. Similarly, the aggregate demand for labor in the traded-sector is given by

$$L_{Tt} = \frac{G_{Ht}}{A_{Tt}} Y_{Tt}, \quad (3.12)$$

where  $G_H = \int_0^1 (P_H(j) / \bar{P}_H)^{-\theta_T} dj$ . Expressions for  $L_N^*$  and  $L_T^*$  can be obtained in a similar way for the foreign country.

### 3.3 The Flexible-Price Equilibrium and the Natural Rate

When price adjustments are flexible, firms' pricing decisions are synchronized, so that the optimal price set by a firm is a constant markup over its contemporaneous marginal cost and that the price index in each sector coincides with the pricing decision of a typical firm in that sector. We now describe the flexible-price equilibrium allocations, that is, the natural rate allocations.

It is easy to show that the natural-rate level of sectoral outputs, in log-deviation forms, are given by

$$\hat{y}_{Tt}^n = \hat{a}_{Tt}, \quad \hat{y}_{Nt}^n = \hat{a}_{Nt}. \quad (3.13)$$

Thus, under flexible prices, each sector's output responds one-for-one with the sector-specific shocks, and there is no inter-sectoral or international spillover effects of shocks on production. It then follows from (3.12) and (3.11) that the natural rates of sectoral employment are constant, that is,  $\hat{l}_{Tt}^n = \hat{l}_{Nt}^n = 0$ .

Next, given the solution for the natural rate of traded output as described in (3.13) and its foreign counterpart, the relation (3.6) implies that the natural rate of the terms of trade is given by

$$\hat{s}_t^n = \hat{a}_{Tt} - \hat{a}_{Tt}^*, \quad (3.14)$$

Thus, an increase in the relative productivity in home's traded sector (relative to the foreign traded sector) tends to lower the relative price of traded goods produced in the home country, and thus leads to worsened terms of trade for that country.

Third, given the solutions for the sectoral outputs and the terms of trade above, we can solve for the natural-rate level of aggregate consumption using (3.10), and the solution is given by

$$\hat{c}_t^n = \alpha \hat{a}_{Tt} + (1 - \alpha) \hat{a}_{Nt} - \alpha(1 - \omega) \hat{s}_t^n. \quad (3.15)$$

Thus, aggregate consumption responds not only to domestic sectoral shocks, but also to movements in the terms of trade since part of the consumption basket consists of imported goods. An improved domestic productivity or terms of trade would raise the natural rate level of consumption. It follows from the intertemporal Euler equation (2.5) that the real interest rate in the flexible-price equilibrium is given by

$$r\hat{r}_t^n = E_t \Delta \hat{c}_{t+1}^n = 0, \quad (3.16)$$

where we have used the solution for  $\hat{c}_t^n$  in (3.15) and the random-walk properties of the shock processes. The solutions for foreign consumption and real interest rate are similar.

Finally, the relative price of non-traded goods in terms of traded goods in the flexible-price equilibrium can be obtained by using the pricing decision equations and the solution for the terms

of trade:

$$\hat{q}_{Nt}^n \equiv \hat{p}_{Nt} - \hat{p}_{Tt} = \hat{a}_{Tt} - \hat{a}_{Nt} - (1 - \omega)\hat{s}_t^n. \quad (3.17)$$

Hence, in the flexible-price equilibrium, the relative price of non-traded goods decreases with the relative productivity of the non-traded sector. Further, an improvement in the terms of trade (i.e., a fall in  $\hat{s}_t^n$ ) would make imported goods relatively cheaper, so that the price of the traded basket would fall and the relative price of non-traded goods would rise.

### 3.4 The Sticky-Price Equilibrium

The sticky price equilibrium is characterized by the optimizing conditions derived in Section 2. Denote  $\tilde{x}_t = \hat{x}_t - \hat{x}_t^n$  the deviation of equilibrium variable  $\hat{x}_t$  under sticky prices from its own natural rate  $\hat{x}_t^n$ , that is, the gap. After log-linearizing, the private sector's optimizing conditions in the home country can be summarized below:

$$\pi_{Nt} = \beta \mathbf{E}_t \pi_{N,t+1} + \kappa_N \tilde{y}_{Nt}, \quad (3.18)$$

$$\pi_{Ht} = \beta \mathbf{E}_t \pi_{H,t+1} + \kappa_T \tilde{y}_{Tt} \quad (3.19)$$

$$\Delta \tilde{y}_{Nt} = \Delta \tilde{y}_{Tt} - \pi_{Nt} + \pi_{Ht} - \Delta \hat{a}_{Nt} + \Delta \hat{a}_{Tt}, \quad (3.20)$$

$$\begin{aligned} \alpha \tilde{y}_{Tt} + (1 - \alpha) \tilde{y}_{Nt} &= \mathbf{E}_t [\alpha \tilde{y}_{T,t+1} + (1 - \alpha) \tilde{y}_{N,t+1}] - \\ &\quad \{ \hat{r}_t - \mathbf{E}_t [\alpha \pi_{H,t+1} + (1 - \alpha) \pi_{N,t+1}] \}, \end{aligned} \quad (3.21)$$

$$(3.22)$$

where the  $\pi$ 's denote the sectoral inflation rates, the  $\tilde{y}$ 's denote the sectoral output gaps, and  $\kappa_i = \frac{(1 - \beta\gamma_i)(1 - \gamma_i)}{\gamma_i}$  is a constant that measures the responsiveness of the pricing decisions to variations in the real marginal cost gap in sector  $i \in \{N, T\}$ . The foreign optimizing conditions are analogous.

Equations (3.18) and (3.19) describe the Phillips-curve relations in the two sectors. These relations are forward-looking in that a sector's period- $t$  inflation rate depends solely upon current and expected future marginal cost gaps, which coincide here with the output gaps, in that sector. Equation (3.20) describes the relation between changes in the expenditures on the two sectors' outputs. Given the Cobb-Douglas aggregation technologies, these expenditures are proportional to each other, as reflected in (3.20). Equation (3.21) is derived from log-linearizing the intertemporal Euler equation (2.5) for the home household, with the consumption gap replaced by the output gaps using the constant-expenditure-share relations. Note that the terms-of-trade gap cancels out as we replace the consumption gap by the output gaps and the consumer price inflation rate by the sectoral inflation rates.

It is instructive to examine the relation between the marginal cost gaps (which equal the output gaps here) and the consumption gap, the terms-of-trade gap, and the relative-price gap. For this purpose, we use (3.8) and (3.9) to obtain

$$\tilde{y}_{Tt} = \tilde{c}_t + (1 - \alpha)\tilde{q}_{Nt} + (1 - \omega)\tilde{s}_t, \quad (3.23)$$

$$\tilde{y}_{Nt} = \tilde{c}_t - \alpha\tilde{q}_{Nt}. \quad (3.24)$$

The marginal cost gap in each sector depends positively on the consumption gap but negatively on the sector's relative price gap. Additionally, the marginal cost in the home country's traded sector depends positively on its terms-of-trade gap, so that a terms-of-trade improvement (i.e., a fall in  $\tilde{s}_t$ ) leads to a fall in the real marginal cost in the home traded sector, but has no direct effect on the marginal cost in the non-traded sector.

Before we proceed to characterize optimal monetary policy, it is necessary to find out whether or not, in a two-sector model like this, the national monetary authority faces a policy trade-off in stabilizing the output gaps and inflation rates. In the absence of such trade-off, optimal independent monetary policy would be able to replicate the efficient flexible-price allocations and there would be no need for cooperation. Woodford (2003, Chapter 3) shows that, in a closed economy with two sectors, if the degree of price stickiness is identical across sectors, then the sectoral Phillips curve relations can be reduced to an aggregate Phillips-curve that is identical to that in a one-sector model, so that the trade-off between price stability and stabilizing output gap fluctuations disappears, regardless of whether or not the sectoral shocks are correlated.

Is this still the case in our two-sector open economy environment? To answer this question, consider the special case with  $\kappa_N = \kappa_T = \kappa$  so that the two sectors have identical durations of price contracts. Define a domestic inflation index as  $\hat{\pi}_{Dt} = \alpha\hat{\pi}_{Ht} + (1 - \alpha)\hat{\pi}_{Nt}$ . Then, by taking a weighted average of the sectoral Phillips curves in (3.18) and (3.19), and use (3.23) and (3.24) to replace the output gaps, we obtain

$$\pi_{Dt} = \beta E_t \pi_{D,t+1} + \kappa\tilde{c}_t + \kappa\alpha(1 - \omega)\tilde{s}_t. \quad (3.25)$$

In the special case of a closed-economy (with  $\omega = 1$ ), this relation reduces to an aggregate Phillips curve that implies no trade-off between output stability and price stability: the national central bank is able to close the output gap by simply setting the domestic inflation index  $\pi_{Dt} = 0$ . In an open economy as the one presented here, however, fluctuations in the terms-of-trade gap act as an endogenous "cost-push shock" that introduces a trade-off between stabilizing the output gap and the domestic inflation index, unless the traded sector is entirely shut off (i.e., with  $\alpha = 0$ ). It turns out that it is in general not possible to implement the flexible price allocations in this open economy.

**Proposition 1.** *In the presence of nominal rigidities in both sectors and sector-specific shocks, it is not possible to implement the flexible-price allocations unless the domestic sectoral shocks are perfectly correlated.*

**Proof:** By contradiction. Suppose that the flexible-price allocations could be replicated. Then the output gaps would both be closed, that is,  $\tilde{y}_{Tt} = \tilde{y}_{Nt} = 0$  for all  $t$ . It follows from (3.18) and (3.19) that  $\pi_{Nt} = \pi_{Ht} = 0$  for all  $t$ . But then, given that the output gaps are all closed, (3.20) implies that  $\Delta\hat{a}_{Tt} - \Delta\hat{a}_{Nt} = \pi_{Nt} - \pi_{Ht}$ , contradicting  $\pi_{Nt} = \pi_{Ht} = 0$  unless  $\Delta\hat{a}_{Tt} = \Delta\hat{a}_{Nt}$  for all  $t$ .

*Q.E.D.*

Although the flexible-price equilibrium allocations are Pareto optimal, the existence of the trade-off between stabilizing the gaps and inflation rates stated in Proposition 1 renders optimal monetary policy second best. In the next section, we define the optimal monetary policy problems and characterize allocations under cooperative and non-cooperative policies.

## 4 Optimal Monetary Policy

Optimal monetary policy entails maximizing a social objective function subject to the private sector's optimizing conditions. A natural welfare criterion in our model is the representative households' expected life-time utility. Following the approach described in Benigno and Woodford (2004), we derive an analytical, quadratic expression for the welfare criterion based on second-order approximations to the representative households' utility functions *and* to the private sectors' optimizing conditions (except for those exact log-linear relations). We substitute all relevant second-order relations into the objective function to obtain a quadratic expression for the welfare objective. Finally, upon obtaining this objective, we solve for the allocations under optimal monetary policy by maximizing the quadratic objective subject to the set of log-linearized equilibrium conditions (3.18)-(3.21) and their foreign counterparts. In this final step, we are essentially solving a linear-quadratic (LQ) problem with rational expectations. The LQ approach has become a popular tool in studying optimal monetary policy in closed economy models with a single sector (e.g., Rotemberg and Woodford (1997)) or multiple sectors (e.g., Erceg, et al. (2000), Huang and Liu (2004b)), and in open economy models with a single traded sector (e.g., Clarida, et al. (2002), Benigno and Benigno (2003), Gali and Monacelli (2002), and Pappa (2004)). We are the first to derive an analytical expression for the welfare objective in an open economy model with multiple



sectors and multiple sources of nominal rigidity, for both a regime with independent central banks (i.e., the Nash regime) and one with cooperating central banks (i.e., the cooperating regime).<sup>8</sup>

## 4.1 Independent Central Banks

A regime with independent central banks is one in which the national monetary authority in each country seeks to maximize the welfare of its own households, subject to the private sector's equilibrium conditions, taking foreign policy variables as given. We refer to this regime as the “Nash regime” and a national central bank under this regime a “Nash central bank.”

Before we proceed to describe optimal monetary policy under the Nash regime, it is important to understand the nature of Nash policy. There are two layers of policy-making issues. The first involves each national policy maker's optimal choice of steady-state subsidy rates, taking the policy instruments (i.e., the subsidy rates) in the other country as given. The second involves each national central bank's optimal choice of the monetary policy instrument (i.e., the nominal interest rate) in the dynamic equilibrium, taking the monetary policy instrument in the other country as given. A Nash equilibrium in such policy games is then the joint best responses. Below, we first describe the problem facing each Nash policy maker who chooses domestic subsidy rates to maximize its own country's welfare, taking the foreign policy as given. We then derive the welfare objective facing each Nash central bank through quadratic approximations to the domestic representative household's utility. Finally, we discuss the optimal monetary problem in the dynamic equilibrium.

### 4.1.1 Optimal Steady-State Subsidy under the Nash Regime

The optimal steady-state subsidy rates in a given country under the Nash regime maximize the country's domestic welfare in the steady state equilibrium, taking as given the domestic resource constraints and the other country's steady-state subsidy rates. Here, we focus on finding the optimal subsidy rates in the home country. The optimal subsidy rates in the foreign country can be found analogously.

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<sup>8</sup>Our approach differs slightly from that adopted in the open-economy papers mentioned here [e.g., Clarida, et al. (2002), Benigno and Benigno (2003), Gali and Monacelli (2002), and Pappa (2004)] in that we do not limit ourselves from the outset to taking first-order approximations to the private sectors' optimizing conditions. An alternative solution method is to take second-order approximations throughout the model and then to compute approximate optimal policy rules through non-linear simulations of the second-order system [e.g., Pesenti and Tille (2004), Sutherland (2002b), Tille (2002), Tscharov (2004)]. A main advantage of our approach, and the standard LQ approach described by Woodford (2003) and Benigno and Woodford (2004) as well, is that it allows us to obtain an analytical and explicit description of the objective function for optimal policy.

In a steady state, the shocks are shut off and pricing decisions are synchronized, so that  $A_N = A_T = 1$ ,  $G_H = G_N = 1$ , and  $L_j = Y_j$  for  $j \in \{N, T\}$ . The home central bank chooses its  $(\tau_T, \tau_N)$  to maximize  $U(C) - V(L)$  subject to the resource constraint (3.10) and the labor market clearing condition  $L = L_N + L_T$ . In the steady-state equilibrium,  $L_j = Y_j$  so that the resource constraint can be rewritten as  $C = \bar{\alpha} L_N^{1-\alpha} [L_T^\omega Y_T^{*1-\omega}]^\alpha$ .

In choosing  $(\tau_T, \tau_N)$ , the home policy maker takes the foreign subsidy rates  $(\tau_T^*, \tau_N^*)$  as given. Since the foreign traded output  $Y_T^*$  is solely determined by foreign subsidy policy, the home policy maker takes  $Y_T^*$  as given. To see how  $Y_T^*$  is determined by foreign subsidy policy, we use the foreign traded sector's pricing equation to get

$$1 + \tau_T^* = \mu_T^* \frac{W^* P^*}{P^* P_F^*} = \frac{\mu_T^* \Psi}{\alpha^*} Y_T^*, \quad (4.1)$$

where the second equality follows from the foreign household's labor supply decision and the constant-expenditure-share condition (3.5) for traded goods. Evidently,  $Y_T^*$  is a function of  $\tau_T^*$  and structural parameters. Thus, as the home policy maker takes  $(\tau_T^*, \tau_N^*)$  as given, it also takes  $Y_T^*$  as given.

The first order conditions for the steady-state optimal policy problem imply that

$$(1 - \alpha)U'(C)C = V'(L)L_N, \quad \alpha\omega U'(C)C = V'(L)L_T. \quad (4.2)$$

With our utility function  $U(C) - V(L) = \log(C) - \Psi L$ , these first-order conditions yields a solution for the steady-state labor allocations:

$$\Psi L_N = 1 - \alpha, \quad \Psi L_T = \alpha\omega, \quad \Psi L = 1 - \alpha + \alpha\omega. \quad (4.3)$$

To find the subsidy rates that are consistent with the optimal steady state allocations under the Nash regime, we use the pricing equations to get

$$1 + \tau_T = \mu_T \frac{W P}{P P_H} = \mu_T \frac{V'(L) Y_T}{U'(C) \alpha C} = \mu_T \frac{V'(L)L_T}{\alpha U'(C)C} = \omega \mu_T, \quad (4.4)$$

where the second equality follows from the labor supply equation and equation (3.4), the third from  $Y_T = L_T$  in the steady state, and the last from the steady-state relation (4.2). Similarly, the optimal subsidy rate in the non-traded sector is given by

$$1 + \tau_N = \mu_N \frac{W P}{P P_N} = \mu_N \frac{V'(L) Y_N}{U'(C) (1 - \alpha)C} = \mu_N. \quad (4.5)$$

The home policy maker under the Nash regime chooses a subsidy rate for the non-traded sector that exactly offsets the steady-state markup distortion in that sector, and it chooses a subsidy rate for the traded sector that lowers the effective markup distortion, but not all the way to neutralize it. The Nash policy maker would like to lower the markup distortion in the

domestic traded sector because the inefficiency stemming from monopolistic competition hurts the home household's welfare; it does not want to completely neutralize the markup distortion in that sector because it is not concerned about the effects of domestic production of traded output on the foreign household. By maintaining some monopoly power in the traded sector, the Nash policy maker allows the traded-goods producers to maintain lower production and charge higher export prices, so as to improve home's terms of trade. Meanwhile, the foreign policy maker takes home subsidy rates as given, and chooses its own subsidy rates in a similar fashion. In consequence, under the Nash subsidy policy, equilibrium allocations are efficient for each individual country, but socially suboptimal from a world-wide view, as Nash policy makers overlook the terms-of-trade externality.

#### 4.1.2 The Welfare Objective for the Nash Central Bank

We characterize the welfare objective for each Nash central bank by taking second-order approximations to its domestic representative household's period utility function. Again, we focus on the home country, and note that the foreign country's problem is similar. A second order approximation to the home household's period utility function yields

$$U_t - U_{ss} = \hat{c}_t - \Psi L \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) + O \left( \|\xi\|^3 \right), \quad (4.6)$$

where  $U_{ss}$  denotes the steady-state period utility, a hatted variable denotes log-deviations of the corresponding level variable from its steady-state value, and  $O \left( \|\xi\|^3 \right)$  represents terms that are of third or higher order in an appropriate bound on the amplitude of the shocks.

In the appendix, we show that the welfare objective function for the home Nash central bank is given by

$$W^{Nash} = E_0 \sum_{t=0}^{\infty} \beta^t U_t = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t^{Nash} + t.i.p. + O \left( \|\xi\|^3 \right), \quad (4.7)$$

where the term *t.i.p.* denotes terms independent of policy and the period loss function is given by

$$L_t^{Nash} = (1 - \alpha)(\tilde{y}_{Nt}^2 + \theta_N \kappa_N^{-1} \pi_{Nt}^2) + \alpha \omega (\tilde{y}_{Tt}^2 + \theta_T \kappa_T^{-1} \pi_{Ht}^2). \quad (4.8)$$

Under the Nash regime, the home central bank solves its optimal monetary policy problem by maximizing the quadratic welfare objective function (4.7) subject to the domestic private sector's optimizing conditions (3.18)- (3.21). The foreign central bank's welfare objective and optimal policy problem are analogous.

The welfare criterion described in (4.7) and (4.8) reveals that a Nash central bank's optimal monetary policy is inward-looking. Specifically, it seeks to minimize the variations in its domestic marginal cost gaps (or output gaps) and inflation rates, so as to bring the equilibrium allocation

close to the natural rate. However, a Nash central bank faces a trade-off between stabilizing domestic marginal cost gaps and inflation rates, so that it cannot implement the flexible-price allocations (Proposition 1), unless the size of one sector approaches zero (i.e.,  $\alpha = 1$  or  $\alpha = 0$ ), or there is only one source of nominal rigidity (i.e.,  $\gamma_T = 0$  or  $\gamma_N = 0$ ), or the shocks are perfectly correlated (i.e.,  $\Delta\hat{a}_{Tt} = \Delta\hat{a}_{Nt}$ ).

In general, allocations under optimal independent monetary policy are second best, and the social welfare under such policy depends on the relative weights in front of each of the four variables that the central bank cares about. The weights assigned to non-traded inflation and output gap are proportional to the sector's size  $(1 - \alpha)$ , while the weights assigned to traded sector inflation and output gap depend on both the sector's size  $(\alpha)$  and the degree of steady-state home bias  $(\omega)$ . Since  $\omega < 1$  in an open economy like this, the weights assigned to the traded-sector variables are less than the sector's size. Thus, the Nash central bank not only tries to exploit the terms-of-trade externality by choosing a lower subsidy rate than necessary to offset the markup distortions in the traded sector, it also cares less about the variabilities of inflation and output gap in that sector. Finally, when the size of each sector is held constant, the weight assigned to a sector's inflation rate increases with the elasticity of substitution between differentiated goods produced in that sector (i.e., increases with  $\theta_j$ ) and with the sector's price-rigidity (i.e., decreases with  $\kappa_j$ ). Yet, a sector with more rigid prices needs not receive a larger weight for its inflation in the loss function, since the weight here is scaled by the relative size of the sector.

To gain further insight of the welfare objective, we use the relations (3.24) and (3.23) to replace the output gaps in the loss function, and we show in the Appendix that the loss function can be rewritten as

$$L_t^{Nash} = \tilde{c}_t^2 + \alpha(1 - \alpha)\tilde{q}_{Nt}^2 + \alpha\omega(1 - \omega)\tilde{s}_t^2 + (1 - \alpha)\theta_N\kappa_N^{-1}\pi_{Nt}^2 + \alpha\omega\theta_T\kappa_T^{-1}\pi_{Ht}^2 + t.i.p.. \quad (4.9)$$

Thus, in addition to variations in the inflation rates and consumption gap, the Nash central bank cares about variations in both the domestic relative price gap and the international relative price gap (i.e., the terms-of-trade gap). The domestic relative price gap receives a weight that is concave in the parameter  $\alpha$  that measures the relative size of the traded sector, and the weight reaches its maximum when  $\alpha = 0.5$ . When the size distribution of sectors is skewed, however, the sector with a greater share receives a larger weight in front of its sectoral inflation rate, and fluctuations in the relative-price gap become less of a concern for optimal policy. The terms-of-trade gap receives a weight that is proportional to  $\alpha$  and is concave in  $\omega$ . In the extreme case with no trade (i.e., with either  $\alpha = 0$  or  $\omega = 1$ ), the Nash central bank would no longer care about the terms-of-trade gap.

### 4.1.3 Optimal Monetary Policy Under the Nash Regime

Under the Nash regime, optimal monetary policy for the home central bank is obtained by maximizing its welfare objective (4.7), subject to the private sector's optimizing conditions summarized by (3.18)-(3.21) and their foreign counterparts, taking the foreign monetary policy instruments as given. Similar for the foreign central bank. A Nash equilibrium is the joint "best responses" in the space of policy instruments.

Assume that a country's monetary policy instrument is its nominal interest rate. Evidently, given the home nominal interest rate  $r_t$ , all variables in the home central bank's loss function (4.8), that is, the sectoral output gaps and inflation rates, can be solved uniquely from the four equations in (3.18)-(3.21). Similarly, given the foreign nominal interest rate  $r_t^*$ , the four variables that the foreign central bank cares about in its loss function can be solved uniquely from the corresponding foreign private sector's optimizing conditions. In other words, taking  $r_t^*$  as given is equivalent to taking foreign private sector's optimizing conditions, and therefore, foreign sectoral output gaps and inflation rates, as given. With no spill-over effects of foreign monetary policy on the variables that the home central bank cares about in its loss function (i.e., the sectoral output gaps and inflation rates), the home central bank seeks to maximize its own welfare objective (4.7) subject to the four domestic optimizing conditions (3.18)-(3.21). Similar for the foreign central bank. In this sense, optimal monetary policy under the Nash regime is inward-looking.<sup>9</sup>

An important issue of concern, in the spirit of Obstfeld and Rogoff (2002), is then: From a global perspective, would the lack of international monetary policy coordination incur substantive welfare losses? Obstfeld and Rogoff find that the answer is perhaps "no" in their model with a single source of nominal rigidity. We revisit this issue below in a context with multiple sources of nominal rigidities and with potential structural differences across countries in the form of asymmetric sizes of traded sectors. For this purpose, we first derive a welfare objective function for the policymakers under the cooperating regime in the next subsection, and then examine the quantitative welfare gains from coordination in Section 5.

## 4.2 Cooperating Central Banks

A regime with cooperating central banks is one in which monetary policy decisions are delegated to a supranational monetary institution (i.e., a social planner), who seeks to maximize a weighted average of national welfare in the two countries, subject to the private sectors' optimizing conditions in both countries. Unlike a Nash central bank, the planner here does not take any country's

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<sup>9</sup>Of course, foreign monetary policy may affect the price level and consumption in the home country through the terms of trade. But these variables do not independently enter the home central bank's loss function.

variables as given. Since the population size is equal across countries, we assume that the planner assigns equal weights (half) to each member country's national welfare.

#### 4.2.1 Optimal Steady-State Subsidy under Cooperation

Under the cooperating regime, the social planner chooses the steady-state subsidy rates in the two countries to maximize the countries' collective welfare  $\frac{1}{2}[U(C) - V(L) + U(C^*) - V(L^*)]$ , subject to the national resource constraints (3.10) and the labor market clearing conditions  $L_T + L_N = L$  and  $L_T^* + L_N^* = L^*$ , with  $Y_j = L_j$  and  $Y_j^* = L_j^*$  imposed. Under our parameterized period utility functions, the first order conditions for the planner's steady-state problem lead to

$$\Psi L_N = 1 - \alpha, \quad \Psi L_T = \alpha\omega + \alpha^*(1 - \omega), \quad (4.10)$$

$$\Psi L_N^* = 1 - \alpha^*, \quad \Psi L_T^* = \alpha^*\omega + \alpha(1 - \omega). \quad (4.11)$$

The optimal subsidy rates under cooperation are then given by

$$1 + \tau_T = \frac{\mu_T}{\alpha}[\alpha\omega + \alpha^*(1 - \omega)], \quad 1 + \tau_N = \mu_N, \quad (4.12)$$

$$1 + \tau_T^* = \frac{\mu_T^*}{\alpha^*}[\alpha^*\omega + \alpha(1 - \omega)], \quad 1 + \tau_N^* = \mu_N^*. \quad (4.13)$$

Evidently, in the case with symmetric structures, that is, with  $\alpha = \alpha^*$ , the optimal subsidy rates in the traded sectors exactly offset the steady-state markup distortions, as do the subsidies in the non-traded sectors, so that the steady-state equilibrium allocations under the cooperating regime coincide with those under perfect competition and are thus Pareto optimal. However, with structural asymmetry, the optimal subsidy rates in the traded sector do not offset the monopolistic markups, although the resulting steady-state allocations are still socially efficient since they solve the social planner's steady-state problem.

To gain some intuition about the planner's optimal subsidy schedule in the presence of structural asymmetry, we assume, without loss of generality, that  $\alpha > \alpha^*$ . Thus, the optimal subsidy rates are such that  $1 + \tau_T < \mu_T$  and  $1 + \tau_T^* > \mu_T^*$ . Evidently, the planner's optimal subsidy schedule entails redistributing some monopoly markup power from traded-sector firms in the foreign country to those in the home country, so as to maintain higher prices of home traded goods relative to foreign traded goods than under symmetric structures. In other words, with structural asymmetry, the planner tries to create an efficient *terms-of-trade bias* in favor of the country that has a larger traded sector.

Why? With  $\alpha > \alpha^*$ , a larger share of consumption expenditure goes to traded consumption in the home country than that in the foreign country. Since the expenditure share of imported goods in the *traded* consumption basket is constant and equal across countries, the home household

needs to import more traded goods than does the foreign household. Yet, since the trade balance is zero in the steady-state equilibrium, the only way to enable the home household to afford more imported goods is to lower the relative price of foreign traded goods, that is, to adjust the terms-of-trade in favor of the home country.

Below, as we derive the welfare objective for the planner, we show that optimal monetary policy under cooperation dictates the planner to balance the desire to set the terms of trade in favor of the country that has a larger traded sector against its need to smooth fluctuations in the terms-of-trade gap.

#### 4.2.2 The Welfare Objective for the Planner under Cooperation

Although the optimal steady-state allocations under the cooperating regime differ from that under the Nash regime, the natural rate allocations in our model are independent of policy regimes, as we have described in Section 3.3. The gaps in the welfare objective functions are thus deviations of the allocations under each policy regime from the same natural rate allocations.

The welfare objective for the social planner is given by

$$W^{Planner} = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [U_t + U_t^*], \quad (4.14)$$

where  $U_t = \log C_t - \Psi L_t$  and  $U_t^* = \log C_t^* - \Psi L_t^*$  are the representative households' period-utility functions. In the appendix, we derive the planner's welfare objective based on second-order approximations to the households' utility functions and to equilibrium conditions. The resulting welfare objective function is then

$$W^{Planner} = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [U_t + U_t^*] = -\frac{1}{4} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t L_t^{Planner} + t.i.p. + O(\|\xi\|^3), \quad (4.15)$$

where the period loss function is given by

$$\begin{aligned} L_t^{Planner} &= (1 - \alpha)(\tilde{y}_{Nt}^2 + \theta_N \kappa_N^{-1} \pi_{Nt}^2) + \tilde{\alpha}(\tilde{y}_{Tt}^2 + \theta_T \kappa_T^{-1} \pi_{Ht}^2) \\ &\quad + (1 - \alpha^*)(\tilde{y}_{Nt}^{*2} + \theta_N^* \kappa_N^{*-1} \pi_{Nt}^{*2}) + \tilde{\alpha}^*(\tilde{y}_{Tt}^{*2} + \theta_T^* \kappa_T^{*-1} \pi_{Ft}^{*2}), \end{aligned} \quad (4.16)$$

with  $\tilde{\alpha} = \alpha\omega + \alpha^*(1 - \omega)$  and  $\tilde{\alpha}^* = \alpha^*\omega + \alpha(1 - \omega)$ .

In the special case with symmetric structures (i.e, with  $\alpha = \alpha^*$ ), we have  $\tilde{\alpha} = \tilde{\alpha}^* = \alpha$ . Thus, the weights to the inflation rates and output gaps in each sector are identical across countries. In this case, as we show above, the planner chooses subsidy rates to fully offset all markup distortions and the flexible-price equilibrium allocations are Pareto optimal. In other words, the planner internalizes the terms-of-trade externality that the independent central banks tend to overlook, and it has no incentive to manipulate the terms of trade in favor of any particular country.



In general, with structural asymmetry (i.e., with  $\alpha \neq \alpha^*$ ), the planner's subsidy rates do not fully offset the markup distortions in the traded sectors. In particular, as we show above, the planner follows a policy that creates a *terms-of-trade bias* in favor of the country that has a larger traded sector. It is important to emphasize that the *terms-of-trade bias* here is different from the terms-of-trade externality that Nash central banks try to exploit: the planner tries to internalize the latter regardless of whether or not the two countries have symmetric structures; but in the case with asymmetry, the planner adjusts the terms of trade in favor of the country that has a larger traded sector.

Under the planner's optimal policy, the *terms-of-trade bias* needs to be balanced against the stabilization goal implicit in the loss function (4.16). To see the dependence of the loss function on variations in the terms-of-trade gap, among other variables, we use (3.23) and (3.24) and their foreign counterparts to replace the output gaps and show in the Appendix that the loss function can be rewritten as

$$\begin{aligned}
L_t^{Planner} = & (1 - \alpha)\theta_N\kappa_N^{-1}\pi_{Nt}^2 + [\alpha\omega + \alpha^*(1 - \omega)]\theta_T\kappa_T^{-1}\pi_{Ht}^2 + \tilde{c}_t^2 + \alpha(1 - \alpha)\tilde{q}_{Nt}^2 \\
& + (1 - \alpha^*)\theta_N^*\kappa_N^{*-1}\pi_{Nt}^{*2} + [\alpha^*\omega + \alpha(1 - \omega)]\theta_T^*\kappa_T^{*-1}\pi_{Ft}^{*2} + \tilde{c}_t^{*2} + \alpha^*(1 - \alpha^*)\tilde{q}_{Nt}^{*2} \\
& + (\alpha + \alpha^*)\omega(1 - \omega)\tilde{s}_t^2.
\end{aligned} \tag{4.17}$$

Evidently, the planner would like to stabilize fluctuations in the terms-of-trade gap, and such incentive increases with aggregate size of the traded sector in the two countries (i.e., with  $\alpha + \alpha^*$ ).

Clearly, the planner's loss function (4.17) is not a simple sum of the Nash central banks' loss functions given by (4.9) and its foreign counterpart. The planner assigns larger weights than do the Nash central banks on the traded-sector inflation rates so as to internalize the terms-of-trade externality that the latter try to exploit. This is a well-known source of welfare gains from international monetary policy coordination under symmetric structures. With asymmetric structures (say,  $\alpha > \alpha^*$ ) and with home bias (i.e.,  $\omega > 0.5$ ), the planner assigns a larger weight on traded inflation in the country with a larger traded sector (since  $\alpha > \alpha^*$  and  $\omega > 0.5$  imply that  $\tilde{\alpha} > \tilde{\alpha}^*$ ). Thus, not only does the planner try to favor the country with a larger traded sector through creating the *terms-of-trade bias*, it also cares more about fluctuations in the traded inflation in that country.

Optimal monetary policy under cooperating central banks is obtained by maximizing the welfare objective (4.15) subject to the private sector's optimizing conditions summarized in (3.18)-(3.21) and their foreign counterparts.

## 5 Gains from Coordination

The analysis above reveals that there are potential gains from coordination, since an independent central bank does not take into account the effects of terms-of-trade movements on its trading partner, while the world planner tries to internalize this terms-of-trade externality when conducting optimal monetary policy. In this section we quantify the welfare gains from coordination and relate the gains to the degree of asymmetry in production and trading structures across countries. We also study the sensitivity of the results to the values of some key parameters in the model.

### 5.1 Parameter Calibration

Since it is difficult to obtain closed-form solutions under optimal monetary policy, we resort to numerical simulations to calculate the welfare outcomes of different policy regimes. For this purpose, we use calibrated parameter values summarized in Table 1.

Since the time frequency in our model is quarterly, we set  $\beta = 0.99$ , so that the steady-state annualized real interest rate is 4 percent. We set  $\omega = 0.7$  to capture steady-state home bias in the traded consumption baskets. Since the steady-state share of imports in the home country's GDP is given by  $\alpha(1 - \omega)$ , if we consider a traded-sector share of  $\alpha = \alpha^* = 0.3$  as a benchmark value, then  $\omega = 0.7$  implies that the steady-state share of imports in GDP is 0.09, roughly corresponding to the sample average of the import share in the U.S. in its trade with the European Union. We set  $\theta_T = \theta_N = 10$ , so that the steady state markup is 11 percent; and  $\gamma_T = \gamma_N = 0.75$ , so that the Calvo pricing contracts in each sector last for four quarters on average. We set the standard deviation of the innovations to sectoral productivity shocks to 0.01. In our baseline experiment, we assume that the shocks are uncorrelated across sectors and across countries.

### 5.2 Symmetric structures

We first consider the special case with symmetric structures across countries, that is, with  $\alpha = \alpha^*$ . In this case, the planner chooses subsidy rates to fully offset the steady-state markup distortions in all sectors and thus the natural-rate allocations under the cooperating regime are Pareto optimal. Since there are two sources of nominal rigidities and domestic shocks are imperfectly correlated in each country, neither independent central banks nor the planner can replicate the natural-rate allocations. There are potential welfare gains from coordination since the planner tries to internalize the terms-of-trade externality that the independent central banks tend to overlook. The question is then: How large is the welfare gain from coordination when the countries have symmetric structures?

Figure 2 plots the welfare losses associated with optimal monetary policy under independent central banks (the solid line) and those under cooperation (the dashed line), where we assume the two countries have symmetric structures. The welfare loss here is measured by the percentage of steady-state consumption equivalence, that is, the percentage increase in steady-state consumption required to keep the households indifferent to move from a world with flexible prices to one with sticky prices and optimal monetary policy. The gains from coordination are the difference between the welfare losses under the Nash regime and those under the cooperating regime.

The figure reveals that there are gains from coordination in our multi-sector model. However, the quantitative size of the gains is small relative to the welfare losses stemming from inefficient fluctuations in domestic relative-price gaps under optimal policy. The magnitude of the welfare losses under optimal policy and of the gains from coordination depends on the size of the traded sector. In the extreme case with  $\alpha = 1$ , the non-traded sector is shut off and the model reduces to the standard, one-sector open economy model; in the other extreme with  $\alpha = 0$ , the traded sector is shut off and the countries in the model become closed-economies. In both these extremes, inefficient movements in domestic relative-price gaps become irrelevant and optimal independent monetary policy can replicate the natural-rate allocations, leaving no scope for welfare gains from coordination.

In the more general case with  $\alpha$  lying between 0 and 1, nominal rigidities in both the traded and the non-traded sectors become relevant for optimal policy. With imperfectly correlated domestic shocks, independent monetary policy cannot attain the natural-rate allocations (Proposition 1), so that there are potential gains from coordination. The gains arise because the planner tries to internalize the terms-of-trade externality that the Nash central banks overlook. As shown in the figure, the gains from coordination are larger when  $\alpha$  takes less extreme values. The gain reaches its peak at  $\alpha = 0.5$ , with a maximum gain of about 0.14% of consumption equivalence. When  $\alpha$  moves away from 0.5, the gain quickly diminishes. With our baseline value of  $\alpha = 0.3$ , the welfare gains from coordination is quite modest at about 0.086% of consumption equivalence.

Figure 2 also bears out the main result established in Proposition 1: optimal policy can replicate the flexible-price equilibrium (so that the welfare loss is zero) only if one sector is shut off (i.e.,  $\alpha = 0$  or  $\alpha = 1$ ) or the domestic shocks are perfectly correlated. In general, with two sources of nominal rigidities within each country and imperfectly correlated domestic shocks, optimal monetary policy faces a non-trivial trade-off. Neither the Nash regime nor the cooperating regime can bring the equilibrium allocations to the efficiency frontier. Further, the welfare losses display a hump shape with respect to  $\alpha$ : as  $\alpha$  rises from 0 to 1, the welfare loss initially rises, reaching a peak at  $\alpha = 0.5$ , and declines thereafter.

Why the hump shape? With a flexible exchange rate, exchange-rate adjustments help insulate the country from foreign shocks. Thus, the hump-shaped relation between the welfare losses and  $\alpha$  primarily reflects the effectiveness of domestic relative-price adjustments in stabilizing the consumption gaps and sectoral inflation rates in face of domestic sector-specific shocks. To make this connection more transparent, it helps to inspect the period loss function (4.9) for an independent central bank. In the loss function, sectoral inflation rates receive weights proportional to the relative sizes of the sectors, whereas the relative-price gap (i.e., the  $\tilde{q}_{Nt}$  term) receives a weight that is concave in  $\alpha$ , reaching its maximum when  $\alpha = 0.5$ . As  $\alpha$  moves away from 0.5, the weight in front of the relative-price gap becomes smaller, so that the monetary authority cares less about fluctuations in the relative-price gap, and it can rely more effectively on relative-price adjustments to insulate the impacts of sector-specific shocks on the consumption gap and the sectoral inflation rates. The further is  $\alpha$  from 0.5, the smaller the weight in front of the relative-price gap becomes, the more effective the monetary authority can use relative-price adjustments to insulate domestic sector-specific shocks, and thus the lower the welfare losses become under optimal policy. Conversely, the closer the value of  $\alpha$  is to 0.5, the greater the weight the relative-price gap receives, the less the policymakers are willing to adjust the relative-price gap, leading to higher welfare losses. A similar logic applies to explaining the hump shape in the welfare losses under cooperation in the case with symmetric structures.

### 5.3 Asymmetric structures

When the countries' have asymmetric structures (i.e.,  $\alpha \neq \alpha^*$ ), there are welfare gains from coordination not only because the planner tries to internalize the terms-of-trade externality overlooked by independent central banks, but the planner's optimal policy also creates a *terms-of-trade bias* that favors the country with a larger traded sector. Further, the *terms-of-trade bias* has to be balanced against the need to stabilize the gaps, including the terms-of-trade gap. In this sense, the presence of the *terms-of-trade bias* when the countries have asymmetric structures leads to a new source of welfare gains from coordination. A natural question is then: How large are such gains?

Figure 3 provides an answer. There, we plot the relative welfare losses under Nash central banks relative to those under cooperating central banks in the  $(\alpha, \alpha^*)$  space. The relative losses here measure the gains from coordination. The gains are small along the diagonal of the space with  $\alpha = \alpha^*$ , as we have just discussed for the case with symmetric structures in the previous subsection. When we move away from the diagonal so that the difference between  $\alpha$  and  $\alpha^*$  enlarges, the gains also increase, until reaching a maximum of 0.62 percent of steady-state consumption at the edges of the grid.

Figure 4 provides an alternative perspective that illuminates the role of asymmetric structures as a new source of gains from coordination. In particular, the figure plots the welfare gains under symmetric structures as a function of  $\alpha$  (the line with circles), and also the gains for different values of  $\alpha^*$  in the range between 0 and 1, with an increment grid of 0.1 (the lines without circles). The magnitude of the gains in general increases with the cross-country difference in structures (i.e., difference in  $\alpha$  and  $\alpha^*$ ). If we take  $\alpha = 0.3$  as a benchmark value for the size of the home traded sector, then the welfare gain rises from 0.086% to 0.139% and then to 0.221% of steady-state consumption equivalence as the value of  $\alpha^*$  increases from 0.3 (symmetric structures) to 0.5 and then to 0.7. The size of these coordination gains is comparable to that from eliminating business cycle fluctuations, as calculated, for example, by Lucas (1987).

## 5.4 Sensitivity

### 5.4.1 Home bias

We have so far considered the welfare gains from coordination with a calibrated value of the degree of steady-state home bias in traded consumption baskets. We have seen in Section 4.1.1 and 4.2.1 that the home-bias parameter  $\omega$  is an important determinant of both the terms-of-trade externality that independent central banks try to exploit and the *terms-of-trade bias* that the planner tries to create in favor of the country that has a larger traded sector. Thus, home bias has also implications on the gains from coordination.

To understand how home bias in traded consumption affects welfare under optimal policies, we revisit the optimal subsidy rates to traded-goods production for both the Nash central bank, as in equation (4.4), and the central planner, as in equations (4.12)-(4.13). As  $\omega$  increases, the optimal subsidy rates to traded-goods production become closer to the sector's steady-state markup distortion; in the limit with  $\omega = 1$ , there is no trade and the optimal subsidy rates exactly offset the markup distortions regardless of the structural differences between the countries. Thus, with a larger value of  $\omega$ , the welfare losses should be smaller under both the Nash and the cooperating regimes; when  $\omega$  approaches 1, the loss under the Nash regime would approach zero, so would the gains from coordination.

Figure 5 confirms this intuition. The figure plots the welfare losses under the two alternative regimes (the upper panel) and the welfare gains from coordination (the lower panel) for  $\omega \in [0.1, 0.9]$ , where we have fixed  $\alpha = 0.3$  and  $\alpha^* = 0.6$  to capture the structural asymmetry between the two countries (these values of the  $\alpha$ 's are also used in the rest of the sensitivity analysis). The figure shows that, as the degree of home bias rises (i.e., as the countries rely less on imported

goods and are thus less exposed to international trade), not only do the welfare losses under both regimes become smaller, but the welfare gains from coordination also decline.

#### 5.4.2 Correlation of Shocks

In our baseline experiments, we have assumed that shocks are uncorrelated both across sectors and across countries. In contrast, the NOEM literature frequently assumes that shocks are perfectly correlated within each country but uncorrelated across countries. We now examine the sensitivity of our results to correlations between the shocks.

Figure 6 displays the welfare gains from coordination as the correlations between sectoral shocks vary in the interval  $[-1, 1]$ . Apparently, variations in cross-country correlations (i.e.,  $\rho_{TT}$  and  $\rho_{NN}$ ) do not generate visible effects on the welfare gains. The gains are much more sensitive to the correlations between domestic shocks (i.e.,  $\rho_{TN}$ ). As domestic shocks become more correlated, the welfare gains become unambiguously smaller. In the extreme case with perfectly correlated domestic shocks, optimal monetary policy under both the Nash regime and the cooperating regime can replicate the flexible-price allocations, and thus there are no gains from coordination, despite of the structural asymmetry across countries. In this sense, our results here extend the findings by Obstfeld and Rogoff (2002) (for the case with perfectly correlated domestic shocks) and by Canzoneri, et al. (2004) (for the case with imperfectly correlated domestic shocks) to an environment that allows for structural asymmetry across countries.

#### 5.4.3 Price stickiness

In our baseline analysis, we have assumed that firms in different sectors face an identical duration of pricing contracts: they both last on average for 4 quarters. We now relax this assumption by allowing the relative length of the pricing contracts to vary. We examine the implications of varying the exogenous price-stickiness on the gains from coordination.

Figure 7 plots the welfare gains as the price-stickiness in one sector varies, holding the stickiness in the other sector fixed at its calibrated value. In particular, the solid line denotes the gains from coordination when  $\gamma_T$  varies in the interval  $[0.1, 0.8]$ , while fixing  $\gamma_N = 0.75$ ; and the dashed line represents the other case when  $\gamma_N$  varies while  $\gamma_T$  is fixed at 0.75. Evidently, holding one sector's price rigidity fixed, the welfare gains increase with the rigidity in the other sector. An exception seems to be that, when  $\gamma_T$  is fixed at 0.75, the gains initially increase with  $\gamma_N$ , and then decline when  $\gamma_N$  exceeds  $\gamma_T = 0.75$ . In general, the gains are more sensitive to variations in traded price rigidity than to non-traded price rigidity. This is so because the welfare gains stem mainly from the terms-of-trade externality and the *terms-of-trade bias*, and higher nominal rigidity and the resulting greater price-dispersions among firms in the traded sector lead to disproportionately

larger distortions in the terms of trade, leaving more room for welfare gains from international monetary policy coordination.

## 6 Conclusions

We have revisited the issue of gains from international monetary policy coordination in a framework that generalizes the standard model in the NOEM literature by introducing both traded and non-traded goods, and more importantly, by allowing for a structural asymmetry across countries in the size of the traded sector. For this purpose, we obtain welfare measures through second-order approximations to the households' utility functions and to the private sector's optimizing conditions. The gains arise from two channels. The first channel is rather standard in the NOEM literature and is independent of the structural asymmetry in the model: if acting independently, a country's central bank tends to overlook the effect of terms-of-trade on the other country's well-being; whereas when the countries cooperate, this terms-of-trade externality would be properly recognized and efforts would be made to internalize it. The second channel is unique to our model and works only through structural asymmetries across countries: the planner's optimal policy under the cooperating regime creates a *terms-of-trade bias* that favors the country with a larger traded sector; and this bias has to be balanced against the need to stabilize fluctuations in the terms-of-trade gap, among other variables in the policy objective. Absent structural asymmetry, the welfare gains from coordination are quantitatively small under calibrated parameters; as the degree of asymmetry enlarges, so do the welfare gains in general. With plausible structural asymmetries, there are sizable gains. Further, holding other things constant, the gains are larger if the countries have a greater share of imported goods in their traded basket, if the domestic shocks are less correlated, or if the duration of pricing contracts is longer.

The *terms-of-trade bias* identified in this paper should not be confused with the usual sense of terms-of-trade externality described in the NOEM literature. In the case with symmetric structures across countries, there is no *terms-of-trade bias* under cooperation and the welfare gains arise solely from internalizing the terms-of-trade externality. Under plausible parameters, such gains are quantitatively small. A stronger case for policy coordination can be made when the countries involved have asymmetric production and trading structures.

In our analysis, we have focused on a particular form of cross-country asymmetries in production and trading structures. To the extent that the structural asymmetry in our model captures some of the differences between developed economies and developing ones, our work sheds some light on the welfare consequences of international monetary policy coordination between countries at different stages of development. Of course, there are other ways to capture cross-country

structural differences and other forms of structural asymmetries that might be relevant for international monetary policy analysis. For instance, countries at different stages of development may have different trend components of traded-sector productivity, they may have different abilities to access international financial markets, or they may have different forms of labor market frictions. Modeling structural differences along these dimensions may have important implications for understanding the consequences of international policy coordination. If future studies along these lines find, as in our current paper, that the gains from coordination might increase with the degree of asymmetry, it would also raise the issue of whether countries have incentive to enter cooperative agreements: it is a non-trivial political-economy question as how to divide the gains among the countries with asymmetric structures. We conjecture that future research along these lines should be both fruitful and promising. The current paper represents a first step toward this direction.

## A Appendix

### A.1 Deriving the Welfare Objective under the Nash Regime

We characterize the welfare objective for a Nash central bank by taking second-order approximations to the representative household's period utility function. A second order approximation to the home household's period utility function is given by (4.6) and is rewritten here for convenience of references:

$$U_t - U_{ss} = \hat{c}_t - \Psi L \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) + O \left( \|\xi\|^3 \right).$$

The first component of the approximated utility function is deviations of consumption from steady state, which are related to deviations of outputs through the aggregate resource constraint (3.10).

The relation is given by

$$\hat{c}_t = (1 - \alpha) \hat{y}_{Nt} + \alpha \omega \hat{y}_{Tt} + \alpha(1 - \omega) \hat{y}_{Tt}^* \quad (\text{A.1.1})$$

The second part of the approximated period utility involves second-order approximations to the labor market clearing condition  $L_t = L_{Nt} + L_{Tt}$ , which is given by

$$\hat{l}_t = \frac{L_N}{L} \hat{l}_{Nt} + \frac{L_T}{L} \hat{l}_{Tt} + \frac{1}{2} \left[ \frac{L_N}{L} \hat{l}_{Nt}^2 + \frac{L_T}{L} \hat{l}_{Tt}^2 - \hat{l}_t^2 \right] + O \left( \|\xi\|^3 \right). \quad (\text{A.1.2})$$

Using this result, we obtain

$$\begin{aligned} \Psi L \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) &= (1 - \alpha) \hat{l}_{Nt} + \alpha \omega \hat{l}_{Tt} + \frac{1}{2} \left( (1 - \alpha) \hat{l}_{Nt}^2 + \alpha \omega \hat{l}_{Tt}^2 \right) + O \left( \|\xi\|^3 \right), \\ &= (1 - \alpha) (\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt}) + \alpha \omega (\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{a}_{Tt}) \\ &\quad + \frac{1}{2} \left( (1 - \alpha) (\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt})^2 + \alpha \omega (\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{a}_{Tt})^2 \right) + O \left( \|\xi\|^3 \right), \end{aligned} \quad (\text{A.1.3})$$



where the first equality follows from the approximated labor market clearing condition (A.1.2), with the steady-state ratios  $L_N/L = 1 - \alpha$  and  $L_T/L = \alpha\omega$  imposed, and the second equality follows from the labor demand equations (3.11) and (3.12).

Subtract (A.1.3) from (A.1.1), and recognizing that the home planner takes foreign output  $\hat{y}_{Tt}^*$  as given, we obtain

$$\begin{aligned} U_t &= -(1 - \alpha)\hat{G}_{Nt} - \alpha\omega\hat{G}_{Ht} - \frac{1}{2} \left[ (1 - \alpha)(\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt})^2 + \alpha\omega(\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{a}_{Tt})^2 \right] \\ &\quad + t.i.p + O(\|\xi\|^3), \\ &= -(1 - \alpha)\hat{G}_{Nt} - \alpha\omega\hat{G}_{Ht} - \frac{1}{2} \left[ (1 - \alpha)\tilde{y}_{Nt}^2 + \alpha\omega\tilde{y}_{Tt}^2 \right] + t.i.p + O(\|\xi\|^3), \end{aligned}$$

where, in obtaining the second equality, we have used the definition of the output gaps  $\tilde{y}_{jt} = \hat{y}_{jt} - \hat{y}_{jt}^n$  with  $\hat{y}_{jt}^n = \hat{a}_{jt}$  being the natural rate output in sector  $j \in \{N, T\}$ , and the fact that the price-dispersion terms  $\hat{G}_{jt}$  are of second order. The notation *t.i.p* represents terms independent of policy, including steady-state terms, shocks, and foreign outputs.

Finally, following Woodford (2003), we can show that the price dispersion terms  $\hat{G}_{Nt}$  and  $\hat{G}_{Ht}$  can be related to variabilities in the sectoral inflation rates. In particular, we have

$$\sum_{t=0}^{\infty} \beta^t \hat{G}_{jt} = \frac{1}{2} \frac{\theta_j \gamma_j}{(1 - \beta\gamma_j)(1 - \gamma_j)} \sum_{t=0}^{\infty} \beta^t \pi_{jt}^2 + t.i.p + O(\|\xi\|^3), \quad j = N, H. \quad (\text{A.1.4})$$

Thus, the home central bank's welfare objective under the Nash regime is given by

$$W^{Nash} = E_0 \sum_{t=0}^{\infty} \beta^t U_t = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t^{Nash} + t.i.p. + O(\|\xi\|^3), \quad (\text{A.1.5})$$

where the period loss function is given by

$$L_t^{Nash} = (1 - \alpha)(\tilde{y}_{Nt}^2 + \theta_N \kappa_N^{-1} \pi_{Nt}^2) + \alpha\omega(\tilde{y}_{Tt}^2 + \theta_T \kappa_T^{-1} \pi_{Ht}^2), \quad (\text{A.1.6})$$

where  $\kappa_j = \gamma_j / ((1 - \beta\gamma_j)(1 - \gamma_j))$  for  $j \in \{N, T\}$ . These correspond to (4.7)-(4.8) in the text.

The foreign Nash central bank's objective can be similarly derived.

## A.2 Deriving the Welfare Objective Under the Cooperating Regime

The welfare objective for the social planner under the cooperating regime is given by (4.14), which we rewrite here for convenience of references:

$$W^{Planner} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [U_t + U_t^*],$$

where  $U_t = \log C_t - \Psi L_t$  and  $U_t^* = \log C_t^* - \Psi L_t^*$  are the representative households' period-utility functions.

A second-order approximation to the home household's period utility function is given by (4.6), the same as in the Nash case. The  $\hat{c}_t$  term is also the same as in the Nash case, and is given by (A.1.1).

The terms involving employment, however, is different from the Nash case since the steady-state allocations are different. In particular, the approximated employment terms in the home household's period utility function are given here by

$$\begin{aligned}\Psi L \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) &= (1 - \alpha) \hat{l}_{Nt} + \tilde{\alpha} \hat{l}_{Tt} + \frac{1}{2} \left\{ (1 - \alpha) \hat{l}_{Nt}^2 + \tilde{\alpha} \hat{l}_{Tt}^2 \right\} + O \left( \|\xi\|^3 \right), \\ &= (1 - \alpha) (\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt}) + \tilde{\alpha} (\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{a}_{Tt}) \\ &\quad + \frac{1}{2} \left\{ (1 - \alpha) (\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt})^2 + \tilde{\alpha} (\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{a}_{Tt})^2 \right\} + O \left( \|\xi\|^3 \right),\end{aligned}\tag{A.2.1}$$

where we have used the steady state conditions (4.10) and the labor demand functions (3.11) and (3.12), and we have defined a constant  $\tilde{\alpha} = \alpha\omega + \alpha^*(1 - \omega)$ . Subtracting (A.2.1) from the expression for consumption in (A.1.1), we obtain the approximated period utility for the home country:

$$\begin{aligned}U_t &= -(1 - \alpha) \hat{G}_{Nt} - \tilde{\alpha} \hat{G}_{Ht} \\ &\quad - \frac{1}{2} \left\{ (1 - \alpha) (\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt})^2 + \tilde{\alpha} (\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{a}_{Tt})^2 \right\} + t.i.p + O \left( \|\xi\|^3 \right), \\ &= -(1 - \alpha) \hat{G}_{Nt} - \tilde{\alpha} \hat{G}_{Ht} - \frac{1}{2} \left\{ (1 - \alpha) \tilde{y}_{Nt}^2 + \tilde{\alpha} \tilde{y}_{Tt}^2 \right\} + t.i.p + O \left( \|\xi\|^3 \right),\end{aligned}$$

where *t.i.p.* denotes terms independent of policy, including constant terms and shocks. Similarly, the approximated period utility function for the foreign country can be obtained as follows:

$$\begin{aligned}U_t^* &= -(1 - \alpha^*) \hat{G}_{Nt}^* - \tilde{\alpha}^* \hat{G}_{Ft}^* \\ &\quad - \frac{1}{2} \left\{ (1 - \alpha^*) (\hat{y}_{Nt}^* + \hat{G}_{Nt}^* - \hat{a}_{Nt}^*)^2 + \tilde{\alpha}^* (\hat{y}_{Tt}^* + \hat{G}_{Ft}^* - \hat{a}_{Tt}^*)^2 \right\} + t.i.p + O \left( \|\xi\|^3 \right), \\ &= -(1 - \alpha^*) \hat{G}_{Nt}^* - \tilde{\alpha}^* \hat{G}_{Ft}^* - \frac{1}{2} \left\{ (1 - \alpha^*) \tilde{y}_{Nt}^{*2} + \tilde{\alpha}^* \tilde{y}_{Tt}^{*2} \right\} + t.i.p + O \left( \|\xi\|^3 \right),\end{aligned}$$

where  $\tilde{\alpha}^* = \alpha^*\omega + \alpha(1 - \omega)$ .

Finally, replacing the  $G$ -terms using (A.1.4), we obtain the planner's welfare objective:

$$W^{Planner} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [U_t + U_t^*] = -\frac{1}{4} E_0 \sum_{t=0}^{\infty} \beta^t L_t^{Planner} + t.i.p. + O \left( \|\xi\|^3 \right),\tag{A.2.2}$$

where the period loss function is given by

$$\begin{aligned}L_t^{Planner} &= (1 - \alpha) (\tilde{y}_{Nt}^2 + \theta_N \kappa_N^{-1} \pi_{Nt}^2) + \tilde{\alpha} (\tilde{y}_{Tt}^2 + \theta_T \kappa_T^{-1} \pi_{Ht}^2) \\ &\quad + (1 - \alpha^*) (\tilde{y}_{Nt}^{*2} + \theta_N^* \kappa_N^{*-1} \pi_{Nt}^{*2}) + \tilde{\alpha}^* (\tilde{y}_{Tt}^{*2} + \theta_T^* \kappa_T^{*-1} \pi_{Ft}^{*2}),\end{aligned}\tag{A.2.3}$$

where  $\tilde{\alpha} = \alpha\omega + \alpha^*(1 - \omega)$  and  $\tilde{\alpha}^* = \alpha^*\omega + \alpha(1 - \omega)$ . These expressions correspond to (4.15)-(4.16) in the text.

### A.3 Loss Function in Terms of Consumption, Relative Prices, and Terms of Trade

We now derive the relation between the period loss functions under the two regimes and the consumption gaps, relative price gaps, and the terms-of-trade gaps.

#### A.3.1 The Loss Function Under the Nash Regime

The period loss function under the Nash regime is described by (4.8). For convenience of reference, we rewrite it here

$$L_t^{Nash} = (1 - \alpha)(\tilde{y}_{Nt}^2 + \theta_N \kappa_N^{-1} \pi_{Nt}^2) + \alpha\omega(\tilde{y}_{Tt}^2 + \theta_T \kappa_T^{-1} \pi_{Ht}^2).$$

From (3.8) and (3.9), the output gaps can be expressed in terms of gaps of consumption, relative prices, and the terms of trade according to the following relations:

$$\tilde{y}_{Tt} = \tilde{c}_t + (1 - \alpha)\tilde{q}_{Nt} + (1 - \omega)\tilde{s}_t, \quad \tilde{y}_{Nt} = \tilde{c}_t + \alpha\tilde{q}_{Nt}, \quad (\text{A.3.1})$$

where  $\tilde{c}_t$ ,  $\tilde{q}_{Nt}$ , and  $\tilde{s}_t$  denote the gaps of consumption, the relative price of non-traded goods, and the terms of trade. Meanwhile, from (), the terms-of-trade gap is related to the relative traded-output gaps in the two countries according to

$$\tilde{s}_t = \tilde{y}_{Tt} - \tilde{y}_{Nt}^*. \quad (\text{A.3.2})$$

Let  $f(\pi_{Nt}^2, \pi_{Ht}^2) = (1 - \alpha)\theta_N \kappa_N^{-1} \pi_{Nt}^2 + \alpha\omega\theta_T \kappa_T^{-1} \pi_{Ht}^2$  denote the composite of the variations in the inflation rates. The period loss function above can then be rewritten as

$$\begin{aligned} L_t^{Nash} &= f(\pi_{Nt}^2, \pi_{Ht}^2) + (1 - \alpha)\tilde{y}_{Nt}^2 + \alpha\tilde{y}_{Tt}^2 - (1 - \omega)\alpha\tilde{y}_{Tt}^2 \\ &= f(\pi_{Nt}^2, \pi_{Ht}^2) + (1 - \alpha)(\tilde{c}_t - \alpha\tilde{q}_{Nt})^2 + \alpha[\tilde{c}_t + (1 - \alpha)\tilde{q}_{Nt} + (1 - \omega)\tilde{s}_t]^2 - (1 - \omega)\alpha\tilde{y}_{Tt}^2 \\ &= f(\pi_{Nt}^2, \pi_{Ht}^2) + \tilde{c}_t^2 + \alpha(1 - \alpha)\tilde{q}_{Nt}^2 + \alpha(1 - \omega)^2\tilde{s}_t^2 + 2\alpha(1 - \omega)[\tilde{c}_t + (1 - \alpha)\tilde{q}_{Nt}]\tilde{s}_t \\ &\quad - (1 - \omega)\alpha(\tilde{y}_{Tt}^* + \tilde{s}_t)^2 \\ &= f(\pi_{Nt}^2, \pi_{Ht}^2) + \tilde{c}_t^2 + \alpha(1 - \alpha)\tilde{q}_{Nt}^2 + \alpha\omega(1 - \omega)\tilde{s}_t^2 - \alpha(1 - \omega)\tilde{y}_{Tt}^{*2}, \end{aligned} \quad (\text{A.3.3})$$

where the second equality follows from (A.3.1), the third from (A.3.2), and the final equality is obtained by collecting terms. Under the Nash regime, the home central bank takes foreign output gap as given, so that  $\tilde{y}_{Tt}^{*2}$  in the loss function is a term independent of policy. Thus, the period loss function for the independent central bank in the home country is given by

$$L_t^{Nash} = (1 - \alpha)\theta_N \kappa_N^{-1} \pi_{Nt}^2 + \alpha\omega\theta_T \kappa_T^{-1} \pi_{Ht}^2 + \tilde{c}_t^2 + \alpha(1 - \alpha)\tilde{q}_{Nt}^2 + \alpha\omega(1 - \omega)\tilde{s}_t^2 + t.i.p., \quad (\text{A.3.4})$$

where the term  $t.i.p. = -\alpha(1 - \omega)\tilde{y}_{Tt}^{*2}$  is independent of home's monetary policy. This corresponds to (4.9) in the text.

Similarly, the loss function for the independent foreign central bank is given by

$$L_t^{*Nash} = (1-\alpha^*)\theta_N^*\kappa_N^{*-1}\pi_{Nt}^{*2} + \alpha^*\omega\theta_T^*\kappa_T^{*-1}\pi_{Ft}^{*2} + \tilde{c}_t^{*2} + \alpha^*(1-\alpha^*)\tilde{q}_{Nt}^{*2} + \alpha^*\omega(1-\omega)\tilde{s}_t^2 + t.i.p.^*, \quad (\text{A.3.5})$$

where the term  $t.i.p.^* = -\alpha^*(1-\omega)\tilde{y}_{Tt}^2$  is independent of foreign's monetary policy.

### A.3.2 The Loss Function Under the Cooperating Regime

The period loss function for the social planner under the cooperating regime given by (4.16) can be rewritten as

$$\begin{aligned} L_t^{Planner} &= (1-\alpha)(\tilde{y}_{Nt}^2 + \theta_N\kappa_N^{-1}\pi_{Nt}^2) + [\alpha\omega + \alpha^*(1-\omega)](\tilde{y}_{Tt}^2 + \theta_T\kappa_T^{-1}\pi_{Ht}^2) \\ &\quad + (1-\alpha^*)(\tilde{y}_{Nt}^{*2} + \theta_N^*\kappa_N^{*-1}\pi_{Nt}^{*2}) + [\alpha^*\omega + \alpha(1-\omega)](\tilde{y}_{Tt}^{*2} + \theta_T^*\kappa_T^{*-1}\pi_{Ft}^{*2}), \\ &= (1-\alpha)(\tilde{y}_{Nt}^2 + \theta_N\kappa_N^{-1}\pi_{Nt}^2) + \alpha\omega(\tilde{y}_{Tt}^2 + \theta_T\kappa_T^{-1}\pi_{Ht}^2) \\ &\quad + (1-\alpha^*)(\tilde{y}_{Nt}^{*2} + \theta_N^*\kappa_N^{*-1}\pi_{Nt}^{*2}) + \alpha^*\omega(\tilde{y}_{Tt}^{*2} + \theta_T^*\kappa_T^{*-1}\pi_{Ft}^{*2}) \\ &\quad + \alpha^*(1-\omega)[\tilde{y}_{Tt}^2 + \theta_T\kappa_T^{-1}\pi_{Ht}^2] + \alpha(1-\omega)[\tilde{y}_{Tt}^{*2} + \theta_T^*\kappa_T^{*-1}\pi_{Ft}^{*2}]. \end{aligned}$$

The first two lines in the last equality correspond to the sum of the national period losses under the Nash regime, and the third line contains terms that are unique to the cooperating regime (but are absent under the Nash regime). Using the national loss functions (A.3.4) and (A.3.5) under the Nash regime, and recognizing the assumption that the social planner under the cooperating regime cares about the variables in both countries, so that the terms  $t.i.p. = -\alpha(1-\omega)\tilde{y}_{Tt}^{*2}$  and  $t.i.p.^* = -\alpha^*(1-\omega)\tilde{y}_{Tt}^2$  in (A.3.4) and (A.3.5) cannot be treated as terms independent of policy, we obtain

$$\begin{aligned} L_t^{Planner} &= (1-\alpha)\theta_N\kappa_N^{-1}\pi_{Nt}^2 + [\alpha\omega + \alpha^*(1-\omega)]\theta_T\kappa_T^{-1}\pi_{Ht}^2 + \tilde{c}_t^2 + \alpha(1-\alpha)\tilde{q}_{Nt}^2 \\ &\quad + (1-\alpha^*)\theta_N^*\kappa_N^{*-1}\pi_{Nt}^{*2} + [\alpha^*\omega + \alpha(1-\omega)]\theta_T^*\kappa_T^{*-1}\pi_{Ft}^{*2} + \tilde{c}_t^{*2} + \alpha^*(1-\alpha^*)\tilde{q}_{Nt}^{*2} \\ &\quad + (\alpha + \alpha^*)\omega(1-\omega)\tilde{s}_t^2, \end{aligned} \quad (\text{A.3.6})$$

which is (4.17) in the text.

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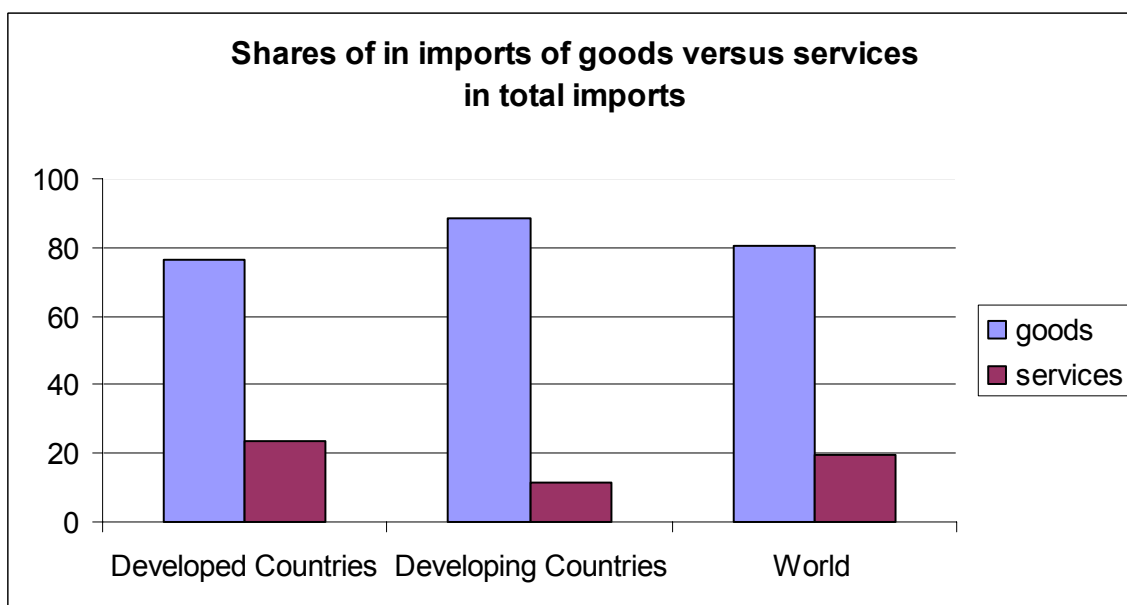
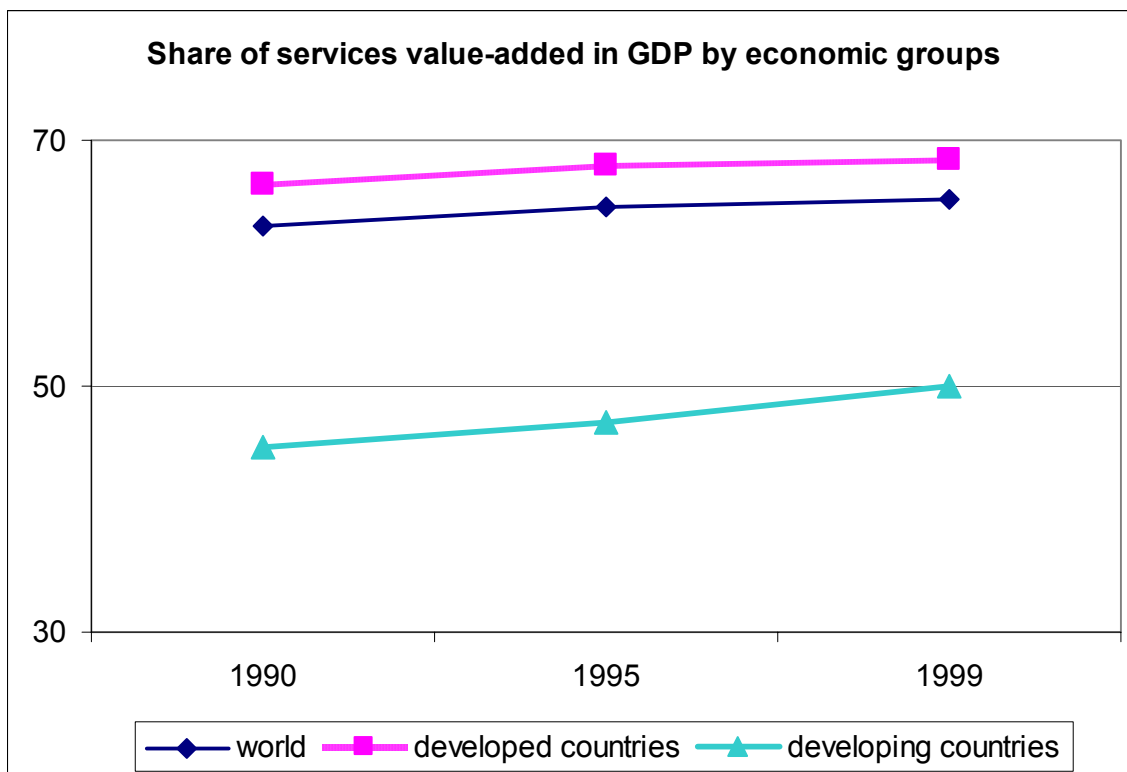
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Table 1. Baseline Parameter Calibration

Preferences:	$U(C, L) = \ln C - \Psi L$	$\beta = 0.99$ $\Psi$ adjusted
Aggregation:	$C = \bar{\alpha} C_T^\alpha C_N^{1-\alpha}$	$\alpha = 0.3$
	$C_T = \bar{\omega} C_H^\omega C_F^{1-\omega}$	$\omega = 0.7$
	$Y_k = \left[ \int_0^1 Y_k(j)^{\frac{\theta_k-1}{\theta_k}} dj \right]^{\frac{\theta_k}{\theta_k-1}}, \theta_k = 10, k \in \{T, N\}$	
Contract duration:		$\gamma_T = 0.75, \gamma_N = 0.75$
Productivity Shocks:		$\sigma_T = 0.01, \sigma_N = 0.01$
		$\rho_{TN} = 0, \rho_{TT} = 0, \rho_{NN} = 0$





Data source: the World Bank.

Figure 1: --- Value-added shares and tradedness of services

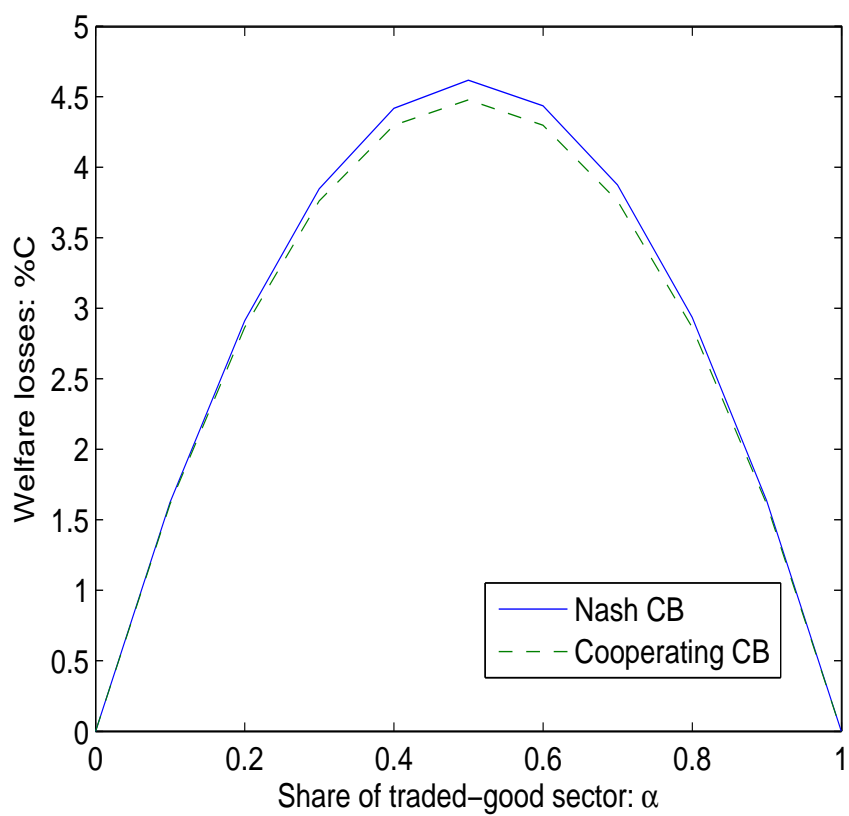


Figure 2:—Welfare losses of alternative monetary policy regimes: symmetric structures.

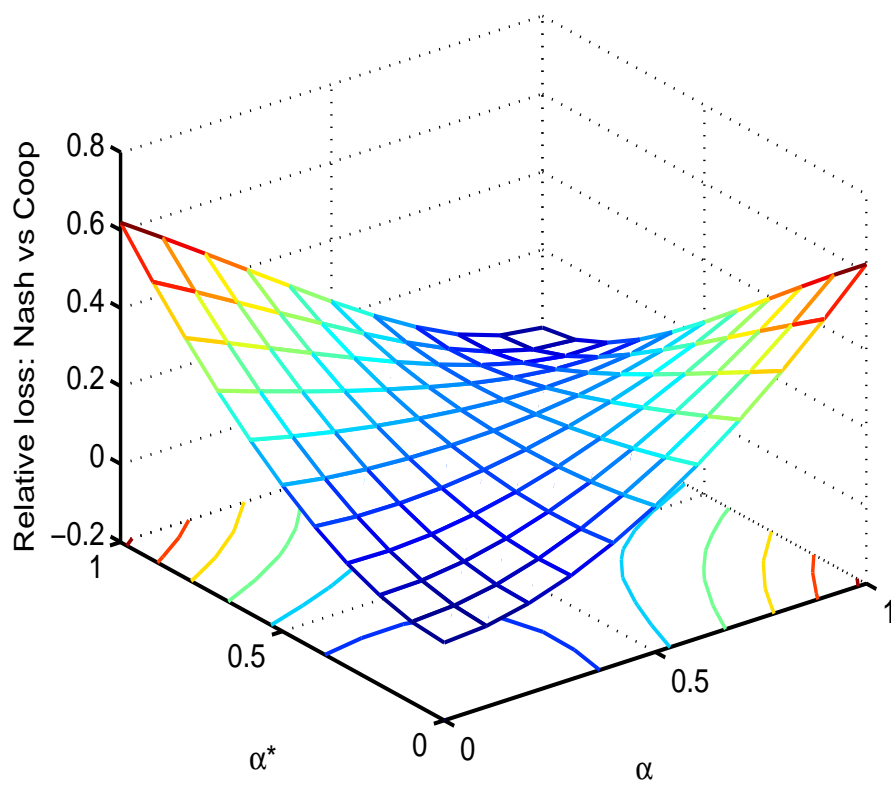


Figure 3:—Welfare gains from coordination: sizes of the traded sectors

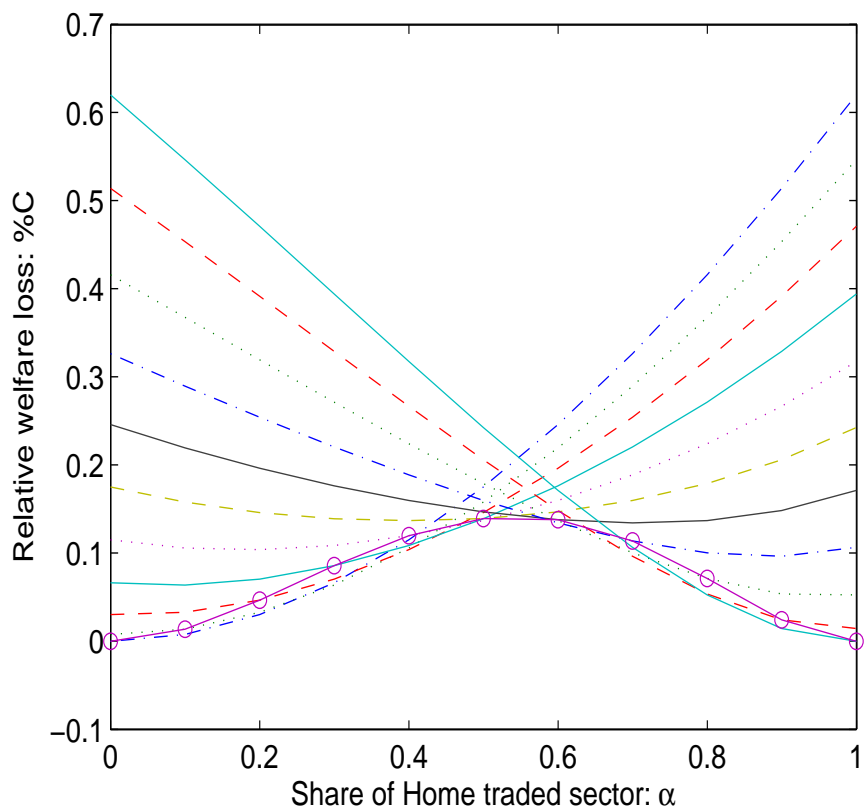


Figure 4:—Welfare gains from coordination: symmetric structures (circled line) versus asymmetric structures (solid lines)

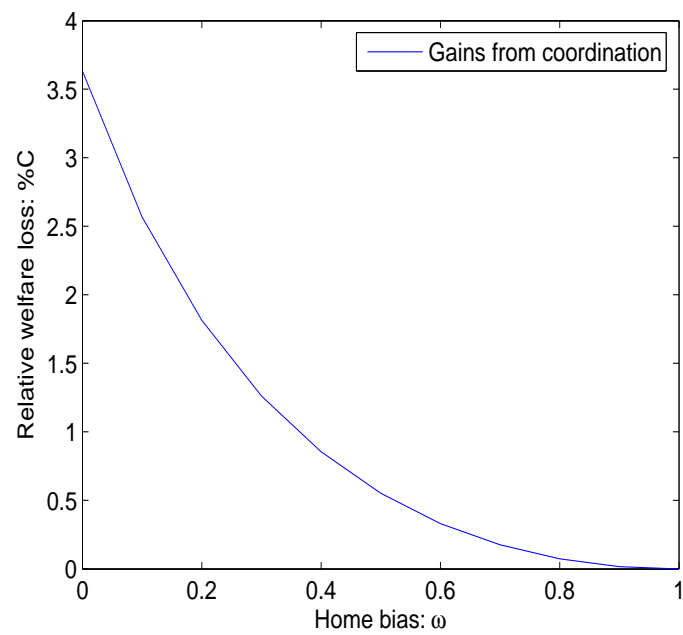
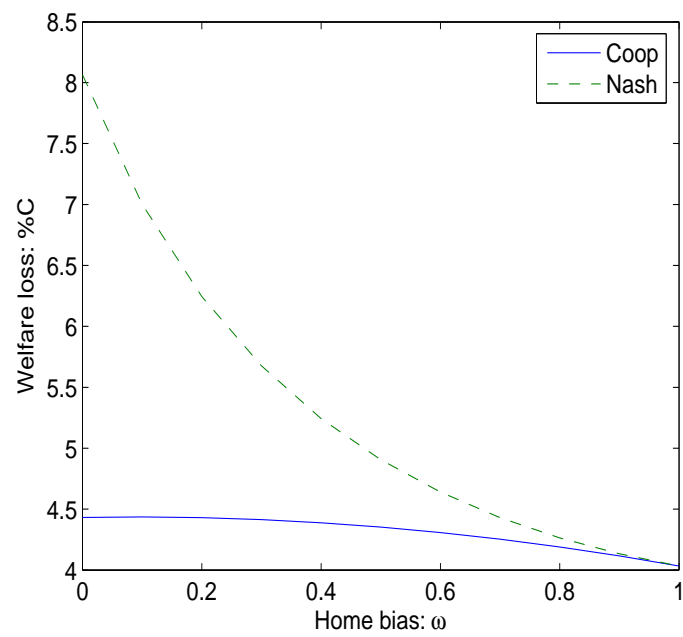


Figure 5:—Welfare losses under alternative regimes and gains from coordination: home bias.

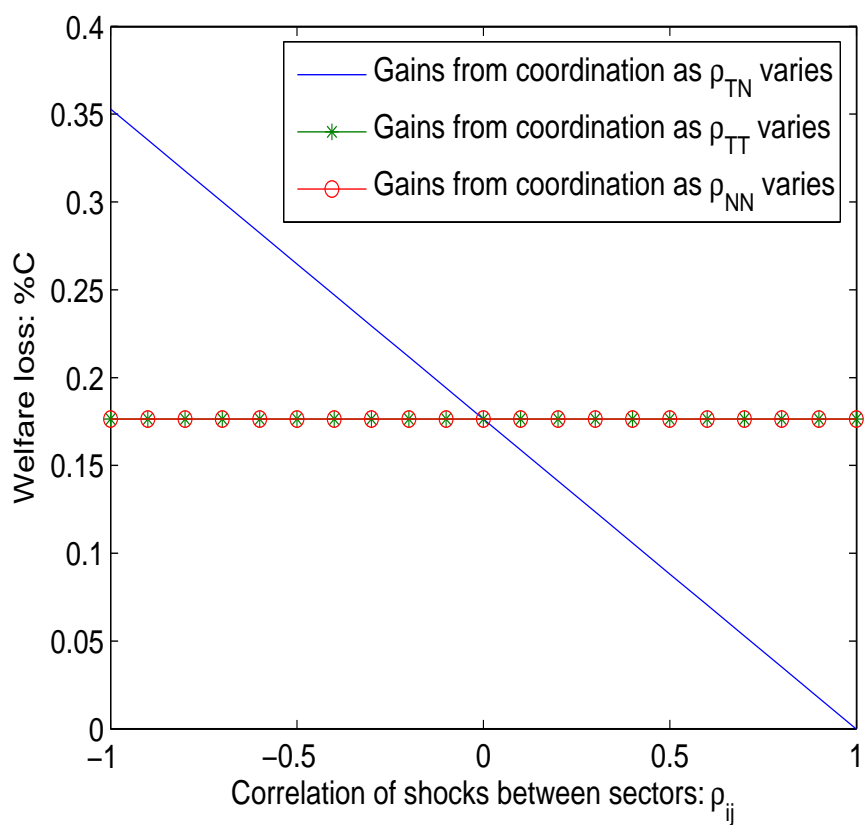


Figure 6:—Welfare gains from coordination: correlations of sectoral shocks.

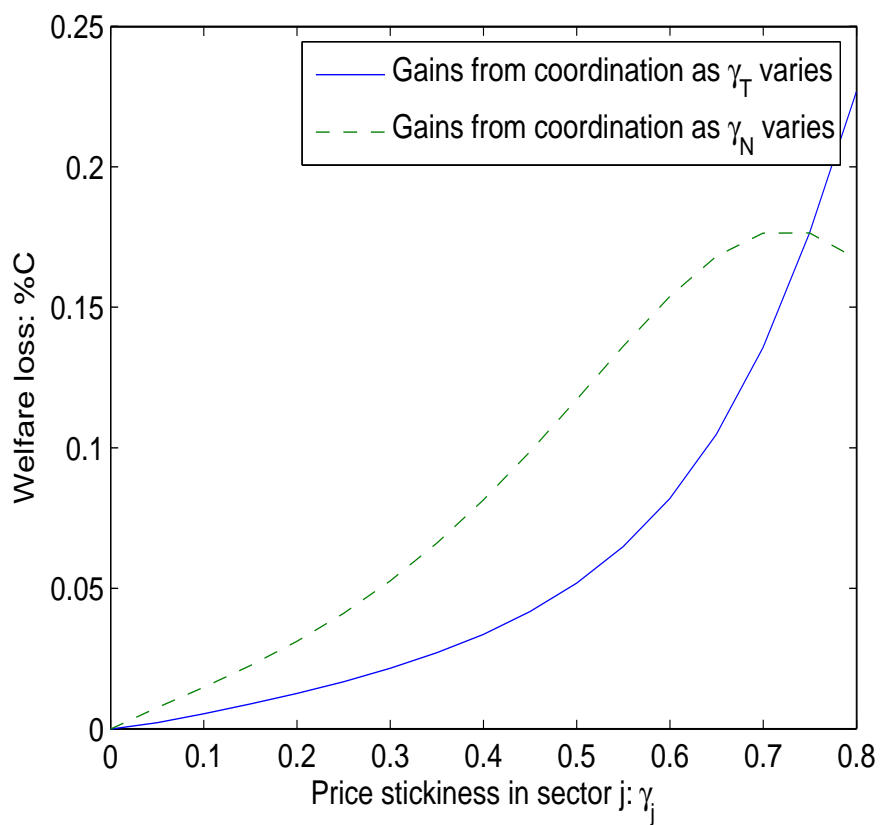


Figure 7:—Welfare gains from coordination: sectoral price stickiness.

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