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« Sequential Location under one-sided Demand Uncertainty »

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Sequential Location under one-sided Demand Uncertainty

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Abstract

By entering new market, firms face uncertainty about their potential demand. We depart from the usual Hotelling duopoly model with sequential entry. Firms can locate outside the city and market conditions are common knowledge. Then we introduce one-sided demand uncertainty. It results that demand uncertainty can be seen as a *differentiation force* when the first entrant faces demand uncertainty and as an *agglomeration force* when it is the second entrant. Finally, firm 2's imperfect information implies higher welfare losses.

Key words: Location, Hotelling, Sequential Duopoly Game, Product Differentiation, Demand Uncertainty
JEL Classification: C72, D43, D81, L13, R30

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1 Introduction

The *"Minimum Differentiation Principle"* introduced by Hotelling (1929) represents a starting point in the theory of location. According to this principle, when two firms propose an homogeneous product and when the demand is inelastic, then firms locate as close to each other as possible, i.e. at the center of the city. Refinements and extensions of this principle lead to a growing literature in Industrial Organization fields. One extension studied in this paper consists of the sequential entry of firms which are imperfectly informed about demand location.

In most markets, products are horizontally differentiated. From an economic point of view, we argue say that products are differentiated if consumers based their decisions not only upon prices but upon other characteristics too. These characteristics include location, design, etc. Most of studies note that the higher homogeneous products, the higher is the competition. This point explains the well-known *"Maximum Differentiation Principle"* of D'Aspremont, Gabszewicz, and Thisse (1979) who demonstrate that in a standard Hotelling framework with quadratic transportation costs and price competition, firms seek to move as far away from the other. Horizontal differentiation represents a mean to relax price competition[1] (see for example Netz and Taylor, 2002, for the retail gasoline industry).

However, the observation of economic activities points out that the *"Maximum Differentiation Principle"* is not a common phenomenon. In some markets, firms are induced to agglomerate despite of the centrifugal force of price competition. As noted by Fujita and Thisse: *"a commercial area involving a large number of stores, restaurants, or theatres is likely to emerge when it offers sufficiently differentiated products, or when the transport cost borne by consumers are low enough, or both",*(Fujita and Thisse, 2002, p.219). Historically, one of the first explanations refers to economies of scale encounter by consumers. When consumers are imperfectly informed about prices, cluster of sellers may facilitate price comparisons (Stahl, 1982b). The same result occurs if, instead of price comparisons, consumers' choices are driven by cost considerations (Eaton and Lipsey, 1979; Stahl, 1982a). By means of the combination of several dimensions of differentiation into the Hotelling model, some researchers have shown that if one dimension overtakes the others, maximum differentiation appears only in this first. This conclusion, named the *"Max-Min Principle"* is another explanation to agglomeration if geographical dimension is a minor one. (see Tabuchi, 1994; Irmen and Thisse, 1998; Ansari, Economides, and Steckel, 1998). Quality uncertainty represents another possible

[1] Same intuition occurs with vertical product differentiation (see Shaked and Sutton, 1982).

explanation of *agglomeration force*. As Bester (1998) and Vettas (1999) show, when consumers face uncertainty over the quality of a good, like in *"experience goods"*, firms are more likely to cluster together. More precisely, a high quality firm may choose to signal its quality to consumers by locating close to its competitors. Or when price is used to ascertain the quality of goods, firms are engaged in drastic quality competition which relaxes differentiation in horizontal dimension. Our approach has a common feature with these two last: uncertainty. Nonetheless, we focus on consequences of demand uncertainty. We tend to examine whether one-sided demand uncertainty can be seen, in some contexts, as a mean of generating agglomeration force when products are not differentiated in other dimensions.

Uncertainty about demand location - or similarly consumer tastes - is a parameter which is usually involved in manager's decision-making process. For example, if a firm decides to establish in a new market or to launch a new product, it encounters the problem of estimating accurately the location of potential buyers or consumer's preferences. Its decisions, often irreversible, will have a critical influence on its profit. Automobile industry is littered with examples of new models which didn't success. French constructors Renault and Peugeot have recently made a flop with their new models *Aventime* and *1007*, respectively. These mistakes in the evaluation of consumers' taste have now dramatic consequences on firms' revenue [2] . Such decisions are made under uncertainty about demand conditions.

Only few models take this parameter into account since location models are usually based under the assumption of complete information about consumers' location or tastes. A first step in this way is the work of Jovanovic (1981) who studies a model of entry and location with n firms which have private information about consumers' location. In another way, Harter (1996) extends Neven's game of sequential entry (Neven, 1987) by introducing demand uncertainty; Firms know only the distribution from which the location of the uniform interval of consumers' taste will be drawn. Balvers and Szerb (1996) introduce demand uncertainty which comes from unobservable aspects of products. Uncertainty resides over the common quality valuation of a firm's product under fixed prices. More recently, Casado-Izaga (2000) investigates uncertainty about consumer tastes with identical firms, by allowing consumer distribution to shift to the right of the unit interval, following a parameter drawn from a uniform distribution on the unit interval $[0, 1]$. The purpose of our model is to extend the studies of Meagher and Zauner (2004, 2005) who analyse the influence of different levels of firms' uncertainty over consumer distribution into a spatial duopoly location-then-price game with simultaneous location.

Yet previous works suffer from different limits. The early studies didn't take

[2] See *Les échos*, February 06, 2007.

into account the possibility for firms to locate outside the "imaginary city", while the most accomplished works of Meagher and Zauner (2004, 2005) didn't achieve to integrate the sequentiality of the game. Or sequentiality is a more realistic description of the way by which firms enter the market. A mean to encompass this problem is to consider the issue of demand uncertainty as an asymmetric problem. In real life, all firms are not equal towards demand uncertainty. Some firms may have private information due to past experiences or costly market studies, while others may be fully uncertain. The case of one-sided demand uncertainty has been largely explored when firms choose their output level, but without location decision (see Liu, 2005 for homogeneous products and Ferreira, Ferreira, and Pinto, 2006 for differentiated products). In this context, when the leader has an informational advantage compared to the follower, the leader does not have necessarily a global advantage over the follower since this last can easily adapt its strategy according to the leader's observable action. Now, if we introduce sequential location choices with one-sided demand uncertainty in a Hotelling framework, several questions arise: What does the advantage of the first mover become when the leader has an informational advantage compared to the follower? And similarly when it is the follower who has an informational advantage about demand location? Finally, what does happen in terms of differentiation and welfare considerations?

Our model takes place in the standard Hotelling duopoly model with quadratic transportation costs. Firms choose sequentially their location, but the location of demand is revealed to the firms before the price subgame. The uncertainty over consumers' taste (demand location) is described by a continuous density function with support $[\underline{M}, \overline{M}]$ contained in the closed interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$. The size of uncertainty is allowed to vary by changing the variance σ^2 of the distribution. In a first step, we investigate the case where demand uncertainty is supported by the incumbent, while the second entrant has private information. In a second step, we reverse the situation and suppose that the first entrant has perfect information about demand location while the second entrant faces demand uncertainty. Results obtained highlight that demand uncertainty can be seen as a *differentiation force* when the first entrant faces demand uncertainty. However, when the second entrant has imperfect information about consumer location, demand uncertainty can be an explanation of *agglomeration force*. This result is close to that obtained by Anderson, de Palma, and Thisse (1992) and de Palma, Ginsburgh, Papageorgiou, and Thisse (1985) who introduce uncertainty at the individual level. Demand uncertainty appears as a mean to intensify the first mover advantage, which is induced to locate closer to its competitor in order to increase its market share. As consequence, firms are engaged in fierce price competition and this statement leads to the worst situation from a social point of view.

In the remainder of this paper, we proceed as follow. In section 2, we expose our benchmark model of location-then-price competition in a Hotteling framework with sequential entry and where firms can locate outside the city. We introduce in section 3 one-sided demand uncertainty firstly for the leader and secondly for the follower. Then we analyse their welfare implications. Finally section 4 concludes.

2 Location-then-price game under demand certainty

In this section, we analyse the usual Hotelling framework in which firms locate sequentially under demand certainty, but they can locate outside the city. This case corresponds to our benchmark model and refers to previous results demonstrated by Lambertini (1994) and Tabuchi and Thisse (1995).

Consider a unit mass of consumers distributed uniformly over the closed interval $[M - \frac{1}{2}, M + \frac{1}{2}]$, with $M \in \mathbb{R}$. Each consumer patronizes one of the two firms indexed $i = 1, 2$ located on $x_i \in \mathbb{R}$. Without loss of generality, we suppose that $x_1 < x_2$, such that it is impossible for firms to be located at the same place[3] . Firms sell a homogeneous good produced with constant marginal costs normalized to 0. As product is not vertically differentiated, the sole difference between consumers' appreciation refers to consumers' taste according to the ideal product in terms of geographic location or product characteristic. Consumers have unit demands. The utility derived by a consumer located on $z \in [M - \frac{1}{2}, M + \frac{1}{2}]$ for patronize firm i is:

$$R - p_i - t(x_i - z)^2 \tag{1}$$

Where p_i represents the price of the good provided by firm i , $t > 0$ and $t(x_i - z)^2$ represents a quadratic transportation costs for consumer z to visit a firm located in x_i. The quadratic expression of transportation costs can be justified on two grounds. On the one hand, for consumers, this assumption appears natural since the loss of utility for a consumer who buys a different product than that desired is increasingly large the further away the product location is. On the other hand, from a theoretical point of view, this assumption enables to obtain the existence of pure-strategy equilibrium prices, a major problem of the usual Hotelling linear framework (D'Aspremont, Gabszewicz, and Thisse, 1979). We assume that $R > 0$ is the basic reservation utility obtained by a consumer patronizing one of the two firms. R is supposed to be large enough so that, in equilibrium, the entire market is covered.

We consider the following game:

[3] If $(x_1, x_2) = (x_1^*, x_2^*)$ (with $x_1 < x_2$) is an equilibrium, then (x_1, x_2)=$(1-x_1^*$,$1-x_2^*)$ is also an equilibrium.

- Stage 1: Firm 1 chooses its location
- Stage 2: Firm 2 chooses its location
- Stage 3: Firms simultaneously compete in prices

Our model takes place in a framework of sequential Hotelling game widely explored since the early works of Hay (1976) and Prescott and Visscher (1977). In this context, firms can choose asymmetric locations and they have the possibility to locate outside the city. We solve the game by backward induction and look for subgame perfect equilibrium (henceforth SPE) when firms maximize their profits and consumers their utilities[4]. We analyse in a first step price competition and thereafter location competition.

2.1 *Equilibrium prices*

We focus on the SPE of our two stages location-then-price competition. We first resolve the subgame equilibrium prices for a given pair of locations (x_1, x_2). Firms' demand is function to the marginal consumer location, in other words the consumer who is indifferent between patronizing firm 1 or firm 2. Let $z \in [M - \frac{1}{2}, M + \frac{1}{2}]$ be the location of the marginal consumer, and so z the demand for firm 1 and $(1 - z)$ the demand for firm 2. This point is determined by the resolution of the following equality:

$$R - p_1 - t(z - x_1)^2 = R - p_2 - t(x_2 - z)^2 \tag{2}$$

Then, the marginal consumer is located at:

$$z = \begin{cases} M + 1/2 & \text{if } p_2 \geq p_1 + t\left(x_1^2 - x_2^2\right) \\ \frac{p_2 - p_1 - t\left(x_1^2 - x_2^2\right)}{2t(x_2 - x_1)} & \text{otherwise} \\ M - 1/2 & \text{if } p_2 \leq p_1 + t\left(x_1^2 - x_2^2\right) + 2t\left(x_2 - x_1\right) \end{cases} \tag{3}$$

If consumers are uniformly distributed over the closed interval $[M - \frac{1}{2}, M + \frac{1}{2}]$, with $M \in \mathbb{R}$, firm 1's profit is given by $\pi_1 = p_1(z - (M - \frac{1}{2}))$ and similarly firm 2's profit is given by $\pi_2 = p_2((M + \frac{1}{2}) - z)$. The equilibrium prices is determined by the solution of the two following first order conditions: $\partial\pi_1/\partial p_1 = 0$ and $\partial\pi_2/\partial p_2 = 0$.

[4] For a review of game theory, see Montet and Serra (2003)

Lemma 1 *For given locations $x_1 < x_2$, the unique equilibrium prices is given by:*

$$p_1^*(x_1, x_2) = \begin{cases} t(-x_1 + x_2)(x_1 - 2M - 1 + x_2) & \text{if } M \leq \frac{x_1+x_2}{2} - \frac{3}{2} \\ -\frac{2}{3}t(x_1 - x_2)\left(\frac{x_1+x_2}{2} - \left(M - \frac{3}{2}\right)\right) & \text{otherwise} \\ 0 & \text{if } M \geq \frac{x_1+x_2}{2} + \frac{3}{2} \end{cases} \tag{4}$$

$$p_2^*(x_1, x_2) = \begin{cases} 0 & \text{if } M \leq \frac{x_1+x_2}{2} - \frac{3}{2} \\ \frac{2}{3}t(x_1 - x_2)\left(\frac{x_1+x_2}{2} - \left(M + \frac{3}{2}\right)\right) & \text{otherwise} \\ t(x_1 - x_2)(x_1 - 2M + 1 + x_2) & \text{if } M \geq \frac{x_1+x_2}{2} + \frac{3}{2} \end{cases}$$

PROOF. See Appendix A

If $M \leq \frac{x_1+x_2}{2} - \frac{3}{2}$, firm 2 is father away from the consumer distribution than firm 1. In equilibrium, firm 1 charges a price such that firm 2 has zero demand which implies $p_2^* = 0$ as the optimal price. The same result occurs for firm 1 when $M \geq \frac{x_1+x_2}{2} + \frac{3}{2}$.

2.2 Equilibrium locations

Given the Nash equilibrium in the price subgame, we can firstly determine the location of the marginal consumer z in equilibrium. We know, if firm 1 captures the entire demand ($p_2^* = 0$ and $M \leq \frac{x_1+x_2}{2} - \frac{3}{2}$), then $z^* = M + \frac{1}{2}$. Conversely, if firm 2 captures the entire demand ($p_1^* = 0$ and $M \geq \frac{x_1+x_2}{2} + \frac{3}{2}$), then $z^* = M - \frac{1}{2}$.

So we can write:

$$z^* = \begin{cases} M + \frac{1}{2} & \text{if } M \leq \frac{x_1+x_2}{2} - \frac{3}{2} \\ \frac{1}{6}(x_1 + 4M + x_2) & \text{if } \frac{x_1+x_2}{2} - \frac{3}{2} < M < \frac{x_1+x_2}{2} + \frac{3}{2} \\ M - \frac{1}{2} & \text{if } M \geq \frac{x_1+x_2}{2} + \frac{3}{2} \end{cases} \tag{5}$$

We face the traditional Stackelberg game in an extended Hotelling framework where each firm can locate outside the closed interval $[M - \frac{1}{2}, M + \frac{1}{2}]$. Note that in this sequential game, firm 1 is the leader and firm 2 the follower. The pair (x_1^*, x_2^*) is an equilibrium locations if their choices constitute a subgame perfect Nash equilibrium, i.e. if $\pi_1^*(x_1^*, x_2^*) \geq \pi_1^*(x_1, x_2^*)$ and $\pi_2^*(x_1^*, x_2^*) \geq \pi_2^*(x_1^*, x_2)$.

As usual, we start by establishing the follower's optimal location and then the leader's one. This backward induction leads to the following proposition

Proposition 1 ***(Equilibrium under Certainty)*** *The unique SPE locations for our location-then-price competition in an extended Hotteling framework is given by:*

$$x_1^* = M \text{ and } x_2^* = M + 1 \tag{6}$$

the equilibrium prices associated to these locations is given by

$$p_1^* = \frac{4}{3}t \text{ and } p_2^* = \frac{2}{3}t \tag{7}$$

and the following equilibrium profits

$$\pi_1^* = \frac{8}{9}t \text{ and } \pi_2^* = \frac{2}{9}t \tag{8}$$

The equilibrium differentiation under certainty Δ^c, is given by $x_2^* - x_1^* = 1$.

PROOF. See Appendix B

This is the traditional result underlined by Lambertini (1994) and Tabuchi and Thisse (1995) when firms can locate outside the city and when only one dimension of differentiation exists. The first entrant locates at the center in order to maximize its profit, whereas second entrant moves away from the center to $(M+1)$ in order to soften price competition. Firm 2 faces the trade-off between increasing its market share by locating closer to firm 1 - and so the market center - but also strengthening price competition. In this case, the negative effect of price competition is stronger than the positive effect of increasing market share and firm 2 is enforced to differentiate itself by locating outside the city.

From socially optimal point of view, the only constraint for the social planner is to minimize total transportation costs, since for inelastic demands pricing decisions do not affect the total welfare. This leads to the following socially optimal locations, $x_1^{soc} = -1/4 + M$ and $x_2^{soc} = 1/4 + M$, and socially equilibrium differentiation $\Delta^{soc} = 1/2$. The equilibrium locations leads to an excessive amount of differentiation compared to the socially optimal locations. We analyse in the following section the equilibrium locations in case of one-sided demand uncertainty.

3 Location-then-price game under one-sided demand uncertainty

In this section, we introduce demand uncertainty in the location game. It is reasonable to assume that it is private information. Moreover, we suppose that firms do not share this information between them. However, we assume that demand uncertainty occurs only for one firm and it is revealed before the price subgame. As previously, two firms enter sequentially into the market and propose a homogeneous product. Here we study the following scenario: firm 1 first enters the market and faces uncertainty about demand location. In that case, firm 1 faces demand uncertainty due to consumers' tastes for example: Do consumers will purchase a new product? Firm 1 doesn't know the location of consumers' ideal point when it chooses its location. Conversely, the second entrant - firm 2 - faces few uncertainties about its demand (for simplicity we suppose that it has no uncertainty). Two hypotheses could explain that: i) we can think that firm 2 observes demand reaction after firm 1 locates and/or ii) firm 2 has private information about the nature of the demand. After firms have chosen their locations, the demand uncertainty is revealed (for firm 1) and firms compete simultaneously in prices [5]. Such a situation can be summarized by the following game in four steps:

- Stage 1: Firm 1 chooses its location under demand uncertainty
- Stage 2: Firm 2 chooses its location under demand certainty
- Stage 3: Demand uncertainty is revealed
- Stage 4: Firms compete simultaneously in prices

In a second step, we change firms' available information. We suppose now that firm 1 is perfectly informed about the demand distribution, but firm 2 faces demand uncertainty.

This kind of framework refers to the works of Meagher and Zauner (2004, 2005) when both firms face uncertainty at the aggregate level but locate simultaneously. Consumers are still uniformly distributed [6] over the closed interval $\left[M-\frac{1}{2}, M+\frac{1}{2}\right]$. The demand uncertainty concerns consumer's preferences. This uncertainty is described by the following assumption: M is a centered random variable distributed on the closed interval $[\underline{M}, \overline{M}]$ contained in $\left[-\frac{1}{2}, \frac{1}{2}\right]$, and according to a continuous density function $f(M)$, with $E(M)=\mu=0$ and $V(M)=\sigma^2$. Varying the variance, let us to change the size of uncertainty.

[5] In real world, prices can be quickly adjusted up or down when market conditions are revealed. We approximate this fact by assuming that firms can immediately adjust their prices and consequently firms compete simultaneously in prices.

[6] For a review of the implications of non uniform density function of consumers in a Hotelling framework with sequential locations, see Anderson, Goeree, and Ramer (1997).

The length of the support of uncertainty $[\underline{M},\overline{M}] \subset \left[-\frac{1}{2},\frac{1}{2}\right]$ is chosen small enough to guarantee that firms can obtain positive demand in each realization of M. In other words, the support of uncertainty $[\underline{M},\overline{M}]$ is supposed contained in the interval where both firms compete $\left[\frac{x_1+x_2}{2}-\frac{3}{2},\frac{x_1+x_2}{2}+\frac{3}{2}\right]$, (see Lemma 1 in Meagher and Zauner, 2004)[7,8].

For simplicity, let $t = 1$. We assume that firms are risk neutral (i.e. they choose locations which maximize their expected profits) and they still locate sequentially. By backward induction, firm 2 chooses the location which maximizes its (expected) profit (when it faces demand uncertainty) then firm 1 chooses its location which maximizes its (expected) profits (when it faces demand uncertainty), given firm 2's optimal location.

3.1 Equilibrium prices

Locations are chosen sequentially: Firstly firm 1 and thereafter firm 2. Whatever the firm which faces demand uncertainty, when firms are located demand is revealed. Price competition is carried out with perfect information for both firms. Firm 1's profit is given by $\pi_1 = p_1(z - (M - \frac{1}{2}))$ and similarly firm 2's profit is given by $\pi_2 = p_2((M + \frac{1}{2}) - z)$.We focus on subgame perfect Nash equilibrium. For that, we depart from the resolution of the price competition game. As demand uncertainty is revealed before the price competition, the resolution of the price subgame is the same than that obtained previously (with $t = 1$).

Lemma 2 *The unique equilibrium prices is given by:*

$$p_1^*(x_1,x_2) = -\frac{2}{3}(x_1-x_2)\left(\frac{x_1+x_2}{2}-\left(M-\frac{3}{2}\right)\right) \tag{9}$$

$$p_2^*(x_1,x_2) = \frac{2}{3}(x_1-x_2)\left(\frac{x_1+x_2}{2}-\left(M+\frac{3}{2}\right)\right)$$

Let see what happens in the location subgame.

---

7 If this condition is not satisfied, Meagher and Zauner (2004) demonstrate that one firm has an incentive to move closer to its competitor until this condition is satisfied.

8 Another implication of the size of the support of the uncertainty is that $\sigma^2 \leq 1/4$.

3.2 Equilibrium locations when first entrant faces demand uncertainty

We turn now to the resolution of the location subgame. As firm 2 locates with perfect information about demand location, its optimal location doesn't change compared to the benchmark case, i.e. $x_2^* = \frac{1}{3}x_1 + 1 + \frac{2}{3}M$. We solve now the optimal location for firm 1, given firm 2's optimal location. Firm 1's optimal location will be that it maximizes its expected profit:

$$\begin{cases} Max\ E(\pi_1^*) \\ with\ x_2^* = \frac{1}{3}x_1 + 1 + \frac{2}{3}M \end{cases}$$

That becomes:

$$\begin{aligned} E(\pi_1^*) &= \int_{\underline{M}}^{\overline{M}} -\frac{32\left(-x_1+M-3\right)^2\left(-x_1+\frac{3}{2}+M\right)}{486} f(M)dM \qquad (10) \\ &= \frac{32}{486}\left[\left(x_1\left(-x_1^2-\frac{9}{2}x_1\right)+\frac{27}{2}\right)\int_{\underline{M}}^{\overline{M}} f\left(M\right)dM + 3x_1\left(x_1+3\right)\int_{\underline{M}}^{\overline{M}} Mf\left(M\right)dM \right. \\ &\quad \left. +\left(-3x_1-\frac{9}{2}+M\right)\int_{\underline{M}}^{\overline{M}} M^2 f\left(M\right)dM\right] \\ &= \frac{16}{243}\sigma^2\left(-3x_1-\frac{9}{2}+M\right)+\frac{8}{9}+\frac{16}{243}x_1\left(-x_1^2-\frac{9}{2}x_1\right) \end{aligned}$$

where we use:

$\int_{\underline{M}}^{\overline{M}} f(M)dM = 1; \int_{\underline{M}}^{\overline{M}} Mf(M)dM = E(M) = \mu = 0; \int_{\underline{M}}^{\overline{M}} M^2 f(M)dM = \mu^2 + \sigma^2 = \sigma^2$

The expected profit function is used to determine Nash equilibrium by solving the first order condition: $\partial E(\pi_1^*)/\partial x_1 = 0$, then we check second order condition. It appears that $x_1^* = -\frac{3}{2} + \frac{1}{2}\sqrt{9-4\sigma^2}$ is the sole optimal location for firm 1. It results the following proposition:

Proposition 2 ***(Equilibrium Locations when Firm 1 Faces Demand Uncertainty)*** *The duopoly location-then-price game when firm 1 faces demand uncertainty has:*

- *the following equilibrium locations*

$$x_1^* = -\frac{3}{2} + \frac{1}{2}\sqrt{9-4\sigma^2} \text{ and } x_2^* = \frac{1}{2} + \frac{1}{6}\sqrt{9-4\sigma^2} + \frac{2}{3}M \qquad (11)$$

- *with associated equilibrium prices*

$$p_1^*\left(x_1^*, x_2^*\right) = -\frac{2}{27}\left(6-\sqrt{9-4\sigma^2}+2M\right)\left(-3-\sqrt{9-4\sigma^2}+2M\right)$$

$$p_2^*\left(x_1^*, x_2^*\right) = \frac{2}{27}\left(6 - \sqrt{9-4\sigma^2} + 2M\right)^2$$

- *the following equilibrium profits*

$$\pi_1^* = \frac{2}{243}\left(6M - 4\sqrt{9-4\sigma^2}M + 4M^2 - 9 - 3\sqrt{9-4\sigma^2} - 4\sigma^2\right)\left(-3 - \sqrt{9-4\sigma^2} + 2M\right)$$

$$\pi_2^* = \frac{2}{243}\left(45 - 12\sqrt{9-4\sigma^2} + 24M - 4\sigma^2 - 4\sqrt{9-4\sigma^2}M + 4M^2\right)\left(6 - \sqrt{9-4\sigma^2} + 2M\right)$$

- *and the equilibrium differentiation* Δ^e *is given by* $x_2^* - x_1^* = 2 - \frac{1}{3}\sqrt{9-4\sigma^2} + \frac{2}{3}M$.

PROOF. See Appendix C

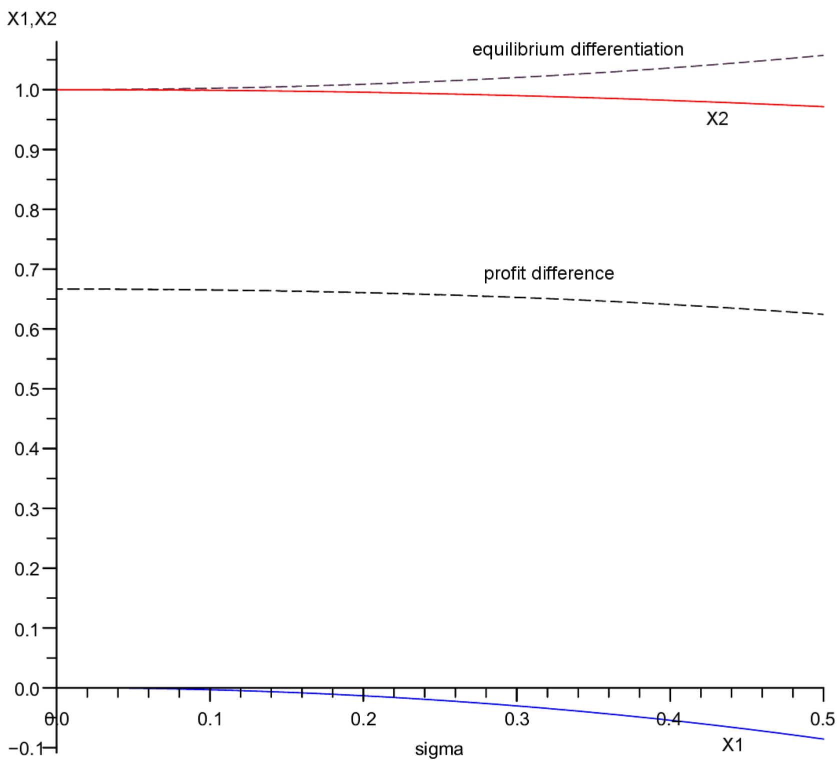

Fig. 1. Equilibrium locations, profit difference and equilibrium differentiation as a function of the standard deviation of the uncertainty σ $(M = 0)$

Figure 1 plots the equilibrium locations, the equilibrium differentiation and the equilibrium profits difference $(\pi_1^* - \pi_2^*)$ as a function of the standard deviation of the uncertainty [9]. Compared to the situation with demand certainty

[9] As expected, when the uncertainty is null $(\sigma = 0)$ firm 1 locates at μ, whatever the value of μ. When $\mu = 0$, we drop down to the previous results.

where firm 1 locates at the center of the market, we note here that uncertainty leads it to move farther away to the center. Conversely, with the increase of uncertainty, firm 2 locates closer to the center, but at a lower rate. These moves are described by the curves representing equilibrium locations and the equilibrium differentiation. However firm 1's decision is taken by anticipating that firm 2 locates with perfect information about consumers' location. With perfect information, firm 1's optimal decision is not to be a long way from the market center but uncertainty acts *slightly* as a *differentiation force*[10]. In fact, firm 1's strategy represents a consequence of the trade-off between increasing the degree of differentiation in order to reduce price competition or decreasing this latter in order to obtain a higher market share. When uncertainty occurs, the rise of the degree of differentiation has a positive effect of reducing price competition which is more pronounced than the negative effect of losing market share. Firm 1 is induced to preserve a part of its hinterland. This result leads to soften price competition and it induces firms to set up higher prices than in the certainty case, which have a positive effect on firms' profit. However, the rise of uncertainty is not in favor of firm 1. The higher the uncertainty, the higher reduced is its profit, compared to the certainty case. Yet this fact is still at a slight level. The benefit on profit is solely for firm 2.

Now, if we compare the equilibrium differentiation in case of demand certainty and when firm 1 entertains demand uncertainty, we observe that the differentiation is higher in the last case. And the gap between these two situations deepens when firm 1's demand uncertainty increases. We drop down to the result of Meagher and Zauner (2004, 2005) exposed in the context of uncertainty for both firms and simultaneous location, i.e. *"in equilibrium, increases in the variance of the uncertainty lead to higher expected equilibrium prices, higher differentiation, and higher profits"*. When uncertainty demand, even *small*, occurs just for the first entrant, this appears as a *differentiation force*. But our outcome is less pronounced than when both firms face uncertainty like in Meagher and Zauner (2004, 2005), Casado-Izaga (2000) or Harter (1996)[11].

3.3 Equilibrium locations when second entrant faces demand uncertainty

Now we reverse the situation and turn to the case where firm 1 is perfectly informed about demand location and firm 2 entertains demand uncertainty[12].

[10] It is noteworthy that whatever the realization of M, the differentiation force remains the same one.

[11] The hypothesis of one-sided demand uncertainty is important since the resolution of the sequential game where both firms face demand uncertainty yields equilibrium locations associated to roots of quartic functions.

[12] Imperfect information can also refer to firm product costs which implies the same type of behaviors for the follower (see for example Boyer, Mahenc, and Moreaux,

Since demand is revealed before the price subgame, the two firms cover the market. Firm 2's optimal location will be that which maximizes its expected profit:

$$E(\pi_2^*) = \int_{\underline{M}}^{\overline{M}} \frac{2}{9(x_2 - x_1)} \left(\left(\frac{1}{2}x_1 - M - \frac{3}{2} + \frac{1}{2}x_2 \right)(x_2 - x_1) \right)^2 f(M)dM$$
$$= -\frac{2}{9(x_1 - x_2)} \left(\frac{1}{4}x_1^2 + \left(-\frac{3}{2} + \frac{1}{2}x_2 \right) x_1 + \sigma^2 - \frac{3}{2}x_2 + \frac{9}{4} + \frac{1}{4}x_2^2 \right) (x_1 - x_2)^2$$

with: $\int_{\underline{M}}^{\overline{M}} f(M)dM = 1$; $\int_{\underline{M}}^{\overline{M}} Mf(M)dM = E(M) = \mu = 0$; $\int_{\underline{M}}^{\overline{M}} M^2 f(M)dM = \mu^2 + \sigma^2 = \sigma^2$

The expected profit function is used to determine Nash equilibrium by solving the first order condition: $\partial E(\pi_2^*)/\partial x_2 = 0$, then we check second order condition. It appears that $x_2^* = -\frac{1}{3}x_1 + 2 - \frac{1}{3}\sqrt{4x_1^2 - 12x_1 + 9 - 12\sigma^2}$ is the optimal location for firm 2.

We solve now the optimal location for firm 1, given firm 2's optimal location, and knowing that firm 1 has perfect information about the demand location. It results the following proposition:

Proposition 3 ***(Equilibrium Locations when Firm*** **2** ***Faces Demand Uncertainty)*** *In the duopoly location-then-price game when firm* 2 *faces demand uncertainty:*

- *the equilibrium locations are* [13]

$$x_1^* = \frac{1}{6}\frac{1}{M-3}((A + 12\sigma M^3\sqrt{B} - 108\sigma M^2\sqrt{B} + 324\sigma M\sqrt{B} + 324\sigma\sqrt{B})^{1/3})$$
$$+\frac{1}{6}\left(12M^3 + 54M^2 + 81 - 132M\sigma^2 + 198\sigma^2 + 22M^2\sigma^2 - 108M + \sigma^4 + M^4\right)$$
$$/((M-3)(A + 12\sigma M^3\sqrt{B} - 108\sigma M^2\sqrt{B} + 324\sigma M\sqrt{B} + 324\sigma\sqrt{B})^{1/3})$$
$$+\frac{1}{6}\frac{-18 - \sigma^2 + 3M + M^2}{M-3}$$

and

$$x_2^* = -\frac{1}{3}x_1^* + 2 - \frac{1}{3}\sqrt{4(x_1^*)^2 - 12x_1^* + 9 - 12\sigma^2}$$

So both firms go farther away to the center with the increase of uncertainty.

2003).

[13] With $A = -1458M + 19764M\sigma^2 + 135M^4 - 540M^3 - 14823\sigma^2 - 297\sigma^4 - \sigma^6 + M^6 + 729 - 9882M^2\sigma^2 - 18M^5 - 33M^2\sigma^4 + 198M\sigma^4 + 2196M^3\sigma^2 - 183M^4\sigma^2 + 1215M^2$ and $B = -3M^4 + 36M^3 + 222M^2\sigma^2 - 162M^2 - 1332M\sigma^2 + 324M + 9\sigma^4 + 1998\sigma^2 - 243$.

- *firm 1 adopts an equilibrium price slightly higher than in the certainty case. Conversely firm 2 decreases drastically its equilibrium price, the decrease being more pronounced with the rise of uncertainty.*
- *the equilibrium differentiation Δ^e given by $x_2^* - x_1^*$ decreases with the rise of uncertainty.*

PROOF. See Appendix D

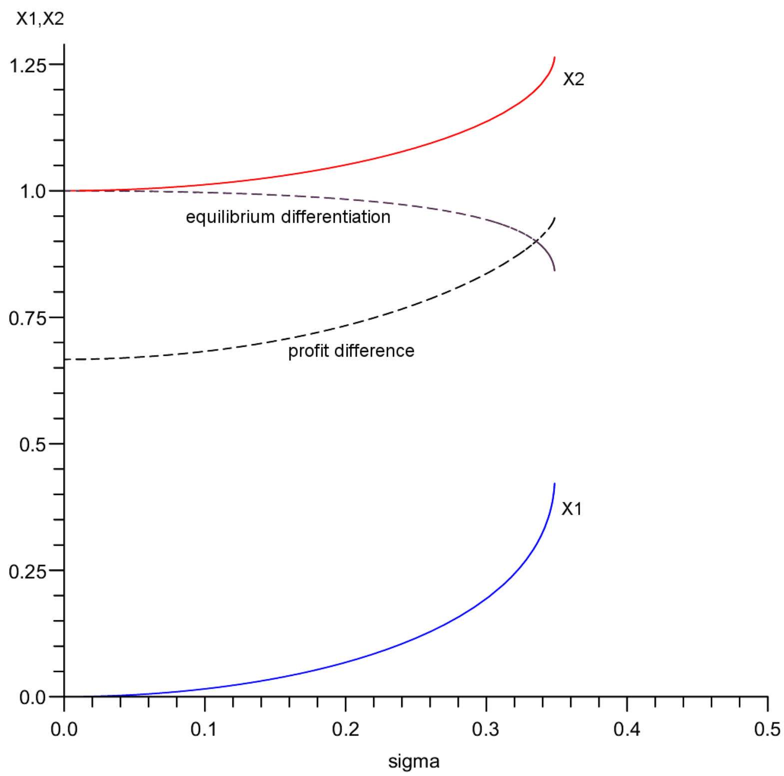

Fig. 2. Equilibrium locations, equilibrium profit difference and equilibrium differentiation as a function of the standard deviation of the uncertainty σ $(M = 0)$

Expressions of equilibrium prices, profits and equilibrium differentiation are unwieldy and cannot be expressed shortly in the proposition. Figure 2 will help us to comment our results [14] . In our model we assume that, at the beginning of the game, firm 1 knows the demand location - or similarly consumer's preferences - and firm 2 ignores it. Being uncertain about the location of consumers' ideal point, firm 2 can act as a follower in its choice of location in order to infer some information about demand from the observation of the leader's location choice. Yet, firm 1 anticipates firm 2's optimal strategy. Moreover, firm 1 possesess a double advantage: The first mover advantage and perfect information about location demand. This will allow it to transfer more

[14] Their analytical expressions are available upon request.

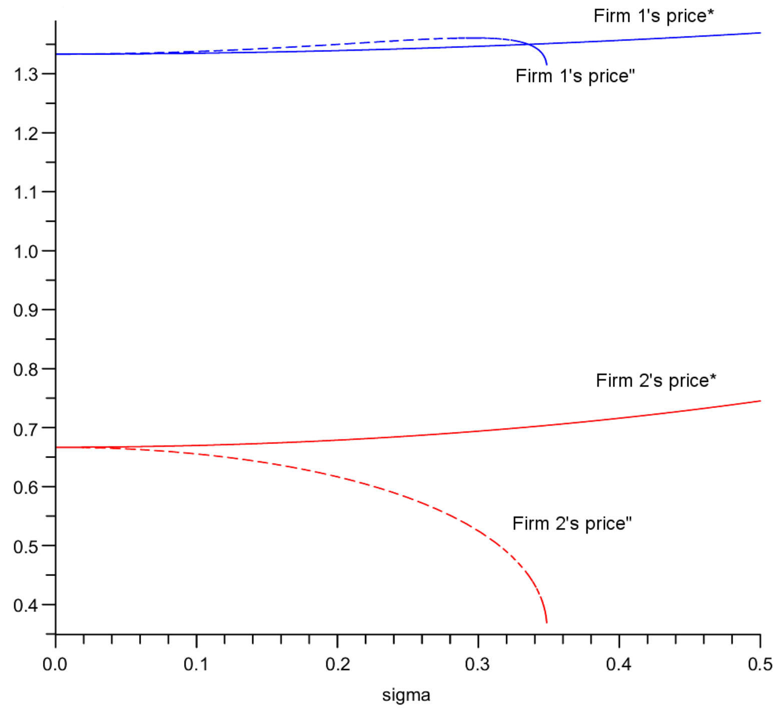

Fig. 3. Equilibrium prices in both cases of uncertainty as a function of the standard deviation of the uncertainty σ ($M = 0$). Note (*) refers to the case where firm 1 faces demand uncertainty and (") when it is firm 2.

intensively the differentiation cost to firm 2. With the increase of uncertainty, firm 1 chooses to locate farther away to the market center, in the aim to induce firm 2 to locate farther away to the demand too, but at a lower rate. These moves lead to a decrease of equilibrium differentiation and consequently firms are engaged in fierce price competition. But due to its first mover advantage which provides its a higher market share, firm 1 is not forced to cut down its price. However, in the price subgame, firm 2 accounts that its equilibrium location is not optimal with respect to the market center and it must cut down its price in order to not lose too much consumers (see figure 3). The advantage of the first mover associated to perfect information about demand location leads to an increase in the profit equilibrium difference $(\pi_1^* - \pi_2^*)$ with the increase of demand uncertainty. The rise of the profit difference is explained at the same time by the increase of firm 1's profit and the decrease of firm 2's profit.

A particularity of the equilibrium locations should be underlined. Depart from a level of uncertainty $\sigma > 0.35$, firm 1's optimal strategy is a complex solution, which leads to an absence of a real equilibrium locations.

Now, if we compare the equilibrium differentiation with demand certainty and when firm 2 entertains demand uncertainty, we observe that the differentiation is lower in the last case. The increase of uncertainty for the second entrant leads to reduce the equilibrium differentiation[15]. This result is opposite to the conclusion of the previous section and Meagher and Zauner (2004, 2005) or Casado-Izaga (2000). Nonetheless, our conclusion is similar to those of de Palma, Ginsburgh, Papageorgiou, and Thisse (1985) who introduce uncertainty at the individual level (Random Utility Model). In our context, one-sided demand uncertainty - for the second entrant - can be seen as an *agglomeration force*.

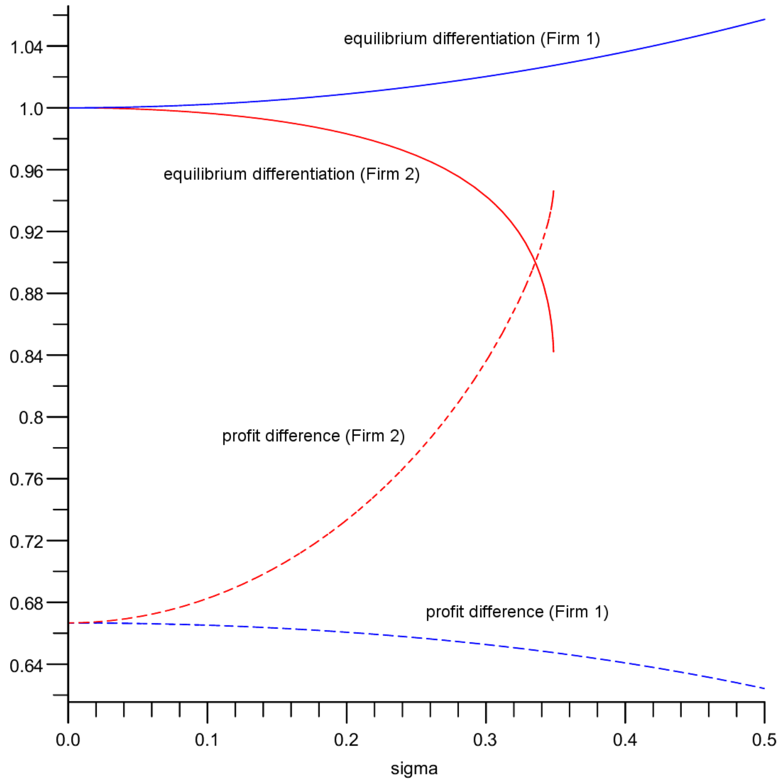

Fig. 4. Equilibrium differentiation and profit difference in both cases as a function of the standard deviation of the uncertainty σ ($M = 0$). Note (Firm 1) refers to the case where firm 1 faces demand uncertainty and (Firm 2) when it is firm 2.

Figure 4 summarizes the outcomes of one-sided demand uncertainty according to the firm imperfectly informed.

3.4 Welfare analysis

Here we are going to study the welfare properties of equilibria. The social planner is supposed to be not better informed than the firms and therefore takes its decisions before the realization of uncertainty. Then it sets prices such

[15] As in the previous case, this result holds whatever the realization of $M \in [-1/2, 1/2]$.

as consumers patronize the nearest firm. Consequently, excluding the problem of covering the market, the objective of a social planner who manages identical firms is to minimize the expected aggregate transportation costs T^u.

$$\min_{(x_1,x_2)} T^u(x_1,x_2) = \min_{(x_1,x_2)} \int_{\underline{M}}^{\overline{M}} \left[\int_{M-1/2}^{\nu} (z-x_1)^2 \, dz + \int_{\nu}^{M+1/2} (x_2-z)^2 \, dz \right] f(M) \, dM$$

where $\nu = (x_1 + x_2)/2$.

The resolution of this problem gives us the socially optimal locations expressed in the following proposition:

Proposition 4 ***(Socially Optimal Locations under Uncertainty)*** *The duopoly location-then-price game under demand uncertainty has the following optimal locations*

$$x_1^{socu} = -\frac{1}{4} - \sigma^2 \text{ and } x_2^{socu} = \frac{1}{4} + \sigma^2$$

The socially optimal differentiation is $\Delta^{socu} = 1/2 + 2\sigma^2$.

PROOF. See Appendix E

Figure 5 plots the equilibrium differentiation in both cases of uncertainty and socially optimal differentiation as a function of the standard deviation of demand uncertainty. When the uncertainty tends to zero, we go back to the results with demand certainty and we observe excessive differentiation ($= 1/2$) compared to the social optimum. However, the increase of uncertainty diminishes the gap between socially optimal differentiation and equilibrium differentiation. This is more true when it is firm 2 which faces demand uncertainty, as we have seen in previous section. Uncertainty tends to reduce the excessive amount of differentiation. When firm 2 faces demand uncertainty, *agglomeration force* pushes firm 1 to come nearer to its competitor and equilibrium differentiation is close to the socially optimal differentiation with high uncertainty. The same conclusion occurs when firm 1 faces demand uncertainty; Nevertheless the diminishing of this gap is due to the increase of the socially optimal differentiation when uncertainty increases rather than the reduction of equilibrium one[16].

However, it will be erroneous to consider that the situation where firm 2 faces demand uncertainty is socially better than when it is firm 1. A comparative

[16] Note that all these results are for a given range of uncertainty $\sigma^2 \leq 1/4$.

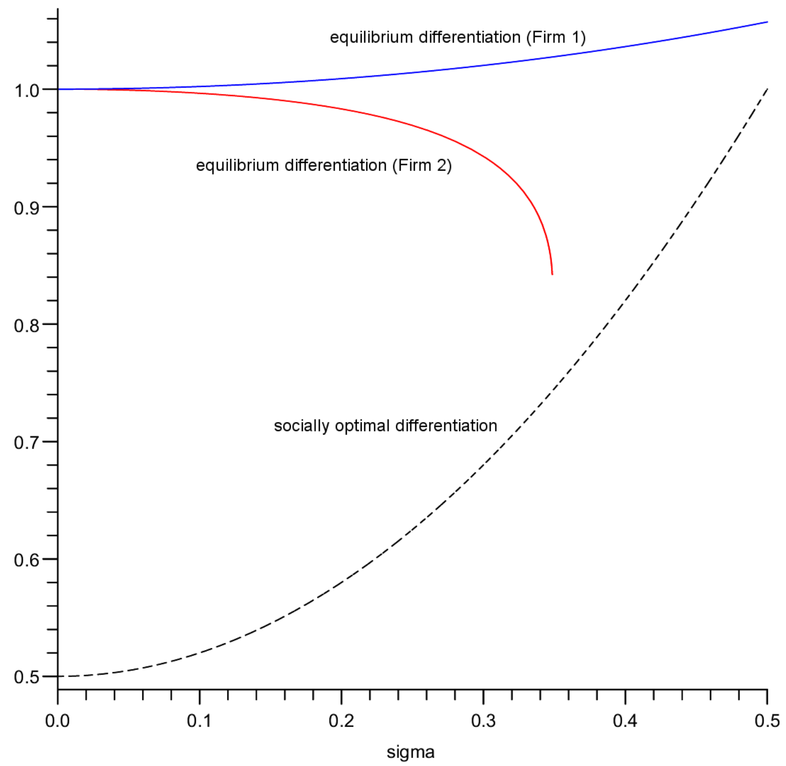

Fig. 5. Socially optimal differentiation and equilibrium differentiation for both cases of uncertainty as a function of the standard deviation of the uncertainty σ ($M = 0$). Note (Firm 1) refers to the case where firm 1 faces demand uncertainty and (Firm 2) when it is firm 2.

study of transportation costs at a given level of uncertainty in these two cases could highlight this fact. These calculus allow us to specify the welfare losses as the difference between the expected transportation costs for the equilibrium locations $T^u_{Nash}(x_1^*, x_2^*)$ and the expected transportation costs for the social optimum $T^{socu}(x_1^{socu}, x_2^{socu})$. The following proposition summarizes the results:

Proposition 5 ***(Welfare Losses under Demand Uncertainty in Both Cases)*** *Under demand uncertainty*

- *the expected transportation costs for the social optimum are*

$$T^{socu}(x_1^{socu}, x_2^{socu}) = \frac{1}{48} + \frac{1}{2}\sigma^2 - \sigma^4$$

- *the expected transportation costs for the equilibrium locations when firm* 1 *faces uncertainty are*

$$T^u_{Nash/Firm1}(x_1^*, x_2^*) = \frac{2}{27}\sigma^2 + \frac{25}{243}\sigma^2\sqrt{9-4\sigma^2} + \frac{5}{36} + \frac{1}{54}\sqrt{9-4\sigma^2}$$

- *the expected transportation costs for the equilibrium locations when firm* 2 *faces uncertainty are*

$$T^{u}_{Nash/Firm2}\left(x_1^*, x_2^*\right) = unwieldy$$

- *the expected welfare losses when firm* 1 *faces uncertainty are*

$$\begin{aligned} EWL_{Firm1} &= T^{u}_{Nash/Firm1}\left(x_1^*, x_2^*\right) - T^{socu}\left(x_1^{socu}, x_2^{socu}\right) \\ &= -\frac{23}{54}\sigma^2 + \frac{25}{243}\sigma^2\sqrt{9-4\sigma^2} + \frac{17}{144} + \frac{1}{54}\sqrt{9-4\sigma^2} + \sigma^4 \end{aligned}$$

- *the expected welfare losses when firm* 2 *faces uncertainty are*

$$\begin{aligned} EWL_{Firm2} &= T^{u}_{Nash/Firm2}\left(x_1^*, x_2^*\right) - T^{socu}\left(x_1^{socu}, x_2^{socu}\right) \\ &= unwieldy \end{aligned}$$

PROOF. See Appendix F

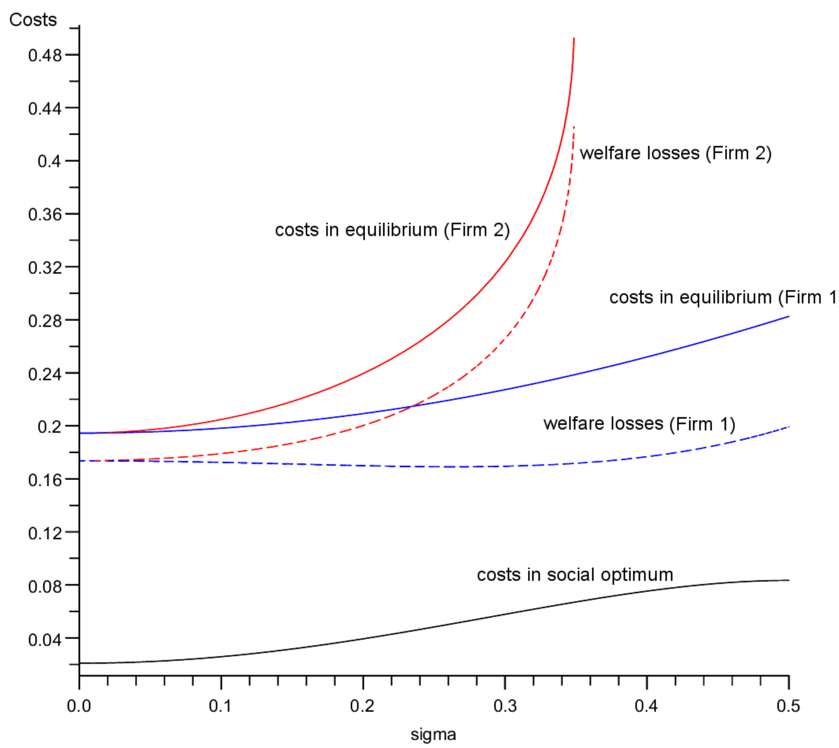

Fig. 6. Expected transportation costs in equilibriums, in the social optimum and welfare losses as a function of the standard deviation σ ($M = 0$). Note (Firm 1) refers to the case where firm 1 faces demand uncertainty and (Firm 2) when it is firm 2.

Figure 6 plots expected transportation costs for the social optimum, equilibria locations and welfare losses when firm 1 faces demand uncertainty (Firm 1) and when it is the firm 2 (Firm 2) as a function of the standard deviation of the uncertainty. As well as expected, transportation costs for competitive equilibria and social optimum one rise with demand uncertainty. Uncertainty is a bad thing for welfare considerations. Nonetheless, the case where firm 2 is the sole to encounter demand uncertainty is the worst from a social point of view. And the welfare losses rise at an exponential rate with uncertainty. In this case, the probability for a consumer to be a long way from a firm increases. Conversely, increasing demand uncertainty has few impacts on the welfare losses when it is firm 1 which faces demand uncertainty. Only a slight increase is noted depart from a high level of uncertainty ($\sigma \approx 0.43$).

4 Conclusion

By entering new market, firms face uncertainty about the nature of the demand. In this paper, we analyse the location of firms in such context. We depart from a simple model where firms locate sequentially and market conditions are common knowledge. Moreover, firms can locate outside the city $\left[M - \frac{1}{2}, M + \frac{1}{2}\right]$. In this situation, the first entrant (firm 1) locates at the market center to maximize its profit and the second entrant (firm 2) moves away to the center to soften price competition. As demonstrated in previous studies, this case yields excessive differentiation compared to the social optimum. Afterwards, we introduce one-sided demand uncertainty in our benchmark model.

In a first step, we assume that only firm 1 faces demand uncertainty whereas firm 2, after the observation of the pioneering firm's location, enters the market with private information about market conditions. In this stackelberg game, the advantage of the first entrant diminishes due to its imperfect information; the demand uncertainty leading it to go farther away to the center of the demand distribution, and to a decrease of its profits. Conversely, firm 2 which has perfect information locates closer to the center and it obtains a higher profit with the increase of uncertainty. Since with the rise of uncertainty, firm 1 moves farther away to the center more quickly than firm 2 goes closer to the center, equilibrium differentiation increases. Demand uncertainty can be seen as a *differentiation force* when only the first entrant faces demand uncertainty. But we have to keep in mind that these outcomes occur at a slight level and for a small range of uncertainty.

In a second step, we assume that only firm 2 faces demand uncertainty whereas firm 1 has private information. In this situation, firm 2 acts as a follower by trying to infer some information about demand location through the observation of the leader's one. But anticipating this behavior, firm 1 goes farther away to the center with the increase of uncertainty, inducing firm 2 to locate farther away to the demand too. Contrary to the previous case, firm 1 moves farther away to the center more quickly than firm 2 – in order to transfer more intensively the differentiation cost – and thus equilibrium differentiation decreases. Consequently, demand uncertainty when only the second entrant is concerned can be seen as an *agglomeration force*. Such strategy is robust with Random Utility Model. In this situation, the advantage of the first mover increases since it sets a similar price and obtains higher profits than in the case of demand certainty.

Finally, the impact of demand uncertainty on welfare is on three kinds. Firstly, the increase of uncertainty reduces the gap between socially optimal differentiation and equilibrium differentiation, whatever the firm who faces demand uncertainty. Secondly, the aim of the social planner being the minimisation of transportation costs, demand uncertainty leads to higher costs - and so to higher welfare losses - than the situation where both firms have perfect information about market conditions. Thirdly, the worst situation for welfare considerations is encounter when firm 2 faces demand uncertainty.

It is noteworthy that these results highlight the role played by a social planner or a regulation agency who publicizes information when it faces itself demand uncertainty. Otherwise, when it is perfectly informed about demand location it appears that a strict control of locations is preferable[17].

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[17] As example, supermarkets location is strictly controlled in several European countries, like in England or France.

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Appendix

A Proof of Lemma 1

In order to determine the equilibrium prices in this framework, we first calculate each firm's best response to its competitor's price. Substituting the demand function z in the profits function, we can write the first order conditions

$$\partial\pi_1/\partial p_1 = 0 \text{ and } \partial\pi_2/\partial p_2 = 0$$

so

$$p_1(p_2) = -\frac{1}{2}t(x_1 - x_2)(x_1 - 2M + 1 + x_2) + \frac{1}{2}p_2 \tag{A.1}$$

$$p_2(p_1) = -\frac{1}{2}t(-x_1 + x_2)(x_1 - 2M - 1 + x_2) + \frac{1}{2}p_1 \tag{A.2}$$

By equalizing these best response functions we find

$$p_1^*(x_1, x_2) = -\frac{2}{3}t(x_1 - x_2)\left(\frac{x_1 + x_2}{2} - \left(M - \frac{3}{2}\right)\right) \tag{A.3}$$

$$p_2^*(x_1, x_2) = \frac{2}{3}t(x_1 - x_2)\left(\frac{x_1 + x_2}{2} - \left(M + \frac{3}{2}\right)\right)$$

If we look now to the corner solution, from (3) we see that firm 2 has zero demand ($z = M + 1/2$) if it charges a price $p_2 \geq p_1 + t(x_1^2 - x_2^2)$, that is to say if its price is higher than that to its competitor plus the transportation costs associated to the distance between firms. In this case, its maximum profit is equal to zero and the price associated to this strategy is $p_2^*(x_1, x_2) = 0$. Then the optimal price for firm 2 is

$$p_2^*(x_1, x_2) \leq 0 \Leftrightarrow \frac{2}{3}t(x_1 - x_2)\left(\frac{x_1 + x_2}{2} - \left(M + \frac{3}{2}\right)\right) \leq 0 \tag{A.4}$$

which implies

$$M \leq \frac{x_1 + x_2}{2} - \frac{3}{2}$$

Given $p_2^*(x_1, x_2) = 0$, firm 1 seeks to maximize its profits. From (A.2), we obtain

$$p_2(p_1) = 0 \Leftrightarrow p_1^* = t(-x_1 + x_2)(x_1 - 2M - 1 + x_2) \tag{A.5}$$

Conversely, firm 1 has zero demand ($z = M - 1/2$) if firm 2 charges a price $p_2 \leq p_1 + t(x_1^2 - x_2^2) + 2t(x_2 - x_1)$. In this case, its maximum profit is equal to zero and the price associated to this strategy is $p_1^*(x_1, x_2) = 0$. Then the optimal price for firm 1 is

$$p_1^*(x_1, x_2) \leq 0 \Leftrightarrow -\frac{2}{3}t(x_1 - x_2)\left(\frac{x_1 + x_2}{2} - \left(M - \frac{3}{2}\right)\right) \leq 0 \qquad \text{(A.6)}$$

which implies

$$M \geq \frac{x_1 + x_2}{2} + \frac{3}{2}$$

Given $p_1^*(x_1, x_2) = 0$, firm 2 seeks to maximize its profits. From (A.1), we obtain

$$p_1(p_2) = 0 \Leftrightarrow p_2^* = t(x_1 - x_2)(x_1 - 2M + 1 + x_2) \qquad \text{(A.7)}$$

B Proof of Proposition 1

The first order condition for firm 2 to maximize its profit ($\partial\pi_2/\partial x_2 = 0$) gives us two solutions:

$$x_2^1 = \frac{1}{3}x_1 + 1 + \frac{2}{3}M \text{ and } x_2^2 = -x_1 + 3 + 2M \qquad \text{(B.1)}$$

Only x_2^1 satisfies second order condition. Substituting x_2 by x_2^1 in (A.3) and (5) allows us to determine firm 1's profit for firm 2's optimal location. For that, we calculate its location which maximizes its profit i.e. $\partial\pi_1^*/\partial x_1 = 0$. It results that for $x_2 = x_2^1$, two solutions appear for firm 1's optimal location:

$$x_1^1 = M \text{ and } x_1^2 = M - 3 \qquad \text{(B.2)}$$

Nonetheless, only x_1^1 satisfies second order condition. x_1^2 as a possible optimal location is avoided. By substituting the optimal location for firm 1 into the optimal location of firm 2, it results that:

$$x_1^* = M \text{ and } x_2^* = M + 1 \qquad \text{(B.3)}$$

The substitution of locations by the optimal locations $\{x_1^*, x_2^*\}$ in (4) and (5) allows us to obtain the following equilibrium prices

$$p_1^* = \frac{4}{3}t \text{ and } p_2^* = \frac{2}{3}t \qquad \text{(B.4)}$$

and the following associated profits:

$$\pi_1^* = \frac{8}{9}t \text{ and } \pi_2^* = \frac{2}{9}t \qquad \text{(B.5)}$$

C Proof of Proposition 2

Firm 1 is risk neutral. Its price equilibrium is given by:

$$p_1^*(x_1, x_2) = -\frac{2}{3}(x_1 - x_2)\left(\frac{x_1 + x_2}{2} - \left(M - \frac{3}{2}\right)\right) \tag{C.1}$$

and its expected profit is given by:

$$E(\pi_1^*) = \int_{\underline{M}}^{\overline{M}} p_1^*(x_1, x_2)(z - (M - \frac{1}{2}))f(M)dM \tag{C.2}$$

$$\begin{aligned}
E(\pi_1^*) &= \int_{\underline{M}}^{\overline{M}} -\frac{32(-x_1 + M - 3)^2\left(-x_1 + \frac{3}{2} + M\right)}{486} f(M)dM \qquad \text{(C.3)}\\
&= \frac{32}{486}\left[\left(x_1\left(-x_1^2 - \frac{9}{2}x_1\right) + \frac{27}{2}\right)\int_{\underline{M}}^{\overline{M}} f(M)\,dM + 3x_1(x_1 + 3)\int_{\underline{M}}^{\overline{M}} Mf(M)\,dM\right.\\
&\quad \left. + \left(-3x_1 - \frac{9}{2} + M\right)\int_{\underline{M}}^{\overline{M}} M^2 f(M)\,dM\right]\\
&= \frac{16}{243}\sigma^2\left(-3x_1 - \frac{9}{2} + M\right) + \frac{8}{9} + \frac{16}{243}x_1\left(-x_1^2 - \frac{9}{2}x_1\right)
\end{aligned}$$

where we use:

$\int_{\underline{M}}^{\overline{M}} f(M)dM = 1$; $\int_{\underline{M}}^{\overline{M}} Mf(M)dM = E(M) = \mu = 0$; $\int_{\underline{M}}^{\overline{M}} M^2 f(M)dM = \mu^2 + \sigma^2 = \sigma^2$

Firm 1 seeks to maximize its expected profit. The resolution of the first order condition ($\partial E(\pi_1^*)/\partial x_1 = 0$) gives us two solutions:

$$x_1^1 = -\frac{3}{2} + \frac{1}{2}\sqrt{9 - 4\sigma^2} \tag{C.4}$$

$$x_1^2 = -\frac{3}{2} - \frac{1}{2}\sqrt{9 - 4\sigma^2} \tag{C.5}$$

But only x_1^1 satisfies second order condition. It results that the optimal location for firm 1 is x_1^1.

D Proof of Proposition 3

Firm 2 is risk neutral. Its price equilibrium is given by:

$$p_2^*(x_1, x_2) = \frac{2}{3}(x_1 - x_2)(\frac{x_1 + x_2}{2} - \left(M + \frac{3}{2}\right)) \tag{D.1}$$

and its expected profit is given by:

$$E(\pi_2^*) = \int_{\underline{M}}^{\overline{M}} p_2^*(x_1, x_2)\left(\left(M + \frac{1}{2}\right) - z\right) f(M) dM \tag{D.2}$$

$$\begin{aligned} E(\pi_2^*) &= \int_{\underline{M}}^{\overline{M}} \frac{2}{9(x_2 - x_1)} \left(\left(\frac{1}{2}x_1 - M - \frac{3}{2} + \frac{1}{2}x_2\right)(x_2 - x_1)\right)^2 f(M) dM \qquad \text{(D.3)} \\ &= -\frac{2}{9(x_1 - x_2)} \left(\frac{1}{4}x_1^2 + \left(-\frac{3}{2} + \frac{1}{2}x_2\right)x_1 + \sigma^2 - \frac{3}{2}x_2 + \frac{9}{4} + \frac{1}{4}x_2^2\right)(x_1 - x_2)^2 \end{aligned}$$

Firm 2 seeks to maximize its expected profit. The resolution of the first order condition ($\partial E(\pi_2^*)/\partial x_2 = 0$) gives us two solutions:

$$x_2^1 = -\frac{1}{3}x_1 + 2 + \frac{1}{3}\sqrt{4x_1^2 - 12x_1 + 9 - 12\sigma^2} \tag{D.4}$$

$$x_2^2 = -\frac{1}{3}x_1 + 2 - \frac{1}{3}\sqrt{4x_1^2 - 12x_1 + 9 - 12\sigma^2} \tag{D.5}$$

But only x_2^2 satisfies second order condition. It results that the optimal location for the firm 2 is x_2^2:

$$x_2^* = -\frac{1}{3}x_1^* + 2 - \frac{1}{3}\sqrt{4(x_1^*)^2 - 12x_1^* + 9 - 12\sigma^2} \tag{D.6}$$

We need now to calculate the optimal location for firm 1, given firm 2's optimal location. For that, we substitute firm 2's optimal location into the optimal location of the marginal consumer (z^*) and into firm 1's price equilibrium. Firm 1's price equilibrium becomes

$$\begin{aligned} p_1^*\left(x_1, x_2^2\right) = &-\frac{2}{3}\left(\frac{4}{3}x_1 - 2 + \frac{1}{3}\sqrt{4x_1^2 - 12x_1 + 9 - 12\sigma^2}\right) \\ &\times \left(\frac{1}{3}x_1 - M + \frac{5}{2} - \frac{1}{6}\sqrt{4x_1^2 - 12x_1 + 9 - 12\sigma^2}\right) \end{aligned}$$

Then we obtain firm 1's profit

$$\pi_1(x_1, x_2^2) = p_1^*(x_1, x_2^2)\left(z^* - \left(M - \frac{1}{2}\right)\right) \tag{D.7}$$

Firm 1 maximizes its profits by resolving the first order condition: $\partial \pi_1(x_1, x_2^2)/\partial x_1 = 0$, which gives us two real solutions:

$$x_1^1 = \frac{1}{2}\frac{18 + \sigma^2 + 3M^2 - 15M}{M - 3} \tag{D.8}$$

$$
\begin{aligned}
x_1^2 = & \frac{1}{6}\frac{1}{M-3}((A+12\sigma M^3\sqrt{B}-108\sigma M^2\sqrt{B}+324\sigma M\sqrt{B}+324\sigma\sqrt{B})^{1/3}) \\
& +\frac{1}{6}\left(12M^3+54M^2+81-132M\sigma^2+198\sigma^2+22M^2\sigma^2-108M+\sigma^4+M^4\right)/((M-3) \\
& (A+12\sigma M^3\sqrt{B}-108\sigma M^2\sqrt{B}+324\sigma M\sqrt{B}+324\sigma\sqrt{B})^{1/3})+\frac{1}{6}\frac{-18-\sigma^2+3M+M^2}{M-3}
\end{aligned}
$$

with

$$
\begin{aligned}
A = & -1458M+19764M\sigma^2+135M^4-540M^3-14823\sigma^2-297\sigma^4-\sigma^6+M^6 \\
& +729-9882M^2\sigma^2-18M^5-33M^2\sigma^4+198M\sigma^4+2196M^3\sigma^2-183M^4\sigma^2+1215M^2
\end{aligned}
$$

and

$B = -3M^4+36M^3+222M^2\sigma^2-162M^2-1332M\sigma^2+324M+9\sigma^4+1998\sigma^2-243$

But only x_1^2 satisfies second order condition. It results that the optimal location for the firm 1 is x_1^2.

E Proof of Proposition 4

With $x_1 < x_2$ and $[\underline{M}, \overline{M}] \subset \left[-\frac{1}{2}, \frac{1}{2}\right]$, the first order condition for x_1 is

$$
\begin{aligned}
\partial T^u(x_1,x_2)/\partial x_1 = & \int_{\underline{M}}^{\overline{M}}\left(\int_{M-1/2}^{v} 2(z-x_1)\,dz\right) f(M)\,dM \qquad \text{(E.1)} \\
= & \left(x_1(-2\nu-1)+\nu^2-\frac{1}{4}\right)\int_{\underline{M}}^{\overline{M}} f(M)\,dM + (2x_1+1)\int_{\underline{M}}^{\overline{M}} Mf(M)\,dM \\
& -1\int_{\underline{M}}^{\overline{M}} M^2 f(M)\,dM \\
= & -\sigma^2-\frac{1}{4}-x_1(2\nu+1)+\nu^2
\end{aligned}
$$

where we use:

$\int_{\underline{M}}^{\overline{M}} f(M)dM = 1$; $\int_{\underline{M}}^{\overline{M}} Mf(M)dM = E(M) = \mu = 0$; $\int_{\underline{M}}^{\overline{M}} M^2 f(M)dM = \mu^2 + \sigma^2 = \sigma^2$ and $\nu = (x_1+x_2)/2$.

Similarly, the first order condition for x_2 is

$$\partial T^{u}\left(x_{1},x_{2}\right)/\partial x_{2}=\int_{\underline{M}}^{\overline{M}}\left(\int_{v}^{M+1/2}2\left(x_{2}-z\right)dz\right)f\left(M\right)dM \tag{E.2}$$
$$=-\sigma^{2}-\frac{1}{4}+x_{2}\left(-2\nu+1\right)+\nu^{2}$$

The resolution of the first order conditions ($\partial T^{u}\left(x_{1},x_{2}\right)/\partial x_{1}=0, \partial T^{u}\left(x_{1},x_{2}\right)/\partial x_{2}=0$) gives us three solutions:

$$x_{1}^{1}=-\frac{1}{4}-\sigma^{2} \text{ and } x_{2}^{1}=\frac{1}{4}+\sigma^{2}$$

$$x_{1}^{2}=-\frac{1}{2}+\frac{1}{2}\sqrt{1-4\sigma^{2}} \text{ and } x_{2}^{2}=\frac{1}{2}+\frac{1}{2}\sqrt{1-4\sigma^{2}}$$

$$x_{1}^{3}=-\frac{1}{2}-\frac{1}{2}\sqrt{1-4\sigma^{2}} \text{ and } x_{2}^{3}=\frac{1}{2}-\frac{1}{2}\sqrt{1-4\sigma^{2}}$$

Second order conditions couldn't discriminate between these solutions. We need to calcul expected transportation costs for both firms. For firm 1

$$T_{firm1}^{u}\left(x_{1},x_{2}\right)=\int_{\underline{M}}^{\overline{M}}\left(\int_{M-1/2}^{v}\left(z-x_{1}\right)^{2}dz\right)f\left(M\right)dM \tag{E.3}$$
$$=\int_{\underline{M}}^{\overline{M}}\left(-\frac{1}{3}\left(-x_{1}+M-\frac{1}{2}\right)^{3}+\frac{1}{3}\left(v-x_{1}\right)^{3}\right)f\left(M\right)dM$$
$$=\left(x_{1}\left(\frac{1}{2}x_{1}+\frac{1}{4}+x_{1}\nu-\nu^{2}\right)+\frac{1}{3}\nu^{3}+\frac{1}{24}\right)\int_{\underline{M}}^{\overline{M}}f\left(M\right)dM$$
$$+\left(-x_{1}^{2}-x_{1}-\frac{1}{4}\right)\int_{\underline{M}}^{\overline{M}}Mf\left(M\right)dM+\left(x_{1}-\frac{1}{3}M+\frac{1}{2}\right)\int_{\underline{M}}^{\overline{M}}M^{2}f\left(M\right)dM$$
$$=\sigma^{2}\left(x_{1}-\frac{1}{3}M+\frac{1}{2}\right)+\frac{1}{24}+x_{1}\left(\frac{1}{2}x_{1}+\frac{1}{4}+x_{1}\nu-\nu^{2}\right)+\frac{1}{3}\nu^{3}$$

where we use:

$\int_{\underline{M}}^{\overline{M}}f(M)dM=1; \int_{\underline{M}}^{\overline{M}}Mf(M)dM=E(M)=\mu=0; \int_{\underline{M}}^{\overline{M}}M^{2}f(M)dM=\mu^{2}+\sigma^{2}=\sigma^{2}$.

And for firm 2

$$T_{firm2}^{u}\left(x_{1},x_{2}\right)=\int_{\underline{M}}^{\overline{M}}\left(\int_{v}^{M+1/2}\left(x_{2}-z\right)^{2}dz\right)f\left(M\right)dM \tag{E.4}$$
$$=\sigma^{2}\left(-x_{2}+\frac{1}{3}M+\frac{1}{2}\right)+\frac{1}{24}+x_{2}\left(\frac{1}{2}x_{2}-\frac{1}{4}-x_{2}\nu+\nu^{2}\right)-\frac{1}{3}\nu^{3}$$

where we use:

$\int_{\underline{M}}^{\overline{M}} f(M)dM = 1; \int_{\underline{M}}^{\overline{M}} Mf(M)dM = E(M) = \mu = 0; \int_{\underline{M}}^{\overline{M}} M^2 f(M)dM = \mu^2 + \sigma^2 = \sigma^2$.

For $\nu = (x_1 + x_2)/2$, the expected aggregate transportation costs, $T^u_{firm1} + T^u_{firm2}$, evaluates at the first solution is $\frac{1}{48} + \frac{1}{2}\sigma^2 - \sigma^4$, and $\frac{1}{12}$ for the second and third solutions. It follows that for $0 \leq \sigma^2 \leq 1/4$, the expected aggregate transportation costs are minimal for the first solution and this set of locations is the social optimal one.

F Proof of Proposition 5

We need now to calculate the expected aggregate transportation costs for the Nash equilibrium locations in both cases of uncertainty. Conversely to the precedent proof, consumers are now supposed to patronize firms with the cheapest price (transportation costs + mill price). The expected aggregate transportation costs for the Nash equilibrium are given by

$$T^u_{Nash}(x_1, x_2) = \int_{\underline{M}}^{\overline{M}} \left(\int_{M-1/2}^{z^*} (z - x_1)^2 \, dz + \int_{z^*}^{M+1/2} (x_2 - z)^2 \, dz \right) f(M) \, dM \tag{F.1}$$

with $z^* = \frac{1}{6}(x_1 + 4M + x_2)$ (see equation (5)). This gives us

$$T^u_{Nash}(x_1, x_2) = \sigma^2 \left(1 + \frac{5}{9}x_1 - \frac{5}{9}x_2\right) + \frac{1}{12} + x_1 \left(\frac{5}{36}x_1 x_2 + \frac{5}{36}x_1^2 + \frac{1}{4} - \frac{5}{36}x_2^2 + \frac{1}{2}x_1\right)$$
$$+ x_2 \left(-\frac{5}{36}x_2^2 - \frac{1}{4} + \frac{1}{2}x_2\right)$$

The evaluation of this expression for the equilibrium locations when firm 1 faces uncertainty gives us

$$T^u_{Nash/Firm1}(x_1^*, x_2^*) = \frac{2}{27}\sigma^2 + \frac{25}{243}\sigma^2\sqrt{9 - 4\sigma^2} + \frac{5}{36} + \frac{1}{54}\sqrt{9 - 4\sigma^2}$$

And when firm 2 faces uncertainty

$$T^u_{Nash/Firm2}(x_1^*, x_2^*) = unwieldy$$

Documents de Recherche parus en 2007[1]

[1] La liste intégrale des Documents de Travail du LAMETA parus depuis 1997 est disponible sur le site internet : http://www.lameta.univ-montp1.fr

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