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## «Misreporting, Retroactive Audit and Redistribution»

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# Misreporting, Retroactive Audit and Redistribution\*

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## Abstract

In this paper, we investigate an audit policy that allows a regulator to control past declarations of an agent who is caught to fraud in the current period or to adopt an action that is not desirable for Society. Coupled with redistribution effects due to the production of a public good, we show that retroactivity has not always the desired effect on the level of evasion or the level of effort, once the agent has decided to deviate from a given objective. Nevertheless, we derive conditions under which retroactivity lessens fraudulent behaviors, in quantity and in value. As a related result, authorities should communicate about how they use the individual contributions but information should not be completely transparent in order to fight efficiently against deviation. Redistribution and retroactivity may have opposite effects on the behavior of the agent when combined together.

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# 1 Introduction

This paper deals with a standard issue of agency, where an agent has to take a decision about a variable that influences the revenue of a principal, but that cannot be observed by him without cost. This variable could be a taxable income to be declared by a taxpayer to the fiscal authorities or a level of environmental effort that firms or farmers should apply. The agent cheats if he does not declare the effective level of revenue or effort. The literature on tax evasion<sup>1</sup> often deals with the optimal design of economic tools to fight against cheating, as in the seminal paper of Allingham and Sandmo (1972) which is considered as a benchmark<sup>2</sup>. The authors focus on an agent who has to decide which amount of his taxable income to declare to the authorities in order to maximize his private expected revenue, knowing the inspection probability and the penalty in case of fraud detection. By imposing penalties on the unpaid taxes rather than on the undeclared revenue as in Allingham and Sandmo, Yitzhaki (1974) shows that an increase in the tax rate always leads to more honesty when the preferences of the agents display decreasing absolute risk aversion. Many contributions followed these two reference papers, dealing either with the optimal levels of fines and audit probabilities or with the design of optimal audit schemes (Witte and Woodbury, 1977; Feinstein, 1991; Collins and Plumlee, 1991; Alm et al., 1992; Jung et al., 1994). In almost all papers the audit policy design takes into account only the current period, neglecting the fact that agents' decisions are linked

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<sup>1</sup>Actually, the literature related to income taxation makes a difference between evasion and fraud. While fraud refers to an out-of-law act, with false declaration, evasion refers to revenues on which no taxes are collected because their owners have found ways to prevent from paying them. In this paper, we will refer to fiscal policy but our results will also be discussed in the context of environmental economics and we will use indifferently both terms fraud and evasion.

<sup>2</sup>The interested reader can also read Srinivasan (1973).

over time. There are exceptions that consider policy design for which the probability of audit depends on the past behavior of the agent. In the context of fiscal evasion, Landsberger and Meilijson (1982) and Greenberg (1984) propose an audit scheme for which the current audit probability is conditioned on the results of past audits. In this scheme detected cheaters are inspected with a different probability than detected honest tax payers. Harrington (1988) proposes a policy where the probability of audit depends on past periods, with an application to environmental policies.

In this paper, we propose an audit scheme that takes into account past periods through retroactive auditing: If an agent is audited in the current period and detected to be cheating, the inspection is extended to a certain number of previous periods. If fraud is detected on previous periods, the agent must reimburse all undeclared revenues, and additionally he is liable for a penalty that applied on the total amount evaded. To the best of our knowledge, no paper on environmental economics and only one paper on tax evasion has considered such type of instrument (Rickard et al., 1982). This is rather surprising, since retroactive audit is quite commonly used in practice by fiscal authorities (France, England, ...). From a theoretical point of view, allowing retroactive audit enlarges the set of instruments available for preventing fraud. This type of policy is of particular interest when the authority (or the principal) is confronted to repeated interactions with a group of tax payers or agents whose declarations or actions are subject to a (partially) random income. This is the case for tax declaration since agents have to decide each year the amount of their current revenue to be declared. Furthermore from one year to another, they may change their behavior, switching from honesty to a little bit of cheating or to strong cheating. The fact that agents face a random income each period is an important assumption. Otherwise, the principal might ultimately discover each agent's income level if there are enough periods. More important even is the assumption that future income is not predictable on the basis of past realizations. Otherwise the authority could know exactly the future incomes of all agents. From a practical point of view these assumptions are reasonable since many taxpayers have

some random component in their income<sup>3</sup>, and we might also think about a population of taxpayers that is subject to entries and exits. Retroactive audit is also likely to improve environmental policy instruments when damage to the environment is due to repeated actions. As an important example, the pollution of groundwater by nitrates due to excessive or inappropriate fertilization by farmers appears only after several years. Besides, because of the cumulative nature of the process, it takes a long time to stabilize the pollution from the moment where more environment-friendly actions are taken. In that case, implementing dynamic environmental policies seems to be well suited for this kind of issue. The ideas developed in this paper hopefully contribute to solve, at least partially, this specific issue.

Our model is based on redistribution. Therefore collected taxes are used to produce public goods, while environmental efforts produce a public benefit. This implies that misreporting is no longer exclusively linked to the probability of audit. Since the agent benefits from the increase in the public good, his evasion strategy takes into account the direct effect on the level of public good. Symmetrically, the behavior of the other agents will affect the private expected revenue of the agent through the level of production of the public good. As a consequence an increase in the probability of audit makes fraud more risky for the agent, but also increases the expected total contribution to the public good since the dishonest agents will be caught more frequently.

One of our main results is that agents' revenue may increase when the audit probability increases because of the public good effect. Besides, retroactivity does not always provide sufficient incentives to be honest since the public good effect increases the expected revenue of the agent and creates a substitution effect. Nevertheless, under some conditions, dishonest agents will more frequently become honest when retroactive audit is implemented. Lastly, the regulator will have to decide which part of collected taxes and penalties will be dedicated to the production of the public good, knowing that the remaining amount will be used for implementing the audit policy in the next

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<sup>3</sup>This is also the case for farmers whose incomes are subject to climate. This case is discussed in the last section of the paper.

period. The optimal allocation of tax revenues depends on the relative efficiency of audit compared to the productivity of the public good, and it maximizes the expected social welfare subject to a financial constraint. In contrast to the standard treatment of tax policies in the literature, our assumptions imply that the behavior of the tax authorities is endogenous, since their current audit policy depend on the past behavior of the agents. All our results are derived with risk-neutral agents.

The paper is organized as follows. The second section displays results obtained when agents take into account the production of a public good under standard, non retroactive, audit. In the third section, we develop a more sophisticated model with both redistribution due to the presence of a public good and retroactive audit. Section four concludes the paper and discusses some interesting extensions of this work, especially in the frame of experimental economics. We also propose a discussion on environmental economics.

## 2 Redistribution and non retroactive audit

In this section and in the following one, we deal with the behavior of a taxpayer. In the last section, we will discuss our results in the context of environmental economics.

Consider  $N$  risk-neutral agents in the economy<sup>4</sup>. Each agent  $i$  earns, at each period  $t$ , a random revenue  $\tilde{w}_i^t$  with realizations  $w_i^t$  in  $\{w_i^1, w_i^2, \dots, w_i^L\}$  whatever the period. The probabilities assigned to each state of nature are denoted  $p_i^l$  with  $\sum_{l=1}^L p_i^l = 1$  and are independent from one period to another. The realized revenue is defined as the taxable income that the agent must declare to the authorities. Hence, if he is honest, the agent declares all his revenue. Nevertheless, he can decide, at each period, to only declare an amount  $x_i^t$ , with  $x_i^t \leq w_i^t$ , so that  $(w_i^t - x_i^t)$  is the amount of taxable revenue that is evaded by Agent  $i$  at date  $t$ . The tax rate imposed by the Authorities is assumed to be constant over time and linear in the declared revenue: It is strictly positive and

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<sup>4</sup>We adopt the assumption of risk-neutrality in order to be able to isolate the impact of retroactivity and redistribution on the behavior of the agent.

denoted as  $\tau$ . Hence the total amount of tax paid by the agent if he is not caught in evading is  $\tau x_i^t$ . At each period, his declaration is audited with probability  $\pi^t$ . If the agent is audited, the tax authorities observe his effective taxable income perfectly. If the agent has evaded some of his income, he must pay back the amount of taxes he tried to evade, that is  $\tau (w_i^t - x_i^t)$ , plus a penalty defined as a rate  $\beta$  applied to the evaded tax. In this section, we assume that audit applies only to the current period  $t$  for which it is implemented: No retroactivity takes place. Let us denote  $T^t$  the total amount of taxes and penalties collected after audit at period  $t$ :

$$T^t = \tau \cdot \left( \sum_{j=1}^N x_j^t + (1 + \beta)\pi^t \cdot \sum_{j=1}^N (w_j^t - x_j^t) \right) \quad (1)$$

Taxes are partly redistributed to the agents through income redistributions or building of public infrastructures and partly used to finance future audits. Precisely, we assume that a share  $\alpha^t$  of the collected taxes and penalties of the current period is invested in the production of a public good in the current period, while the remaining is allocated to the financing of audit in the next period. Hence, denoting  $n^t$  the number of audits in period  $t$  and letting  $C$  be the fixed unit cost of an audit, the regulator's budget constraint is given by:

$$\alpha^t T^t + n^{t+1} \cdot C = T^t$$

This equality allows us to define the probability of audit as a function of the past collected taxes and penalties:

$$\pi^t(T^{t-1}) = \frac{T^{t-1} \cdot (1 - \alpha^{t-1})}{C \cdot N} \quad (2)$$

This hypothesis has two important features. First, it is consistent with reality knowing that the budget of a government for Period  $t + 1$  is usually adopted at the end of Period  $t$ . Consequently, the budget is fixed for Period  $t + 1$  and it can only be increased through external funds<sup>5</sup>. This fact allows us to derive comparative statics (see Section 3)

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<sup>5</sup>Such non anticipated funds could be, for instance, the fruits of a sudden increase in the gross interior product at the end of Period  $t$ .

about the relation between the income declaration of the agent and the audit probability. Second, Condition 2 highlights an important strategic aspect: If audit would be only financed out of the declared incomes, agents would have an interest in coordinating each other actions on the zero-declaration-equilibrium. Hence fiscal income would be zero and no audit could take place in the next period.

The probability of audit  $\pi^t$  is common knowledge at the beginning of period  $t$ . The production function of the public good is  $g(\alpha^t T^t)$  with  $g'(\cdot) > 0$ ,  $g''(\cdot) < 0$ .

Because we are dealing with a public good, each agent benefits from the production function  $g(\cdot)$  without preventing the other agents from consuming the same level  $g(\cdot)$ . Furthermore, the level of public good enjoyed by Agent  $i$  depends both on his declaration and on the declarations of all other agents. The redistributive effect has an impact on the strategy of each agent since they make a trade-off between an expected increase of their revenue through evasion and bearing a reduction in the level of public good.

Lastly, we denote  $r$  the discount rate of wealth through time.

### Optimal Strategy of Fraud

After having observed his revenue for period  $t$  but before knowing the realization of  $\alpha^t$  and of  $T^t$ , Agent  $i$  will declare the amount  $x_i^t$  which maximizes his expected net revenue  $\mathbb{X}_i^t$  over the remaining periods, i.e. from period  $t$  to the final period noted  $To^6$ :

$$\max_{x_i^t} \mathbb{X}_i^t = X_i^t + E \left[ \sum_{s=1}^{To-t} e^{-rs} \cdot X_i^{t+s} \right] \quad (3)$$

with (1), (2) and

$$X_i^{t+s} = w_i^{t+s} + E [g(\alpha^{t+s} T^{t+s})] - \tau [x_i^{t+s} + \pi^{t+s}(1 + \beta)(w_i^{t+s} - x_i^{t+s})], \quad \forall s \geq 0 \quad (4)$$

The second term in the current wealth  $X_i^t$  is the gain expected from redistribution, namely from the production of the public good. It is evaluated in expectation because

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<sup>6</sup>The past cumulated revenue is constant for the agent at date  $t$ , so that we chose not to incorporate it in the objective function, for sake's simplicity and without implication on the result as shown by Rickard et al. (1982).



the agent does not know at the beginning of the period how many taxes will be collected from the other agents. The third term is the tax that Agent  $i$  pays to the authorities with respect to the declared revenue  $x_i^t$  plus the amount of penalties and the unpaid taxes recovered by the authorities in the case of audit. If the agent decides not to evade, this last term is equal to  $\tau w_i^t$ . The second term in (3) is the expected discounted wealth of future periods. Since the audit probability for the next period depends on the current collected taxes, the decision to evade in the current period has an impact on the future wealth and, thus, on the future strategy. We have to deal with a dynamic decision process.

In the course of the text, we use the following notation:

$$\forall i, \forall s, \quad \Delta_i^{t+s} = w_i^{t+s} - x_i^{t+s} \quad (5)$$

The solution  $x_i^t$  to Program (3) satisfies  $0 < x_i^t < w_i^t$  if and only if:

$$E [\alpha^t g'(\alpha^t T^t)] + E \left[ a.e^{-r}.\alpha^{t+1}.g'(\alpha^{t+1}T^{t+1}).\sum_{j=1}^N \Delta_j^{t+1} \right] = 1 + E [a.e^{-r}.\Delta_i^{t+1}] \quad (6)$$

with  $a = \tau(1 + \beta)\frac{(1-\alpha^t)}{C.N}$ . Details of the calculus are given in Appendix.

Expression (6) can be interpreted in terms of standard marginal cost and benefit. The left-hand-side term corresponds to the expected marginal benefit of honesty. For each additional euro declared, the agent obtains some benefit from the increase in the production of the public good in the current period. He also obtains an expected gain from the marginal production of the public good in the next period because a proportion  $(1 - \alpha^t)$  of the declared euro will be invested in the next audit policy. The right-hand-side term is the expected marginal cost of honesty. It is equal to the declared euro “lost” by the agent if declared plus the increase in the threat of being audited, and detected, in the next period if he decides to cheat in  $t + 1$  (i.e. if  $\Delta_i^{t+1} > 0$ ). Notice that the expected net revenue  $X_i^t$  of the agent is non linear in  $x_i^t$ . Hence with our setting, it is possible to obtain interior solutions in contrast to Rickard et al. (1982)<sup>7</sup>. Furthermore

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<sup>7</sup>Actually, the authors focus on the rate of evasion, which can only be equal to 1 (100% of the taxable

since each period is affected by the previous one through the audit probability (recall Equation (2)), the decision process is a markovian process in this model.

The results of this simple model are summarized as Proposition 1 hereafter.

**Proposition 1**

*i) Other things being equal, the redistribution process through the production of a public good induces a decrease of evasion compared to a system without redistribution.*

*ii) If the penalty rate is sufficiently high, an increase in the probability of audit leads to a increase in the declared income. Precisely:*

$$\frac{dx_i^t}{d\pi^t} > 0 \text{ iff } \beta > \frac{\pi^t}{1 - \pi^t}$$

*iii) If the penalty rate is sufficiently high, a richer agent declares a higher revenue.*

*Precisely:*

$$\frac{dx_i^t}{dw_i^t} > 0 \text{ iff } \beta > \frac{\pi^t}{1 - \pi^t}$$

**Proof.** See Appendix.

The solution of Program (3) is not a scalar but a best response function  $x_i^t(\cdot)$  of Agent  $i$  depending on the strategies of the other agents. Hence, we obtain a unique Nash equilibrium for each given vector of strategies of the other agents.

Our approach allows us to enhance the important role played by the public good provision on the agents' behavior. Taking into account the public good induces two opposite effects: First, the agent has additionnal incentives to be honest to allow the authorities to produce more public good. Second, it induces the agent to evade a little bit more since he knows that the declared incomes of the other agents also contribute to the production of the public good. This is Point ii) of Proposition 1: More audit yields more expected tax and, consequently, more expected public good for the agent, which revenue is evaded or 0 (the agent is honest) at optimum in their model.

may compensate the increase in the expected penalty in the case of detected evasion<sup>8</sup>. Thus the penalty rate must be sufficiently high to push the agent to evade less if, for an exogenous reason, the audit probability increases: investing in the public good must be more profitable at the margin than the activity of fraud.

Until now, we have implicitly assumed that the production of the public good is a profitable activity. Actually, one important aim of the authorities is to ensure that the production of the public good is socially welfare improving at each period. Hence we must be sure that producing  $g(\cdot)$  is profitable. We focus on this point in Section 4. In the following section, we still assume that a public good is produced and we propose to analyze retroactive audit as a way for the authorities to give, under some conditions, more incentives to agents to reduce evasion and also to obtain more funds for financing audit.

### 3 Retroactive audit

Assume now that, when an agent has evaded at period  $t$  and is caught, then the Authorities pursue the audit on a number  $k$  of periods preceding the current one, with  $0 \leq k \leq t - 1$ . In such a situation, the agent will have to reimburse all evaded taxes and to pay the penalty  $\beta$  for each euro evaded from period  $t - k$  to period  $t$ .

Let us denote  $z_i^t$  the amount that Agent  $i$  declares in a retroactive system and  $Z_i^t$  his expected net revenue at date  $t$ . Agent  $i$  still earns a taxable income  $w_i^t$  which is random at the beginning of each period  $t$ .

The total amount of taxes and penalties collected by the authorities at the end of period  $t$  is now

$$\Gamma^t = \tau \cdot \left[ \sum_{j=1}^N z_j^t + (1 + \beta)\pi^t \sum_{j=1}^N \left( (w_j^t - z_j^t) + \mathbf{1}_{\{z_j^t < w_j^t\}} \sum_{q=t-k}^{t-1} \max(w_j^q - \tilde{z}_j^q; 0) \right) \right], \quad (7)$$

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<sup>8</sup>If  $g''(\cdot) = 0$ , the ratio  $dx_i^t/d\pi^t$  obtained in Appendix is no longer defined. This is consistent with our setting: if the marginal productivity of the public good were linear, the optimum would always be a corner solution: either the agent would evade all his revenue or nothing.

with  $\widehat{z}_j^q$  being the amount of income effectively taken into account by the authorities when evaluating the taxes due from period  $q$  (with  $q < t$ ): If  $\widehat{z}_j^q$  equals  $z_j^q$  with  $z_j^q < w_j^q$ , this means that the agent has successfully cheated in period  $q$  and the remaining non declared revenue  $w_j^q - z_j^q$  may still be confiscated in period  $t$  if a retroactive audit takes place. If  $\widehat{z}_j^q$  equals  $w_j^q$ , either the agent was honest in period  $q$  or he was caught and there is no more undeclared revenue in period  $q$  so that  $\max(.,.)$  equals zero. Audit of the preceding periods takes place only if the agent has cheated in the current period. This is taken into account by using the indicator function  $\mathbf{1}_{\{z_j^t < w_j^t\}}$ , which is worth 1 if the agent has cheated (i.e. if  $z_j^t < w_j^t$ ) and zero otherwise.

As in the previous section, the probability of audit is defined by:

$$\pi^t(\Gamma^{t-1}) = \frac{\Gamma^{t-1} \cdot (1 - \alpha^{t-1})}{C.N} \quad (8)$$

Let us denote  $\Lambda_i^{t+s}$  the incomes, evaluated at Period  $t$ , that Agent  $i$  will have hidden in the periods preceding  $t + s$  and that will be discovered in period  $t + s$  if a retroactive audit takes place.

Hence with

$$\Delta_i^{t+s} = w_i^{t+s} - z_i^{t+s}, \quad \forall s, \forall i, \quad (9)$$

we have at Period  $t$  (for  $k \geq 2$ )<sup>9</sup>:

$$\Lambda_i^{t+s} = \begin{cases} \text{i) } \mathbf{1}_{\{\Delta_i^t > 0\}} \sum_{m=t-k}^{t-1} \max(w_i^m - \widehat{z}_i^m; 0) & \text{if } s = 0 \\ \text{ii) } \mathbf{1}_{\{\Delta_i^{t+1} > 0\}} \left[ \sum_{m=t+1-k}^{t-1} \max(w_i^m - \widehat{z}_i^m; 0) + \Delta_i^t \cdot (1 - \pi^t) \right] & \text{if } s = 1 \\ \text{iii) } \mathbf{1}_{\{\Delta_i^{t+s} > 0\}} \left[ \sum_{m=t+s-k}^{t-1} \max(w_i^m - \widehat{z}_i^m; 0) \right. \\ \quad \left. + \sum_{m=t}^{t+s-1} \Delta_i^m \cdot \prod_{l=0}^{t+s-1-m} \left( (1 - \pi^{m+l}) + \pi^{m+l} \cdot \mathbf{1}_{\{\Delta_i^{m+l} = 0\}} \right) \right], \forall 1 < s \leq k-1 \\ \text{iv) } \mathbf{1}_{\{\Delta_i^{t+s} > 0\}} \left[ \sum_{m=t}^{t+s-1} \Delta_i^m \cdot \prod_{l=0}^{t+s-1-m} \left( (1 - \pi^{m+l}) + \pi^{m+l} \cdot \mathbf{1}_{\{\Delta_i^{m+l} = 0\}} \right) \right], \forall s \geq k \end{cases}, \quad (10)$$

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<sup>9</sup>For  $k = 1$  we have

$$\Lambda_i^{t+s} = \begin{cases} \mathbf{1}_{\{\Delta_i^t > 0\}} \cdot \max(w_i^{t-1} - \widehat{z}_i^{t-1}; 0) & \text{if } s = 0 \\ \mathbf{1}_{\{\Delta_i^{t+s} > 0\}} \left[ (w_i^{t+s-1} - z_j^{t+s-1}) (1 - \pi^{t+s-1}) \right], \forall s \geq 1 \end{cases}$$

In Equality iii) of (10) evaded taxes in the future will be collected only if they were not yet confiscated before, which happens with probability  $\prod_{l=0}^{t+s-1-m} \left( (1 - \pi^{m+l}) + \pi^{m+l} \cdot \mathbf{1}_{\{\Delta_i^{m+l}=0\}} \right)$ . For instance, hidden taxes in Period  $m = t$  are still available in period  $t + s$  only if they have not been confiscated as a consequence of earlier audit at the previous dates, i.e. with probabilities  $(1 - \pi^{t+l})$ , with  $l = 0, \dots, s - 1$ , or if the agent was honest in Period  $t + l$  (so that no retroactivity takes place): this explains the term  $\pi^{m+l} \cdot \mathbf{1}_{\{\Delta_i^{m+l}=0\}}$ . Now we are able to define precisely the total taxes collected by the authorities at date  $t + s$  but evaluated at  $t$ :

$$\Gamma^{t+s} = \tau \cdot \sum_{j=1}^N [z_j^{t+s} + \pi^{t+s}(1 + \beta) (\Delta_j^{t+s} + \Lambda_j^{t+s})], \forall s \geq 0 \quad (11)$$

The maximization program of Agent  $i$  is finally

$$\max_{z_i^t} \mathbb{Z}_i^t = Z_i^t + E \left[ \sum_{s=1}^{T_0-t} e^{-rs} \cdot Z_i^{t+s} \right] \quad (12)$$

with

$$Z_i^{t+s} = w_i^{t+s} + E [g(\alpha^{t+s} \Gamma^{t+s})] - \tau [z_i^{t+s} + \pi^{t+s}(1 + \beta) (\Delta_i^{t+s} + \Lambda_i^{t+s})], \quad \forall s \geq 0, \quad (13)$$

(9), (10) and (11).

Besides, the cost of an audit occurring at date  $t$  is now  $c^t(k)$ .  $c$  is a function of  $k$  with  $c'(k) > 0$  and  $c(0) = C$  (no retroactivity): It becomes increasingly costly for the regulator as he investigates more remote periods. This information will be useful in the next section, when we will focus on the constraint of the regulator.

Because of the indicator function, the expected net revenue (12) of the agent displays a discontinuity at point  $z_i^t = w_i^t$ . Thus we have to analyze two separate situations: The amount of revenue  $z_i^t$  that the agent declares if he decides to evade, and the conditions under which the agent becomes honest, i.e. by moving from  $z_i^t < w_i^t$  to  $z_i^t = w_i^t$ .

The first question is investigated by focusing on the first order condition related to the situation in which the agent evades, that is when  $\mathbf{1}_{\{z_i^t < w_i^t\}} = 1$ .

**Lemma 1** *When Agent  $i$  decides to evade, the optimal declared amount of revenue  $z_i^t$  satisfies the following first order condition:*

$$\begin{aligned}
& E [\alpha^t g'(\alpha^t \Gamma^t)] + E \left[ a \cdot e^{-r} \cdot E [\alpha^{t+1} g'(\alpha^{t+1} \Gamma^{t+1})] \cdot \sum_{j=1}^N (\Delta_j^{t+1} + \Lambda_j^{t+1}) \right] \\
& + \frac{(1 + \beta)}{[1 - \pi^t(1 + \beta)]} E [P_1^k] + E \left[ a \cdot (Q_2^{k+1} - O_2^{k+1}) \right] \\
& = 1 + E [a \cdot e^{-r} \cdot (\Delta_i^{t+1} + \Lambda_i^{t+1})],
\end{aligned} \tag{14}$$

with, for  $k \geq 1$ ,

$$\begin{aligned}
O_2^{k+1} &= \sum_{s=2}^{k+1} e^{-rs} \cdot \pi^{t+s} \cdot \frac{\partial \Lambda_i^{t+s}}{\partial \pi^{t+1}}, \\
P_1^k &= \sum_{s=1}^k e^{-rs} \cdot \pi^{t+s} \cdot (1 - E [\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})]) \cdot \mathbf{1}_{\{\Delta_i^{t+s} > 0\}} \cdot \prod_{l=0}^{s-1} \left( (1 - \pi^{t+l}) + \pi^{t+l} \cdot \mathbf{1}_{\{\Delta_i^{t+l} = 0\}} \right), \\
Q_2^{k+1} &= \sum_{s=2}^{k+1} e^{-rs} \cdot E [\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})] \cdot \pi^{t+s} \cdot \sum_{j=1}^N \frac{\partial \Lambda_j^{t+s}}{\partial \pi^{t+1}} \text{ and} \\
a &= \tau(1 + \beta) \frac{(1 - \alpha^t)}{c(k) \cdot N}.
\end{aligned}$$

**Proof.** See Appendix. ■

Let us interpret each term in (14). If  $k = 0$  (no retroactivity),  $O$ ,  $P$ ,  $Q$  and  $\Lambda_i^{t+s}$  equal zero and we obtain the first order condition (6) of the model without retroactivity.

Consider now the case  $k > 0$ . Still here, the audit probability of period  $t + 1$  depends on the strategy chosen by the agent in period  $t$  (see Equation (8)). The difference with the model without retroactivity is that not only period  $t + 1$  is concerned, but also all forthcoming periods in which a retroactive audit of period  $t + 1$  may take place: this explains the operators  $\sum_{s=1}^k$  and  $\sum_{s=2}^{k+1}$  in  $O_2^{k+1}$ ,  $P_1^k$  and  $Q_2^{k+1}$ .

The right-hand-side term in (14) is the expected marginal cost of declaring one additional unit of revenue. It is similar to the marginal cost in the previous model, except that the impact of retroactivity must be added. This explains the added term  $\Lambda_i^{t+1}$  in the brackets. Recall that for each unit of revenue that is declared in  $t$ , a part  $(1 - \alpha^t)$  will finance the audit policy in Period  $t + 1$  (determined by the audit probability  $\pi^{t+1}$ ).

The left-hand-side term is the expected marginal benefit of declaring one additional Euro. The first line deals with the direct redistributive effect. The declared unit is split into the production of the public good today (Period  $t$ ) and the probability of audit tomorrow (recall Equ. (8)), thus increasing the production of the public good also tomorrow (in Period  $t + 1$ ). Here again, the fact that retroactivity takes place in the case of an audit is represented by the additional term  $\sum_{j=1}^N \Lambda_j^{t+1}$  in the brackets. This first line is close to the one in (6). It is immediate to see that retroactivity increases both the marginal cost and the marginal benefit in Period  $t$  and  $t + 1$ . This prevents us to conclude about its immediate total effect on the behavior of the agent. Indeed, while retroactivity increases the benefit of being "more" honest because it permits it to collect more taxes for the production of the public good, it also increases the threat of being caught to cheat in the next period by increasing  $\pi^{t+1}$ .

Nevertheless more can be said by looking at the second line of (14). For  $k > 0$ ,  $P$  is positive, while  $O$  and  $Q$  are negative. Indeed, we show in Section 4 that the expression  $1 - E[\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})]$  in  $P$  is always positive in optimum for any  $s$ . Moreover,  $\frac{\partial \Lambda_j^{t+s}}{\partial \pi^{t+1}}$  in  $O$  and  $Q$  is negative<sup>10</sup> for any agent  $j$ : an increase in the audit probability in Period  $t + 1$  decreases the chances to confiscate  $\Delta_i^t$  or  $\Delta_i^{t+1}$  in the future. The term in  $P$  deals with the impact of the penalty on the expected net revenue of the agent. It is positive: the fact that the agent will have to pay, in addition to the hidden taxes, a penalty in the case of an audit increases the marginal benefit of declaring one additional unit of revenue. The expression in  $O$  and in  $Q$  represents the impact of declaring more today on the expected net revenue in the future (Periods  $t + 1$  until periods  $t + k + 1$ ). It corresponds to the future impact of retroactivity on the marginal consumption of the public good minus the future impact of retroactivity on the revenues that may be confiscated. Actually, the

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<sup>10</sup>Indeed, we have from (10) that,  $\forall i$ ,

$$\begin{aligned} \frac{\partial \Lambda_i^{t+s}}{\partial \pi^{t+1}} = & \mathbf{1}_{\{\Delta_i^{t+s} > 0\}} \left[ \Delta_i^t \cdot \prod_{l=1}^{s-1} \left( (1 - \pi^{t+l}) + \pi^{t+l} \cdot \mathbf{1}_{\{\Delta_i^{t+l} = 0\}} \right) \cdot \left( \mathbf{1}_{\{\Delta_i^{t+1} = 0\}} - 1 \right) \right. \\ & \left. + \Delta_i^{t+1} \cdot \prod_{l=1}^{s-2} \left( (1 - \pi^{t+1+l}) + \pi^{t+1+l} \cdot \mathbf{1}_{\{\Delta_i^{t+1+l} = 0\}} \right) \cdot \left( \mathbf{1}_{\{\Delta_i^{t+1} = 0\}} - 1 \right) \right], \end{aligned}$$

which is negative or equal to zero.

term, in  $Q$ , is relative to the fact that the revenues evaded in periods  $t + s$  ( $s \geq 1$ ) have less chances to be confiscated in periods  $t + s + m$  (with  $m \geq 1$ ) because the probability to be confiscated immediately, than means during the period where they are hidden by the agent, increases since more funds are allocated to the audit policy. This effect is obtained by *contagion*. Part of the declared revenue today is affected to the audit policy for tomorrow, which increases the threat of being caught cheating. But, in doing so, it lessens the chances of this evaded revenue to be confiscated in the future and allocated to the public good in the future.

Finally the terms in  $O$ ,  $P$  and  $Q$  display the fact that the taxes collected in Period  $t + s$  depend on the strategy of the Agent in previous periods. They appear in the first order condition (14), while being absent from the condition obtained in the model without retroactivity. Still notice that  $O$  equals zero if the agent does not evade in Period  $t + 1$  (in footnote 9, we would have  $\Delta_i^{t+1} = 0$ ,  $\mathbf{1}_{\{\Delta_i^{t+1}=0\}} = 1$  and  $\mathbf{1}_{\{\Delta_i^{t+1}=0\}} - 1 = 0$ ).

Such as it stands, we are not able to conclude about the level of  $z_i^t$  in optimum. Is it higher or lower than the amount of revenue  $x_i^t$  that is declared in the absence of retroactivity? Proposition 2 goes a step further.

**Proposition 2** *Consider a risk neutral agent who decides to evade a positive amount of revenue. Assume that  $\frac{\partial \pi^{t+1}}{\partial T^t} = \frac{\partial \pi^{t+1}}{\partial \Gamma^t}$ .*

*An efficient policy provides a positive rate  $\beta$  of penalty and announces a constant and strictly positive productivity of the public good. Formally, such an admissible policy satisfies  $g'(\cdot) = g > 0$  and  $\beta$  is such that*

$$\frac{(1 + \beta)}{1 - \pi^t(1 + \beta)} \geq - \frac{E \left[ a \cdot \left( Q_2^{k+1} - O_2^{k+1} + e^{-r} \cdot \left( E[\alpha^{t+1}g] \cdot \sum_{j=1}^N \Lambda_j^{t+1} - \Lambda_i^{t+1} \right) \right) \right]}{E[P_1^k]} \quad (15)$$

**Proof.** See Appendix ■

Agents must be aware of the production of public good so that they have incentives to declare more revenue in order to contribute to its production. Nevertheless, they should not know all the characteristics of the production function, especially if it displays some concavity. Indeed they may not be willing to declare more than a given level of revenue



because of the decreasing productivity. In that sense, the government could announce that each agent will benefit in a proportional way from the production of the public good.

Furthermore, if the impact of the increase in the future audit probability following the declaration of one additional unit is negative (that means, it induces incentives to cheat more today), the government should increase the penalty to a level that counterbalance the willingness of the agent to hide this unit. This is the case if the expression at right in (15) is positive<sup>11</sup>. If retroactivity has a net positive effect on the behavior of the agent, then this expression is negative and Inequality (15) is always satisfied.

As a limit case, if the agent is audited at each period, retroactivity will never play a role since all the revenue would be confiscated at each period:  $\Lambda_i^{t+s} = 0 \forall s, \forall j$ . Then  $O, P$  and  $Q$  become equal to zero and (14) is reduced to (6). Nevertheless, this is only possible either when the probability of audit equals one<sup>12</sup> or if the agents are particularly *unlucky!*

Finally, once Agent  $i$  decides to evade at date  $t$ , retroactivity has only a (direct) effect on the amount of revenue he decides to evade through the contagion effect on the future. The fact that past revenue may be confiscated has no effect. Indeed, he cannot change anything in the preceding periods in the case of an audit. This result is interesting for it enhances that, essentially, the future is important here: it is concerned by retroactivity and the agent can affect it through the strategy he decides to adopt today.

Retroactivity affects also his willingness to move from a dishonest behavior ( $z_i^t < w_i^t$ ) to honesty ( $z_i^t = w_i^t$ ).

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<sup>11</sup>Recall that the left member must always be positive: Indeed  $1 - \pi^t(1 + \beta)$  must be positive otherwise the agent would always have an interest in sheating. This implies that the value of  $\beta$  is upper bounded by  $1 - \pi^t/\pi^t$ .

<sup>12</sup>In that case, the agents never sheat because of the penalty.

**Proposition 3** *Agent  $i$  evades less frequently under retroactive audit than under non-retroactive audit if a constant marginal productivity of the public good is announced.*

*If all other agents are honest, he has always less incentives to cheat whatever the productivity of the public good.*

**Proof.** See Appendix. ■

According to Proposition 3 retroactivity may induce less frequent evasion behavior. Actually the consumption of the public good adds some wealth to the agent and it may counterbalance the first effect if the productivity of the public good is high. As a limit case, if the agent considers a fixed impact of the public good on his expected net revenue, he will always evade less frequently than in a case without retroactivity. From a policy viewpoint, the authorities should communicate on how they reinvest the collected taxes because, thanks to Proposition 1, we know that it may induce less fraud. However, they should also restrict the information released, especially when the productivity of the public good is high. If the agent is aware of the productivity, he will make a tradeoff between more penalty if audited and more public good since everyone may be audited and retroactivity takes place.

### **Impact of the audit probability on the agent's expected revenue**

To obtain all effects of  $\pi^t$  on the agent's behavior, we have to derivate his expected net revenue with respect to  $\pi^t$  in order to make them appear<sup>13</sup>. Agent  $i$ 's program is given by (12).

Recalling that  $\partial Z_i^t / \partial z_i^t$  equals zero in optimum, we obtain:

$$\frac{dZ_i^t}{d\pi^t} = -\tau(1 + \beta) \cdot (\Delta_i^t + \Lambda_i^t) + E \left[ \alpha^t \cdot \frac{\partial \Gamma^t}{\partial \pi^t} \cdot g'(\alpha^t \Gamma^t) \right] \quad (16)$$

From (11),  $\partial \Gamma^t / \partial \pi^t > 0$ , so that the sign of (16) is undetermined. Nevertheless, if collected taxes and penalties were allocated to another sector and not to the agents that paid them, the public good effect would vanish (i.e. the second term) and the sign

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<sup>13</sup>Even if this technique is the one used to obtain the first order conditions of an optimization program, this is not what we are doing here since  $\pi^t$  is imposed to the agent. He cannot decide it.

of  $dZ_i^t/d\pi^t$  would be immediate: an increase in the probability of audit would always deteriorate the expected net revenue of the agent, other things being equal. Only the negative effect through the penalty would have an impact on  $Z$ .

When the agents know that taxes are allocated to the public good, the impact of a harsher audit policy is not so straightforward. The first term in Equality (16) is positive while the second term is negative. The total impact on  $Z_i^t$  is summarized in the proposition hereafter.

**Proposition 4** *A harsher audit policy may improve the expected net revenue of a dishonest agent. It is always the case if the agent is honest.*

When the authorities increase the probability of audit, they increase the chances of Agent  $i$  being caught if he has evaded, but they also increase his expected net revenue through the increase in the total amount of public good. Hence a dishonest agent may have an interest in a harsher audit policy: His expected net revenue may increase if the individuals who evade are more frequently caught. This positive impact is systematic if the agent is honest. Formally the first term in Equality (16) disappears in that case:  $\Delta_i^t = 0$  and  $\Lambda_i^t = 0$ .

## 4 Optimal investment decision of the regulator

Up to now, we implicitly assumed that the public good is always produced. Nevertheless, the regulator has to cope with both the financing constraint and the maximization of the social welfare. In such a situation, he must decide on the optimal rate  $\alpha^{t*}$  of the collected taxes and penalties that he will engage in the production of the public good. The remaining part will be dedicated to the audit activity in the next period. The decision of the government is an endogenous variable in this model: it depends on the behavior of the agents.

The regulator must also decide the length  $k$  of retroactivity in the second model. We assume that he decides the level of  $\alpha^t$  after audit has taken place, so that, when

determining  $\alpha^t$ , the regulator considers realized values for  $g(\cdot)$ . But  $k$  is decided at the beginning of the period. Denote  $h(\cdot)$  the function that measures the efficiency of the future audit, knowing the rate  $(1 - \alpha^t)$  of the current taxes dedicated to it. If  $h(\cdot)$  is the identity function this means that one Euro of current taxes invested in the future audit yields one Euro of benefit. Formally, we assume that  $0 < h((1 - \alpha^t)T^t) \leq (1 - \alpha^t).T^t$  and  $0 < h'(\cdot) < 1$ . The level of efficiency is assumed to be exogenous and deterministic here.

### Audit without retroactivity

Let us denote  $\mathbb{W}^t$  the expected social welfare at date  $t$  when no retroactivity takes place. The maximization program of the regulator is

$$\max_{\alpha^t} \mathbb{W}^t = \sum_{j=1}^N \mathbb{X}_j^t - n^t.C + h((1 - \alpha^t)T^t) \quad (17)$$

subject to

$$\alpha^t T^t + n^{t+1}.C = T^t \quad (18)$$

with  $\mathbb{X}_j^t$  defined by (4).<sup>14</sup>

The decision of the regulator at date  $t$  will have an impact on the future, through the proportion  $(1 - \alpha^t)$  of current taxes and penalties reserved for the financing of the future audits.

The first order condition of Program (17-18) yields:

$$0 < \alpha^{t*} < 1 \text{ iff } E [T^t.g'(\alpha^{t*}T^t)] = \frac{1}{N}.T^t.h'((1 - \alpha^{t*}).T^t) \quad (19)$$

Since  $T^t$  is known from the regulator when he decides  $\alpha^{t*}$ , it is equivalent to

$$0 < \alpha^{t*} < 1 \text{ iff } g'(\alpha^{t*}T^t) = \frac{1}{N}.h'((1 - \alpha^{t*}).T^t)$$

It is worth noticing that the left-hand-side term concerns Period  $t$ , while the right-hand-side term is related to Period  $t + 1$  since  $h(\cdot)$  is the efficiency of the audit in the next period.

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<sup>14</sup>See page 6 for a discussion of Constraint (18).

The government has to make a trade-off between producing the public good in the current period and fighting against fraud in the next period. From (18) and (19) we have more precisely:

$$\begin{aligned}
0 < \alpha^{t*} = 1 - \frac{n^{t+1} \cdot C}{T^t} < 1 &\text{ iff } g'(\alpha^{t*} T^t) = \frac{1}{N} \cdot h'((1 - \alpha^{t*}) T^t) \\
\alpha^{t*} = 1 &\text{ if } g'(\alpha^{t*} T^t) \geq \frac{1}{N} \cdot h'((1 - \alpha^{t*}) T^t) \\
\alpha^{t*} = 0 &\text{ if } g'(\alpha^{t*} T^t) \leq \frac{1}{N} \cdot h'((1 - \alpha^{t*}) T^t)
\end{aligned}$$

The second and the third case must be discussed. If  $\alpha^{t*} = 1$ , then all the agents may have an interest in hiding all their revenue in the next period since no audit will be financed. A direct consequence is that no taxes will be available for the production of the public good: we will have at equilibrium in Period  $t + 1$ :  $T^{t+1} = 0$  and  $g(0) = 0$ . Hence a high production of the public good in the current period may lead to no public good in the next period!

In the opposite third case,  $\alpha^{t*} = 0$  and no public good is produced in the current period. This is especially the case if the audit technology is so efficient that all funds are invested in audit for the next period. Nevertheless, taxes are not needed and the fiscal rate  $\tau$  should be equal to zero at equilibrium.

### **Retroactive audit**

In the case with retroactivity, the regulator must choose  $\alpha^t$  and the length  $k$  of retroactivity<sup>15</sup> that are solution to the following program:

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<sup>15</sup>Actually, the regulator will seek information in the past only if the agent is kept frauding at date  $t$  after an audit. Hence the total cost of audit should be composed of the cost  $C$  for the current period and a function depending on  $k$  and multiplied by  $\mathbf{1}_{\{\Delta_t^i > 0\}}$ . Because this indicator function induces a discontinuity at date  $t$  when derivating with respect to  $k$ , we choose to work with a continuous function, namely  $c(k)$  which occurs for each audit. The important point here is that this function is increasing with  $k$  such as the discontinuous function would be.

$$\max_{\alpha^t, k} \mathbb{W}^t(k) = \sum_{j=1}^N \mathbb{Z}_j^t - n^t \cdot c(k) + h((1 - \alpha^t)\Gamma^t) \quad (20)$$

subject to

$$\alpha^t \Gamma^t + n^{t+1} \cdot c(k) = \Gamma^t \quad (21)$$

The private revenues  $\mathbb{Z}_j^t$  are considered with respect to either realized values of income or expected values, depending on which parameter,  $\alpha^t$  or  $k$ , is to be determined.

Recall that by definition we have  $c'(k) > 0$ . The first order conditions for  $\alpha^{t*}$  induce

$$\begin{aligned} 0 < \alpha^{t*} = 1 - \frac{n^{t+1} \cdot c(k)}{\Gamma^t} < 1 & \text{ iff } g'(\alpha^{t*} \Gamma^t) = \frac{1}{N} \cdot h'((1 - \alpha^{t*})\Gamma^t) \\ \alpha^{t*} = 1 & \text{ if } g'(\alpha^{t*} \Gamma^t) \geq \frac{1}{N} \cdot h'((1 - \alpha^{t*})\Gamma^t) \\ \alpha^{t*} = 0 & \text{ if } g'(\alpha^{t*} \Gamma^t) \leq \frac{1}{N} \cdot h'((1 - \alpha^{t*})\Gamma^t) \end{aligned} \quad (22)$$

and, for  $k^*$ ,

$$\begin{aligned} 0 < k^* < t - 1 & \text{ iff} \\ NE \left[ \sum_{j=1}^N \frac{\partial \Lambda_j^t}{\partial k} \cdot [\alpha^t g'(\alpha^t \Gamma^t) + (1 - \alpha^t) h'((1 - \alpha^t)\Gamma^t)] \right] - E \left[ \frac{\partial \Lambda_i^t}{\partial k} \right] \\ & = \frac{n^t \cdot c'(k)}{\tau(1 + \beta)\pi^t} \end{aligned} \quad (23)$$

From (10) we have that  $\frac{\partial \Lambda_j^t}{\partial k} > 0 \forall j$ . The first term in (23) displays the positive effect of retroactivity for Society: more expected collected taxes induce more money for the production of the public good, and the intensity of this positive effect depends simultaneously on the productivity of the public good and on the efficiency of the audit policy. The second term is the negative effect of an increase in  $k$  directly borne by the agents, that means the increase in the taxes (and penalties) collected in the current period and coming from past behaviors. The ratio in the right-hand-side term deals with the marginal cost of audit.

## 5 Conclusion

In this paper, we have shown that if taxpayers take into account the financing of the production of public goods through taxes, they tend to evade less of their income than in a system without information on redistribution. Besides, the fact that agents incorporate in their expected revenue the consumption of a public good, i.e. a redistribution of collected taxes, has different effects on their behavior. First, they are more likely to report truthfully because the collected taxes are redistributed in units of the public good. Second, there is a negative effect, leading to more tax evasion since an additional Euro can either be obtained from an increase of the consumption of the public good or an increase in the evaded income, for a given audit probability. Besides, we have also shown that a richer agent increases the amount of declared income if the penalty rate is sufficiently high. But this does not mean that the percentage of fraud decreases since our model is based on monetary amounts of incomes and not on shares of taxable incomes.

Another interesting result deals with the impact of the audit probability on the agents' behavior. In a standard model without redistribution, an increase in the audit probability leads to a deterioration of the financial situation of the agent. In our model this is not always the case because the agent can have a benefit from a harsher audit policy, through the increase of the total expected taxes and penalties collected and used to produce the public good. These results hold both in the simple model and in the model with retroactivity.

Retroactivity may enter in conflict with the way the agent considers the redistribution process in his expected wealth. Precisely, retroactivity always leads to more expected collected taxes and penalties, thus to more expected public good provision. If the productivity of the public good is high, the agent has an incentive to decrease his level of declared revenue. Nevertheless, we state sufficient conditions under which retroactivity leads to more honest income reporting. Moreover our results suggest the opportunity for the government to use communication as an additional tool for fighting fiscal fraud. Indeed it may be worth not divulging all the information about the productivity of

the production function of the public good; especially, if agents believe that the productivity is constant, they always have an incentive to declare more revenue in a system with retroactivity. Taking a point of view of “positive economics”, our model also suggests that such a fiscal system is efficient if agents are boundedly rational, which might correspond to the use of a constant marginal productivity.

Our results are of particular interest with respect to fiscal policy because incomes have to be reported periodically, and current income declarations are not independent from past declarations. These characteristics can also be found in environmental issues, for example when pollution is not sudden, but appears after some accumulation of emissions. In these situations, it is not the emission by one source that leads to the pollution of a site, but the repeated emissions of several polluters. And when the pollution is discovered, the rehabilitation of the polluted area may take a long time and may also call for repeated actions through time. This is especially the case for groundwater polluted by agricultural fertilizers, which contain nitrates. Lands are fertilized one or several times each year, depending on the type of good that is cultivated (meal, corn, potatoes, ...) and also on the type of land. It is particularly difficult for a regulator to observe the agricultural practices of farmers without some costly investigations so that we have to deal with moral hazard in a dynamic principal-agent relationship. Several papers have stressed the usefulness of random audit schemes (Mookherjee and Png, 1990, 1992; Picard, 1996, 2000). Here they would depend on the physical properties of the cultivated land and also on the past climate. The cost of an audit in such a context deals with the cost of obtaining consistent data through a precise analysis of the land and of the infiltration process of nitrates in the groundwater. Most of the existing economic studies deal with static models and audit that would depend on the past of the agent, here the farmer, was never analyzed up to now (except by Harrington (1988)). This can be done with our model. Instead of declaring an amount of income agents announce a level of effort, that would fit with a given agricultural practice. For instance, fertilizing in several times during the year, reducing the quantities of fertilizers, cultivating some plants during the winter that capture the exceeding nitrates still present in the soil, etc. Hence the



regulator will have to audit in order to be sure that the environmental advice that was given to the farmers at the beginning of the period was respected. This audit should provide information about the agricultural practices during the current period but also on the past periods if the level of pollution in the soil cannot be attributed to actions undertaken only during the current period. The way the regulator communicates with the farmers is also particularly important here: the farmer must know how his efforts affect environmental quality and how much he benefits from the increase in the quality of the groundwater. This point is not so easy as with tax policy because the farmer does not need high quality water for irrigation so that he may not be convinced by a redistribution process. Nevertheless, he may be sensitive to environmental insights if this could have a positive impact on his expected revenue. Hence such an argument calls for the implementation of subsidies that would be connected to the results of the audit. Environmentally friendly actions by farmers should be financially supported, at least at the beginning of a process of changing agricultural practice or technology. This means that in our model, it would be interesting to consider not only monetary sanctions in the case of a deviation but also retributions, because agents are not necessarily frauding when they do not follow some given advice and do worse than required.

There are several interesting possible developments of the previous model. First, introducing imperfect auditing schemes may be useful. In many practical applications the audit does not always detect shirkers. Therefore, if the aim is to obtain a given target level of fraud, it will be necessary to compensate the imperfectness of the audit by a larger penalty on a longer retroactivity period. Second, risk aversion may play an important role. Indeed retroactivity may have a larger impact when agents are risk averse. Other interesting theoretical developments should deal with infinite horizons, continuous time and also discounting. This is not taken into account in our model.

Finally it would be useful to test the impact of retroactivity on “real people”. Fraud prevention policies are widespread, but while the cost of the policy is known with some accuracy, the benefits are hardly known. Nevertheless, many countries rely on it. It would be much more efficient to have a detailed account about the impact of retroactivity.

We suggest therefore that an experimental investigation would be useful (this work is in progress).

## APPENDIX

### First order condition (6)

Thanks to a differentiation of (3) and with  $\frac{\partial T^t}{\partial x_i^t} = -\frac{\partial X_i^t}{\partial x_i^t}$  (from (1) and (4)), we have in optimum:

$$\begin{aligned} & \frac{\partial \mathbb{X}_i^t}{\partial x_i^t} = 0 \\ \Leftrightarrow & \frac{\partial T^t}{\partial x_i^t} \cdot \left[ \frac{\partial X_i^t}{\partial T^t} + e^{-r} \cdot E \left( \frac{\partial X_i^{t+1}}{\partial \pi^{t+1}} \cdot \frac{\partial \pi^{t+1}}{\partial T^t} + \frac{\partial X_i^{t+1}}{\partial T^{t+1}} \cdot \frac{\partial T^{t+1}}{\partial \pi^{t+1}} \cdot \frac{\partial \pi^{t+1}}{\partial T^t} \right) \right] = -\frac{\partial X_i^t}{\partial x_i^t} \\ \Leftrightarrow & \frac{\partial X_i^t}{\partial T^t} + e^{-r} \cdot E \left( \frac{\partial \pi^{t+1}}{\partial T^t} \cdot \left( \frac{\partial X_i^{t+1}}{\partial \pi^{t+1}} + \frac{\partial X_i^{t+1}}{\partial T^{t+1}} \cdot \frac{\partial T^{t+1}}{\partial \pi^{t+1}} \right) \right) = 1 \end{aligned}$$

From (4), (1), (2) and with (5) we have  $\frac{\partial X_i^t}{\partial T^t} = E[\alpha^t g'(\alpha^t T^t)]$ ,  $\frac{\partial \pi^{t+1}}{\partial T^t} = \frac{(1-\alpha^t)}{C.N}$ ,  $\frac{\partial X_i^{t+1}}{\partial \pi^{t+1}} = -\tau(1+\beta)\Delta_i^{t+1}$  and  $\frac{\partial T^{t+1}}{\partial \pi^{t+1}} = \tau(1+\beta) \cdot \sum_{j=1}^N \Delta_j^{t+1}$ . Finally:

$$E[\alpha^t g'(\alpha^t T^t)] + e^{-r} \cdot E \left[ \frac{(1-\alpha^t)}{C.N} \cdot \tau(1+\beta) \left( -\Delta_i^{t+1} + E \left[ \alpha^{t+1} \cdot g'(\alpha^{t+1} T^{t+1}) \cdot \sum_{j=1}^N \Delta_j^{t+1} \right] \right) \right] = 1$$

With  $a = \tau(1+\beta)\frac{(1-\alpha^t)}{C.N}$ , we obtain the first order condition (6).

Q.E.D. ■

### Proof of Proposition 1.

**i)** Consider (6). With  $g'(\cdot) > 0$ , the expected marginal benefit of honesty is higher than in a system with no redistribution, while the expected marginal cost remains unchanged.

**ii)** A total differentiation of (6) w.r.t.  $\pi^t$  and  $x_i^t$  yields

$$\begin{aligned} & \left[ (1 - (1+\beta)\pi^t) \cdot \tau \cdot \left( E[\alpha^{t2} \cdot g''(\alpha^t T^t)] + E \left[ e^{-r} \cdot a^2 \cdot (\alpha^{t+1})^2 \cdot g''(\alpha^{t+1} T^{t+1}) \left( \sum_{j=1}^N \Delta_j^{t+1} \right)^2 \right] \right) \right] dx_i^t \\ & + \left[ \tau \cdot (1+\beta) \cdot \left( E \left[ \alpha^{t2} g''(\alpha^t T^t) \cdot \sum_{j=1}^N \Delta_j^t \right] + E \left[ e^{-r} \cdot a^2 \cdot (\alpha^{t+1})^2 \cdot g''(\alpha^{t+1} T^{t+1}) \left( \sum_{j=1}^N \Delta_j^{t+1} \right)^2 \cdot \sum_{j=1}^N \Delta_j^t \right] \right) \right] d\pi^t = 0 \end{aligned}$$

⇔

$$\frac{dx_i^t}{d\pi^t} = \frac{-(1+\beta) \cdot \left( E \left[ \alpha^{t2} g''(\alpha^t T^t) \cdot \sum_{j=1}^N \Delta_j^t \right] + E \left[ e^{-r} \cdot a^2 \cdot (\alpha^{t+1})^2 \cdot g''(\alpha^{t+1} T^{t+1}) \left( \sum_{j=1}^N \Delta_j^{t+1} \right)^2 \cdot \sum_{j=1}^N \Delta_j^t \right] \right)}{(1 - (1+\beta)\pi^t) \cdot \tau \cdot \left( E [\alpha^{t2} \cdot g''(\alpha^t T^t)] + E \left[ e^{-r} \cdot a^2 \cdot (\alpha^{t+1})^2 \cdot g''(\alpha^{t+1} T^{t+1}) \left( \sum_{j=1}^N \Delta_j^{t+1} \right)^2 \right] \right)}$$

The numerator is positive. The term into brackets at the denominator is negative.

Hence this ratio is positive if and only if  $1 - (1+\beta)\pi^t < 0$ , i.e.  $\beta > \frac{\pi^t}{1-\pi^t}$ .

iii) A differentiation of (6) w.r.t.  $w_i^t$  and  $x_i^t$  yields

$$\left[ (1 - (1+\beta)\pi^t) \cdot \tau \cdot \left( E [\alpha^{t2} \cdot g''(\alpha^t T^t)] + E \left[ e^{-r} \cdot a^2 \cdot (\alpha^{t+1})^2 \cdot g''(\alpha^{t+1} T^{t+1}) \left( \sum_{j=1}^N \Delta_j^{t+1} \right)^2 \right] \right) \right] dx_i^t + \left[ \tau \cdot (1+\beta)\pi^t \cdot \left( E [\alpha^{t2} \cdot g''(\alpha^t T^t)] + E \left[ e^{-r} \cdot a^2 \cdot (\alpha^{t+1})^2 \cdot g''(\alpha^{t+1} T^{t+1}) \left( \sum_{j=1}^N \Delta_j^{t+1} \right)^2 \right] \right) \right] dw_i^t = 0$$

$$\frac{dx_i^t}{dw_i^t} = \frac{-(1+\beta)\pi^t \cdot \left( E [\alpha^{t2} \cdot g''(\alpha^t T^t)] + E \left[ e^{-r} \cdot a^2 \cdot (\alpha^{t+1})^2 \cdot g''(\alpha^{t+1} T^{t+1}) \left( \sum_{j=1}^N \Delta_j^{t+1} \right)^2 \right] \right)}{(1 - (1+\beta)\pi^t) \cdot \tau \cdot \left( E [\alpha^{t2} \cdot g''(\alpha^t T^t)] + E \left[ e^{-r} \cdot a^2 \cdot (\alpha^{t+1})^2 \cdot g''(\alpha^{t+1} T^{t+1}) \left( \sum_{j=1}^N \Delta_j^{t+1} \right)^2 \right] \right)}$$

The numerator is positive, while the denominator is positive if and only if  $1 - (1+\beta)\pi^t < 0$ .

Q.E.D. ■

**First order condition (14).**

With  $Z_i^t = f \left( Z_i^t(z_i^t, \Gamma^t(z_i^t)); \sum_{s=1}^{k+1} Z_i^{t+s}(\pi^{t+1}, \Lambda_i^{t+s}(z_i^t, \pi^{t+1}), \Gamma^{t+s}(\Lambda_{j=1..N}^{t+s})) \right)$ , and

$\pi^{t+1} = f(\Gamma^t(z_i^t))$ , a differentiation of (12) leads in optimum to:

$$\frac{dZ_i^t}{dz_i^t} = 0$$

$\Leftrightarrow$

$$\begin{aligned} 0 &= \frac{\partial Z_i^t}{\partial z_i^t} + \frac{\partial \Gamma^t}{\partial z_i^t} \cdot \left[ \frac{\partial Z_i^t}{\partial \Gamma^t} + E \left[ \sum_{s=1}^{k+1} e^{-rs} \cdot \left( \frac{\partial Z_i^{t+s}}{\partial \pi^{t+1}} + \frac{\partial Z_i^{t+s}}{\partial \Gamma^{t+s}} \cdot \frac{\partial \Gamma^{t+s}}{\partial \pi^{t+1}} \right) \cdot \frac{\partial \pi^{t+1}}{\partial \Gamma^t} \right] \right] \\ &+ E \left[ \sum_{s=1}^k e^{-rs} \cdot \left( \left( \frac{\partial Z_i^{t+s}}{\partial \Lambda_i^{t+s}} + \frac{\partial Z_i^{t+s}}{\partial \Gamma_i^{t+s}} \cdot \frac{\partial \Gamma_i^{t+s}}{\partial \Lambda_i^{t+s}} \right) \cdot \frac{\partial \Lambda_i^{t+s}}{\partial z_i^t} \right) \right] \end{aligned}$$

With  $-\frac{\partial Z_i^t}{\partial z_i^t} = \frac{\partial \Gamma^t}{\partial z_i^t} = \tau [1 - \pi^t(1 + \beta)]$  (from (11) and (13)) we have

$$\begin{aligned} &\frac{\partial Z_i^t}{\partial \Gamma^t} + E \left[ \sum_{s=1}^{k+1} e^{-rs} \cdot \frac{\partial Z_i^{t+s}}{\partial \Gamma^{t+s}} \cdot \frac{\partial \Gamma^{t+s}}{\partial \pi^{t+1}} \cdot \frac{\partial \pi^{t+1}}{\partial \Gamma^t} \right] \\ &+ \frac{1}{\tau [1 - \pi^t(1 + \beta)]} \cdot E \left[ \sum_{s=1}^k e^{-rs} \cdot \left( \frac{\partial Z_i^{t+s}}{\partial \Lambda_i^{t+s}} + \frac{\partial Z_i^{t+s}}{\partial \Gamma_i^{t+s}} \cdot \frac{\partial \Gamma_i^{t+s}}{\partial \Lambda_i^{t+s}} \right) \cdot \frac{\partial \Lambda_i^{t+s}}{\partial z_i^t} \right] \\ &= 1 - E \left[ \sum_{s=1}^{k+1} e^{-rs} \cdot \frac{\partial Z_i^{t+s}}{\partial \pi^{t+1}} \cdot \frac{\partial \pi^{t+1}}{\partial \Gamma^t} \right] \end{aligned}$$

From (8), (11), (13), and with (9) and (10) we have that

$$\begin{aligned} \frac{\partial Z_i^{t+s}}{\partial \Gamma^{t+s}} &= E [\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})] \quad \forall s \geq 0, \\ \frac{\partial \Gamma^{t+s}}{\partial \pi^{t+1}} &= \tau \cdot (1 + \beta) \pi^{t+s} \cdot \sum_{j=1}^N \frac{\partial \Lambda_j^{t+s}}{\partial \pi^{t+1}} \quad \forall s > 1 \text{ and } \frac{\partial \Gamma^{t+1}}{\partial \pi^{t+1}} = \tau \cdot (1 + \beta) \cdot \sum_{j=1}^N (\Delta_j^{t+1} + \Lambda_j^{t+1}), \\ \frac{\partial \pi^{t+1}}{\partial \Gamma^t} &= \frac{(1 - \alpha^t)}{C \cdot N}, \\ \frac{\partial Z_i^{t+s}}{\partial \Lambda_i^{t+s}} &= -\tau (1 + \beta) \pi^{t+s}, \\ \frac{\partial \Gamma^{t+s}}{\partial \Lambda_i^{t+s}} &= \tau (1 + \beta) \pi^{t+s}, \\ \frac{\partial \Lambda_i^{t+s}}{\partial z_i^t} &= -\mathbf{1}_{\{\Delta_i^{t+s} > 0\}} \cdot \prod_{l=0}^{s-1} \left( (1 - \pi^{t+l}) + \pi^{t+l} \cdot \mathbf{1}_{\{\Delta_i^{t+l} = 0\}} \right) \quad \forall s \geq 1, \\ \frac{\partial Z_i^{t+s}}{\partial \pi^{t+1}} &= -\tau \cdot (1 + \beta) \pi^{t+s} \cdot \frac{\partial \Lambda_i^{t+s}}{\partial \pi^{t+1}} \quad \forall s > 1 \text{ and } \frac{\partial Z_i^{t+1}}{\partial \pi^{t+1}} = -\tau \cdot (1 + \beta) \cdot (\Delta_i^{t+1} + \Lambda_i^{t+1}). \end{aligned}$$

Thus we have :

$$\begin{aligned}
& E [\alpha^t g'(\alpha^t \Gamma^t)] \\
& + E \left[ \tau(1 + \beta) \cdot \frac{(1 - \alpha^t)}{C \cdot N} \cdot \left( \sum_{s=2}^{k+1} e^{-rs} \cdot E [\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})] \cdot \pi^{t+s} \cdot \sum_{j=1}^N \frac{\partial \Lambda_j^{t+s}}{\partial \pi^{t+1}} \right. \right. \\
& \left. \left. + e^{-r} \cdot E [\alpha^{t+1} g'(\alpha^{t+1} \Gamma^{t+1})] \cdot \sum_{j=1}^N (\Delta_j^{t+1} + \Lambda_j^{t+1}) \right) \right] \\
& + \frac{\tau(1 + \beta) E \left[ \sum_{s=1}^k e^{-rs} \cdot \pi^{t+s} \cdot (1 - E [\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})]) \cdot \mathbf{1}_{\{\Delta_i^{t+s} > 0\}} \cdot \prod_{l=0}^{s-1} \left( (1 - \pi^{t+l}) + \pi^{t+l} \cdot \mathbf{1}_{\{\Delta_i^{t+l} = 0\}} \right) \right]}{\tau [1 - \pi^t (1 + \beta)]} \\
= & 1 + E \left[ \tau \cdot (1 + \beta) \frac{(1 - \alpha^t)}{c(k) \cdot N} \cdot \left( \sum_{s=2}^{k+1} e^{-rs} \cdot \pi^{t+s} \cdot \frac{\partial \Lambda_i^{t+s}}{\partial \pi^{t+1}} + e^{-r} \cdot (\Delta_i^{t+1} + \Lambda_i^{t+1}) \right) \right]
\end{aligned}$$

With  $a = \tau(1 + \beta) \cdot \frac{(1 - \alpha^t)}{c(k) \cdot N}$  we obtain:

$$\begin{aligned}
& E [\alpha^t g'(\alpha^t \Gamma^t)] \\
& + E \left[ a \cdot \sum_{s=2}^{k+1} e^{-rs} \cdot E [\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})] \cdot \pi^{t+s} \cdot \sum_{j=1}^N \frac{\partial \Lambda_j^{t+s}}{\partial \pi^{t+1}} \right] \\
& + E \left[ a \cdot e^{-r} \cdot E [\alpha^{t+1} g'(\alpha^{t+1} \Gamma^{t+1})] \cdot \sum_{j=1}^N (\Delta_j^{t+1} + \Lambda_j^{t+1}) \right] \\
& + \frac{(1 + \beta) E \left[ \sum_{s=1}^k e^{-rs} \cdot \pi^{t+s} \cdot (1 - E [\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})]) \cdot \mathbf{1}_{\{\Delta_i^{t+s} > 0\}} \cdot \prod_{l=0}^{s-1} \left( (1 - \pi^{t+l}) + \pi^{t+l} \cdot \mathbf{1}_{\{\Delta_i^{t+l} = 0\}} \right) \right]}{[1 - \pi^t (1 + \beta)]} \\
= & 1 + E \left[ a \cdot \sum_{s=2}^{k+1} e^{-rs} \cdot \pi^{t+s} \cdot \frac{\partial \Lambda_i^{t+s}}{\partial \pi^{t+1}} + a \cdot e^{-r} \cdot (\Delta_i^{t+1} + \Lambda_i^{t+1}) \right]
\end{aligned}$$

With the following simplified notations

$$Q_2^{k+1} = \sum_{s=2}^{k+1} e^{-rs} \cdot E [\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})] \cdot \pi^{t+s} \cdot \sum_{j=1}^N \frac{\partial \Lambda_j^{t+s}}{\partial \pi^{t+1}},$$

$$P_1^k = \sum_{s=2}^k e^{-rs} \cdot \pi^{t+s} \cdot (1 - E [\alpha^{t+s} g'(\alpha^{t+s} \Gamma^{t+s})]) \cdot \mathbf{1}_{\{\Delta_i^{t+s} > 0\}} \cdot \prod_{l=0}^{s-1} \left( (1 - \pi^{t+l}) + \pi^{t+l} \cdot \mathbf{1}_{\{\Delta_i^{t+l} = 0\}} \right),$$

$$O_2^{k+1} = \sum_{s=2}^{k+1} e^{-rs} \cdot \pi^{t+s} \cdot \frac{\partial \Lambda_i^{t+s}}{\partial \pi^{t+1}},$$

and a last arrangement, we obtain the first order condition (14).

Q.E.D. ■

## Proof of Proposition 2.

By subtracting (6) from (14) we have:

$$\begin{aligned}
& E \left[ \alpha^t (g'(\alpha^t \Gamma^t) - g'(\alpha^t T^t)) \right] \\
& + E \left[ a.e^{-r} . E \left[ \alpha^{t+1} (g'(\alpha^{t+1} . \Gamma^{t+1}) - g'(\alpha^{t+1} . T^{t+1})) \right] . \sum_{j=1}^N \Delta_j^{t+1} \right] \\
& + E \left[ a.e^{-r} . \left( E \left[ \alpha^{t+1} g'(\alpha^{t+1} . \Gamma^{t+1}) \right] . \sum_{j=1}^N \Lambda_j^{t+1} - \Lambda_i^{t+1} \right) \right] \\
& + \frac{(1 + \beta)}{[1 - \pi^t(1 + \beta)]} E \left[ P_1^k \right] + E \left[ a. (Q_2^{k+1} - O_2^{k+1}) \right] \tag{24}
\end{aligned}$$

From the second order conditions, we will have that  $z_i^t \geq x_i^t$  if and only if Expression (24) is positive.

Because  $g''(\cdot) \leq 0$  by assumption and  $\Gamma^t \geq T^t$  because of retroactivity, the term in the first line is negative. So does the term in the second line for the same reasons. The sign of the third line is undetermined. In the fourth line, the expression in  $P$  is positive (see in the text), while the sign of  $E \left[ a. (Q_2^{k+1} - O_2^{k+1}) \right]$  is undetermined<sup>16</sup>.

Now, consider that  $g'(\cdot) = g$ . Then expression (24) becomes

$$\begin{aligned}
& E \left[ a.e^{-r} . \left( E \left[ \alpha^{t+1} g'(\alpha^{t+1} . \Gamma^{t+1}) \right] . \sum_{j=1}^N \Lambda_j^{t+1} - \Lambda_i^{t+1} \right) \right] \\
& + \frac{(1 + \beta)}{[1 - \pi^t(1 + \beta)]} E \left[ P_1^k \right] + E \left[ a. (Q_2^{k+1} - O_2^{k+1}) \right]
\end{aligned}$$

It is positive, thus yielding  $z_i^t \geq x_i^t$  if and only if  $\frac{(1+\beta)}{1-\pi^t(1+\beta)} \geq -\frac{E[a.(Q_2^{k+1}-O_2^{k+1})+e^{-r}.(E[\alpha^{t+1}g].\sum_{j=1}^N\Lambda_j^{t+1}-\Lambda_i^{t+1})]}{E[P_1^k]}$ .

Q.E.D. ■

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<sup>16</sup>If we consider identical agents, then we can show that  $E \left[ a. (Q_2^{k+1} - O_2^{k+1}) \right]$  is positive. Nevertheless, such an assumption would weaken our approach. Recall that we want to consider the case where agents are heterogenous, so that the regulator does not know who is lying and who is honest within the population.

### Proof of Proposition 3

By denoting  $\Gamma^{t,h}$  (respectively  $\Gamma^{t,e}$ ) the collected taxes and penalties in the case where Agent  $i$  is honest (respectively evades), he evades if and only if:

$$E [g(\alpha^t \Gamma^{t,h})] < \tau \cdot \Delta_i^t + E [g(\alpha^t \Gamma^{t,e})] - \pi^t \tau (1 + \beta) (\Delta_i^t + \Lambda_i^t)$$

$\Leftrightarrow$

$$E [g(\alpha^t \Gamma^{t,h})] - E [g(\alpha^t \Gamma^{t,e})] < \tau \cdot \Delta_i^t - \pi^t \tau (1 + \beta) (\Delta_i^t + \Lambda_i^t) \quad (25)$$

We have to compare this situation to the one without retroactive penalty. In a simple system, Agent  $i$  evades if and only if (for the comparison we use  $x_i^t \equiv z_i^t$ ):

$$E [g(\alpha^t T^{t,h})] - E [g(\alpha^t T^{t,e})] < \tau \cdot \Delta_i^t - \pi^t \tau (1 + \beta) \cdot \Delta_i^t \quad (26)$$

Recall that  $\Gamma^{t,h} = T^{t,h} + \pi^t \tau (1 + \beta) \sum_{j=1}^N \Lambda_j^t$  (and similarly for  $\Gamma^{t,e} = T^{t,e}$ ). A limited development of each term in  $\Gamma^{t,\cdot}$  in the neighborhood of  $T^{t,\cdot}$  and a rearrangement of (25) leads to the fact that Agent  $i$  evades in a system with retroactive penalty if and only if:

$$\begin{aligned} & E [g(\alpha^t T^{t,h})] - E [g(\alpha^t T^{t,e})] \\ & < \tau \Delta_i^t - \pi^t \tau (1 + \beta) (\Delta_i^t + \Lambda_i^t) \\ & \quad + E \left[ \alpha^t \pi^t \tau (1 + \beta) \cdot (g'(\alpha^t T^{t,e}) - g'(\alpha^t T^{t,h})) \cdot \sum_{j=1}^N \Lambda_j^t \right] \end{aligned} \quad (27)$$

The left-hand-side term in (26) is identical to the one of (27). The right-hand-side term in (27) differs from the one of (26) by the term

$$\begin{aligned} & \pi^t \tau (1 + \beta) \left\{ E \left[ \alpha^t \cdot (g'(\alpha^t T^{t,e}) - g'(\alpha^t T^{t,h})) \cdot \sum_{j \neq i}^N \Lambda_j^t \right] - \Lambda_i^t + E [\alpha^t \cdot \Lambda_i^t \cdot g'(\alpha^t T^{t,e})] \right\} \\ = & \pi^t \tau (1 + \beta) \left\{ E \left[ \alpha^t \cdot (g'(\alpha^t T^{t,e}) - g'(\alpha^t T^{t,h})) \cdot \sum_{j \neq i}^N \Lambda_j^t \right] + \Lambda_i^t \cdot (E [\alpha^t \cdot g'(\alpha^t T^{t,e})] - \pi^t \tau (1 + \beta) \Delta_i^t) \right\} \end{aligned} \quad (28)$$

From the (next) section 4, the second term is always negative. The first one is positive because  $g'(\cdot) < 0$  and  $T^{t,e} < T^{t,h}$ . If  $g'(\alpha^t T^{t,e}) - g'(\alpha^t T^{t,h})$  is not too high, Expression (28) may be negative so that retroactivity lessens the interest of Agent  $i$  to evade. As

a limit case, if the agent considers  $g'(\cdot)$  as a constant, the first term disappears and he always evades less frequently in the retroactive system than in a simple one.

Lastly, if all other agents are honest in period  $t$ , then  $\sum_{j \neq i}^N \Lambda_j^t = 0$ , and the first term in (28) disappears, so that the right-hand-side term in (27) is lower than the one in (26). This yields the second result in Proposition 3. Q.E.D.

## References

- Allingham, M.G. and A. Sandmo**, “Income Tax Evasion: A Theoretical Analysis,” *Journal of Public Economics*, 1972, 1, 323–338.
- Alm, J., B.R. Jackson, and M. McKee**, “Estimating the Determinants of Taxpayer Compliance with Experimental Data,” *National Tax Journal*, 1992, 45, 107–114.
- Collins, J.H. and R.D. Plumlee**, “The Taxpayer’s Labor and Reporting Decision: The Effect of Audit Schemes,” *The Accounting Review*, 1991, 66, 559–576.
- Feinstein, J.S.**, “An Econometric Analysis of Income Tax Evasion and its Detection,” *Rand Journal of Economics*, 1991, 22, 14–35.
- Greenberg, J.**, “Avoiding Tax Avoidance: A (Repeated) Game-Theoretic Approach,” *Journal of Economic Theory*, 1984, 32, 1–13.
- Harrington, W.**, “Enforcement Leverage when Penalties are restricted,” *Journal of Public Economics*, 1988, 37, 29–53.
- Jung, Y.H., A. Snow, and G.A. Trandel**, “Tax Evasion and the Size of the Underground Economy,” *Journal of Public Economics*, 1994, 54, 391–402.
- Landsberger, M. and I. Meilijson**, “Incentive Generating State Dependent Penalty System,” *Journal of Public Economics*, 1982, 19, 333–352.
- Mookherjee, D. and I. Png**, “Enforcement Costs and the Optimal Progressivity of Income Taxes,” *Journal of Law, Economics and Organization*, 1990, 6, 411–431.



- **and** –, “Monitoring vis-a-vis Investigation in Enforcement of Law,” *American Economic Review*, 1992, 82, 556–565.
- Picard, P.**, “Auditing Claims in Insurance Market with Fraud : the Credibility Issue,” *Journal of Public Economics*, 1996, 63, 411–431.
- , “On the Design of Optimal Insurance Policies under Manipulation of Audit Cost,” *International Economic Review*, 2000, 41, 1049–1071.
- Rickard, J.A., A.M. Russel, and T.D. Howroyd**, “A Tax Evasion Model with Allowance for Retroactive Penalties,” *The Economic Record*, 1982, 58, 379–385.
- Srinivasan, T.N.**, “Tax Evasion: A Model,” *Journal of Public Economics*, 1973, 2, 339–346.
- Witte, R.D. and D.F. Woodbury**, “The Effects of Tax Laws and Tax Administration on Tax Compliance: The Case of the US Individual Income Tax,” *Quarterly Review of Economics and Business*, 1977, 17, 21–31.
- Yitzhaki, S.**, “A Note on Income Tax Evasion: A Theoretical Analysis,” *Journal of Public Economics*, 1974, 3, 201–202.

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