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Contracts to Insure Catastrophe Risk :
Implications for Crop Risk Management »

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**USING PARTICIPATING AND FINANCIAL CONTRACTS TO INSURE CATASTROPHE RISK:
IMPLICATIONS FOR CROP RISK MANAGEMENT**

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Abstract

This paper proposes a combination of participating and financial contracts in order to hedge catastrophic risk. Assuming unfair policies and the existence of a basis risk, we prove the optimal coverage is realized using: first, a participating contract, which covers the idiosyncratic part of the risk under a variable premium; second, a financial contract, which hedges the systemic part of the risk under a fixed premium. The necessary intermediation of insurance companies in the conception of such contracts is emphasized as well as the impact of unfair premia. From then, potential implications for crop risk management are examined.

Keywords

Catastrophe risk, Crop insurance, Optimal hedging, Securitization

Résumé

Cet article propose une combinaison de contrats d'assurance participatifs et financiers dans le but de couvrir un risque catastrophique. En supposant que les polices d'assurances ne pas surévaluées, nous prouvons que la couverture optimale est réalisée en deux étapes : dans un premier temps, en souscrivant un contrat d'assurance participatif qui couvre la composante individuelle du risque avec une prime ajustable ; dans un second temps, en souscrivant un contrat financier qui couvre la composante systémique du risque avec une prime fixe. Le modèle démontre la nécessaire intermédiation des compagnies d'assurance dans la conception de tels contrats et évalue l'impact des primes surtarifées. Dès lors, des applications potentielles pour le développement de l'assurance récolte sont examinées.

Mots-clefs

Assurance agricole, Catastrophes naturelles, Couverture optimale, Sécurisation

1 Introduction

In recent years, more and more developed countries have modernized their insurance system against natural events, especially in the agricultural sector, by redistributing the roles of the main actors involved in catastrophe insurance: the States, the (re)insurers and the farmers. Historically, the States used to manage a catastrophe fund in order to face the different catastrophes. In theory, their guarantee is unlimited but for budgetary constraints, the indemnifications are restricted in practice. Moreover, catastrophes are more and more frequent and damageable (Munich Re Group, 2006). By encouraging and controlling private insurance, the States limit their implication and try to improve efficiency in the coverage of catastrophe events.

Encouraged by a favourable legislative environment, private or mutual insurers tend to offer more and more catastrophic coverage. In fact, the potential market is considerable and they have an intermediation role to play. However, there exist several limits to the ability of private insurance and reinsurance to fund catastrophic losses. The two first critics are upon the financial reserve the insurers must cup with (Jaffee and Russell, 1997) and agency conflicts (Froot, 2000). Although our model doesn't focus on these two sources of inefficiency, it offers ways to reduce them. The existence of transaction and administrative costs also limit the efficiency of the different contracts and we introduce them in our formulation.

Marshall (1974) noticed that a mechanism of risk sharing exists in which individuals are direct actors. This mechanism makes possible an allocation of collective risk, mutual participating contract: *"Mutual and participating stock insurance companies issue contracts which include, besides the obligation to indemnify loss, a dividend to the consumer which depends on the overall performance of the company"* (pp. 483). A participating insurance policy is a policy in which the insured completely covers the idiosyncratic component of his individual risk but he receives a dividend or respectively pays an extra premium, if the aggregation of the insurers' contracts is profitable, respectively insufficient. Although individuals covered by this kind of contract gain

the same "dividend", and not a dividend proportional to their risk tolerance, this kind of mechanism seems to be able to yield a more efficient repartition of the risk than a single insurance contract that only permits to diversify individual risk. This hypothesis is explored and validated by Dionne and Doherty (1993) and their model includes both individual and social (or systemic) risk. Thus, insurers can propose contracts determined according to the individual risk only or that include a dividend conditioned to the realization of a particular social state. In this case, resource allocation is Pareto-superior.

In his famous article, Raviv (1979) demonstrates that when losses are correlated, which is typically the case during a natural event, the optimal design of an insurance contract is based on a risk decomposition in two elements: a systemic risk (not diversifiable) and an individual or idiosyncratic risk (diversifiable). This distinction leads to risk sharing through mutualisation: the former component becomes completely insurable, so it is covered. Then, the latter has to be hedged through securitization. Arrow (1996) underlines the fact that risk transfer contracts observed in the real world mainly cover individual risks: each individual agent doesn't want to be risk-bearer and this function rests upon insurance companies.

Risk securitization is an alternative to insurance because it allows the insurer to transfer an excess risk to financial markets. It is a useful tool to break up risky contracts into less risky ones, rather than to deal with its totality. Doherty and Schlesinger (2002) show that this distinction yields an increase in the policyholders' welfare. Then, the idea is to substitute a financial contract to the "classical" non-participating contract in order to better hedge the systemic risk. Mahul (2001 and 2002) develops this reasoning and proves that the introduction of financial contracts is a market-enhancing instrument. However, most of financial contracts are elaborated at a "global" level, so that the stakeholder is subject to an important basis risk due to the imperfect correlation between the index and its real losses. Added to unfair premia, this may explain why such contracts are not

subscribed if they are not subsidized. Our model considers the introduction of a basis risk in order to measure its influence on potential losses.

Participating policies are nowadays used in cars and health insurance. In France, since 2003, the decrease in road accidents due to reinforced policy controls was quickly passed on the premium level of mutual benefit companies. In December 2006, a French insurer (*Mutuelles du Mans Assurances*) proposed a specific participating health insurance: with a 15% premium increase compared to standard contracts, the contribution is divided in two parts. The insurer collects the first one whereas the second one is put apart and operates as a reimbursable reserve. During the following year, if health expenses are low or null, then the insurer reimburses all or part of this reserve. By this way, the solidarity principle is corrected by the individual risk but, by construction, this kind of contract is made for low-risk individuals or non-catastrophic losses. In the agricultural sector, the States progressively decided to replace their global catastrophic coverage funds by an individual private and subsidized insurance. These contracts are generally restricted to catastrophic events, so their coverage subject to a basis risk is not incentive.

Then, the purpose of this article is to examine whether financial markets and their interactions with the insurance markets can help better absorb correlated risk. In this article, we refer to the models of Doherty and Schlesinger (2002) and Mahul (2002) who distinguish idiosyncratic and systemic risks and introduce participating and non-participating contracts. We choose to extend Mahul's one because its structure allows determining values for the indemnities and premia of the two kinds of contracts. Indeed, we replace non-participating contracts with financial ones that are in fact exchanged on real markets. Moreover, we introduce a basis risk associated to the financial indexes to capture their relative inefficiency. We also consider unfair insurance and financial contracts in order to examine the consequences of additional loading factors on the coverage efficiency. These two factors may explain why crop insurance is not subscribed nowadays unless it is highly supported by public funds. Finally, we prove that the combination of participating and

financial contracts is optimal to cover both individual and catastrophic losses and requires the intermediation of insurance companies.

The paper is organized as follows. First, we develop our model and show its validity to solve our problems compared to other approaches. Then, the optimum design of insurance contracts is calculated with a generalization of Mahul (2002) relaxing the assumption of fair insurance. In particular, we introduce in our analysis positive loading factors and financial indexes closely correlated with the systemic risk but subject to a basis risk. Then, we prove that the combination of both participating and financial contracts offers an optimal coverage that eliminates the basis risk and provides a sustainable solution for the insurer and the stakeholder. Implications for crop risk insurance contracts are then examined.

2 The model

2.1 General notations

The model is developed within the framework of the expected utility theory. A risk-averse firm has an initial non-random welfare w_0 subject to a risk of loss $\tilde{l} \in [0, w_0]^2$. We assume this loss can be separated into two components: an individual one, \tilde{x} and a systemic one, $\tilde{\varepsilon}$. Thus, we have:

$$\tilde{l} = l(\tilde{x}, \tilde{\varepsilon}), \text{ with } \tilde{x} \geq 0, \tilde{\varepsilon} \geq 0, l_x \geq 0, l_\varepsilon \geq 0 \text{ and } E(\tilde{\varepsilon}) = 0. \quad (1)$$

We also assume that \tilde{l} , \tilde{x} and $\tilde{\varepsilon}$ are commonly identified by everyone. Among the stakeholders, \tilde{x}_i are independent and are not correlated with the common risk $\tilde{\varepsilon}$.

For example, let's consider a pool in which all the members are located in a floodable watershed. The \tilde{l} risk is then defined as the individual exposure to flood risk. The $\tilde{\varepsilon}$ risk would be the common uncertainty that affects all the members of the pool, *i.e.* flood intensity. Over the years,

² The \sim symbol indicates random variables whereas variables without tilde are realizations of random variables.

$\tilde{\varepsilon}$ is very often equal to zero as no flood event occurs but in some rare occasions, $\tilde{\varepsilon}$ is widely positive. The \tilde{x} risk would be the impact of local parameters on the individual losses, which can be considered as independent among the members of the risk pool. The \tilde{x} risk seems to be partially diversifiable at the risk pool level provided the size of the insurer's portfolio is sufficiently large so that the law of large numbers applies. However, it's not the case for the $\tilde{\varepsilon}$ risk unless this risk is spread with other groups exposed to different catastrophes or transferred to financial markets.

2.2 Form of the loss function

The form of this diversification is closely linked to the form of the loss function $l = l(x, \varepsilon)$. We consider two standard cases: additive and multiplicative loss functions. This distinction is economically justified because the additive loss function assumes that a given catastrophic event corresponds to an equal amount of catastrophic loss ε for each firm, whatever its initial risk. Considering a multiplicative loss function takes into account the "initial" idiosyncratic risk so that a catastrophe engenders proportional losses to individual risk.

We first look at the additive form: $l = l(x, \varepsilon) = x + \varepsilon$. In this case, we assume that losses can be decomposed into two additive risks. For example, $\varepsilon = 10$ implies that all individual losses increase of 10 monetary units due to the occurrence of a catastrophe. Then, Mahul (2001) shows the insurance coverage can be replicated by the combination of a "classical" non-participating insurance policy and a futures contract to cover the systemic component of the risk. In theory, the stakeholders can realize this combination by themselves, which is potentially interesting in order to reduce transaction costs using financial markets.

Let's consider now a multiplicative form for \tilde{l} : $l = l(x, \varepsilon) = x(1 + \varepsilon)$. For example, $\varepsilon = 0.10$ implies that all individual losses increase by 10% because of the occurrence of a catastrophe. In this case, Doherty and Schlesinger (2002) indicate that the intervention of an insurer is necessary.

Securitization comes from the pooling of the individual risks into the insurer's portfolio. The optimal design is then the combination a variable participating contract for the stakeholder and a futures contract for the insurer in order to cover its aggregate risk of dividends.

2.3 The contracts

We consider that each individual or firm can construct a variable participating insurance policy by buying two kinds of contracts: a non-participating policy and a fully participating policy. Each policy is defined by a schedule, *i.e.* a premium and an indemnity.

By definition, the indemnity of the participating contract depends both on the idiosyncratic risk and on the realization ε of the systemic risk:

$$I(x, \varepsilon) \geq 0, \quad \forall x, \varepsilon \tag{2}$$

The premium is variable and depends on the occurrence of systemic risk. The individual risk is assumed to be insurable without any transaction cost and the insurer's portfolio is supposed to be large enough so that the law of large numbers and the mutualisation principle apply. The premium is then defined as the mathematical expectation of the systemic risk conditional indemnity, that is:

$$P(\varepsilon) = (1 + \theta_1) E(I(\tilde{x}, \varepsilon)), \quad \forall \varepsilon, \theta_1 \geq 0 \tag{3}$$

Under the assumption of risk-neutral insurers, the market price of the risk, as defined by Schlesinger (1999), is represented by the loading factor θ_1 . In fact, premium P is potentially subject to *ex-post* adjustments, which supposes there's no default risk for the policyholders. It can be fixed for catastrophes of mean size and revised at the end of the year to reflect the occurrence or the non-occurrence of an event. Thus, the systemic risk is not covered by this kind of contract, which is only able to insure the idiosyncratic component of the risk.

As in the former case, the indemnity of non-participating contract is written as a function of the idiosyncratic and systemic risks:

$$J(x, \varepsilon) \geq 0, \quad \forall x, \varepsilon \quad (4)$$

The premium is fixed *ex-ante* and defined by the mathematical expectation of the idiosyncratic and systemic risk conditional indemnity.

A loading factor θ_2 also applies because the premium is fixed *ex-ante*, that is:

$$Q = (1 + \theta_2) E(J(\tilde{x}, \tilde{\varepsilon})), \quad \theta_2 \geq 0 \quad (5)$$

It includes transaction costs in the calculus of the premium in addition to damage expected expenditures.

Classically, the fixed premium of the non-participating contract is a tool to insure the systemic risk of a firm. This implies that the insurer accepts to bear risk. To secure its contract and cover the ε risk, it is possible to buy financial contract based on catastrophe indexes.

2.4 Index and Basis Risk

To link catastrophe events and economic losses, indexes have been computed in the major financial centres, such as CBOT in Chicago or LIFFE in London. 90% of weather derivatives have an underlying asset based on temperature. The most famous are the heating/cooling degree-days³, based on cumulative temperatures. There also exist derivatives based on rainfall but their market is still confidential. These indices are assumed to be closely related to climatic catastrophic losses so that the systemic component $\tilde{\varepsilon}$ can be rewritten as a function of a loss

³ A degree-day gauges the amount of heating or cooling needed for a building using 65-Fahrenheit degrees as a baseline. To compute heating/cooling degree-days, take the average temperature for a day and subtract the reference temperature of 65-Fahrenheit degrees. If the difference is positive, it is called a "Cooling Degree Days". If the difference is negative, it is called a "Heating Degree Days". The magnitude of the difference is the number of days and this information is utilized to calculate the individual needs.

index called \tilde{z} . This index is generally computed at a global scale so that it's not perfectly correlated to the individual systemic component of risk $\tilde{\varepsilon}$. Thus, each financial contract written on \tilde{z} is exposed to a basis risk \tilde{b} whose consequences are examined in the paper.

For simplifying reasons, we assume we can write $\tilde{\varepsilon}$ as:

$$\tilde{\varepsilon} = \tilde{\varepsilon}(\tilde{z}, \tilde{b}) = \psi + \varphi \tilde{z} + \tilde{b}, \text{ with } \psi \geq 0, \varphi \geq 0, E(\tilde{b}) = 0 \quad (6)$$

The basis risk, \tilde{b} , is assumed to be independent of the index \tilde{z} , the risk pool's systemic component $\tilde{\varepsilon}$ and the idiosyncratic risk \tilde{x} . For example, \tilde{x} denotes the aggregate loss of a regional pool and the \tilde{z} index represents the national aggregate losses. In order to simplify the notations and without loss of generality, we assume now that $\psi = 0$ and $\varphi = 1$.

Using the properties of \tilde{b} in (6) leads to the following equality:

$$E(\tilde{\varepsilon}) = E(\tilde{z}) \quad (6')$$

Then, we replace $\tilde{\varepsilon}$ by \tilde{z} and the indemnity depends now on the catastrophe index, under the basis risk \tilde{b} .

$$J(x, \varepsilon) = J(x, \varepsilon(z, b)) \equiv J(x, z, b) \geq 0, \quad \forall x, z, b \quad (7)$$

Of course, the premium remains unchanged. The loading factor θ_2 includes administrative costs and the insurer's risk aversion against the basis risk \tilde{b} .

$$Q = (1 + \theta_2) E(J(\tilde{x}, \tilde{z}, \tilde{b})), \quad \theta_2 \geq 0 \quad (8)$$

Finally, the combination of a participating and a non-participating insurance policy, called variable participating policy, is sold at price $[P(\varepsilon)+Q]$ and procures an indemnity equal to $[I(x, \varepsilon)+J(x, z, b)]$ when the effective values of the individual and systemic components and of the index are respectively x , ε , z and b .

We can write final wealth as:

$$\tilde{w} = w_0 - l(\tilde{x}, \tilde{\varepsilon}) + I(\tilde{x}, \tilde{\varepsilon}) - P(\tilde{\varepsilon}) + J(\tilde{x}, \tilde{z}, \tilde{b}) - Q \quad (9)$$

The stakeholder has a twice-differentiable von Neumann-Morgenstern utility function $u(\cdot)$, with $u' > 0$ and $u'' < 0$.

The problem of the risk-averse firm is to determine the optimal indemnity and the premium of both participating and non-participating policies that maximize the expected utility of its final wealth under the constraints (2), (3), (7) and (8):

$$\underset{I, J, P, Q}{Max} Eu(w_0 - l(\tilde{x}, \tilde{\varepsilon}) + I(\tilde{x}, \tilde{\varepsilon}) - P(\tilde{\varepsilon}) + J(\tilde{x}, \tilde{z}, \tilde{b}) - Q) \quad (10)$$

3 Validation of the model compared to standard literature

The model we propose is designed to recover classical problems about participating contracts. Proposition 1 demonstrates the possible equivalence between our model's formulation and Doherty and Schlesinger's (2002) one⁴.

⁴ In this proposition, we only look at the formulation of Doherty and Schlesinger's (2002) because their paper is not developed under the framework of expected utility. Thus, our model is not a direct extension of theirs.

Proposition 1: Under a multiplicative loss function $l(\tilde{x}, \tilde{\varepsilon}) = (1 + \tilde{\varepsilon})\tilde{x}$ and fair premia, resolving equation 10: $\text{Max}_{I, J, P, Q} Eu(w_0 - l(\tilde{x}, \tilde{\varepsilon}) + I(\tilde{x}, \tilde{\varepsilon}) - P(\tilde{\varepsilon}) + J(\tilde{x}, \tilde{\varepsilon}) - Q)$ is equivalent to resolving the following problem: $\text{Max}_{\alpha, \beta} Eu(w_0 - T(x, \varepsilon) - (1 - \alpha)(1 + \varepsilon)x)$, with $T(x, \varepsilon) = \alpha E(x)[1 + (1 - \beta)\varepsilon]$.

Where: α is the proportion of loss indemnified by the insurer and β is a stakeholder choice variable denoting the degree of participation, with $\beta = 1$ denoting a fixed premium and $\beta = 0$ denoting full participation.

Proof is given in Appendix 1. Our model proposes to determine four optimal values for the premia and indemnities of the two separate participating and non-participating contracts. On the contrary, traditional approaches tend to determine the proportions of indemnified losses and the degree of participation, which leads to coinsurance. The principles of the two formulations are also different because our model maximizes the difference between initial wealth, the individual loss plus the result of the coverage (indemnities minus premia) while Doherty and Schlesinger's maximizes the difference between initial wealth and the premia paid plus non-covered losses. Our model appears to be more general because determining I, J, P and Q yields α and β . With some adaptations, we demonstrate the possible equivalence between the formulations. We also provide an extension of Mahul (2002) as the values of the indemnities and premia of the participating and non-participating contracts are not expressed in the same way.

4 Optimal insurance contracts design

Proposition 2 shows the design of optimal fully participating and non-participating contracts based on the use of financial markets.

Proposition 2: *The optimal indemnity of fully participating and non-participating contracts I^* and J^* , solutions to problem (10), take the form:*

- (i) *If $(1+\theta_1)E(I(\tilde{x},\tilde{\varepsilon})) < (1+\theta_2)E(J(\tilde{x},\tilde{\varepsilon}))$, then there exist $D_1 \geq 0$ and $D_2 \geq 0$ such that*

$$J^*(x, z, b) \equiv J^*(x, \varepsilon) = \text{Max}(P(\varepsilon) - D_2; 0) \text{ and } I^*(x, \varepsilon) = \text{Max}(I(x, \varepsilon) - D_1; 0).$$
- (ii) *If $(1+\theta_1)E(I(\tilde{x},\tilde{\varepsilon})) > (1+\theta_2)E(J(\tilde{x},\tilde{\varepsilon}))$, then there exist $D_3 \geq 0$ such that*

$$J^*(x, z, b) \equiv J^*(x, \varepsilon) = \text{Max}(I(x, \varepsilon) - D_3; 0) \text{ and } I^*(x, \varepsilon) = 0.$$

Proof is given in Appendix 2. The main point to consider before pricing the different contracts is about the existence of the participating contract as the non-participating contract is systematically used to cover the systemic risk. We readapt Mahul (2002) introducing transaction costs on both contracts and a financial contract instead of a standard non-participating contract.

In fact, the price of the participating contract may be lower than the one of the non-participating contract, and this contract would then exist as stated in point (i). It may occur if the administrative costs plus the risk premium of the non-participating policy are above the administrative costs plus the risk premium of the participating policy. In fact, the price of the non-participating policy closely depends on the capacity of the insurer to share the systemic risk, which justifies point (i). This can be done by securitization on the financial markets, taking into account a basis risk. By definition, this risk is not diversifiable and it is passed to the stakeholders through a large premium rate. Thus, following Arrow (1974) and Raviv (1979), the rationale is to buy a participating contract to cover the idiosyncratic risk above a deductible D_1 , which filters the systemic risk. Then, the former is fully covered by a non-participating contract above a deductible D_2 . The important thing to notice is that the indemnity of the non-participating contract depends on the premium of the participating contract.

In practice, the insurers prefer to anticipate the occurrence of a catastrophe and then they artificially increase the premium in order to avoid a possible ex-post default from the stakeholders. If no event occurs, then a dividend is distributed. This mechanism increases *ex-ante* the cost of subscribing a participating contract, even if the probability to recover money is not negligible, which leads to point (ii): this kind of contract is not subscribed and insurance is displayed only with the non-participating contract. This case is standard in the literature (and in practice). Then, the stakeholder is fully covered above a deductible D_3 .

We only look at the case when a participating policy is subscribed and Proposition 3 characterizes the optimal level of deductibility.

Proposition 3: *Supposing that participating contracts exist, i.e. $(1 + \theta_1)EI(\tilde{x}, \tilde{\varepsilon}) < (1 + \theta_2)EJ(\tilde{x}, \tilde{\varepsilon})$:*

- (i) *The optimal deductible D_1 of the participating policy equals zero if the premium is actuarially fair, i.e. $\theta_1 = 0$, whereas it is positive if the premium is unfair, i.e. $\theta_1 > 0$.*
- (ii) *The optimal deductible D_2 of the non-participating policy equals zero if the premium is actuarially fair, i.e. $\theta_2 = 0$, whereas it is positive if the premium is unfair, i.e. $\theta_2 > 0$.*

Proof is given in Appendix 3. This result is conforming to standard insurance literature. With fair premia, the deductible and the loading factor are null and with unfair premia, the two are strictly positive. Propositions 2 and 3 characterize the optimal insurance strategy with participating and non-participating contracts.

Finally, both participating and non-participating policies can be combined to construct what is usually called a variable participating insurance contract, whose indemnity and premium are respectively:

$$A(x, \varepsilon(z)) = I(x, \varepsilon) + J(x, z, b) \quad (11)$$

$$B(\varepsilon) = P(\varepsilon) + Q \quad (12)$$

This differs from Mahul (2002) because the indemnity of the non-participating policy depends on a financial index instead of pure systemic risk. Inserting the optimal values of I , J , P and Q in (11) and (12) gives:

$$A(x, \varepsilon) = \text{Max}[l(x, \varepsilon) - D_1; 0] + \text{Max}[P(\tilde{\varepsilon}) - D_2; 0] \quad (13)$$

$$B(\varepsilon) = (1 + \theta_2)E(\text{Max}[l(\tilde{x}, \varepsilon) - D_1; 0]) + (1 + \theta_2)E(\text{Max}[P(\tilde{\varepsilon}) - D_2; 0]) \quad (14)$$

The financial index doesn't clearly appear in the former expressions because it is hidden by the ε value. The optimal strategy of coverage with a variable participating contract permits to see the interest of such formulation.

5 Optimal coverage strategy

5.1 Back to real market, introducing index-based securities

In practice, two types of contracts are sold on real world markets, a participating one and a non-participating one exclusively based on a financial index z closely correlated to the pure systemic risk ε . Thus, the indemnity of the financial contract is written as follows:

$$K(z) \geq 0, \quad \forall z \quad (15)$$

Remembering that we have assumed for $\tilde{\varepsilon} : \tilde{\varepsilon} = \tilde{z} + \tilde{b}$ and $E(\tilde{b}) = 0$.

Keeping the loading factor θ_2 , which still includes transaction and administrative costs and the insurer's risk aversion against the financial risk \tilde{b} , the fixed premium Q becomes:

$$Q = (1 + \theta_2) E(K(\tilde{z})) \quad (16)$$

In the former expression, the lack of the idiosyncratic risk in the equations (15) and (16) is justified by its coverage using a participating contract.

Using Proposition 2 and 3 for the participating policy, the indemnity and the premium of contracts sold on real markets become:

$$I^*(x, \varepsilon) = \text{Max}(I(x, \varepsilon) - D_1; 0) \quad (17)$$

$$P(\varepsilon) = (1 + \theta_1) E(\text{Max}(I(\tilde{x}, \varepsilon) - D_1; 0)) \quad (18)$$

With: $D_1 = 0$ if $\theta_1 = 0$ and $D_1 > 0$ otherwise.

For the non-participating policy, we similarly obtain:

$$K^*(z) = \text{Max}(P(\varepsilon) - D_2; 0) \quad (19)$$

$$Q = (1 + \theta_2) E(\text{Max}(P(\tilde{\varepsilon}) - D_2; 0)) \quad (20)$$

With: $D_2 = 0$ if $\theta_2 = 0$ and $D_2 > 0$ otherwise.

The aim is now to examine the optimal strategy of coverage, using first a participating contract, second a non-participating contract, and third the combination of both types of contracts.

5.2 The participating contract

In this section, we still consider premia are unfair and consequently $\theta_l = 0$. First selecting the additive form of the loss function, *i.e.* $l = l(x, \varepsilon) = x + \varepsilon$, equations (17) and (18) relative to the participating contract become:

$$I^*(x, \varepsilon) = x + \varepsilon - D_1 \quad (21)$$

$$P(\varepsilon) = (1 + \theta_1) (E(\tilde{x}) + \varepsilon - D_1) \quad (22)$$

The stakeholder's final loss is then equal to yield loss plus the difference between the premium and the indemnity of the participating contract:

$$\begin{aligned} Loss_{PC}^+ &= l(x, \varepsilon) + P(\varepsilon) - I(x, \varepsilon) = x + \varepsilon + P(\varepsilon) - \text{Max}[x + \varepsilon - D_1; 0] \\ &= \begin{cases} x + \varepsilon + P(\varepsilon) & \text{if } x \leq D_1 - \varepsilon \\ D_1 + P(\varepsilon) & \text{if } x \geq D_1 - \varepsilon \end{cases} \end{aligned} \quad (23)$$

Similarly, selecting the multiplicative form of the loss-function, *i.e.* $l = l(x, \varepsilon) = x(1 + \varepsilon)$, equations (17) and (18) become:

$$I^*(x, \varepsilon) = x(1 + \varepsilon) - D_1 \quad (24)$$

$$P(\varepsilon) = (1 + \theta_1) (E(\tilde{x})(1 + \varepsilon) - D_1) \quad (25)$$

The stakeholder's final loss is then equal to yield loss plus the difference between the premium and the indemnity of the participating contract:

$$\begin{aligned}
Loss_{PC}^x &= l(x, \varepsilon) + P(\varepsilon) - I(x, \varepsilon) = x(1 + \varepsilon) + P(\varepsilon) - \text{Max}[x(1 + \varepsilon) - D_1; 0] \\
&= (1 + \varepsilon) \left(x - \text{Max}[x - D_1 / (1 + \varepsilon); 0] \right) + P(\varepsilon) \\
&= \begin{cases} x(1 + \varepsilon) + P(\varepsilon) & \text{if } x \leq D_1 / (1 + \varepsilon) \\ D_1 + P(\varepsilon) & \text{if } x \geq D_1 / (1 + \varepsilon) \end{cases}
\end{aligned} \tag{26}$$

Under the subscription of a participating contract, the policyholder's loss always depends on the systemic component ε but for large idiosyncratic losses, *i.e.* $x \geq D_1 - \varepsilon$ for additive losses and $x \geq D_1 / (1 + \varepsilon)$ for multiplicative losses, we observe that it only depends on ε . Thus, the participating contract offers a perfect coverage against the idiosyncratic risk but it completely filters the systemic risk, which is not covered at all.

Proposition 4 gives loss value obtained after the subscription of a participating contract.

Proposition 4: *Eliciting $P(\varepsilon)$ under the assumption that the contract is not sold at a fair price, *i.e.* $\theta_1 > 0$ and consequently $D_1 > 0$ (Proposition 3-i), we get:*

- (i) $Loss_{PC}^x = E(\tilde{x}) + \varepsilon + \theta_1 (E(\tilde{x}) + \varepsilon - D_1)$
- (ii) $Loss_{PC}^x = E(\tilde{x})(1 + \varepsilon) + \theta_1 (E(\tilde{x})(1 + \varepsilon) - D_1)$

Proof is given in Appendix 4. If the pricing were fair, the stakeholder would have been fully protected against its individual risk whereas the systemic risk would have not been insured. However, the existence of θ_1 increases the policyholder's loss proportionally to insured losses, *i.e.* taking into account the deductible. This is a major source of inefficiency, whose consequences are cumulative when combining different contracts, as we will see later.

With fair premia, Proposition 4 can be rewritten as follows:

Corollary 4: *Eliciting $P(\varepsilon)$ under the assumption that the contract is sold at a fair price, i.e. $\theta_1=0$ and consequently $D_1 = 0$ (Proposition 3-i), we get:*

$$(i) \quad Loss_{PC}^+ = E(\tilde{x}) + \varepsilon$$

$$(ii) \quad Loss_{PC}^\times = E(\tilde{x})(1 + \varepsilon)$$

Proof is trivial. In this particular case, we clearly see that the non-participating coverage is only used to protect against the systemic risk. We examine now the optimal strategy of coverage against the systemic risk using a financial non-participating contract.

5.3 The financial non-participating contract

The interest is now to focus on the replication of the financial non-participating contract, which is given by Proposition 5. The impact of the existence of loading factors is taken into account.

Proposition 5:

(i) *The optimal non-participating contract can be replicated by purchasing $(1 + \theta_1)$ call options at a strike price equal to $E(\tilde{x}) - \frac{D_1(1 + \theta_1) + D_2}{1 + \theta_1}$, subject to the basis risk \tilde{b} , for an additive loss function.*

(ii) *The optimal non-participating contract can be replicated by purchasing a number of $(1 + \theta_1)E(\tilde{x})$ call options at a strike price equal to $1 - \frac{D_1(1 + \theta_1) + D_2}{(1 + \theta_1)E(\tilde{x})}$, subject to the basis risk \tilde{b} , for a multiplicative loss function.*

Proof is given in Appendix 5.

By definition of z in (6) and of $K^*(z)$ in (19), the optimal replication strategy of the financial non-participating contract is given by:

- (i) $K^+(z) = (1 + \theta_1) \text{Max} \left[z + E(\tilde{x}) - \frac{D_1(1 + \theta_1) + D_2}{1 + \theta_1} + b; 0 \right]$, for an additive loss function.
- (ii) $K^\times(z) = (1 + \theta_1) E(\tilde{x}) \text{Max} \left[z + 1 - \frac{D_1(1 + \theta_1) + D_2}{(1 + \theta_1) E(\tilde{x})} + b; 0 \right]$, for a multiplicative loss function.

The presence of the hedge ratio $E(\tilde{x})$ in the second formula (for a multiplicative loss function) indicates that the policyholder is not able to replicate by himself the financial contract and needs the intermediation of an insurance company in order to construct its variable participating contract. This comes from the fact that the optimal hedge ratio should be equal to the random variable \tilde{x} but this stochastic value is not available on real-world financial markets. Then, the role of the insurance company is to eliminate this idiosyncratic risk through mutualisation and select a hedge ratio equal to the expectation of the different idiosyncratic risks of its portfolio.

Proposition 6 considers now the coverage of resulting loss after the subscription of a financial contract.

Proposition 6: *Using the property of z given by (6'), loss after the subscription of the financial non-participating contract can be optimally covered by:*

- (i) *Selling $(1 + \theta_1)$ unbiased futures contracts on z , subject to the basis risk b , for an additive loss function.*
- (ii) *Selling $(1 + \theta_1) E(\tilde{x})$ unbiased futures contracts on z , subject to the basis risk b , for a multiplicative loss function.*

Proof is given in Appendix 6.

Loss after the subscription of the financial non-participating contract is equal to:

(i) $Loss_{NPC}^+ = Q - K^*(z) = (1 + \theta_1) \{ [E(\tilde{z}) - z - b] + \theta_2 E[K^+(z)] \}$, for an additive loss function.

(ii) $Loss_{NPC}^\times = Q - K^*(z) = (1 + \theta_1) E(\tilde{x}) \{ [E(\tilde{z}) - z - b] + \theta_2 E[K^\times(z)] \}$, for a multiplicative loss function.

In each case, losses are increased by the loading ratio θ_2 multiplying the expectation of the indemnity $K(z)$. This corresponds to the lack of indemnification associated to the supplementary premium of the non-participating contract. Moreover, the existence of coefficient θ_1 multiplies total loss and proportionally increases the coverage cost. Thus, the combination of two different unfair contracts generates multiples additional costs for the policyholders, which can explain firms' defiance towards insurance.

Corollary 6 looks at the standard case, when the price of the financial contract is fair. It allows seeing more clearly some other implications of our model.

Corollary 6: *Assuming all contracts are sold at a fair price, i.e. $D_1 = D_2 = 0$ and $\theta_1 = \theta_2 = 0$, loss after the subscription of the financial non-participating contract can be optimally covered by:*

(i) *Selling one unbiased futures contracts on z , subject to the basis risk b , for an additive loss function.*

(ii) *Selling $E(\tilde{x})$ unbiased futures contracts on z , subject to the basis risk b , for a multiplicative loss function.*

Proof is trivial.

We obtain a quite standard result in the literature (Mahul, 2002), *i.e.* an unbiased coverage with futures contracts only subject to a basis risk:

$$(i) \quad Loss_{NPC}^+ = Q - K^*(z) = [E(\tilde{z}) - z - b], \text{ for an additive loss function.}$$

$$(ii) \quad Loss_{NPC}^\times = Q - K^*(z) = E(\tilde{x})[E(\tilde{z}) - z - b], \text{ for a multiplicative loss function.}$$

In other words, the efficiency of the coverage depends on the correlation between $E(\tilde{z})$ and ε . Moreover, the basis risk is proportional to the expectation of the idiosyncratic risk, for a multiplicative loss function.

5.4 The variable participating contract

As defined before, the variable participating contract is the combination of a participating contract and a non-participating contract. The strength of such a strategy is to get a more efficient coverage, as shown by Proposition and Corollary 7.

Proposition 7: *Using the property of z given by (6'), total loss after the subscription of the variable non-participating contract is equal to:*

$$(i) \quad Loss_{PC+NPC}^+ = (1 + \theta_1) \left[E(\tilde{x}) + E(\tilde{z}) + \theta_2 E(K^+(z)) - \frac{\theta_1}{1 + \theta_1} D_1 \right], \text{ for an additive loss function.}$$

$$(ii) \quad Loss_{PC+NPC}^\times = (1 + \theta_1) E(\tilde{x}) \left[1 + E(\tilde{z}) + \theta_2 E(K^\times(z)) - \frac{\theta_1}{(1 + \theta_1) E(\tilde{x})} D_1 \right], \text{ for a multiplicative loss function.}$$

Proof is given in Appendix 7. This result provides two major advantages of our combination. First, the variable participating contract neutralizes the basis risk generated by the use of financial products. Second, both idiosyncratic and systemic risks are covered and the initial loss is replaced by the expectations of the idiosyncratic and financial components. In counterpart, the systemic risk is replaced by a financial risk and there still remains heavy transaction costs.

Under usual assumptions, Corollary 7 offers a perfect coverage.

Corollary 7: *Assuming the financial contract is sold at a fair price, i.e. $D_1 = D_2 = 0$ and $\theta_1 = \theta_2 = 0$, total loss after the subscription of the variable participating contract is equal to:*

$$(i) \quad \text{Loss}_{PC+NPC}^+ = E(\tilde{x}) + E(\tilde{z}), \text{ for an additive loss function.}$$

$$(ii) \quad \text{Loss}_{PC+NPC}^\times = E(\tilde{x})[1 + E(\tilde{z})], \text{ for a multiplicative loss function.}$$

Proof is trivial. Referring to classical assumption adopted in the literature, this combination of the participating and the financial (non-participating) contracts creates a perfect unbiased coverage. In particular, one should notice there is no covariance term associated to a multiplicative loss function. This gives an argumentation in favour of the subscription of both participating and non-participating contracts by exposed stakeholders. Index-based securities exist and are frequently the only one subscribed despite the basis risk and the incomplete coverage of the idiosyncratic risk. Proposition 7 affirms the theoretical interest to use participating contracts in complement of index-based non-participating contracts.

Assuming fair premia, the standard result is:

$$(i) \quad l^+(x, \varepsilon) = x + \varepsilon + b \rightarrow E(\tilde{x}) + E(\tilde{z}) \tag{27}$$

$$(ii) \quad l^\times(x, \varepsilon) = x(1 + \varepsilon) = x(1 + z + b) \rightarrow E(\tilde{x})(1 + E(\tilde{z}))$$

Initial losses are transformed in an interesting way for the stakeholder, providing transaction costs are eliminated. The aim is now to study how this interesting result can be applied to crop insurance contracts.

6 Implications for crop insurance contracts

In developed countries, crop insurance contracts are more and more proposed to the farmers in substitution to global and/or emergency indemnification fund. Faced to structural deficits of their catastrophic funds, the USA reformed their system in 1996 with the "Fair Act" introducing an improperly named "revenue insurance", which is in reality a crop revenue insurance. The insurers propose different contracts whose premia are 60% subsidized so that 70% of the agricultural surfaces are now covered against climatic risks.

France decided to adopt a similar system in 2005 and extended it in 2006 and 2007 encouraging private insurance. Public subvention is fixed equal to 35% of the premia and it is coupled with a deductible equal to 25% of insured capital. A recent ministry report confirms 60% of the agricultural surfaces are now covered with crop insurance. In practice, such contracts are now designed to cover against severe weather events (drought, floods, *etc.*) but there are still restricted to field crops.

However, such systems face three main difficulties: First, the number of policyholders is not optimal because one third of the farms are not protected. It is both damageable for risk mutualisation and the premia level. Second, the states intervention is necessary to guarantee the solvency of this new kind of insurance. Third, there is the problem of reinsurance because it is almost known that the global market of agricultural insurance cannot face the amount of damages of a catastrophic year.

Our theoretical framework provides answers to these three majors limits of crop insurance systems. Let's consider losses take a multiplicative form, *i.e.* $l = l(x, \varepsilon) = x(1 + \varepsilon)$, with our usual notations. This realistic hypothesis means that losses increase by $\varepsilon\%$ due to the occurrence of a catastrophe⁵. Applied to agricultural crop insurance, $\tilde{\varepsilon}$ is yield shortfall caused by local weather

⁵ The reasoning that will follow can strictly be applied to additive losses with similar results as proved by Proposition 6.

events and \tilde{x} is long-term average individual crop loss based on crop price at harvest. $\tilde{\varepsilon}$ and \tilde{x} are supposed independent.

Facing crop yield risks, the farmer can subscribe both participating and non-participating contracts⁶. As proved with Proposition 2, he will select first the participating contract to cover only its idiosyncratic risk. Therefore, in our approach, the non-participating coverage is only used to protect against the systemic risk. We assume there exists an individual crop yield index⁷ noted z that is closely correlated to the systemic risk, as defined by equation (6). For example, this index can be based upon cumulated degree-days to cover yield shortfall after harvest, monthly precipitations to cover drought and daily precipitations to cover floods. In counterpart, there exist a basis risk \tilde{b} due to the imperfect linkage between the index and the reality, which remains if this contract is the only one subscribed (Proposition 6).

Proposition 5 showed that the optimal combination of both participating and financial contracts supposes an intermediation of the insurer. It is also an encouragement for insurers to propose crop insurance contracts based on financial products, which transfer the systemic risk to financial markets and contribute to resolve the reinsurance limitations. In fact, the variable participating contract appears to be the only one able to face the basis risk and to cover the different risks. Then, there are more incentives for farmers to cover their losses with such policies.

In addition, our analysis considers unfair insurance introducing loading factors. With Proposition 3, transaction costs and risk premia imply the existence of deductibles. Proposition 7 indicates that these three former elements increase final loss after indemnification. Moreover, the combination of contracts generates a multiplicative effect of the loading factors on this loss, which reduces the performance of the coverage. This weakness appears to be a justification of the States' intervention in crop insurance regimes. To induce the farmers subscribe variable

⁶ We assume both contracts exist according to Proposition 2.

⁷ It is typically the case in the U.S. system in which the farmers can subscribe different financial contracts corresponding to their "portfolio" of cultures.

participating contracts, their role should be first to encourage the creation of integrated variable participating policies that would cover both idiosyncratic and systemic risk. Then, they should subsidize separately both participating and financial contracts so that their combination would be fairly priced for farmers. The final objective would be to get the advantages of Corollary 7, *i.e.* a perfect unbiased coverage.

7 Conclusion

The introduction of crop insurance policies for the management of climatic disasters in agriculture is generalized all over developed countries. Since current contracts are defined with an ex-ante premium, variable participating policies seem to offer an alternative way to promote insurance in the agricultural sector.

They are interesting for both sides of the insurance market and the States:

- The policyholders take full benefits from the combination of the two contracts, which insures their idiosyncratic risk and completely securitizes their systemic one through a financial index.
- The insurers also minimize their potential losses by simply covering the idiosyncratic risk through a variable premium and securitizing the systemic risk on financial markets. They are also reinforced in their role of intermediation of climatic events.
- The States limit their intervention to the subsidization of the premia of both participating and financial contracts. Moreover, their intervention can help the policies to be more fairly priced.

Then, the perspectives are promising as more and more countries decide to reform their agricultural coverage against climatic events. Variable participating policies are a credible way to increase the number of policyholders and enhance this recent market because they take into account the whole risk, catastrophic and individual. Further research should investigate practical approaches and test the possibility for farmers to subscribe variable participating contracts instead of current ones.

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Appendix 1 – Proof of Proposition 1

We start from our original problem:

$$\underset{I, J, P, Q}{Max} Eu(w_0 - l(\tilde{x}, \tilde{\varepsilon}) + I(\tilde{x}, \tilde{\varepsilon}) - P(\tilde{\varepsilon}) + J(\tilde{x}, \tilde{\varepsilon}) - Q) \quad (A1)$$

The equivalence is obtained by simplifications and the elicitation of α , the proportion of loss indemnified by the insurer (coinsurance) and β denoting the degree of participation. α is defined by the following equality:

$$I(\tilde{x}, \tilde{\varepsilon}) + J(\tilde{x}, \tilde{\varepsilon}) = \alpha \times l(\tilde{x}, \tilde{\varepsilon}) \quad (A2)$$

β is the degree of participation. It can be elicited when replacing P and Q by their value when premia are fair. We also consider a multiplicative loss function:

$$l(\tilde{x}, \tilde{\varepsilon}) = (1 + \tilde{\varepsilon})\tilde{x} \quad (A3)$$

Under our notations and assuming no loading factors, it comes:

$$P(\tilde{\varepsilon}) = \alpha(1 - \beta)E[l(x, \varepsilon)] = \alpha(1 - \beta)E(x)(1 + \varepsilon) \quad (A4)$$

$$Q = \alpha\beta E(x) \quad (A5)$$

Replacing (A2), (A3), (A4) and (A5) in (A1) permits to obtain the following maximization:

$$\underset{\alpha, \beta}{Max} Eu(w_0 - T(x, \varepsilon) - (1 - \alpha)(1 + \varepsilon)x), \text{ with } T(x, \varepsilon) = \alpha E(x)[1 + (1 - \beta)\varepsilon] \quad (A6)$$

Where α and β are now to be determined instead of I, J, P and Q .

Appendix 2 – Proof of Proposition 2

Problem (10) can be solved using Karush-Kuhn-Tucker conditions for $I(x, \varepsilon)$ and $J(x, z, b)$

because their first derivatives appear neither in the objective function nor in the constraints.

$$\begin{aligned}
 & \underset{I, J, P, Q}{\text{Max}} \quad Eu(w_0 - I(\tilde{x}, \tilde{\varepsilon}) + I(\tilde{x}, \tilde{\varepsilon}) - P(\tilde{\varepsilon}) + J(\tilde{x}, \tilde{z}, \tilde{b}) - Q) \\
 & \text{subject to :} \\
 & \left\{ \begin{array}{l} I(x, \varepsilon) \geq 0 \text{ associated with } \lambda_1(x, \varepsilon) \\ P(\varepsilon) = (1 + \theta_1)E(I(x, \varepsilon)) \text{ associated with } \mu_1(\varepsilon) \\ J(x, z, b) \geq 0 \text{ associated with } \lambda_2(x, z) \\ Q = (1 + \theta_2)E(I(x, \varepsilon)) \text{ associated with } \mu_2 \end{array} \right. \quad (\text{B1})
 \end{aligned}$$

Where: $\lambda_1, \lambda_2, \mu_1$ and μ_2 are Lagrange multipliers.

The first-order condition associated to the indemnity of the participating contract is:

$$\frac{\partial L}{\partial I(x, \varepsilon)} = u'(w_0 - I(x, \varepsilon) + I(x, \varepsilon) - P(\varepsilon) + J(x, z, b) - Q) + \lambda_1(x, \varepsilon) - (1 + \theta_1) = 0 \quad (\text{B2})$$

A supplementary condition for the maximisation is associated to the Lagrange multiplier of the indemnity of the participating contract:

$$\lambda_1(x, \varepsilon) \left\{ \begin{array}{l} = 0 \text{ if } I(x, \varepsilon) > 0 \\ \geq 0 \text{ otherwise} \end{array} \right. \quad (\text{B3})$$

Considering a positive indemnification, (B2) can be rewritten as:

$$u'(w_0 - I(x, \varepsilon) + I(x, \varepsilon) - P(\varepsilon) + J(x, z, b) - Q) = \mu_1(\varepsilon)(1 + \theta_1) \quad (\text{B4})$$

At the optimum, the first derivative of the utilitarian function is supposed to be constant for each level of systemic (or financial) risk, remembering that participating contracts filters this kind of risk. Thus, for given w_0 and Q , it comes that:

$$I(x, \varepsilon) + J(x, z, b) = l(x, \varepsilon) + P(\varepsilon), \forall \varepsilon : I(x, \varepsilon) > 0 \quad (\text{B5})$$

We use the same reasoning for non-participating contracts. The first-order condition associated to the indemnity of the non-participating contract is:

$$\frac{\partial L}{\partial J(x, z, b)} = u'(w_0 - l(x, \varepsilon) + I(x, \varepsilon) - P(\varepsilon) + J(x, z, b) - Q) + \lambda_2(x, z, b) - \mu_2(1 + \theta_2) = 0 \quad (\text{B6})$$

A supplementary condition for the maximisation is associated to the Lagrange multiplier of the indemnity of the non-participating contract:

$$\lambda_2(x, z, b) \begin{cases} = 0 \text{ if } J(x, z, b) > 0 \\ \geq 0 \text{ otherwise} \end{cases} \quad (\text{B7})$$

Considering a positive indemnification, (B6) can be rewritten as:

$$u'(w_0 - l(x, \varepsilon) + I(x, \varepsilon) - P(\varepsilon) + J(x, z, b) - Q) = \mu_2(1 + \theta_2) \quad (\text{B8})$$

At the optimum, the first derivative of the utilitarian function is supposed to be constant for each level of systemic (or financial) risk for each state of the world where J is paid, considering the non-participating contract protects against both idiosyncratic and systemic risks. Therefore, for given w_0 and Q , it comes that:

$$I(x, \varepsilon) + J(x, z, b) = l(x, \varepsilon) + P(\varepsilon), \forall (x, z, b) : J(x, z, b) > 0 \quad (\text{B9})$$

Then, we must consider the stakeholder's choice. The first question is whether they include in their insurance policy participating contracts. In fact, there the "classical" non-participating

contract is always selected, as it is the only one that covers systemic risk. The second subject is about the form of the contract. Following Arrow (1974) and Raviv (1979), when two risks x and ε (as defined in our paper) are insurable, then the insurance policy with a deductible on the aggregate losses is optimal.

For the subscription of the participating contract, two cases exist:

- The premium of the non-participating contract is higher than for the participating contract:

$$Q > P(\varepsilon) \Leftrightarrow (1 + \theta_1)EI(\tilde{x}, \tilde{\varepsilon}) < (1 + \theta_2)EJ(\tilde{x}, \tilde{\varepsilon}) \quad (\text{B10})$$

Then, to cover the idiosyncratic risk x , the cheapest contract is selected, *i.e.* the participating one, and the premium is defined taking into account total loss minus a deductible D_1 , as follows:

$$I^*(x, \varepsilon) = \text{Max}\{I(x, \varepsilon) - D_1; 0\} \quad (\text{B11})$$

With respect to (B5), the premium of the non-participating contract depends on the variable premium of the participating contract minus a deductible D_2 :

$$J^*(x, z, b) \equiv J^*(x, \varepsilon) = \text{Max}\{P(\varepsilon) - D_2; 0\} \quad (\text{B12})$$

- The premium of the participating contract is higher than for the non-participating contract:

$$P(\varepsilon) > Q \Leftrightarrow (1 + \theta_1)E(I(\tilde{x}, \tilde{\varepsilon})) > (1 + \theta_2)E(J(\tilde{x}, \tilde{\varepsilon})) \quad (\text{B13})$$

Then, to cover the idiosyncratic risk x , the cheapest contract is selected, *i.e.* the non-participating one. This implies that the participating contract is nether chosen:

$$I^*(x, \varepsilon) = 0 \quad (\text{B14})$$

Its premium is neither calculated and full insurance is only provided by the non-participating contract above a deductible D_3 :

$$J^*(x, z, b) \equiv J^*(x, \varepsilon) = \text{Max}\{I(x, \varepsilon) - D_3; 0\} \quad (\text{B15})$$

This is the standard result in literature when only non-participating contracts exist.

Appendix 3 – Proof of Proposition 3

The optimisation (B1) is now operated on Q and $P(\varepsilon)$ to find the optimal level of deductibles D_1 and D_2 . For practical purposes, we define:

$$\psi(x, \varepsilon) = w_0 - I(x, \varepsilon) + I(x, \varepsilon) - P(\varepsilon) + J(x, z, b) - Q \quad (\text{C1})$$

- For the non-participating contract, the first-order condition is:

$$\frac{\partial L}{\partial Q} = Eu'(\psi(\tilde{x}, \tilde{\varepsilon})) - \mu_2 = 0 \quad (\text{C2})$$

Replacing the value of μ_2 in (B6) gives the following equality:

$$\lambda_2(x, z, b) = -u'(\psi(x, \varepsilon)) + Eu'(\psi(\tilde{x}, \tilde{\varepsilon}))(1 + \theta_2) \quad (\text{C3})$$

Taking the expectation of λ_2 yields:

$$E\lambda_2(\tilde{x}, \tilde{z}, \tilde{b}) = \theta_2 \times Eu'(\psi(\tilde{x}, \tilde{\varepsilon})) \quad (\text{C4})$$

Consequently, $\theta_2 = 0$ implies that $E\lambda_2(\tilde{x}, \tilde{z}, \tilde{b}) = 0$. Then, $\lambda_2(x, z, b) = 0, \forall (x, z, b)$ because $\lambda_2(x, z, b) \geq 0$. Using (B7), it means that $J(x, z, b) > 0$. Thus, $D_2 = 0$. Similarly, $\theta_2 > 0$ implies $D_2 > 0$.

- For the participating contract, the optimisation is quite different because the premium is variable and depends on ε . Thus, it is not possible to compute the first-order condition of problem (B1) by deriving the Lagrange function. The solution is to replace $I(\tilde{x}, \tilde{\varepsilon})$ by its value found in Proposition 2:

$$I(\tilde{x}, \tilde{\varepsilon}) = \text{Max}(l(\tilde{x}, \tilde{\varepsilon}) - D_1; 0) \quad (\text{C5})$$

Problem (B1) becomes:

$$\text{Max}_{D_1} Eu'(w_0 - l(\tilde{x}, \tilde{\varepsilon}) + \text{Max}(l(\tilde{x}, \tilde{\varepsilon}) - D_1; 0) - P(\tilde{\varepsilon}) + J(\tilde{x}, \tilde{z}, \tilde{b}) - Q) \quad (\text{C6})$$

The first-order condition of this problem is:

$$\frac{\partial L}{\partial D_1} = Eu'(\psi(\tilde{x}, \tilde{\varepsilon})) + \lambda_1(x, \varepsilon) - \mu_1(\varepsilon)(1 + \theta_1) = 0 \quad (\text{C7})$$

Replacing the value of μ_1 in (B2) and rearranging the expectation operator gives the following equality:

$$E\lambda_1(\tilde{x}, \tilde{\varepsilon}) = \theta_1 \times Eu'(\psi(\tilde{x}, \tilde{\varepsilon})) \quad (\text{C8})$$

Consequently, $\theta_1 = 0$ implies $E\lambda_1(\tilde{x}, \tilde{\varepsilon}) = 0$ and $\lambda_1(x, \varepsilon) = 0, \forall(x, \varepsilon)$ because $\lambda_1(x, \varepsilon) \geq 0$. Using (B3), it means that $I(x, \varepsilon) > 0$. Then, $D_1 = 0$. Similarly, $\theta_1 > 0$ implies $D_1 > 0$.

Appendix 4 – Proof of Proposition 4

Eliciting $P(\varepsilon)$ under the assumption that the contract is not sold at a fair price, i.e. $\theta_1 > 0$ and consequently $D_1 > 0$ (Proposition 3-i) gives:

$$\begin{aligned} Loss_{pc}^+ &= l(x, \varepsilon) + P(\varepsilon) - I(x, \varepsilon) \\ &= x + \varepsilon + (1 + \theta_1) E(\text{Max}[x + \varepsilon - D_1; 0]) - \text{Max}[x + \varepsilon - D_1; 0] \\ &= E(\tilde{x}) + \varepsilon + \theta_1 (E(\tilde{x}) + \varepsilon - D_1) \end{aligned} \quad (\text{D1})$$

$$\begin{aligned} Loss_{pc}^\times &= l(x, \varepsilon) + P(\varepsilon) - I(x, \varepsilon) \\ &= x(1 + \varepsilon) + (1 + \theta_1) E(\text{Max}[x(1 + \varepsilon) - D_1; 0]) - \text{Max}[x(1 + \varepsilon) - D_1; 0] \\ &= E(\tilde{x})(1 + \varepsilon) + \theta_1 (E(\tilde{x})(1 + \varepsilon) - D_1) \end{aligned} \quad (\text{D2})$$

This leads to points (i) and (ii).

Appendix 5 – Proof of Proposition 5

Supposing first an additive loss function, i.e. $l(x, \varepsilon) = x + \varepsilon$, then, by definition of z in (6), of $K^*(z)$ in (19) and $P(\varepsilon)$ in (22), the optimal strategy of replication of the non-participating contract is given by:

$$\begin{aligned} K^*(z) &= \text{Max}[P(\varepsilon) - D_2; 0] = \text{Max}[(1 + \theta_1)(E(\tilde{x}) + \varepsilon - D_1) - D_2; 0] \\ &= (1 + \theta_1) \text{Max}\left[z + E(\tilde{x}) - \frac{D_1(1 + \theta_1) + D_2}{1 + \theta_1} + b; 0\right] \end{aligned} \quad (\text{E1})$$

This leads to point (i).

Supposing now a multiplicative loss function, *i.e.* $l(x, \varepsilon) = x(1 + \varepsilon)$, then, by definition of z in (6), of $K^*(z)$ in (19) and $P(\varepsilon)$ in (24), the optimal strategy of replication of the non-participating contract is given by:

$$\begin{aligned} K^*(z) &= \text{Max}[P(\varepsilon) - D_2; 0] = \text{Max}[(1 + \theta_1)(E(\tilde{x})(1 + \varepsilon) - D_1) - D_2; 0] \\ &= (1 + \theta_1)E(\tilde{x}) \text{Max}\left[z + 1 - \frac{D_1(1 + \theta_1) + D_2}{(1 + \theta_1)E(\tilde{x})} + b; 0\right] \end{aligned} \quad (\text{E2})$$

Appendix 6 – Proof of Proposition 6

Let's consider the additive case with $l(x, \varepsilon) = x + \varepsilon$. By definition of $K^+(z)$ in Proposition 5, Q in (20), $P(\varepsilon)$ in (22) and using the definition of z in (6) and (6'), we obtain:

$$\begin{aligned} \text{Loss}_{NPC}^+ &= Q - K^+(z) = (1 + \theta_2)E[\text{Max}(P(\tilde{\varepsilon}) - D_2; 0)] - \text{Max}(P(\tilde{\varepsilon}) - D_2; 0) \\ &= (1 + \theta_2)E\left[(1 + \theta_1)(E(\tilde{x}) + z + b - D_1 - D_2 / (1 + \theta_1))\right] \\ &\quad - (1 + \theta_1)(E(\tilde{x}) + z + b - D_1 - D_2 / (1 + \theta_1)) \\ &= (1 + \theta_1)\left\{(E(\tilde{z}) - z - b) + \theta_2 E(K^+(z))\right\} \end{aligned} \quad (\text{F1})$$

This is point (i).

Let's consider now the multiplicative case with $l(x, \varepsilon) = x(1 + \varepsilon)$. By definition of $K^\times(z)$ in Proposition 5, Q in (20), $P(\varepsilon)$ in (25) and using the definition of z in (6) and (6'), we obtain:

$$\begin{aligned} \text{Loss}_{NPC}^\times &= Q - K^\times(z) = (1 + \theta_2)E[\text{Max}(P(\tilde{\varepsilon}) - D_2; 0)] - \text{Max}(P(\tilde{\varepsilon}) - D_2; 0) \\ &= (1 + \theta_2)E\left[(1 + \theta_1)E(\tilde{x})(z + 1 - D_1 / E(\tilde{x}) - D_2 / E(\tilde{x})(1 + \theta_1) + b)\right] \\ &\quad - (1 + \theta_1)E(\tilde{x})(z + 1 - D_1 / E(\tilde{x}) - D_2 / E(\tilde{x})(1 + \theta_1) + b) \\ &= (1 + \theta_1)E(\tilde{x})\left\{(E(\tilde{z}) - z - b) + \theta_2 E(K^\times(z))\right\} \end{aligned} \quad (\text{F2})$$

This is point (ii).

Appendix 7 – Proof of Proposition 7

Defining the variable participating contract as the simple combination of a participating and a non-participating contract, total loss is equal in case to the sum of the losses of the two contracts.

For additive losses, it is the sum of (D1) and (F1), *i.e.*:

$$\begin{aligned}
 Loss_{NC+NPC}^+ &= E(\tilde{x}) + \varepsilon + \theta_1 (E(\tilde{x}) + \varepsilon - D_1) + (1 + \theta_1) (E(\tilde{z}) - z - b + \theta_2 E[K^+(z)]) \\
 &= (E(\tilde{x}) + \varepsilon)(1 + \theta_1) - \theta_1 D_1 + (1 + \theta_1) (E(\tilde{z}) - z - b + \theta_2 E[K^+(z)]) \\
 &= (1 + \theta_1) \left[E(\tilde{x}) + E(\tilde{z}) + \theta_2 E[K^+(z)] - \frac{\theta_1 D_1}{1 + \theta_1} \right]
 \end{aligned} \tag{G1}$$

This leads to point (i).

For multiplicative losses, total loss is equal to the sum of (D2) and (F2), *i.e.*:

$$\begin{aligned}
 Loss_{NC+NPC}^\times &= E(\tilde{x})(1 + \varepsilon) + \theta_1 (E(\tilde{x})(1 + \varepsilon) - D_1) + E(\tilde{x})(1 + \theta_1) \{ (E(\tilde{z}) - z - b) + \theta_2 E[K^\times(z)] \} \\
 &= E(\tilde{x})(1 + \varepsilon)(1 + \theta_1) - \theta_1 D_1 + E(\tilde{x})(1 + \theta_1) (E(\tilde{z}) - z - b) + E(\tilde{x})(1 + \theta_1) \theta_2 E[K^\times(z)] \\
 &= E(\tilde{x})(1 + \theta_1) \left[1 + E(\tilde{z}) + \theta_2 E[K^\times(z)] - \frac{\theta_1 D_1}{(1 + \theta_1) E(\tilde{x})} \right]
 \end{aligned} \tag{G2}$$

This leads to point (ii).

Documents de Recherche parus en 2008¹

DR n°2008 - 01 : Geoffroy ENJOLRAS, Robert KAST
« Using Participating and Financial Contracts to Insure Catastrophe
Risk : Implications for Crop Risk Management »

¹ La liste intégrale des Documents de Travail du LAMETA parus depuis 1997 est disponible sur le site internet : <http://www.lameta.univ-montp1.fr>

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