Correspondence Analysis of Two Transition Tables

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Abstract

The case of two transition tables is considered, that is two square asymmetric matrices of frequencies where the rows and columns of the matrices are the same objects observed at three different time points. Different ways of visualizing the tables, either separately or jointly, are examined. We generalize an existing idea where a square matrix is decomposed into symmetric and skew-symmetric parts to two matrices, leading to a decomposition into four components: (1) average symmetric, (2) average skew-symmetric, (3) symmetric difference from average, and (4) skew-symmetric difference from average. The method is illustrated with an artificial example and an example using real data from a study of changing values over three generations.

1 Introduction

Correspondence analysis (CA) is a technique for visualizing tables of frequencies as well as nonnegative data on a commensurate set of ratio-scale variables. There are a number of different ways of defining correspondence analysis, many of them based on the singular value decomposition (SVD). The reader interested in more details is referred to Greenacre (1984) or Blasius and Greenacre (1994), for example.

A table which deserves special treatment is the case when the table is square and has rows and columns referring to the same objects, for example tables of social mobility or transitions between a set of states. In many cases there is a problem with the visualization of such a table by correspondence analysis because the diagonal elements of the matrix are large and tend to contribute excessively to the solution, overshadowing other more subtle features in the table.

Greenacre (1996) proposed separate analyses of the symmetric and skew-symmetric parts of the table, based on the decomposition proposed by Constantine & Gower (1978) and Gower (1980). He showed how this idea could be implemented in the correspondence analysis framework by setting up a block matrix of the table reproduced twice down the diagonal and in transposed form in the off-diagonal positions. The simple correspondence analysis of this block matrix yielded exactly the symmetric and skew-symmetric analyses in one joint analysis, with the principal axes of the two analyses appearing interlaced in the solution and in order of importance.

The purpose of this paper is to go one step further by analyzing two square tables simultaneously in the same style. These two tables could be mobility tables, for example, from grandfather to father and from father to son, that is two successive transition tables. Or the two tables could arise as a subdivision of a table according to a binary variable such as gender (male/female). In the former example we would be interested in comparing the transition in the first generational change with the second, in the latter example we would be interested in comparing the difference in the transitions between males and females.

In Section 2 we give a brief technical summary of correspondence analysis applied to square matrices. In Section 3, we consider two artificial square matrices, constructed to represent two generational changes, showing how conventional correspondence analysis and the approach of Greenacre (1996) can be used to visualize these matrices separately.

In Section 4 we consider two ways to analyze these matrices jointly. The first way is the regular way of stacking correspondence tables (see, for example, Blasius (1994) and Greenacre (1994)). The second way is an extension of the approach of Greenacre (1996) where symmetric and skew-symmetric components are of interest. We conclude with an application to real data in Section 5.

2 Methodology

2.1 Correspondence analysis

Correspondence analysis can be defined as a method for weighted least-squares approximation of a frequency matrix. In general, suppose that the data matrix $\mathbf{N}$ has been divided by its grand total n to obtain $\mathbf{P} = \mathbf{N}/n$, called the *correspondence matrix*. Suppose $\mathbf{P}$ has row and column sums $\mathbf{r}$ and $\mathbf{c}$ respectively, and that $\mathbf{D}_r$ and $\mathbf{D}_c$ are diagonal matrices with the elements of $\mathbf{r}$ and $\mathbf{c}$ on the diagonal. For a contingency table $\mathbf{N}$, $\mathbf{P}$ would be the (sample) discrete bivariate distribution and $\mathbf{r}$ and $\mathbf{c}$ the marginal distributions. CA can be defined as the reduced-rank matrix approximation of $\mathbf{P}$ by weighted least squares, minimizing the following expression:

$$\text{trace}[\mathbf{D}_r^{-1}(\mathbf{P} - \hat{\mathbf{P}})\mathbf{D}_c^{-1}(\mathbf{P} - \hat{\mathbf{P}})^\mathsf{T}] = \sum_i \sum_j \frac{(p_{ij} - \hat{p}_{ij})^2}{r_i c_j} \qquad (1)$$

for a matrix $\hat{\mathbf{P}}$ of given reduced rank. We know that the best rank 1 approximation is given by $\hat{\mathbf{P}} = \mathbf{r}\mathbf{c}^\mathsf{T}$, called the *trivial solution*, so that we can equivalently consider the approximation of the centered matrix $\mathbf{P} - \mathbf{r}\mathbf{c}^\mathsf{T}$. The solution for any low rank is given by the singular value decomposition (SVD): of the matrix of standardized residuals $\mathbf{D}_r^{-1/2}(\mathbf{P} - \mathbf{r}\mathbf{c}^\mathsf{T})\mathbf{D}_c^{-1/2}$ (see, for example, Blasius and Greenacre, 1994):

$$\mathbf{D}_r^{-1/2}(\mathbf{P} - \mathbf{r}\mathbf{c}^\mathsf{T})\mathbf{D}_c^{-1/2} = \mathbf{U}\mathbf{D}_\alpha\mathbf{V}^\mathsf{T} \qquad \text{where } \mathbf{U}^\mathsf{T}\mathbf{U} = \mathbf{V}^\mathsf{T}\mathbf{V} = \mathbf{I} \qquad (2)$$

For constructing CA maps, the *principal coordinates* of the row and column points are given by $\mathbf{F} = \mathbf{D}_r^{-1/2}\mathbf{U}\mathbf{D}_\alpha$ and $\mathbf{G} = \mathbf{D}_c^{-1/2}\mathbf{V}\mathbf{D}_\alpha$ respectively. For example, to plot the rows and columns in two dimensions, the rank 2 solution given by the first two columns of $\mathbf{F}$ and $\mathbf{G}$ are used. The resulting plot is called the *symmetric map*, as opposed to other so-called *asymmetric maps* (see Greenacre 1984, 1993), described below.

An alternative way of defining CA is as a weighted least-squares approximation of the row or column *profiles* of the table. A profile is a row or column of the matrix divided by its corresponding sum. For example, the row profiles are the rows of the matrix $\mathbf{D}_r^{-1}\mathbf{P}$, in which case CA can be defined as the approximation of the row profiles by points in a low-dimensional subspace. Distances and scalar products in the space are computed using the *chi-square metric*, a weighted Euclidean metric using $\mathbf{D}_c^{-1}$ as the weighting matrix. Furthermore, the row profiles are weighted by the respective elements of $\mathbf{r}$, called the *row masses*. The objective function in this case is:

$$\text{trace}[\mathbf{D}_r(\mathbf{D}_r^{-1}\mathbf{P} - \hat{\mathbf{Q}})\mathbf{D}_c^{-1}(\mathbf{D}_r^{-1}\mathbf{P} - \hat{\mathbf{Q}})^\mathsf{T}] = \sum_i r_i \sum_j \frac{(p_{ij}/r_i - \hat{q}_{ij})^2}{c_j} \qquad (3)$$

Again we have a trivial solution because it turns out that the row vector $\mathbf{c}^\mathsf{T}$ comes closest to all the row profiles in terms of weighted least sum-of-squared distances, so that it is equivalent to approximate the centered profiles $\mathbf{D}_r^{-1}\mathbf{P}-\mathbf{1c}^\mathsf{T}$. Again this problem is solved using the SVD of the matrix $\mathbf{D}_r^{1/2}(\mathbf{D}_r^{-1}\mathbf{P}-\mathbf{1c}^\mathsf{T})\mathbf{D}_c^{-1/2}$, which is identical to the matrix of standardized residuals decomposed previously, so the solution is as before. Because we think of the matrix as a set of rows, it is often convenient to visualize the results using the asymmetric map. In this case the row profiles would be plotted using principal coordinates $\mathbf{F}$ as before, but the columns would be in so-called *standard coordinates* $\mathbf{Y}=\mathbf{GD}_\alpha^{-1}$. These column points are the projections of the unit vectors onto the optimal subspace, and together $\mathbf{F}$ and $\mathbf{Y}$ constitute a biplot of the frequency table (Greenacre, 1992).

In both definitions of CA the *total inertia* of the table, a measure of the table's total variation, is equal to the weighted sum-of-squares of the centered matrix being approximated:

$$\text{total inertia} = \sum_i \sum_j (p_{ij} - r_i c_j)^2/(r_i c_j) \tag{4}$$

The inertia accounted for by the rank K^* solution (or K^*-dimensional solution) is equal to the weighted sum-of-squares of the matrix approximation, which is equal to $\sum_{k=1}^{K^*}\alpha_k^2$. The minimum value of (1) (or (3)), which is the residual inertia not accounted for, is equal to the remaining sum-of-squared singular values: $\sum_{k=K^*+1}^{K}\alpha_k^2$.

The special case considered here is the application of CA to square tables where the rows and columns refer to the same set of objects. For a transition table where the rows refer to the first time point and the columns the second (e.g., father and son), the row profiles are the relative frequencies of change from time one to time two. The problem with the CA of such tables is, as pointed out by Greenacre (1996), the predominant role played by the diagonal of the table, indeed the *symmetric part* of the table as a whole.

2.2 Symmetric and skew-symmetric components

Greenacre (1996) adapted the ideas of Constantine & Gower (1978) and Gower (1980) to the decomposition of the correspondence matrix into symmetric and skew-symmetric components:

$$\mathbf{P} = \mathbf{S} + \mathbf{T} \tag{5}$$

To solve the centring problem, Greenacre considered the average of the row and column margins $\mathbf{w}=\frac{1}{2}(\mathbf{r}+\mathbf{c})$ as the centre, so that the decomposition is actually:

$$\mathbf{P} - \mathbf{ww}^\mathsf{T} = \mathbf{S} - \mathbf{ww}^\mathsf{T} + \mathbf{T} \tag{6}$$

with the metric also being defined as $\mathbf{D}_w^{-1}$. The corresponding decomposition of inertia is thus

$$\sum_i \sum_j (p_{ij} - w_i w_j)^2/(w_i w_j) = \sum_i \sum_j (s_{ij} - w_i w_j)^2/(w_i w_j) + \sum_i \sum_j t_{ij}^2/(w_i w_j) \qquad (7)$$

A convenient way to perform the CA on the separate matrix components was shown to be the simple CA algorithm applied to the block matrix:

$$\widetilde{\mathbf{N}} = \begin{bmatrix} \mathbf{N} & \mathbf{N}^\mathsf{T} \\ \mathbf{N}^\mathsf{T} & \mathbf{N} \end{bmatrix} \qquad (8)$$

For a $p \times p$ matrix $\mathbf{N}$, the CA of the block matrix yields $2p-1$ dimensions, $p-1$ of which coincide with the symmetric part of $\mathbf{N}$. These dimensions correspond to coordinate matrices which have vectors of coordinates reproduced twice, i.e. of the form:

$$\begin{bmatrix} \mathbf{f} \\ \mathbf{f} \end{bmatrix}$$

Clearly only one block of coordinates needs to be used for plotting, so that there is one set of points displayed.

The other p dimensions (or $p-1$ if p is an odd number) correspond to pairs of equal singular values (i.e., equal principal inertias), and are the so-called *bimensions* of the skew-symmetric component, with pairs of row and column principal coordinate vectors of the form:

$$\text{rows}: \begin{bmatrix} \mathbf{f}_1 & \mathbf{f}_2 \\ -\mathbf{f}_1 & -\mathbf{f}_2 \end{bmatrix} \qquad \text{columns}: \begin{bmatrix} \mathbf{f}_2 & -\mathbf{f}_1 \\ -\mathbf{f}_2 & \mathbf{f}_1 \end{bmatrix}$$

Each block of coordinates is a 90 degree rotation of the other. To apply the usual rules of interpretation it would be necessary to plot row and column points. But, as before, only one set of points needs to be plotted, say the first block of row points, since the 90 degree relationship means that we can use the triangle area rule of interpretation of the points (see Greenacre, 1996, and the applications below).

In practice we map the first two dimensions of the symmetric part and the first two dimensions of the skew-symmetric part, in separate maps. Thus in terms of parsimony, the symmetric/skew-symmetric decomposition has the same number of displayed points as an ordinary CA of the original table which would have two sets of points in one map.

2.3 Two repeated tables

We can generalize the above ideas to the case where we have two tables. The idea is as follows (see also Greenacre, 1998). Suppose the two tables are $\mathbf{M}$ and $\mathbf{N}$ respectively, each $p \times p$ and each crosstabulating the same set of n individuals. We set up the following $4p \times 4p$ block matrix:

$$\begin{bmatrix} \mathbf{M} & \mathbf{N} & \mathbf{M}^\mathsf{T} & \mathbf{N}^\mathsf{T} \\ \mathbf{N} & \mathbf{M} & \mathbf{N}^\mathsf{T} & \mathbf{M}^\mathsf{T} \\ \mathbf{M}^\mathsf{T} & \mathbf{N}^\mathsf{T} & \mathbf{M} & \mathbf{N} \\ \mathbf{N}^\mathsf{T} & \mathbf{M}^\mathsf{T} & \mathbf{N} & \mathbf{M} \end{bmatrix} \tag{9}$$

This matrix has a 2×2 block pattern, where the matrix

$$\begin{bmatrix} \mathbf{M} & \mathbf{N} \\ \mathbf{N} & \mathbf{M} \end{bmatrix}$$

plays the role of the single matrix we had before in (8). So the CA of (9) yields the analyses of the symmetric part:

$$\begin{bmatrix} \mathbf{M} & \mathbf{N} \\ \mathbf{N} & \mathbf{M} \end{bmatrix} + \begin{bmatrix} \mathbf{M}^\mathsf{T} & \mathbf{N}^\mathsf{T} \\ \mathbf{N}^\mathsf{T} & \mathbf{M}^\mathsf{T} \end{bmatrix} = \begin{bmatrix} \mathbf{M}+\mathbf{M}^\mathsf{T} & \mathbf{N}+\mathbf{N}^\mathsf{T} \\ \mathbf{N}+\mathbf{N}^\mathsf{T} & \mathbf{M}+\mathbf{M}^\mathsf{T} \end{bmatrix} \tag{10}$$

and the skew-symmetric part:

$$\begin{bmatrix} \mathbf{M} & \mathbf{N} \\ \mathbf{N} & \mathbf{M} \end{bmatrix} - \begin{bmatrix} \mathbf{M}^\mathsf{T} & \mathbf{N}^\mathsf{T} \\ \mathbf{N}^\mathsf{T} & \mathbf{M}^\mathsf{T} \end{bmatrix} = \begin{bmatrix} \mathbf{M}-\mathbf{M}^\mathsf{T} & \mathbf{N}-\mathbf{N}^\mathsf{T} \\ \mathbf{N}-\mathbf{N}^\mathsf{T} & \mathbf{M}-\mathbf{M}^\mathsf{T} \end{bmatrix} \tag{11}$$

(in both cases we omit the division by 2 of the sum and the difference, which in case does not affect the correspondence analysis).

Now (10) and (11) are themselves block matrices, and Greenacre (1998) has shown the similar, but more general, result that the analysis of any block matrix of the form:

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix}$$

yields an analysis of the sum $\mathbf{A}+\mathbf{B}$ and the difference $\mathbf{A}-\mathbf{B}$. Hence the analyses of (10) and (11) yield two components each:

$$(\mathbf{M}+\mathbf{M}^\mathsf{T}) + (\mathbf{N}+\mathbf{N}^\mathsf{T}) \quad \text{and} \quad (\mathbf{M}+\mathbf{M}^\mathsf{T}) - (\mathbf{N}+\mathbf{N}^\mathsf{T})$$

and

$$(\mathbf{M}-\mathbf{M}^\mathsf{T}) + (\mathbf{N}-\mathbf{N}^\mathsf{T}) \quad \text{and} \quad (\mathbf{M}-\mathbf{M}^\mathsf{T}) - (\mathbf{N}-\mathbf{N}^\mathsf{T})$$

which gives a total of four components altogether:

1. $(\mathbf{M}+\mathbf{M}^\mathsf{T}) + (\mathbf{N}+\mathbf{N}^\mathsf{T})$ (average symmetric part)
2. $(\mathbf{M}+\mathbf{M}^\mathsf{T}) - (\mathbf{N}+\mathbf{N}^\mathsf{T})$ (difference in the symmetric parts)
3. $(\mathbf{M}-\mathbf{M}^\mathsf{T}) + (\mathbf{N}-\mathbf{N}^\mathsf{T})$ (average skew-symmetric part)
4. $(\mathbf{M}-\mathbf{M}^\mathsf{T}) - (\mathbf{N}-\mathbf{N}^\mathsf{T})$ (difference in the skew-symmetric parts)

In Section 4.2 we shall show how these components appear in practice and how they are interpreted.

3 An illustrative example

3.1 Construction of artificial data set

In order to illustrate the methodology, we constructed two artificial tables with a known structure. The advantage of this exercise is that we can observe how the method handles certain transitional patterns which have been introduced intentionally into the data. The tables have six rows and six columns and can be imagined as the sequence of transitions between three generations, grandfather, father and son. In the real data set we present later, the rows and columns represent values which are regarded as the most important by each generation, so that the tables summarize the change in values from one generation to the next.

Tables 1 and 2 show the two tables **M** and **N**. In general, we refer to the single set of categories as A, B, C, D, E and F. When referring to specific rows or columns, we label the columns of **M** and **N** as `A`, `B`, `C`, `D`, `E` and `F`, while the six rows of the first table **M** are labelled `a` to `f`, and the six rows of the second table **N**, `aa` to `ff`. Notice that the columns of Table 1 actually correspond to the rows of Table 2.

The way we constructed the transition matrices is as follows:

- A moves with equal probability to one of the six categories, in both transitions.
- B mostly stays B, with some flow to A, in both transitions.
- C moves almost exclusively to D, in both transitions.
- D stays almost entirely D, in both transitions.
- E mostly stays E but also changes to F, in both transitions.
- F stays mostly at F but also changes to E in the first transition, but in the second transition changes mostly to E.

If we think of the categories as different values regarded as most important by each generation, then the frequencies in row b of Table 1 show that, out of the 20 grandfathers who regarded B as the most important value, 6 of their children (the fathers) now regard A as most important, 12 still regard B as most important, while 1 each regard C and F as most important. We will now see how different analyses of these two matrices reveal the patterns of transition and the changes taking place between the generations.

M	A	B	C	D	E	F	*sum*
a	4	4	4	4	4	4	20
b	6	12	1	0	0	1	20
c	0	1	2	16	0	1	20
d	1	1	0	20	0	0	22
e	0	1	0	1	14	10	26
f	1	0	1	0	10	14	26

Table 1: Generational change from grandfathers to fathers

N	A	B	C	D	E	F	*sum*
aa	4	2	2	2	2	0	12
bb	6	11	1	0	0	1	19
cc	0	0	1	7	0	0	8
dd	1	1	0	38	1	0	41
ee	1	0	1	1	16	9	28
ff	1	2	1	0	23	3	30

Table 2: Generational change from fathers to sons

3.2 Simple correspondence analyses of M and N

The two-dimensional CA map of **M** is shown in Figure 1 (see Appendix, where all maps and numerical output are given). The quality of the display is excellent, with 93.2% of the total inertia being displayed. Since we tend to think of a transition matrix such as this one as a set of rows, with the row profiles giving the transition probabilities conditional on each row margin, we first interpret the positions of the column points, i.e. the "receiving" categories in the transition. Categories `E` and `F` form a cluster on the left, opposing `D` on the right. `A` and `B` form another cluster high on the vertical axis, while C is the most central point and thus does not play a strong role in determining the spread of the row profiles. Looking at the row points now, we can see that `e` and `f` are strongly clustered at bottom left, and `c` and `d` at bottom right. Along the vertical axis the point `b` is far out while `a` is close to the centre. All of these positions concord with the way the matrix has been constructed. C is not receiving anything in particular, hence it is close to the centre, while a is not moving to anything in particular, hence also close to the centre. The position of all the other row points in the direction of a receiving point (column) agrees with the high frequencies in Table 1.

Figure 2 shows a map of **N** in Table 2, and this is hardly different from Figure 1.

Even though the quality of the map is excellent (90.2% of the inertia displayed), there is no sign of the major difference between the two transitions, i.e. the major change of F to E in the second transition whereas in the first transition F moved to both E and F. Looking at the table of contributions for dimensions 3 and 4 of this analysis we can see that this effect does become clear in these later dimensions (compare the signs of the principal coordinates of E and F in this analysis with those for dimensions 3 and 4 in the analysis of $\mathbf{M}$ given after Figure 1 — in the analysis of $\mathbf{M}$ the signs are the same for the rows (`e`,`f`) and columns (`E`, `F`), whereas in the analysis of $\mathbf{N}$ the signs are opposite).

In both analyses the total inertia is of the same order of magnitude, 1.361 and 1.546 respectively. These are high values which is consistent with the strong patterns of association in the matrices, typical for transition matrices of this kind.

3.3 Symmetric and skew-symmetric analyses

Following Greenacre (1996) we consider, for $\mathbf{M}$ and $\mathbf{N}$ in turn, the analysis which separates the symmetric and skew-symmetric components of the transition matrix. Figures 3 and 4 show the symmetric and skew-symmetric maps respectively. Notice that there is only one set of points in each map. The points in the symmetric analysis of Figure 3 will be close if there is flow *between* the categories, irrespective of the direction of the flow. Hence `E` and `F` are close, `B` is closest to `A`, and now `C` appears towards the bottom left along with `D` because C generally moves to D. The fact that there is no flow back from D to C is not shown in this map. The skew-symmetric map in Figure 4, in fact, tells us exactly the direction of flows if they are not symmetric. This map has a slightly different style of interpretation, since it is the area of the triangle between two points and the centre of the display which approximates the values in the skew-symmetric matrix. Thus we see that `C` and `D` form a large triangle with the centre, indicating a large asymmetric flow between C and D, which we know to be from C to D, hence the anti-clockwise sense of the interpretation indicated in the map. The fact that A is C's biggest "feeder" is also indicated in Figure 4.

Figures 5 and 6 show the symmetric and skew-symmetric maps for $\mathbf{N}$. The symmetric map is very similar to that of $\mathbf{M}$ (compare Figures 5 and 3). The skew-symmetric map, however, is different, showing not only the one-directional flow from C to D, but also the flow from F to E which is strong in the second transition. Other triangles in Figure 6 which indicate asymmetric flows, from F to A, from A to C and from E to C, are all correct in their directions even though the magnitude of the flow is quite small.

4 Joint Analysis of the Tables

4.1 Correspondence analysis of stacked tables

We now look at the usual way of visualizing two data matrices jointly, that is either concatenating them side by side, or stacking them one on top of another. Since our interpretation of the transition matrices is in terms of row profiles, the preferred way of combining the tables is to stack them. Figure 7 shows the CA map of the 12×6 matrix

$$\begin{bmatrix} \mathbf{M} \\ \mathbf{N} \end{bmatrix}$$

The map, showing 88.2% of the inertia, is a simple combination of the maps in Figures 1 and 2. Hence this is no real improvement on the separate analyses of $\mathbf{M}$ and $\mathbf{N}$. Looking at the coordinates of the points on the third dimension in the tables of contributions, we do see, however, that the change in flow from category F is depicted in this dimension: "receiver" categories `E` and `F` are located on opposite sides of this dimension, and the first transition row `f` is on the side of `F` while the second transition row `ff` is on the opposite side towards `E`. Again, we have to look at dimensions beyond the first two to see this difference.

4.2 Correspondence analysis of two tables in block matrix form

In order to generalize the ideas of Section 3.3 to the joint analysis of the two transition tables, we have seen in Section 2.3 that this involves the correspondence analysis of a block matrix of 16 tables:

$$\begin{bmatrix} \mathbf{M} & \mathbf{N} & \mathbf{M}^\mathsf{T} & \mathbf{N}^\mathsf{T} \\ \mathbf{N} & \mathbf{M} & \mathbf{N}^\mathsf{T} & \mathbf{M}^\mathsf{T} \\ \mathbf{M}^\mathsf{T} & \mathbf{N}^\mathsf{T} & \mathbf{M} & \mathbf{N} \\ \mathbf{N}^\mathsf{T} & \mathbf{M}^\mathsf{T} & \mathbf{N} & \mathbf{M} \end{bmatrix}$$

¿From the pattern of signs in the subvectors of each vector of principal coordinates we can deduce whether the dimensions refer to (1) the average symmetric component, (2) the difference in the symmetric components, (3) the average skew-symmetric component, or (4) the difference in the skew-symmetric components.

Figures 8, 9 and 10 and the accompanying tables show maps for the first three pairs of dimensions. In the first two maps only one set of points needs to be plotted. Attention has to be given to which component is being displayed, since the style of interpretation is different in each case.

Dimensions 1 and 2 both have principal coordinates in a $[+,+,+,+]$ pattern, so Figure 8 is a visualization of the table $\mathbf{M}+\mathbf{N}+\mathbf{M}^\mathsf{T}+\mathbf{N}^\mathsf{T}$ wich is the average symmetric component. This map is practically the same as Figure 3 and 5.

Dimensions 3 and 4 have principal coordinates in a $[+,+,-,-]$ pattern, so Figure 9 is a visualization of the table

$$\mathbf{M}+\mathbf{N}-\mathbf{M}^{\mathsf{T}}-\mathbf{N}^{\mathsf{T}}=(\mathbf{M}-\mathbf{M}^{\mathsf{T}})+(\mathbf{N}-\mathbf{N}^{\mathsf{T}}),$$

i.e. the average skew-symmetric component. This map is a compromise between Figures 4 and 6 and shows C and D in outlying positions because of the unidirectional flow from C to D in both transitions. The relatively high flow from A to C in both transition is also show here, but not the main difference between Figures 4 and 6 which is the increase in flow from F to E in the second transition.

Dimensions 5 and 6 have principal coordinates in a $[+,-,+,-]$ pattern, so Figure 10 displays the component

$$\mathbf{M}-\mathbf{N}+\mathbf{M}^{\mathsf{T}}-\mathbf{N}^{\mathsf{T}}=(\mathbf{M}+\mathbf{M}^{\mathsf{T}})-(\mathbf{N}+\mathbf{N}^{\mathsf{T}}),$$

i.e., the difference between the symmetric parts. Notice, however, that the row points and column points do not have exactly the same pattern of signs on both dimensions. While the sixth dimension has identical row and column coordinates, on the fifth dimension the column coordinates are the negative of the row coordinates and thus have pattern $[-,+,-,+]$ relative to the row pattern. There is a reflection of the row and column points in the fifth dimension, called an *inverse dimension* (see, for example, Greenacre, 1984, page 240) which is also reminiscent of the axis of reflection found in the unfolding of a symmetric matrix (Gower and Greenacre, 1996). In this case only we need to plot both sets of points in order to be able to reconstruct differences in the symmetric parts.

There are two main features in Figure 10. The first is the positive association between row C and column D, identically row D and column C since we are looking at a symmetric matrix. This is the larger flow between C and D in the first transition compared to the second, hence a positive difference. At the same time row and column C and row and column D are on opposite sides of the map and thus estimate negative differences. In the case of C this is not an accurate estimate, whereas for D there is an increase from 20 to 38 on the diagonal for D which is being displayed correctly as a negative difference. The second feature is the association between row F and column F at the bottom of the map. This is the larger stability of F in the first transition (14 stay F), compared to the second (3 stay F), also leading to a large positive difference.

If we were to continue with the interpretation, dimensions 7 and 8 would be the main axes of the difference in the skew-symmetric components. Notice that the difference in two skew-symmetric matrices is itself skew-symmetric, hence the occurrence of the duplicate inertias and the consequent interpretation using the triangle areas. Since we would be looking at very small percentages of inertia (1.42% and 1.42%) we stop the interpretation here.

5 Application to changing values over three generations

To show an application of our analysis to real data, consider the transition matrices given in Table 3 and 4, obtained from García et al. (1997). These data concern the values that three generations recognized as the most important. The data collection took place in the School of Economics at the University of Murcia. The students were asked to obtain the information from their families. The ten values from which they —and their parents and grandparents— could choose were: honesty, abbreviated as (`ho`), family (`fa`), culture (`cu`), responsability (`re`), happiness (`ha`), solidarity (`so`), freedom (`fr`), loyalty (`lo`), industriousness (`in`) and tolerance (`to`). The crosstabulations have been calculated for the group of 171 students that responded at all three time points. Both transition tables are rather sparse because of the low sample size relative to the number of cells in the tables, so we should be careful in checking that the features that we observe in the maps actually exist in the original data. One way of ensuring correct interpretation of the maps is to look at the contributions to the inertia of each axis, and to restrict our interpretation to those points which make large contributions.

Figures 12 and 13 show the asymmetric maps for the simple CAs of the respective tables. In the first transition, the column points ("receiving" values) with contributions to axes higher than average are `LO` on the first axis and `FR, SO, HO, FA` on the second. The value loyalty (as a column, `LO`) opposes most of the other values, which spread out along the vertical axis. The position of industriousness (as a row, `in`, accounting for 66% of the first inertia) on the right reflects the fact that 3 out of 6 (or 50%) of grandfathers who considered industriousness as the most important value correspond to fathers who consider loyalty as most important. Clearly, we are working here with very small features of the crosstabulation. Nevertheless, in each row of Table 3 we can confirm the highest frequency in the corresponding pair of points in the map, for example `ha` lies towards `HO` and `lo` towards `SO`. The map is accurate, but is an accurate picture of low frequencies.

Figure 13 shows the generational transition from fathers to sons. The picture is rather different, again affected by low frequencies and thus fairly unstable. The important "receiving" values are `SO, RE, IN, TO`. Notice that the total inertia of the table is higher than in the first case, in other words there is more movement towards specific values. For example, `so` is on the left towards `SO`, showing the high frequency of "stayers" for the loyalty value (9 out of 18). Again, the highest frequencies in each row can generally be confirmed in the map. The position of `IN` at the top of the map is outlying because there is only one son who chooses this value – it is debatable whether this is a feature which is worth displaying, but it again illustrates the fact that we are dealing with small features in general in this example. The group of responsability (RE), honesty (HO) and happiness (HA) on the negative side of the vertical axis form a group and are "attracting" values in the previous generation of honesty, happiness, tolerance and family. As a general comment, there seems to be a higher level of consistency in this second table, which could be attributed to a higher similarity between fathers and sons in modern Spain, compared to the fathers and grandfathers.

Looking at the symmetric parts of each transition in Figures 14 and 16, we see the cluster of values honesty, family and happiness in both maps. These values remain linked over both generation changes. The positions of solidarity and loyalty are interchanged in the two maps: first solidarity appears with responsability and tolerance and then separate in the scond map, while loyalty appears with the same two values in the second map, but is separate in the first. The frequencies that we are working with are so small that interpreting the skew-symmetric components in Figures 15 and 17 is very difficult.

The stacked analysis in Figure 18 is more successful. Rows from the first transition are indicated by 1 and from the second by 2. There are three important groups of "receiving" values: solidarity (`SO`) at top left, friendship (`FR`) and loyalty (`LO`) on the right, and family (`FA`) and honesty (`HO`) at bottom left. Many of the changes between the first to the second transition are towards the right of the display, that is towards the "receiving" values of friendship and loyalty, in particular the values `in, fr, lo, re, fa` all move from the left, often from top left, towards these values. This means that in the father–to–son transition there are changes from other values towards friendship and loyalty. The points `so1` and `so2` show a change towards the solidarity value, that is, grandfathers with solidarity values changed to other values, whereas fathers with solidarity value have many sons with this value too. As a comment, the points `HA` and `ha2` feature strongly on the positive side of axis 3 (not shown here, but the numerical results are given), which indicates the fact that from parents to sons the happiness value stays fairly stable.

Because of the difficulty of interpreting the individual symmetric and skew-symmetric analyses, it is even more difficult to interpret the components of the block matrix constructed from both transition tables. We have included all the results in the Appendix for completeness. Dimensions 1 and 2, with a $[+,+,+,+]$ pattern display the main features of the average symmetric table. Dimensions 4 and 5, with a $[+,+,-,-]$ pattern, account for the major dimensions of the average skew-symmetric part. Dimensions 3 and 8, with a pattern $[+,-,+,-]$, are related to the difference in skew-symmetric parts, while dimensions 6 and 7, with a $[+, -, +, -]$ pattern, are related to the difference in skew-symmetric parts. In this example, where the difference components are quite strong compared to the symmetric parts, the decomposition into symmetric and skew-symmetric components is not so successful from a practical point of view, and the stacked analysis is much easier to understand and to interpret.

6 Discussion and conclusions

We have showed several ways of tackling the analysis and visualization of a pair of transition tables. When the tables are dominated by their symmetric parts, then the decomposition into symmetric and skew-symmetric parts makes sense, so that we can interpret the skew-symmetry without the overwhelming influence of the symmetric part.

We have generalized this idea to the case of two tables being analyzed jointly, in which case there are four components of interest: the average symmetric part, the average skew-symmetric part, the difference between the symmetric parts and the difference betweeen the two skew-symmetric parts. This strategy functions optimally when the individual symmetric parts are strong, but not necessarily similar. The total inertia of the two tables is distributed over the four components and this can be useful in quantifying the amount of variance attributable to these four sources. The analysis may be executed by applying a regular correspondence analysis to the matrices set up in a block table where the table and its transpose are included four times in a pattern where no matrix is repeated twice in the same row or column. The map of each component involves only one set of points at a time, with the exception of the difference in symmetric parts where inverse dimensions are possible and where the row and column points are reflections of each other, thereby allowing reconstruction of negative differences on the diagonal of the symmetric matrix.

An alternative and more common way to analyze the tables jointly is a simple stacking of the tables, leading to two points for each row and one point for each column. Our initial experience with these different approaches is that the four-component approach is useful when the symmetric parts of the tables are strong. In all the usual approaches the deviations from symmetry, which are the interesting flows in the tables, would be masked and difficult to see. On the other hand, the four-component model is more complicated to interpret and different rules of interpretation apply to the maps of different components. Applying this method to the real data in this paper, where the tables were quite sparse in data, was particularly difficult. The more regular approach of stacking the tables is useful when the overall differences between the tables is strong, but will not depict the flows accurately in a planar map when the tables have strong symmetric components.

GP to P	HO	FA	CU	RE	HA	SO	FR	LO	IN	TO	
ho	6	9	1	3	3	5	3	2	1	2	35
fa	9	5	0	4	5	4	0	0	2	3	32
cu	0	0	0	1	1	0	0	0	0	0	2
re	0	0	0	0	1	2	1	0	0	0	4
ha	5	3	1	2	3	1	0	0	0	0	15
so	2	1	0	1	0	0	1	1	0	0	6
fr	3	2	0	0	3	1	3	0	1	1	14
lo	0	0	0	2	2	3	1	0	1	0	9
in	0	0	1	1	0	1	0	3	0	0	6
to	1	0	0	1	1	1	0	2	0	0	6

Table 3: Changing values from grandfathers to fathers

P to S	HO	FA	CU	RE	HA	SO	FR	LO	IN	TO	
ho	5	0	0	5	9	0	6	1	0	0	26
fa	2	1	0	0	10	1	3	3	0	0	20
cu	0	0	0	0	0	1	2	0	0	0	3
re	0	0	1	0	4	0	6	3	0	1	15
ha	1	1	0	0	12	0	4	1	0	0	19
so	2	0	0	1	4	9	2	0	0	0	18
fr	1	0	1	0	1	0	4	1	0	1	9
lo	0	0	0	0	1	0	3	2	0	2	8
in	0	1	0	0	0	1	2	0	1	0	5
to	1	1	0	1	2	0	1	0	0	0	6

Table 4: Changing values from fathers to sons

7 References

Blasius, J. (1994), "Correspondence analysis in social science research," in M.J. Greenacre and J. Blasius (eds.), *Correspondence Analysis in the Social Sciences.* London: Academic Press, pp. 23–52.

Blasius, J. & Greenacre, M.J. (1994), "Computation of correspondence analysis," in M.J. Greenacre and J. Blasius (ed.), *Correspondence Analysis in the Social Sciences.* London: Academic Press, pp. 53–78.

Constantine, A.G. and Gower, J.C. (1978), "Graphical representation of asymmetry," *Applied Statistics,* **27**, 297–304.

García, J.G., Frutos, I.P. & Clavel, J.G. (1997), "Una cadena de Markov para el estudio de los valores sociales," *Cuadernos de Realidades Sociales,* **49-50**, 207–229.

Gower, J.C. (1980), "Problems in interpreting asymmetrical relationships," in F.A. Bisby, J.G. Vaughan and C.A. Wright (eds.), *Chemosystematics: Principles and Practice.* London: Academic Press, pp. 399–409.

Gower, J.C. & Greenacre, M.J. (1980), "Unfolding a symmetric matrix," *Journal of Clasiification,* **13**, 81–105.

Greenacre, M.J. (1984), *Theory and Applications of Correspondence Analysis*, London: Academic Press.

Greenacre, M.J. (1992), "Biplots in correspondence analysis," *Journal of Applied Statistics,* **20**, 251–269.

Greenacre, M.J. (1994), "Multiple and joint correspondence analysis," in M.J. Greenacre and J. Blasius (ed.), *Correspondence Analysis in the Social Sciences.* London: Academic Press, pp. 141–161.

Greenacre, M.J. (1996), "An adaptation of correspondence analysis for square tables," UPF Economics Working Paper number 189, submitted for publication.

Greenacre, M.J. (1998), "Visualization of repeated matrices," UPF Economics Working Paper, to appear.

Appendix: Output and Maps

FIGURE 1:

Asymmetric map of dimension 1 and 2 of artificial data, matrix M.

```
+-----------------------------------------+
|                    :                    |
|                    :                    |
|                    :                    |
|                    :*B                  |
|                    A*b                  |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    *a*C                 |
| : : : : : : : : : : : : : : : : : : : :|
|             f      :                    |
|          F**e      :       *c*d         |
|          *E        :         *D         |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
+-----------------------------------------+
```

INERTIAS AND PERCENTAGES OF INERTIA

```
 1 0.729724  53.60%  *****************************************************
 2 0.539105  39.60%  ***************************************
 3 0.065548   4.82%  ****
 4 0.020880   1.53%  *
 5 0.006064   0.45%
   --------
   1.361321
```

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR
1	a	986	174	46	-65	12	1	363	368	43	454	574	546	-108	33	98
2	b	1000	145	288	39	1	0	1629	981	713	-218	18	106	44	1	13
3	c	984	145	144	1062	831	224	-428	135	49	152	17	51	-9	0	1
4	d	994	159	213	1243	849	338	-456	114	62	-231	29	130	40	1	12
5	e	1000	188	156	-904	727	211	-461	189	74	-234	48	157	-200	36	361
6	f	1000	188	153	-935	792	226	-412	153	59	62	3	11	239	52	515

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR
1	A	968	87	92	28	1	0	1176	957	223	109	8	16	63	3	17

2	B	995	138	208	100	5	2	1409	964	507	-230	26	112	-27	0	5
3	C	994	58	45	142	19	2	309	90	10	957	865	811	-145	20	58
4	D	1000	297	349	1162	843	550	-498	155	137	-59	2	16	5	0	0
5	E	1000	203	167	-931	773	241	-444	175	74	-116	12	42	-208	39	422
6	F	999	217	138	-831	800	206	-351	142	50	37	2	4	219	55	498

FIGURE 2:

Asymmetric map of dimension 1 and 2 of artificial data, matrix N.

```
+---------------------------------------+
|                   :                   |
|                   :                   |
|                *B :                   |
|                *bb:                   |
|                *A :                   |
|                   :                   |
|                   :                   |
|                  *aa                  |
|                 *C:                   |
|                   :                   |
|                   :                   |
| : : : : : : : : : : : : : dd*cc : : : |
|           *ff    :          *D        |
|          F**ee    :                   |
|           *E      :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
+---------------------------------------+
```

INERTIAS AND PERCENTAGES OF INERTIA

```
 1 0.816286  52.81%  ****************************************************
 2 0.577099  37.34%  *************************************
 3 0.076998   4.98%  *****
 4 0.065789   4.26%  ****
 5 0.009446   0.61%  *
   --------
   1.545618
```

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR
1	aa	990	87	69	-151	19	2	-781	496	92	-600	293	407	473	182	295
2	bb	1000	138	271	-399	52	27	-1674	921	669	244	20	107	-138	6	40
3	cc	922	58	59	1133	822	91	190	23	4	-221	31	37	265	45	62
4	dd	998	297	285	1193	960	518	208	29	22	73	4	21	-87	5	34
5	ee	1000	203	160	-817	549	166	610	306	131	312	80	257	280	64	242
6	ff	1000	217	157	-856	659	195	469	198	83	-247	55	172	-314	89	327

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR
1	A	972	94	104	-297	51	10	-1203	845	236	-221	29	60	281	46	113
2	B	996	116	202	-361	48	18	-1549	890	482	240	21	87	-315	37	175
3	C	931	43	34	-229	43	3	-431	152	14	-668	364	252	674	372	300
4	D	1000	348	337	1203	965	616	227	34	31	25	0	3	-19	0	2
5	E	1000	304	221	-840	629	263	601	322	191	-155	21	95	-176	27	143
6	F	1000	94	101	-879	468	89	529	169	46	641	249	503	432	113	267

FIGURE 3:

Dimensions 1 and 2 of block table: symmetric part of artificial data, matrix M.

```
+--------------------------------------+
|                  :                   |
|                  :                   |
|                  : *B                |
|                  :                   |
|                  :                   |
|                  :                   |
|                  :                   |
|                  :                   |
|                  :*A                 |
|                  :                   |
|: : : : : : : : : : : : : : : : : : : |
|                  :                   |
|         F*E      :       *C          |
|                  :                   |
|                  :         *D        |
|                  :                   |
|                  :                   |
|                  :                   |
|                  :                   |
|                  :                   |
|                  :                   |
|                  :                   |
+--------------------------------------+
```

FIGURE 4:

Dimensions 3 and 4 of block table: skew-symmetric part of artificial data, matrix M.

```
+-----------------------------------------+
|                    :                    |
|                    :                    |
|                    :                    |
|                   *C                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :   *A               |
|            *D     F*E                   |
|: : : : : : : : : : *B: : : : : : : : : :|
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
|                    :                    |
+-----------------------------------------+
```

```
INERTIAS AND PERCENTAGES OF INERTIA
-----------------------------------

 1 0.692567  44.89%  *************************************************
 2 0.451659  29.28%  ********************************
 3 0.169292  10.97%  ************
 4 0.169292  10.97%  ************
 5 0.024023   1.56%  **
 6 0.012091   0.78%  *
 7 0.012091   0.78%  *
 8 0.006896   0.45%
 9 0.003328   0.22%
10 0.000761   0.05%
11 0.000761   0.05%
   --------
   1.542761
```

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	a	885	65	24	117	24	1	500	446	36	-199	71	15	392	274	59	67	8	12	185	61	185
2	b	994	71	103	238	25	6	1469	959	337	100	4	4	21	0	0	-42	1	5	-95	4	53
3	c	979	51	80	834	286	51	-312	40	11	-1256	647	472	-67	2	1	70	2	10	-77	2	25
4	d	995	114	156	1062	535	186	-564	151	80	-2	0	0	-806	308	438	-39	1	7	45	1	19
5	e	982	98	72	-974	830	134	-296	77	19	-106	10	6	-40	1	1	-244	52	242	116	12	110
6	f	990	101	65	-913	847	122	-268	73	16	-52	3	2	-22	0	0	230	54	223	114	13	109
7	A	885	65	24	117	24	1	500	446	36	199	71	15	-392	274	59	67	8	12	-185	61	185
8	B	994	71	103	238	25	6	1469	959	337	-100	4	4	-21	0	0	-42	1	5	95	4	53
9	C	979	51	80	834	286	51	-312	40	11	1256	647	472	67	2	1	70	2	10	77	2	25
10	D	995	114	156	1062	535	186	-564	151	80	2	0	0	806	308	438	-39	1	7	-45	1	19
11	E	982	98	72	-974	830	134	-296	77	19	106	10	6	40	1	1	-244	52	242	-116	12	110

12	F	990	101	65	-913	847	122	-268	73	16	52	3	2	22	0	0	230	54	223	-114	13	109

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	A	901	65	24	117	24	1	500	446	36	392	274	59	199	71	15	67	8	12	-208	78	234
2	B	992	71	103	238	25	6	1469	959	337	21	0	0	-100	4	4	-42	1	5	-75	3	33
3	C	977	51	80	834	286	51	-312	40	11	-67	2	1	1256	647	472	70	2	10	7	0	0
4	D	996	114	156	1062	535	186	-564	151	80	-806	308	438	2	0	0	-39	1	7	-67	2	42
5	E	986	98	72	-974	830	134	-296	77	19	-40	1	1	106	10	6	-244	52	242	133	16	143
6	F	983	101	65	-913	847	122	-268	73	16	-22	0	0	52	3	2	230	54	223	75	6	47
7	a	901	65	24	117	24	1	500	446	36	-392	274	59	-199	71	15	67	8	12	208	78	234
8	b	992	71	103	238	25	6	1469	959	337	-21	0	0	100	4	4	-42	1	5	75	3	33
9	c	977	51	80	834	286	51	-312	40	11	67	2	1	-1256	647	472	70	2	10	-7	0	0
10	d	996	114	156	1062	535	186	-564	151	80	806	308	438	-2	0	0	-39	1	7	67	2	42
11	e	986	98	72	-974	830	134	-296	77	19	40	1	1	-106	10	6	-244	52	242	-133	16	143
12	f	983	101	65	-913	847	122	-268	73	16	22	0	0	-52	3	2	230	54	223	-75	6	47

FIGURE 5:

Dimensions 1 and 2 of block table: symmetric part of artificial data, matrix N.

```
+-----------------------------------+
|                 :                 |
|              *B :             *   |
|                 :                 |
|                 :                 |
|                 :                 |
|                 :                 |
|               *A:                 |
|                 :                 |
|                 :                 |
|                 :                 |
|                 :    *C           |
| : : : : : : : : : : : : : : : : ::|
|          *F     :        *D       |
|          *E     :                 |
|                 :                 |
|                 :                 |
|                 :                 |
|                 :                 |
|                 :                 |
|                 :                 |
|   *             :                 |
|                 :                 |
+-----------------------------------+
```

FIGURE 6:

Dimensions 4 and 5 of block table: symmetric part of artificial data, matrix N.

```
+-------------------------------------------+
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :*D                   |
|: : : : : : : : : : : : : : : : : : : : : :|
|                   *B:                     |
|                     :  *E   *C            |
|                  *F*A                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
|                     :                     |
+-------------------------------------------+
```

INERTIAS AND PERCENTAGES OF INERTIA

```
 1 0.755416  45.58%  **********************************************
 2 0.512704  30.93%  *******************************
 3 0.091353   5.51%  ******
 4 0.075203   4.54%  *****
 5 0.075203   4.54%  *****
 6 0.056832   3.43%  ****
 7 0.056832   3.43%  ****
 8 0.014709   0.89%  *
 9 0.009491   0.57%  *
10 0.009491   0.57%  *
11 0.000209   0.01%
   --------
   1.657445
```

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	a	822	42	28	-156	22	1	793	570	51	28	1	0	373	126	77	-124	14	8	-313	89	72
2	b	962	63	114	-246	20	5	1653	915	338	48	1	2	53	1	2	-169	10	24	213	15	51
3	c	856	25	25	569	199	11	55	2	0	34	1	0	286	50	28	798	392	215	-588	213	154
4	d	958	161	148	1150	867	282	-275	50	24	-11	0	0	-231	35	114	95	6	19	29	1	2
5	e	998	127	106	-857	530	123	-521	196	67	358	93	178	267	51	120	284	58	136	313	71	219
6	f	898	82	79	-847	448	77	-354	78	20	-598	223	319	383	91	159	-300	56	97	-35	1	2
7	A	822	42	28	-156	22	1	793	570	51	28	1	0	-373	126	77	124	14	8	313	89	72
8	B	962	63	114	-246	20	5	1653	915	338	48	1	2	-53	1	2	169	10	24	-213	15	51
9	C	856	25	25	569	199	11	55	2	0	34	1	0	-286	50	28	-798	392	215	588	213	154
10	D	958	161	148	1150	867	282	-275	50	24	-11	0	0	231	35	114	-95	6	19	-29	1	2
11	E	998	127	106	-857	530	123	-521	196	67	358	93	178	-267	51	120	-284	58	136	-313	71	219

12	F	898	82	79	-847	448	77	-354	78	20	-598	223	319	-383	91	159	300	56	97	35	1	2

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	A	822	42	28	-156	22	1	793	570	51	-28	1	0	-124	14	8	-373	126	77	314	89	72
2	B	963	63	114	-246	20	5	1653	915	338	-48	1	2	-169	10	24	-53	1	2	220	16	54
3	C	650	25	25	569	199	11	55	2	0	-34	1	0	798	392	215	-286	50	28	-101	6	5
4	D	989	161	148	1150	867	282	-275	50	24	11	0	0	95	6	19	231	35	114	-219	31	136
5	E	927	127	106	-857	530	123	-521	196	67	-358	93	178	284	58	136	-267	51	120	-13	0	0
6	F	998	82	79	-847	448	77	-354	78	20	598	223	319	-300	56	97	-383	91	159	-403	101	233
7	a	822	42	28	-156	22	1	793	570	51	-28	1	0	124	14	8	373	126	77	-314	89	72
8	b	963	63	114	-246	20	5	1653	915	338	-48	1	2	169	10	24	53	1	2	-220	16	54
9	c	650	25	25	569	199	11	55	2	0	-34	1	0	-798	392	215	286	50	28	101	6	5
10	d	989	161	148	1150	867	282	-275	50	24	11	0	0	-95	6	19	-231	35	114	219	31	136
11	e	927	127	106	-857	530	123	-521	196	67	-358	93	178	-284	58	136	267	51	120	13	0	0
12	f	998	82	79	-847	448	77	-354	78	20	598	223	319	300	56	97	383	91	159	403	101	233

FIGURE 7:

Assymetric map of stacked tables of both transition, dimension 1 and 2.

```
+----------------------------------------+
|                   :                    |
|                   :                    |
|                  *B                    |
|                  A:                    |
|               bb**b                    |
|                   :                    |
|                   :                    |
|                   *aa                  |
|                   *C                   |
|                  *a                    |
| : : : : : : : : : : : : : : : : : : : |
|                   :         d          |
|             *ff   :        c****D      |
|           F**e    :          dd        |
|           E*f     :                    |
|                   :                    |
|                   :                    |
|                   :                    |
|                   :                    |
|                   :                    |
|                   :                    |
|                   :                    |
+----------------------------------------+
```

The following points could not be printed due to overlap: cc ee

INERTIAS AND PERCENTAGES OF INERTIA

```
 1 0.771914  51.06%  **************************************************
 2 0.560982  37.11%  ************************************
 3 0.099991   6.61%  ******
 4 0.065708   4.35%  ****
 5 0.013093   0.87%  *
```

1.511687

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR
1	a	982	87	26	-161	58	3	-389	338	23	159	56	22	-487	530	314
2	b	999	72	139	-201	14	4	-1668	960	359	13	0	0	267	25	79
3	c	959	72	58	1032	874	100	245	49	8	110	10	9	-175	25	34
4	d	994	80	84	1225	937	155	254	40	9	-26	0	1	160	16	31
5	e	999	94	69	-856	661	89	557	280	52	127	14	15	221	44	70
6	f	998	94	93	-902	546	99	514	178	44	639	274	384	33	1	2
7	aa	906	43	34	-32	1	0	-754	478	44	-179	27	14	-690	400	315
8	bb	1000	69	128	-203	15	4	-1651	968	335	23	0	0	220	17	51
9	cc	974	29	32	1189	840	53	337	67	6	24	0	0	-334	66	49
10	dd	997	149	165	1232	905	292	356	75	33	-52	2	4	158	15	56
11	ee	998	101	67	-835	701	92	543	297	53	-22	0	0	-10	0	0
12	ff	998	109	104	-880	534	109	409	115	32	-711	349	550	10	0	0

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR
1	A	945	91	95	-153	15	3	-1197	899	231	-5	0	0	-223	31	69
2	B	992	127	197	-146	9	3	-1488	943	501	-52	1	3	300	38	173
3	C	956	51	39	-39	1	0	-390	131	14	104	9	6	-972	814	730
4	D	1000	322	332	1199	923	601	344	76	68	-6	0	0	41	1	8
5	E	1000	254	203	-879	641	254	540	242	132	-376	117	359	18	0	1
6	F	1000	156	134	-830	531	139	443	151	54	637	312	632	89	6	19

FIGURE 8:

Dimensions 1 and 2 of block table of both transitions: average symmetric part.

```
+---------------------------------------+
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|          *E       :         *D        |
|          *F       :                   |
|                   :      *C           |
| : : : : : : : : : : : : : : : : : : : :|
|                   :                   |
|                   :                   |
|                   *A                  |
|                   :                   |
|                   :                   |
|                   :                   |
|                   :                   |
|                   *B                  |
|                   :                   |
|                   :                   |
+---------------------------------------+
```

FIGURE 9:

Dimensions 3 and 4 of block table of both transitions: average skew-symmetric part.

```
+----------------------------------------------+
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                   *A :                       |
|                     *F*B                     |
|: : : : : : : : : : :*E : : : : : : : : : : :|
|                      :                       |
|           *C         :  *D                   |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
|                      :                       |
+----------------------------------------------+
```

FIGURE 10:

Dimensions 5 and 6 of block table of both transitions: difference in symmetric parts.

```
+---------------------------------------------+
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      *E                     |
|                      *B                     |
|: : : : : : : : *D: : *A:*C : : : : : : : : :|
|                      :                      |
|                      :                      |
|                      *F                     |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
+---------------------------------------------+
```

FIGURE 11:

Dimensions 7 and 8 of block table of both transitions: difference in skew-symmetric parts.

```
+---------------------------------------------+
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      E**C                   |
|: : : : : : : : : F*D**B:*A : : : : : : : : :|
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
|                      :                      |
+---------------------------------------------+
```

```
INERTIAS AND PERCENTAGES OF INERTIA
-----------------------------------

 1 0.726579  41.63%  **************************************************
 2 0.496140  28.43%  **********************************
 3 0.103911   5.95%  *******
 4 0.103909   5.95%  *******
```

```
 5 0.076464   4.38%  *****
 6 0.075283   4.31%  *****
 7 0.024848   1.42%  **
 8 0.024848   1.42%  **
 9 0.020222   1.16%  *
10 0.020222   1.16%  *
11 0.015397   0.88%  *
12 0.012092   0.69%  *
13 0.011780   0.67%  *
14 0.011780   0.67%  *
15 0.007513   0.43%  *
16 0.005272   0.30%
17 0.003690   0.21%
18 0.003689   0.21%
19 0.000787   0.05%
20 0.000459   0.03%
21 0.000206   0.01%
22 0.000206   0.01%
23 0.000081   0.00%
   --------
   1.745377
```

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	a	752	28	15	63	4	0	-697	518	27	-317	107	27	-307	100	25	-26	1	0	-142	21	7
2	b	965	34	49	58	1	0	-1560	951	164	-135	7	6	102	4	3	17	0	0	53	1	1
3	c	851	19	25	746	240	15	142	9	1	519	116	49	-1002	433	184	-327	46	27	-126	7	4
4	d	970	69	81	1103	593	115	473	109	31	501	122	166	224	24	33	498	121	223	-21	0	0
5	e	851	56	41	-940	691	68	402	126	18	40	1	1	-78	5	3	11	0	0	189	28	27
6	f	855	45	39	-914	555	52	305	62	8	-46	1	1	-54	2	1	12	0	0	-594	235	210
7	aa	752	28	15	63	4	0	-697	518	27	-317	107	27	-307	100	25	26	1	0	142	21	7
8	bb	965	34	49	58	1	0	-1560	951	164	-135	7	6	102	4	3	-17	0	0	-53	1	1
9	cc	851	19	25	746	240	15	142	9	1	519	116	49	-1002	433	184	327	46	27	126	7	4
10	dd	970	69	81	1103	593	115	473	109	31	501	122	166	224	24	33	-498	121	223	21	0	0
11	ee	851	56	41	-940	691	68	402	126	18	40	1	1	-78	5	3	-11	0	0	-189	28	27
12	ff	855	45	39	-914	555	52	305	62	8	-46	1	1	-54	2	1	-12	0	0	594	235	210
13	A	752	28	15	63	4	0	-697	518	27	317	107	27	307	100	25	-26	1	0	-142	21	7
14	B	965	34	49	58	1	0	-1560	951	164	135	7	6	-102	4	3	17	0	0	53	1	1
15	C	851	19	25	746	240	15	142	9	1	-519	116	49	1002	433	184	-327	46	27	-126	7	4
16	D	970	69	81	1103	593	115	473	109	31	-501	122	166	-224	24	33	498	121	223	-21	0	0
17	E	851	56	41	-940	691	68	402	126	18	-40	1	1	78	5	3	11	0	0	189	28	27
18	F	855	45	39	-914	555	52	305	62	8	46	1	1	54	2	1	12	0	0	-594	235	210
19	AA	752	28	15	63	4	0	-697	518	27	317	107	27	307	100	25	26	1	0	142	21	7
20	BB	965	34	49	58	1	0	-1560	951	164	135	7	6	-102	4	3	-17	0	0	-53	1	1
21	CC	851	19	25	746	240	15	142	9	1	-519	116	49	1002	433	184	327	46	27	126	7	4
22	DD	970	69	81	1103	593	115	473	109	31	-501	122	166	-224	24	33	-498	121	223	21	0	0
23	EE	851	56	41	-940	691	68	402	126	18	-40	1	1	78	5	3	-11	0	0	-189	28	27
24	FF	855	45	39	-914	555	52	305	62	8	46	1	1	54	2	1	-12	0	0	594	235	210

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	A	752	28	15	63	4	0	-697	518	27	-307	100	25	317	107	27	26	1	0	-142	21	7
2	B	965	34	49	58	1	0	-1560	951	164	102	4	3	135	7	6	-17	0	0	53	1	1
3	C	851	19	25	746	240	15	142	9	1	-1002	433	184	-519	116	49	327	46	27	-126	7	4

4	D	970	69	81	1103	593	115	473	109	31	224	24	33	-501	122	166	-498	121	223	-21	0	0
5	E	851	56	41	-940	691	68	402	126	18	-78	5	3	-40	1	1	-11	0	0	189	28	27
6	F	855	45	39	-914	555	52	305	62	8	-54	2	1	46	1	1	-12	0	0	-594	235	210
7	AA	752	28	15	63	4	0	-697	518	27	-307	100	25	317	107	27	-26	1	0	142	21	7
8	BB	965	34	49	58	1	0	-1560	951	164	102	4	3	135	7	6	17	0	0	-53	1	1
9	CC	851	19	25	746	240	15	142	9	1	-1002	433	184	-519	116	49	-327	46	27	126	7	4
10	DD	970	69	81	1103	593	115	473	109	31	224	24	33	-501	122	166	498	121	223	21	0	0
11	EE	851	56	41	-940	691	68	402	126	18	-78	5	3	-40	1	1	11	0	0	-189	28	27
12	FF	855	45	39	-914	555	52	305	62	8	-54	2	1	46	1	1	12	0	0	594	235	210
13	a	752	28	15	63	4	0	-697	518	27	307	100	25	-317	107	27	26	1	0	-142	21	7
14	b	965	34	49	58	1	0	-1560	951	164	-102	4	3	-135	7	6	-17	0	0	53	1	1
15	c	851	19	25	746	240	15	142	9	1	1002	433	184	519	116	49	327	46	27	-126	7	4
16	d	970	69	81	1103	593	115	473	109	31	-224	24	33	501	122	166	-498	121	223	-21	0	0
17	e	851	56	41	-940	691	68	402	126	18	78	5	3	40	1	1	-11	0	0	189	28	27
18	f	855	45	39	-914	555	52	305	62	8	54	2	1	-46	1	1	-12	0	0	-594	235	210
19	aa	752	28	15	63	4	0	-697	518	27	307	100	25	-317	107	27	-26	1	0	142	21	7
20	bb	965	34	49	58	1	0	-1560	951	164	-102	4	3	-135	7	6	17	0	0	-53	1	1
21	cc	851	19	25	746	240	15	142	9	1	1002	433	184	519	116	49	-327	46	27	126	7	4
22	dd	970	69	81	1103	593	115	473	109	31	-224	24	33	501	122	166	498	121	223	21	0	0
23	ee	851	56	41	-940	691	68	402	126	18	78	5	3	40	1	1	11	0	0	-189	28	27
24	ff	855	45	39	-914	555	52	305	62	8	54	2	1	-46	1	1	12	0	0	594	235	210

FIGURE 12:

Asymmetric map of dimension 1 and 2 of value transitions from grandfathers to fathers.

```
+----------------------------------------------------------------+
|              :                                                 |
|       *FR    :                                                 |
|              :                                                 |
|             *SO                                                |
|          *re:                                                  |
|              :                                                 |
|              :                                                 |
|   *IN      *lo                                                 |
|              :                                                 |
|       *HA    :                                                 |
|              :                                                 |
|       *fr   *cu                                                |
|              :              *to           *in                  |
|: : : : : : :*ho: *RE : : : : : : : : : : : : : : : : : : : *LO |
|         *fa  :    *so                                          |
|              :                                                 |
|            *ha                                                 |
|   *TO        :                                                 |
|              :                                                 |
|        *FA   :                                                 |
|        *HO   :                          *CU                    |
|              :                                                 |
+----------------------------------------------------------------+
```

INERTIAS AND PERCENTAGES OF INERTIA

```
 1 0.321182  44.41%  ************************************************
 2 0.149594  20.69%  ***********************
 3 0.102624  14.19%  ****************
 4 0.047096   6.51%  *******
 5 0.044292   6.12%  *******
 6 0.023009   3.18%  ****
 7 0.021053   2.91%  ***
 8 0.011429   1.58%  **
 9 0.002874   0.40%
   --------
```

0.723152

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	ho	948	271	44	-27	6	1	-87	65	14	230	457	140	-167	241	161	-14	2	1	-144	178	243
2	fa	994	248	86	-313	389	76	-209	173	72	-213	180	110	-46	8	11	239	227	320	64	16	44
3	cu	891	16	61	-64	1	0	257	23	7	-1290	585	252	670	158	148	-485	83	82	-341	41	78
4	re	909	31	91	-208	20	4	1287	784	343	248	29	19	-232	25	35	-250	30	44	208	20	58
5	ha	974	116	73	-116	29	5	-431	407	144	-218	104	54	-44	4	5	-424	394	472	128	36	82
6	so	888	47	52	350	150	18	-249	76	19	405	201	74	531	346	278	-32	1	1	-303	113	186
7	fr	938	109	91	-452	336	69	224	83	36	438	316	203	294	142	199	-17	1	1	190	60	171
8	lo	952	70	101	-148	21	5	877	731	358	-424	171	122	-77	6	9	20	0	1	-157	24	75
9	in	985	47	300	2129	972	656	36	0	0	48	0	1	-187	8	35	-30	0	1	141	4	40
10	to	917	47	100	1072	739	167	129	11	5	-235	35	25	347	77	119	270	47	77	106	7	23

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	HO	897	202	78	-213	163	28	-423	641	241	31	4	2	116	48	58	18	1	2	107	41	101
2	FA	967	155	72	-239	169	28	-377	422	148	255	192	98	-172	88	97	-65	12	15	-169	84	192
3	CU	923	23	89	1168	496	99	-415	62	27	62	1	1	-611	136	185	-742	200	289	276	28	77
4	RE	986	116	67	191	88	13	-5	0	0	-524	662	311	133	43	44	-55	7	8	-277	185	388
5	HA	930	147	61	-264	231	32	204	138	41	-302	303	131	158	82	78	-150	74	74	174	100	193
6	SO	932	140	90	38	3	1	559	669	292	-77	13	8	-334	239	330	64	9	13	17	1	2
7	FR	993	70	123	-283	63	17	668	350	208	775	472	408	309	75	141	-189	28	56	-82	5	20
8	LO	998	62	337	1947	964	732	-18	0	0	211	11	27	189	9	47	232	14	75	38	0	4
9	IN	683	39	39	-442	266	24	308	130	25	-114	18	5	-39	2	1	443	267	172	23	1	1
10	TO	850	47	43	-425	268	26	-248	91	19	135	27	8	-137	28	19	532	420	297	104	16	22

FIGURE 13:

Asymmetric map of dimension 1 and 2 of value transitions from fathers to sons.

```
+-------------------------------------------------+
|                           :                     |
|       *IN                 :                     |
|                           :                     |
|                           :                     |
|                           :                     |
|                           :                     |
|                           :                     |
|                           :                     |
|                           :                     |
|              *in          :                     |
|                           :                     |
|                           :                *TO  |
|                           :                     |
|                           :          lo**CU     |
|                           :                     |
|                  *cu      :        *fr          |
|   *SO                     :                     |
|                           :  *FR re**LO         |
|                           :                     |
|         *so            *FA:                     |
| : : : : : : : : : : : : : : : : : : : : : : : : |
|                           : *fa                 |
|                           HA**ha                |
|                       HO* *to                   |
|                           :*ho                  |
|                           :                     |
|                        *RE:                     |
|                           :                     |
+-------------------------------------------------+
```

INERTIAS AND PERCENTAGES OF INERTIA

```
 1 0.412250  33.79%  **************************************************
 2 0.295251  24.20%  ************************************
 3 0.242505  19.87%  *****************************
 4 0.137862  11.30%  *****************
 5 0.063311   5.19%  ********
 6 0.028236   2.31%  ***
 7 0.023095   1.89%  ***
 8 0.012775   1.05%  **
 9 0.004924   0.40%  *
   --------
   1.220208
```

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	ho	996	202	110	63	6	2	-604	549	249	87	11	6	517	402	391	28	1	2	-133	27	126
2	fa	869	155	40	122	47	6	-229	166	28	90	26	5	-442	618	220	60	11	9	23	2	3
3	cu	697	23	37	-652	220	24	678	238	36	-406	85	16	162	14	4	-367	70	49	-367	70	111
4	re	950	116	70	630	537	112	430	251	73	-211	60	21	-71	7	4	-254	88	119	-73	7	22
5	ha	914	147	63	203	79	15	-343	226	59	232	103	33	-509	496	277	-49	5	6	-51	5	14
6	so	998	140	237	-1303	819	575	130	8	8	-588	167	199	-53	1	3	42	1	4	56	2	16
7	fr	968	70	73	645	325	70	585	267	81	-308	74	27	344	92	60	-445	154	218	266	55	174
8	lo	997	62	119	888	336	119	858	314	155	-457	89	53	120	6	6	767	251	577	-20	0	1
9	in	998	39	207	-911	128	78	1422	311	265	1898	554	576	182	5	9	32	0	1	-93	1	12

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
10	to	910	47	44	-53	2	0	-544	258	47	575	288	63	275	66	26	147	19	16	563	276	522

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	HO	900	93	41	-163	49	6	-540	544	92	-11	0	0	382	273	99	-2	0	0	134	34	59
2	FA	960	31	71	-249	22	5	140	7	2	1419	723	257	-333	40	25	189	13	17	658	155	475
3	CU	957	16	60	993	208	37	934	184	46	-526	59	18	367	28	15	-1390	408	473	574	70	181
4	RE	985	54	94	-232	25	7	-903	385	150	122	7	3	1080	551	459	186	16	30	-38	1	3
5	HA	980	333	80	107	39	9	-396	532	177	78	21	8	-336	383	272	2	0	0	-38	5	17
6	SO	1000	93	271	-1709	822	659	467	61	69	-628	111	151	-130	5	11	35	0	2	35	0	4
7	FR	876	256	51	226	210	32	319	418	88	14	1	0	155	99	45	-156	99	98	-110	49	109
8	LO	851	85	61	700	564	101	327	124	31	-234	63	19	-232	62	33	175	35	41	-58	4	10
9	IN	978	8	158	-1418	81	38	2616	276	180	3853	599	475	489	10	13	129	1	2	-550	12	83
10	TO	981	31	113	1188	316	106	1257	354	166	-727	119	68	345	27	27	830	154	337	229	12	58

FIGURE 14:

Dimensions 1 and 2 of block table: symmetric part of value transition, grandfathers to fathers.

```
+---------------------------------------------------------------+
|                              :                                |
|                              :                            *IN |
|                              :                                |
|   *LO                        :                                |
|                              :                                |
|                              :                                |
|                              :                                |
|                              :                                |
|                              :                                |
|                              :                                |
|                              :                                |
|         *CU                  :                                |
|                              : *RE    *SO                     |
|                              :                                |
|                              :           *TO                  |
|                              :                                |
|: : : : : : : : : : : : : :*FR: : : : : : : : : : : : : : : : :|
|                              :                                |
|                              :                                |
|                              :*HO  *HA                        |
|                        *FA   :                                |
|                              :                                |
+---------------------------------------------------------------+
```

FIGURE 15:

Dimensions 3 and 4 of block table: skew-symmetric part of value transition, fathers to sons.

```
+---------------------------------------+
|           :                           |
|           :                    *IN    |
|           :                           |
|           :                           |
|           :                   *TO     |
|           :                           |
|           :  *FA                      |
|           :   *HO                     |
|           :                           |
|   *FR     :                           |
|           :                           |
| : : : : : : :*HA: : : : : : : : : : : |
|           :                           |
|           :                           |
|           :                           |
|           :                       *CU |
|           :                     *LO   |
|           :   *SO                     |
|           :                           |
|           :                           |
|           :        *RE                |
|           :                           |
+---------------------------------------+
```

INERTIAS AND PERCENTAGES OF INERTIA

```
 1 0.171766  20.02%  **************************************************
 2 0.142276  16.58%  *****************************************
 3 0.142276  16.58%  *****************************************
 4 0.096497  11.24%  ****************************
 5 0.049428   5.76%  **************
 6 0.047201   5.50%  **************
 7 0.047201   5.50%  **************
 8 0.045472   5.30%  *************
 9 0.030450   3.55%  *********
10 0.021586   2.52%  ******
11 0.021586   2.52%  ******
12 0.011710   1.36%  ***
13 0.011710   1.36%  ***
14 0.009352   1.09%  ***
15 0.007270   0.85%  **
16 0.002263   0.26%  *
17 0.000057   0.01%
18 0.000057   0.01%
19 0.000024   0.00%
   --------
   0.858182
```

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	ho	742	118	28	-27	3	0	231	260	45	-148	106	18	-198	190	48	-194	183	90	-2	0	0
2	fa	593	101	50	241	137	34	331	260	78	-93	20	6	-245	142	63	119	34	29	13	0	0
3	cu	639	10	40	807	185	37	-324	30	7	-894	227	54	287	23	8	609	105	73	487	68	49

4	re	841	37	47	-76	5	1	-726	483	136	-338	104	29	179	29	12	-388	138	112	-298	81	69
5	ha	316	66	31	-233	133	21	-66	11	2	-109	29	6	-180	80	22	6	0	0	-161	63	36
6	so	875	47	40	-332	148	30	-484	315	76	-144	28	7	195	51	18	426	244	170	-255	87	64
7	fr	206	45	39	105	15	3	103	14	3	321	139	32	-30	1	0	48	3	2	158	33	23
8	lo	975	33	105	1017	377	198	-396	57	36	-829	250	159	719	188	176	-120	5	10	515	97	185
9	in	939	21	83	-1102	363	151	728	158	79	-801	192	96	824	203	150	-106	3	5	-260	20	31
10	to	799	23	37	-430	136	25	474	165	37	-750	415	92	73	4	1	140	14	9	-294	64	43
11	ho	742	118	28	-27	3	0	-231	260	45	148	106	18	-198	190	48	-194	183	90	2	0	0
12	fa	593	101	50	241	137	34	-331	260	78	93	20	6	-245	142	63	119	34	29	-13	0	0
13	cu	639	10	40	807	185	37	324	30	7	894	227	54	287	23	8	609	105	73	-487	68	49
14	re	841	37	47	-76	5	1	726	483	136	338	104	29	179	29	12	-388	138	112	298	81	69
15	ha	316	66	31	-233	133	21	66	11	2	109	29	6	-180	80	22	6	0	0	161	63	36
16	so	875	47	40	-332	148	30	484	315	76	144	28	7	195	51	18	426	244	170	255	87	64
17	fr	206	45	39	105	15	3	-103	14	3	-321	139	32	-30	1	0	48	3	2	-158	33	23
18	lo	975	33	105	1017	377	198	396	57	36	829	250	159	719	188	176	-120	5	10	-515	97	185
19	in	939	21	83	-1102	363	151	-728	158	79	801	192	96	824	203	150	-106	3	5	260	20	31
20	to	799	23	37	-430	136	25	-474	165	37	750	415	92	73	4	1	140	14	9	294	64	43

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	HO	746	118	28	27	3	0	148	106	18	231	260	45	-198	190	48	194	183	90	28	4	2
2	FA	889	101	50	-241	137	34	93	20	6	331	260	78	-245	142	63	-119	34	29	353	296	267
3	CU	571	10	40	-807	185	37	894	227	54	-324	30	7	287	23	8	-609	105	73	33	0	0
4	RE	808	37	47	76	5	1	338	104	29	-726	483	136	179	29	12	388	138	112	230	48	41
5	HA	397	66	31	233	133	21	109	29	6	-66	11	2	-180	80	22	-6	0	0	243	144	82
6	SO	787	47	40	332	148	30	144	28	7	-484	315	76	195	51	18	-426	244	170	10	0	0
7	FR	253	45	39	-105	15	3	-321	139	32	103	14	3	-30	1	0	-48	3	2	245	81	57
8	LO	878	33	105	-1017	377	198	829	250	159	-396	57	36	719	188	176	120	5	10	43	1	1
9	IN	951	21	83	1102	363	151	801	192	96	728	158	79	824	203	150	106	3	5	-328	32	48
10	TO	737	23	37	430	136	25	750	415	92	474	165	37	73	4	1	-140	14	9	-47	2	1
11	HO	746	118	28	27	3	0	-148	106	18	-231	260	45	-198	190	48	194	183	90	-28	4	2
12	FA	889	101	50	-241	137	34	-93	20	6	-331	260	78	-245	142	63	-119	34	29	-353	296	267
13	CU	571	10	40	-807	185	37	-894	227	54	324	30	7	287	23	8	-609	105	73	-33	0	0
14	RE	808	37	47	76	5	1	-338	104	29	726	483	136	179	29	12	388	138	112	-230	48	41
15	HA	397	66	31	233	133	21	-109	29	6	66	11	2	-180	80	22	-6	0	0	-243	144	82
16	SO	787	47	40	332	148	30	-144	28	7	484	315	76	195	51	18	-426	244	170	-10	0	0
17	FR	253	45	39	-105	15	3	321	139	32	-103	14	3	-30	1	0	-48	3	2	-245	81	57
18	LO	878	33	105	-1017	377	198	-829	250	159	396	57	36	719	188	176	120	5	10	-43	1	1
19	IN	951	21	83	1102	363	151	-801	192	96	-728	158	79	824	203	150	106	3	5	328	32	48
20	TO	737	23	37	430	136	25	-750	415	92	-474	165	37	73	4	1	-140	14	9	47	2	1

FIGURE 16:

Dimensions 1 and 4 of block table: symmetric part of value transition, fathers to sons.

```
+------------------------------------+
|                        :           |
|                        :  *HA      |
|                        :           |
|                        :   *FA     |
|   *SO                  :  *HO      |
|                        :           |
|: : : : : : : : : : : : : : : : : : |
|                        :      *TO  |
|                        :   *RE     |
|                        :      *LO  |
|                        : *FR       |
|                        :           |
|                        :           |
|                    *CU :           |
|                        :           |
|                        :           |
|                        :           |
|                        :           |
|                        :           |
|                        :           |
|               *IN      :           |
|                        :           |
+------------------------------------+
```

FIGURE 17:

Dimensions 2 and 3 of block table: skew-symmetric part of value transition, fathers to sons.

```
+-----------------------------------------------+
|                                   :           |
|                                   :   *TO     |
|                                   :           |
|                      *IN          :           |
|     *FA                           :           |
|             *HO                   :           |
|                         *SO       :           |
| : : : : : : : : : : : : : : : : : : : : : : : |
|                                   :           |
|                                   :           |
|                                   :           |
|                                   :           |
|                                   :           |
|                                   :           |
|   *RE                          *CU:           |
|                                   :           |
|                                   :           |
|                                   :           |
|                              *HA  :           |
|                                   :       *FR |
|                                  *LO          |
|                                   :           |
+-----------------------------------------------+
```

```
INERTIAS AND PERCENTAGES OF INERTIA
-----------------------------------

 1 0.334597  23.11%  **************************************************
 2 0.228473  15.78%  **********************************
```

```
 3 0.228473  15.78%  **********************************
 4 0.155379  10.73%  ***********************
 5 0.108644   7.50%  ****************
 6 0.069906   4.83%  **********
 7 0.069906   4.83%  **********
 8 0.054623   3.77%  ********
 9 0.048451   3.35%  *******
10 0.033812   2.34%  *****
11 0.031246   2.16%  *****
12 0.031246   2.16%  *****
13 0.013982   0.97%  **
14 0.013982   0.97%  **
15 0.013708   0.95%  **
16 0.009214   0.64%  *
17 0.001844   0.13%
18 0.000074   0.01%
19 0.000074   0.01%
   --------
   1.447632
```

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	ho	749	74	42	213	55	10	-145	26	7	-644	502	134	126	19	7	151	28	15	314	120	104
2	fa	677	47	49	226	34	7	-178	21	6	-867	497	153	218	31	14	-374	92	60	-50	2	2
3	cu	483	10	21	-286	26	2	357	41	5	-85	2	0	-701	159	31	513	85	24	-720	168	72
4	re	731	43	55	259	36	9	380	78	27	-928	463	161	-227	28	14	318	54	40	-366	72	82
5	ha	939	120	64	187	46	13	612	486	197	-144	27	11	454	268	160	-202	53	45	214	60	79
6	so	936	58	105	-1514	876	398	-74	2	1	-274	29	19	143	8	8	173	11	16	167	11	23
7	fr	760	81	51	143	22	5	658	474	154	200	44	14	-432	204	98	118	15	11	28	1	1
8	lo	574	37	44	504	148	28	734	313	87	-13	0	0	-306	54	22	193	22	13	-250	36	33
9	in	919	12	50	-681	75	16	-258	11	3	-371	22	7	-1379	306	142	-1591	407	271	-782	98	102
10	to	293	19	20	454	139	12	-371	93	12	119	10	1	-172	20	4	192	25	7	102	7	3
11	ho	749	74	42	213	55	10	145	26	7	644	502	134	126	19	7	151	28	15	-314	120	104
12	fa	677	47	49	226	34	7	178	21	6	867	497	153	218	31	14	-374	92	60	50	2	2
13	cu	483	10	21	-286	26	2	-357	41	5	85	2	0	-701	159	31	513	85	24	720	168	72
14	re	731	43	55	259	36	9	-380	78	27	928	463	161	-227	28	14	318	54	40	366	72	82
15	ha	939	120	64	187	46	13	-612	486	197	144	27	11	454	268	160	-202	53	45	-214	60	79
16	so	936	58	105	-1514	876	398	74	2	1	274	29	19	143	8	8	173	11	16	-167	11	23
17	fr	760	81	51	143	22	5	-658	474	154	-200	44	14	-432	204	98	118	15	11	-28	1	1
18	lo	574	37	44	504	148	28	-734	313	87	13	0	0	-306	54	22	193	22	13	250	36	33
19	in	919	12	50	-681	75	16	258	11	3	371	22	7	-1379	306	142	-1591	407	271	782	98	102
20	to	293	19	20	454	139	12	371	93	12	-119	10	1	-172	20	4	192	25	7	-102	7	3

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	HO	633	74	42	213	55	10	-644	502	134	145	26	7	126	19	7	151	28	15	56	4	3
2	FA	798	47	49	226	34	7	-867	497	153	178	21	6	218	31	14	-374	92	60	-431	123	124
3	CU	373	10	21	-286	26	2	-85	2	0	-357	41	5	-701	159	31	513	85	24	-425	58	25
4	RE	735	43	55	259	36	9	-928	463	161	-380	78	27	-227	28	14	318	54	40	376	76	86
5	HA	889	120	64	187	46	13	-144	27	11	-612	486	197	454	268	160	-202	53	45	85	9	12
6	SO	971	58	105	-1514	876	398	-274	29	19	74	2	1	143	8	8	173	11	16	-344	45	98
7	FR	818	81	51	143	22	5	200	44	14	-658	474	154	-432	204	98	118	15	11	-232	59	63
8	LO	570	37	44	504	148	28	-13	0	0	-734	313	87	-306	54	22	193	22	13	-238	33	30
9	IN	822	12	50	-681	75	16	-371	22	7	258	11	3	-1379	306	142	-1591	407	271	-109	2	2

10	TO	423	19	20	454	139	12	119	10	1	371	93	12	-172	20	4	192	25	7	-451	137	56
11	HO	633	74	42	213	55	10	644	502	134	-145	26	7	126	19	7	151	28	15	-56	4	3
12	FA	798	47	49	226	34	7	867	497	153	-178	21	6	218	31	14	-374	92	60	431	123	124
13	CU	373	10	21	-286	26	2	85	2	0	357	41	5	-701	159	31	513	85	24	425	58	25
14	RE	735	43	55	259	36	9	928	463	161	380	78	27	-227	28	14	318	54	40	-376	76	86
15	HA	889	120	64	187	46	13	144	27	11	612	486	197	454	268	160	-202	53	45	-85	9	12
16	SO	971	58	105	-1514	876	398	274	29	19	-74	2	1	143	8	8	173	11	16	344	45	98
17	FR	818	81	51	143	22	5	-200	44	14	658	474	154	-432	204	98	118	15	11	232	59	63
18	LO	570	37	44	504	148	28	13	0	0	734	313	87	-306	54	22	193	22	13	238	33	30
19	IN	822	12	50	-681	75	16	371	22	7	-258	11	3	-1379	306	142	-1591	407	271	109	2	2
20	TO	423	19	20	454	139	12	-119	10	1	-371	93	12	-172	20	4	192	25	7	451	137	56

FIGURE 18:

Assymetric map of stacked table of both transition.

```
+---------------------------------------------------------------------------------------------------+
|                                              :                                                    |
|               *SO                            :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                          *re1:                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                 *so2         :         *cu2                                       |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                 *lo1         :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                             *FR                    |
|                                    *in2      :             *HA                                    |
|                                              :                                                    |
|                                              :              *ha2                                  |
|   *IN                                        :            *fa2                                    |
|                                              :                                                    |
|: : : : : : : : : : : : : : : : : : : : : :*cu1 ho2**to2: : : : : : : : *re2: : : : : : : : : :  |
|                                         *fr1 :                                                    |
|                                              :                     *fr2                           |
|                            *fa1  *ho1     *to1                    *in1  *lo2                      |
|                                              :                                                    |
|                          *RE                 :                                                    |
|                               *ha1           :                                                    |
|                                              :                                                    |
|                                          *so1:                                                *LO |
|                                              :                                                    |
|                                              :                                                    |
|                       *HO                    :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :                                                    |
|                                              :      *TO                                           |
|            *FA                               :                                                    |
|                                              :                             *CU                    |
|                                              :                                                    |
+---------------------------------------------------------------------------------------------------+
```

```
INERTIAS AND PERCENTAGES OF INERTIA
-----------------------------------

 1 0.292832  28.51%  **************************************************
 2 0.194394  18.93%  *********************************
 3 0.181938  17.72%  *******************************
 4 0.141067  13.74%  ************************
 5 0.066453   6.47%  ***********
 6 0.055161   5.37%  *********
```

```
7 0.048601   4.73%  ********
8 0.027381   2.67%  *****
9 0.019180   1.87%  ***
  --------
  1.027008
```

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	ho1	884	136	60	-439	425	89	-286	180	57	-268	158	53	-108	26	11	-192	82	76	-79	14	15
2	fa1	944	124	74	-647	683	177	-286	133	52	15	0	0	-98	16	8	37	2	3	260	110	152
3	cu1	797	8	22	-105	4	0	-42	1	0	775	202	26	892	268	44	950	304	105	238	19	8
4	re1	975	16	27	-148	12	1	1266	894	128	-250	35	5	-74	3	1	-186	19	8	-142	11	6
5	ha1	959	58	47	-519	325	53	-486	286	71	143	25	7	273	90	31	-127	20	14	-420	213	186
6	so1	600	23	21	-131	18	1	-623	419	46	-169	31	4	221	53	8	224	54	18	-150	24	9
7	fr1	895	54	21	-178	79	6	-122	37	4	112	31	4	-537	722	111	-78	15	5	62	10	4
8	lo1	835	35	46	-464	160	26	755	424	102	-107	8	2	145	16	5	500	186	131	238	42	36
9	in1	958	23	99	751	128	45	-290	19	10	-1401	447	251	1262	362	262	-62	1	1	-69	1	2
10	to1	876	23	31	223	36	4	-39	1	0	-432	135	24	899	587	133	31	1	0	400	116	67
11	ho2	968	101	52	200	75	14	-55	6	2	487	447	131	177	59	22	440	365	294	-90	15	15
12	fa2	960	78	45	486	393	63	98	16	4	380	241	62	198	65	21	-358	213	149	140	32	27
13	cu2	904	12	30	366	50	5	1010	380	61	-434	70	12	-764	217	48	388	56	26	-593	131	74
14	re2	993	58	66	1020	892	206	-31	1	0	-256	56	21	-191	31	15	19	0	0	-123	13	16
15	ha2	972	74	73	543	289	74	172	29	11	755	560	231	-17	0	0	-305	92	103	34	1	2
16	so2	942	70	105	-449	130	48	1070	738	411	-226	33	20	198	25	19	-146	14	22	-62	2	5
17	fr2	889	35	50	801	436	76	-218	32	8	-447	136	38	-463	145	53	182	23	17	-417	118	110
18	lo2	907	31	72	982	403	102	-256	27	10	-594	148	60	-568	135	71	104	5	5	673	189	254
19	in2	588	19	47	-335	45	7	379	58	14	-406	66	18	-990	396	135	229	21	15	-49	1	1
20	to2	826	23	10	-109	27	1	-239	128	7	502	565	32	46	5	0	129	37	6	-169	64	12

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	HO	779	147	72	-453	407	103	-393	307	117	146	42	17	57	6	3	-1	0	0	-89	16	21
2	FA	900	93	97	-666	415	141	-559	293	150	-61	3	2	-251	59	41	-350	114	171	-131	16	29
3	CU	813	19	67	596	101	24	-595	100	35	-1045	309	116	412	48	23	-140	6	6	-942	251	312
4	RE	967	85	79	-394	164	45	-196	40	17	87	8	4	547	317	181	643	437	531	34	1	2
5	HA	978	240	115	281	161	65	159	52	31	575	674	436	122	31	26	-153	48	85	78	12	26
6	SO	993	116	171	-599	237	142	938	581	526	-482	154	149	131	11	14	-126	10	28	-1	0	0
7	FR	981	163	118	596	478	198	199	53	33	-36	2	1	-496	330	284	213	61	111	-208	58	128
8	LO	967	74	144	945	446	224	-305	46	35	-715	255	207	584	171	178	-158	12	28	271	37	98
9	IN	592	23	64	-835	245	55	59	1	0	-250	22	8	-748	197	92	345	42	42	491	85	102
10	TO	795	39	74	145	11	3	-524	141	55	-533	145	60	-755	292	157	23	0	0	634	206	283

FIGURE 19:

Dimensions 1 and 2 of block tables of both transitions: average symmetric part.

```
+---------------------------------------------------+
|                              :                    |
|              *IN             :                    |
|                              :                    |
|                              :                    |
|                              :                    |
|                              :                    |
|                              :                *LO |
|   *SO                        :                    |
|                              *CU                  |
|                              :                    |
|                              :  *FR               |
|                              :    *RE             |
|                              :                    |
|                              : *TO                |
|: : : : : : : : : : : : : : : : : : : : : : : : : :|
|                              :                    |
|                              :                    |
|                              : *HO                |
|                              :  *FA               |
|                              :                    |
|                           *HA:                    |
|                              :                    |
+---------------------------------------------------+
```

FIGURE 20:

Dimensions 4 and 5 of block tables of both transitions: average skew-symmetric part.

```
+--------------------------------------------------------+
|                     :                                  |
|                     :              *RE                 |
|                     :                                  |
|                     :                                  |
|                     :                                  |
|                     :                                  |
|                     :      *LO                         |
|                     :        *CU                       |
|                     :                                  |
|                     :                                  |
|                *HA  :*SO                               |
|   *FR               :                                  |
|                     :                                  |
|                     :                                  |
|                     :                                  |
|                     :                                  |
|                     :             *TO              *IN |
|                     :                                  |
| : : : : : : : : : : : : : : : : : : : :*HO: : : : : : :|
|                     :                                  |
|                     :                    *FA           |
|                     :                                  |
+--------------------------------------------------------+
```

FIGURE 21:

Dimensions 6 and 7 of block tables of both transitions: difference in skew-symmetric parts.

```
+-------------------------------------------------+
|                                 :               |
|                                 :    *TO        |
|                                 :               |
|                                 :               |
|                      *SO        :               |
|                                 *IN             |
|                                 :               |
|                                 :               |
| : : : : : : : : : : : *HO : : : : : : : : : : : |
|                                 :               |
|                                 :               |
|   *RE           *FA             :               |
|                                 :               |
|                                 :               |
|                                 :               |
|                                 :               |
|                                 :   *CU  *FR    |
|                             *HA :               |
|                                 :               |
|                                 :               |
|                                 :           *LO |
|                                 :               |
+-------------------------------------------------+
```

INERTIAS AND PERCENTAGES OF INERTIA

```
 1 0.169275  12.70%  **************************************************
 2 0.110566   8.30%  *********************************
 3 0.092615   6.95%  ***************************
 4 0.088864   6.67%  **************************
 5 0.088864   6.67%  **************************
 6 0.081471   6.11%  ************************
 7 0.081471   6.11%  ************************
 8 0.079127   5.94%  ***********************
 9 0.064303   4.83%  *******************
10 0.062862   4.72%  *******************
11 0.062862   4.72%  *******************
12 0.046138   3.46%  **************
13 0.033352   2.50%  **********
14 0.033111   2.48%  **********
15 0.032353   2.43%  **********
16 0.022579   1.69%  *******
17 0.021312   1.60%  ******
18 0.021312   1.60%  ******
19 0.021236   1.59%  ******
20 0.017553   1.32%  *****
21 0.017553   1.32%  *****
22 0.016366   1.23%  *****
23 0.010504   0.79%  ***
24 0.010504   0.79%  ***
25 0.009582   0.72%  ***
26 0.008452   0.63%  **
27 0.008452   0.63%  **
28 0.006234   0.47%  **
29 0.005252   0.39%  **
30 0.004780   0.36%  *
31 0.001844   0.14%  *
32 0.000588   0.04%
```

33	0.000588	0.04%
34	0.000349	0.03%
35	0.000205	0.02%
36	0.000015	0.00%
37	0.000015	0.00%
38	0.000001	0.00%
39	0.000001	0.00%

	1.332510	

ROW CONTRIBUTIONS

I	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	ho	561	48	21	83	12	2	-187	61	15	386	259	77	6	0	0	363	229	71	2	0	0
2	fa	568	37	29	108	11	3	-252	61	21	570	315	129	82	6	3	395	151	65	154	23	11
3	cu	140	5	14	-2	0	0	329	28	5	271	19	4	-436	50	10	165	7	1	371	36	8
4	re	370	20	26	216	27	5	130	10	3	201	24	9	-657	252	97	279	45	17	140	12	5
5	ha	554	47	27	-98	12	3	-404	209	69	-55	4	1	-295	112	46	-108	15	6	397	202	90
6	so	642	26	43	-1045	495	169	425	82	43	143	9	6	-302	41	27	9	0	0	-174	14	10
7	fr	423	31	23	105	12	2	205	44	12	-151	24	8	-261	71	24	-358	133	45	367	140	52
8	lo	525	17	30	683	202	48	476	98	36	30	0	0	-466	94	43	133	8	3	532	123	61
9	in	348	8	25	-607	90	18	790	153	46	237	14	5	-76	1	1	587	85	32	-127	4	2
10	to	174	11	13	98	6	1	50	2	0	308	60	11	-71	3	1	266	45	9	-304	59	12
11	ho	561	48	21	-83	12	2	-187	61	15	-386	259	77	6	0	0	363	229	71	-2	0	0
12	fa	568	37	29	-108	11	3	-252	61	21	-570	315	129	82	6	3	395	151	65	-153	23	11
13	cu	140	5	14	2	0	0	329	28	5	-271	19	4	-436	50	10	165	7	1	-371	36	8
14	re	370	20	26	-216	27	5	130	10	3	-201	24	9	-657	252	97	279	45	17	-140	12	5
15	ha	554	47	27	98	12	3	-404	209	69	55	4	1	-295	112	46	-108	15	6	-397	202	90
16	so	642	26	43	1045	495	169	425	82	43	-143	9	6	-302	41	27	9	0	0	174	14	10
17	fr	423	31	23	-105	12	2	205	44	12	151	24	8	-261	71	24	-358	133	45	-367	140	52
18	lo	525	17	30	-683	202	48	476	98	36	-30	0	0	-466	94	43	133	8	3	-532	123	61
19	in	348	8	25	607	90	18	790	153	46	-237	14	5	-76	1	1	587	85	32	127	4	2
20	to	174	11	13	-98	6	1	50	2	0	-308	60	11	-71	3	1	266	45	9	304	59	12
21	HO	561	48	21	83	12	2	-187	61	15	386	259	77	-6	0	0	-363	229	71	-2	0	0
22	FA	568	37	29	108	11	3	-252	61	21	570	315	129	-82	6	3	-395	151	65	-153	23	11
23	CU	140	5	14	-2	0	0	329	28	5	271	19	4	436	50	10	-165	7	1	-371	36	8
24	RE	370	20	26	216	27	5	130	10	3	201	24	9	657	252	97	-279	45	17	-140	12	5
25	HA	554	47	27	-98	12	3	-404	209	69	-55	4	1	295	112	46	108	15	6	-397	202	90
26	SO	642	26	43	-1045	495	169	425	82	43	143	9	6	302	41	27	-9	0	0	174	14	10
27	FR	423	31	23	105	12	2	205	44	12	-151	24	8	261	71	24	358	133	45	-367	140	52
28	LO	525	17	30	683	202	48	476	98	36	30	0	0	466	94	43	-133	8	3	-532	123	61
29	IN	348	8	25	-607	90	18	790	153	46	237	14	5	76	1	1	-587	85	32	127	4	2
30	TO	174	11	13	98	6	1	50	2	0	308	60	11	71	3	1	-266	45	9	304	59	12
31	HO	561	48	21	-83	12	2	-187	61	15	-386	259	77	-6	0	0	-363	229	71	2	0	0
32	FA	568	37	29	-108	11	3	-252	61	21	-570	315	129	-82	6	3	-395	151	65	154	23	11
33	CU	140	5	14	2	0	0	329	28	5	-271	19	4	436	50	10	-165	7	1	371	36	8
34	RE	370	20	26	-216	27	5	130	10	3	-201	24	9	657	252	97	-279	45	17	140	12	5
35	HA	554	47	27	98	12	3	-404	209	69	55	4	1	295	112	46	108	15	6	397	202	90
36	SO	642	26	43	1045	495	169	425	82	43	-143	9	6	302	41	27	-9	0	0	-174	14	10
37	FR	423	31	23	-105	12	2	205	44	12	151	24	8	261	71	24	358	133	45	367	140	52
38	LO	525	17	30	-683	202	48	476	98	36	-30	0	0	466	94	43	-133	8	3	532	123	61
39	IN	348	8	25	607	90	18	790	153	46	-237	14	5	76	1	1	-587	85	32	-127	4	2
40	TO	174	11	13	-98	6	1	50	2	0	-308	60	11	71	3	1	-266	45	9	-304	59	12

COLUMN CONTRIBUTIONS

J	NAME	QLT	MAS	INR	k=1	COR	CTR	k=2	COR	CTR	k=3	COR	CTR	k=4	COR	CTR	k=5	COR	CTR	k=6	COR	CTR
1	HO	652	48	21	83	12	2	-187	61	15	-386	259	77	363	229	71	-6	0	0	-230	92	31
2	FA	664	37	29	108	11	3	-252	61	21	-570	315	129	395	151	65	-82	6	3	-350	119	55
3	CU	106	5	14	-2	0	0	329	28	5	-271	19	4	165	7	1	436	50	10	75	1	0
4	RE	614	20	26	216	27	5	130	10	3	-201	24	9	279	45	17	657	252	97	-663	256	107
5	HA	364	47	27	-98	12	3	-404	209	69	55	4	1	-108	15	6	295	112	46	-97	12	5
6	SO	654	26	43	-1045	495	169	425	82	43	-143	9	6	9	0	0	302	41	27	-241	26	19
7	FR	325	31	23	105	12	2	205	44	12	151	24	8	-358	133	45	261	71	24	201	42	16
8	LO	432	17	30	683	202	48	476	98	36	-30	0	0	133	8	3	466	94	43	264	30	15
9	IN	344	8	25	-607	90	18	790	153	46	-237	14	5	587	85	32	76	1	1	2	0	0
10	TO	124	11	13	98	6	1	50	2	0	-308	60	11	266	45	9	71	3	1	114	8	2
11	HO	652	48	21	-83	12	2	-187	61	15	386	259	77	363	229	71	-6	0	0	230	92	31
12	FA	664	37	29	-108	11	3	-252	61	21	570	315	129	395	151	65	-82	6	3	350	119	55
13	CU	106	5	14	2	0	0	329	28	5	271	19	4	165	7	1	436	50	10	-75	1	0
14	RE	614	20	26	-216	27	5	130	10	3	201	24	9	279	45	17	657	252	97	663	256	107
15	HA	364	47	27	98	12	3	-404	209	69	-55	4	1	-108	15	6	295	112	46	97	12	5
16	SO	654	26	43	1045	495	169	425	82	43	143	9	6	9	0	0	302	41	27	240	26	19
17	FR	325	31	23	-105	12	2	205	44	12	-151	24	8	-358	133	45	261	71	24	-201	42	16
18	LO	432	17	30	-683	202	48	476	98	36	30	0	0	133	8	3	466	94	43	-264	30	15
19	IN	344	8	25	607	90	18	790	153	46	237	14	5	587	85	32	76	1	1	-2	0	0
20	TO	124	11	13	-98	6	1	50	2	0	308	60	11	266	45	9	71	3	1	-114	8	2
21	ho	652	48	21	83	12	2	-187	61	15	-386	259	77	-363	229	71	6	0	0	230	92	31
22	fa	664	37	29	108	11	3	-252	61	21	-570	315	129	-395	151	65	82	6	3	350	119	55
23	cu	106	5	14	-2	0	0	329	28	5	-271	19	4	-165	7	1	-436	50	10	-75	1	0
24	re	614	20	26	216	27	5	130	10	3	-201	24	9	-279	45	17	-657	252	97	663	256	107
25	ha	364	47	27	-98	12	3	-404	209	69	55	4	1	108	15	6	-295	112	46	97	12	5
26	so	654	26	43	-1045	495	169	425	82	43	-143	9	6	-9	0	0	-302	41	27	240	26	19
27	fr	325	31	23	105	12	2	205	44	12	151	24	8	358	133	45	-261	71	24	-201	42	16
28	lo	432	17	30	683	202	48	476	98	36	-30	0	0	-133	8	3	-466	94	43	-264	30	15
29	in	344	8	25	-607	90	18	790	153	46	-237	14	5	-587	85	32	-76	1	1	-2	0	0
30	to	124	11	13	98	6	1	50	2	0	-308	60	11	-266	45	9	-71	3	1	-114	8	2
31	ho	652	48	21	-83	12	2	-187	61	15	386	259	77	-363	229	71	6	0	0	-230	92	31
32	fa	664	37	29	-108	11	3	-252	61	21	570	315	129	-395	151	65	82	6	3	-350	119	55
33	cu	106	5	14	2	0	0	329	28	5	271	19	4	-165	7	1	-436	50	10	75	1	0
34	re	614	20	26	-216	27	5	130	10	3	201	24	9	-279	45	17	-657	252	97	-663	256	107
35	ha	364	47	27	98	12	3	-404	209	69	-55	4	1	108	15	6	-295	112	46	-97	12	5
36	so	654	26	43	1045	495	169	425	82	43	143	9	6	-9	0	0	-302	41	27	-241	26	19
37	fr	325	31	23	-105	12	2	205	44	12	-151	24	8	358	133	45	-261	71	24	201	42	16
38	lo	432	17	30	-683	202	48	476	98	36	30	0	0	-133	8	3	-466	94	43	264	30	15
39	in	344	8	25	607	90	18	789	153	46	237	14	5	-587	85	32	-76	1	1	2	0	0
40	to	124	11	13	-98	6	1	50	2	0	308	60	11	-266	45	9	-71	3	1	114	8	2