Seeking θ's desperately: estimating the distribution of consumers under increasing block rates[1]

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Abstract

This paper shows that the distribution of observed consumption is not a good proxy for the distribution of heterogeneus consumers when the current tariff is an increasing block tariff. We use a two step method to recover the "true" distribution of consumers. First, we estimate the demand function induced by the current tariff. Second, using the demand system, we specify the distribution of consumers as a function of observed consumption to recover the true distribution. Finally, we design a new two-part tariff which allows us to evaluate the equity of the existence of an increasing block tariff.

Key words: Heterogeneous Demand, Nonlinear Pricing.
J.E.L. classification: L15, L95.

1 Introduction

The advantages of nonlinear pricing stem from heterogeneity among consumers. So for the purposes of nonlinear pricing it is necessary to know the preferences and the frequency distribution of tastes the consumers have for the product. Where the taste variable is readily measured, such as income, it may be easy to construct the distribution of types directly from observed data. In the case where the taste variable is not directly observed and the current tariff is such that consumers with different preferences demand different amounts of the good, we can use the distribution of observed consumption to approximate the distribution of consumers types. This has been the standard way to estimate the distribution of types in the literature.[1]

In some utilities the regulators frequently institute increasing block rate pricing. This is widely practiced in water utilities, where the regulators *"use the price policy with the dual objectives of improving equity among customers and achieving some reduction in total water use"* (Agthe & Billings (1987)).

In this paper we show that under this price schedule and when the taste variable is not readily measured, we cannot use the distribution of consumption as a good proxy of the distribution of consumer types because the tariff induces a pooling equilibrium. That is, some consumers of different types demand the same amount of good. Then, in order to recover the distribution of types we need to know which levels of consumption are demanded for more than one type of consumer, and how many consumers demand the same quantity. In some sense, we can say that these individuals are hidden and we need "seek them desperately".

We recover the distribution of consumer types for a local water service from the city of Vigo (Spain) where the tariff includes a minimum of consumption "free of charge" and marginal prices that are increasing in output. To that purpose, we model the preferences, following Mitchell (1978), in a simple way. We assume that all the differences between consumers can be represented by a taste parameter. More specifically, we assume that all the consumers have the same reservation price and the differences across them comes from the level of satiation.

To estimate the "true" distribution of types we use a two step method. First, using aggregate data we estimate the demand function induced by the current tariff. In particular, we estimate the demand relation for each block as function of the marginal prices of that block and the marginal prices of the adjacent blocks as price variables. This allows us to obtain the reservation price. Second, using this value we specify the frequencies of consumption as a function of the taste distribution parameters. We use the observed frequencies of consumption to estimate these parameters. This allows us to obtain an estimation of the number of consumers induced to demand the same amount of water.

[1]See Brown & Sibley (1986) or Wilson (1993).

Once the true distribution of consumer types has been estimated, we consider the effect of introducing a two part tariff. We impose that this new tariff must neither reduce the revenue of the firm nor increase the aggregate level of water consumption. That is, all the potential welfare gains come from the reallocation of the actual levels of consumption and payments between consumers. These welfare gains give us an idea about the "equity" of the existence of an increasing block tariff. In particular, we show that the introduction of a two part tariff improves the welfare of the small consumers, because it expand the consumption of the big consumers which allows to reduce the fixed fee paid by the small consumers.

Most studies in the literature about evaluating the welfare effects of reforming pricing schedules in utilities use or estimate aggregate demand or per capita demand because of the difficulty of obtaining individual demand functions. The procedure is to assign the average consumer in the population of consumers ("representative" consumer) the average prices and quantities observed and an average price elasticity of demand. Then, using a distribution of consumer tastes separately estimated from the observed consumption or approximated by an income distribution, they obtain the individual demand functions (see Brown and Sibley, 1986). In each of these studies, therefore, the relation between consumption pattern and tariff is specified a priori rather than determined empirically. In this study we show that the consumer's consumption pattern depends on the current tariff, so the distribution of consumers and the demand function must be estimated jointly.

On the other hand, in the literature the efficiency gains from reforming prices arise because aggregate consumption increases. In this study, however, the potential welfare gains come from the reallocation of the actual levels of consumption, given the existence of a constraint on the level of aggregate consumption.

This paper is organized as follows. Section 2 presents the model. In section 3 we estimate the parameters of the demand function and the true distribution of the consumer types. In section 4 we consider the effect of introducing a two-part tariff. Finally in section 5 we conclude.

2 The Model

The tariff of the water utility of the city of Vigo (Spain) has a fixed fee A, which includes q_0 units free of charge and the rent of the meter, M. When the level of consumption exceeds q_0, the consumer must pay the rent of the meter M plus p_i for each unit. The price p_i depends on the level of consumption:

$$p_i = \begin{cases} p_1 & \text{if } q_0 < q \leq q_1 \\ p_2 & \text{if } q_1 < q \leq q_2 \\ p_3 & \text{if } q_2 < q \end{cases}$$

where $p_3 > p_2 > p_1 > 0$, and $q_2 > q_1 > q_0$. That is, the marginal price is increasing in output. There is another way to interpret the tariff. If the level of consumption exceeds q_0, the consumers pay A plus an additional payment of $p_i - p_0$ for the "free of charge" units and p_i for the $q - q_0$ remaining units. Formally, we can write the total payment, $T(q)$, as

$$T(q) = \begin{cases} A & \text{if } q \leq q_0 \\ A + (p_1 - p_0)q_0 + p_1(q - q_0) & \text{if } q_0 < q \leq q_1 \\ A + (p_2 - p_0)q_0 + p_2(q - q_0) & \text{if } q_1 < q \leq q_2 \\ A + (p_3 - p_0)q_0 + p_3(q - q_0) & \text{if } q_2 < q \end{cases}$$

where p_0 is the "price" of the first q_0 units, defined as $p_0 = \frac{(A-M)}{q_0}$.

The consumers preferences can be represented by:

$$V_\theta(q, T) = \begin{cases} U(q, \theta) - T & \text{if } q > 0 \\ 0 & \text{if } q = 0, \end{cases}$$

where q is the level of consumption, T is the total payment and θ is taste parameter that varies across consumers (we will call it the consumer's type). We will assume a quadratic utility function in consumption

$$U(q, \theta) = \alpha q - \frac{1}{2\theta} q^2,$$

where α, the reservation price, is equal for all the consumers. Note that consumers are different because they have different levels of satiation, $\alpha\theta$.

To obtain the demands we must solve the consumer problem, $\max_q \{U(q, \theta) - T(q)\}$. It is useful to define the consumer who is sure that is consuming in the limit of each interval of prices; formally:

$$\theta_0 = \frac{q_0}{\alpha}, \quad \theta_k(p_k) = \frac{q_k}{\alpha - p_k}, \quad k = 1, 2. \tag{1}$$

Note that θ_0 is independent of the prices because the consumer who demand q_0 pays a fixed fee. The existence of a "free of charge" level of consumption q_0, induce all the consumers with type $\theta \leq \theta_0$ to consume his level of satiation $\alpha\theta$. We expect that consumers with $\theta > \theta_0$ will consume a higher amount of the good. Nevertheless, some of them prefer to consume q_0 given that, if they consume more, they must pay more for the first q_0 units. In general, when the price of the good changes from p_k to p_{k+1}, there are some consumers of type $\theta > \theta_k$, which prefer to consume q_k because in any other case they will pay more for the first q_k units and the increase in the bill is higher than the additional welfare obtained by increasing the consumption of the good (area A is greater than area B in Figure 1).

[Insert Figure 1]

Let θ_k^* be the consumer who is indifferent between consuming q_k or his best choice if he decide to demand $q > q_k$, $q_k^* = \theta_k^*(\alpha - p_{k+1}) > q_k$. Formally θ_k^* satisfies that:

$$U(q_k, \theta_k^*) - p_k q_k = U(\theta_k^*(\alpha - p_{k+1}), \theta_k^*) - p_{k+1}\theta_k^*(\alpha - p_{k+1}).$$

Tedious algebra allows us to obtain

$$\theta_k^* = \frac{(\alpha - p_k) \pm \sqrt{2\Delta p_k \left(\alpha - p_k - \frac{\Delta p_k}{2}\right)}}{(\alpha - p_{k+1})^2} q_k, \quad k = 0, 1, 2, \tag{2}$$

where $\Delta p_k = p_{k+1} - p_k$.[2]. Then all the consumers with types $\theta \in [\theta_k, \theta_k^*]$ prefer to consume q_k. Then we can write the individual consumer demands as follows:

$$q(\theta) = \begin{cases} \theta\alpha & if \quad \theta < \theta_0 \\ q_0 & if \quad \theta_0 \leq \theta \leq \theta_0^* \\ \theta(\alpha - p_1) & if \quad \theta_0^* < \theta < \theta_1 \\ q_1 & if \quad \theta_1 \leq \theta \leq \theta_1^* \\ \theta(\alpha - p_2) & if \quad \theta_1^* < \theta < \theta_2 \\ q_2 & if \quad \theta_2 \leq \theta \leq \theta_2^* \\ \theta(\alpha - p_3) & if \quad \theta_2^* < \theta, \end{cases} \tag{3}$$

Note that we have a pooling equilibrium. In particular, all consumers with type $\theta \in [\theta_0, \theta_0^*]$ consume exactly the maximum quantity free of charge. So the model predicts a high concentration of consumers in q_0. Figure 2 shows the distribution of consumers of water, from January 1992 to April 1993. There is a high concentration around the $30m^3$, exactly the level of consumption free of charge.

[Insert Figure 2]

3 Model estimation

Last section shows that different types of consumers are induced by the current tariff to consume the same amount of water. This implies that we can not use the observed consumption distribution as a proxy for the distribution of types. Nevertheless, it is possible to specify the consumption distribution as a function of the actual distribution of types. Given the relation (3) between consumption and type, the probability distribution of the variable q in the space of consumptions depends on

[2]We always take the solution with the positive sign, because it is easy to verify that $\theta_k^{*(-)} \leq \theta_k \leq \theta_k^{*(+)}$ for $k = 1, 2$ and if α is big enough (specifically, greater than 226.12) it is also true for $k = 0$ ($\theta_k^{*(-)}$ and $\theta_k^{*(+)}$ are the solutions of (2) with negative and positive signs of the square root). It will be show later that α must be greater than 226.12.

the types distribution and the parameters α, θ_0^*, θ_1^* and θ_2^*. Let F_q be the distribution function of q, defined as

$$F_q(q) = \begin{cases} F_\theta(q/\alpha) & if \quad q < q_0 \\ F_\theta(\theta_0^*) & if \quad q_0 \leq q < \theta_0^*(\alpha - p_1) \\ F_\theta(q/(\alpha - p_1)) & if \quad \theta_0^*(\alpha - p_1) \leq q < q_1 \\ F_\theta(\theta_1^*) & if \quad q_1 \leq q < \theta_1^*(\alpha - p_2) \\ F_\theta(q/(\alpha - p_2)) & if \quad \theta_1^*(\alpha - p_2) \leq q < q_2 \\ F_\theta(\theta_2^*) & if \quad q_2 \leq q < \theta_2^*(\alpha - p_3) \\ F_\theta(q/(\alpha - p_3)) & if \quad \theta_1^*(\alpha - p_2) \leq q, \end{cases} \tag{4}$$

where F_θ is the distribution function of θ.

The reservation price and the number of hidden consumers are calculated in two steps. First, we estimate the reservation price α from a demand system. Given α (or its estimation), F_θ can be estimated (jointly with $\theta_k^*, k = 0, 1, 2$) from the observed consumption distribution F_q. Then, the hidden consumers in each interval of consumption can be quantified.

3.1 Estimating the reservation price α

Let us start aggregating the individual demands for each interval of consumption. Thus, we can write the total demand in each interval (Q_k, $k = 0, 1, 2, 3$) as a function of the current prices for this interval and the prices of the adjacent intervals. Formally,

$$\begin{aligned} Q_0(p_0, p_1) &= N\left(\int_{\underline{\theta}}^{\theta_0} \alpha\theta f(\theta)d\theta + \int_{\theta_0}^{\theta_0^*(p_0,p_1)} q_0 f(\theta)d\theta\right), \\ Q_1(p_0, p_1, p_2) &= N\left(\int_{\theta_0^*(p_0,p_1)}^{\theta_1} \theta(\alpha - p_1)f(\theta)d\theta + \int_{\theta_1}^{\theta_1^*(p_1,p_2)} q_1 f(\theta)d\theta\right), \\ Q_2(p_1, p_2, p_3) &= N\left(\int_{\theta_1^*(p_1,p_2)}^{\theta_2} \theta(\alpha - p_2)f(\theta)d\theta + \int_{\theta_2}^{\theta_2^*(p_2,p_3)} q_2 f(\theta)d\theta\right), \\ Q_3(p_2, p_3) &= N\int_{\theta_2^*(p_2,p_3)}^{\overline{\theta}} \theta(\alpha - p_3)f(\theta)d\theta, \end{aligned} \tag{5}$$

where N is the size of the whole population. Given that the aggregated demands by interval are multiplicatively separable, it is useful to normalize Q_k by the total number of consumers N. So, we define the normalized demands $D_i(\cdot)$ as $D_k = Q_k/N, k = 0, 1, 2, 3$.

The system allows us to obtain α from $\frac{\partial D_0}{\partial p_0}$ and $\frac{\partial D_1}{\partial p_0}$, given that[3]

$$\frac{\partial D_1}{\partial p_0} = -\theta_0^*(\alpha - p_1) f(\theta_0^*) \frac{d\theta_0^*}{dp_0},$$

and

$$\frac{\partial D_0}{\partial p_0} = q_0 \frac{d\theta_0^*}{dp_0} f(\theta_0^*),$$

it follows that

$$-\frac{\partial D_1/\partial p_0}{\partial D_0/\partial p_0} = \frac{\theta_0^*(\alpha - p_1)}{q_0}.$$

Remember that (2) implies that θ_0^* is

$$\theta_0^* = \frac{(\alpha - p_0) + \sqrt{2\Delta p_0 \left(\alpha - p_0 - \frac{\Delta p_0}{2}\right)}}{(\alpha - p_1)^2} q_0. \tag{6}$$

Then, α can be calculated from the expression

$$\Phi(\alpha) = \frac{\partial D_1/\partial p_0}{\partial D_0/\partial p_0} + \frac{(\alpha - p_0) + \sqrt{2\Delta p_0 \left(\alpha - p_0 - \frac{\Delta p_0}{2}\right)}}{(\alpha - p_1)} = 0 \tag{7}$$

where $\Delta p_0 = p_1 - p_0$.

In order to estimate the partial derivatives of the normalized demands in each interval of consumption, we use per capita levels of consumption in each interval, $\bar{x}_k$, for several areas in the city differing in the proportion of drained water $\overline{q}_k^t$. We can compute the normalized demands as

$$d_k = \bar{x}_k \frac{N_k}{N},$$

where N_k is the number of consumers in each interval. We have calculated the average prices as

$$p_k^t = p_k^t(\text{supply}) + \gamma_k^t p_k^t(\text{drain}),$$

the sum of the supply and drain prices weighted by the proportion of drained water in each area and interval γ_k^t (the areas in use and its coefficients are listed in Appendix 6.2.2). In that way we build a sample from which we can estimate a linear approximation to system (5) where the coefficients are the partial derivatives of D_i with respect to the prices. The linear system of equations is

$$\begin{bmatrix} d_0^t \\ d_1^t \\ d_2^t \\ d_3^t \end{bmatrix} = \begin{bmatrix} \beta_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \beta_{00} & \beta_{01} & 0 & 0 \\ \beta_{10} & \beta_{11} & \beta_{12} & 0 \\ 0 & \beta_{21} & \beta_{22} & \beta_{23} \\ 0 & 0 & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} p_0^t \\ p_1^t \\ p_2^t \\ p_3^t \end{bmatrix} + \begin{bmatrix} \varepsilon_0^t \\ \varepsilon_1^t \\ \varepsilon_2^t \\ \varepsilon_3^t \end{bmatrix}, \tag{8}$$

[3]The partial derivatives are calculated in Appendix 6.1.

where β_{ij} is the partial derivative of D_i with respect to p_j. Note that the water demand has an independent term only in the first interval of consumption (β_0). From the estimation of the system, we can estimate α replacing $\dfrac{\partial D_1/\partial p_0}{\partial D_0/\partial p_0}$ by β_{01}/β_{00} in equation (7).

The coefficients in (8) are subject to several restrictions required by the model and by the observed data. The restrictions are as follows:

- **Symmetry.** In Appendix 6.1 it is proved that $\beta_{01} = \beta_{10}$, $\beta_{12} = \beta_{21}$ and $\beta_{23} = \beta_{32}$.
- **Sign of the coefficients.** It is also proved in Appendix 6.1 that demands in each interval are decreasing functions of the own price and increasing functions of the neighboring prices. So,

$$\begin{array}{lll} \beta_{ij} < 0 & \text{if} & i = j, \\ \beta_{ij} > 0 & \text{if} & i \neq j. \end{array}$$

- The saturated agents' consumption is less than or equal to that observed in the first interval of consumption, so

$$\beta_0 \leq d_0(p_0, p_1).$$

- $H = -(\beta_{01}/\beta_{00}) > 1$ because $\Phi(\alpha) = 0$ implies

$$H = \frac{(\alpha - p_0) + \sqrt{2\Delta p \left(\alpha - p_0 - \frac{\Delta p_0}{2}\right)}}{(\alpha - p_1)} > \frac{\alpha - p_0}{\alpha - p_1} > 1.$$

- **Value of α.** The estimation of α has to be compatible with the consumptions under the present rate. So, we define the consumer with the lowest valuation of the good ($\underline{\theta}$) as the one who buy a cubic meter of water. His gross profit is $\frac{\alpha}{2}$. Now, he is paying the minimum bill (T=2280 pesetas), therefore

$$\alpha \geq 4560.$$

  If such a restriction holds then only the positive sign in the squared root involved in the definition of θ_0^* is compatible with the fact that $\theta_0 < \theta_0^*$.

- **Restrictions on the proportion of hidden consumers.** The estimation of the proportion of hidden consumers has to be lower than the observed proportion of people consuming just the quantities that separate two intervals. So, it has to be that

$$\begin{aligned} 0 \leq \int_{\theta_0}^{\theta_0^*} f(\theta)d\theta &= \frac{23,67 - \beta_0}{30} \leq 0.3536, \\ 0 \leq \int_{\theta_1}^{\theta_1^*} f(\theta)d\theta &= \frac{43.34 + K_1(\alpha - 70.5)}{70} \leq 0.0081, \\ 0 \leq \int_{\theta_2}^{\theta_2^*} f(\theta)d\theta &= \frac{97.02 + K_2(\alpha - 83.5)}{200} \leq 0.0002, \end{aligned}$$

where K_i (defined in Appendix 6.1) are functions of β.

- From the definition of K_3 and the expression of the aggregated demand in the third interval, it follows that

$$\frac{d_3(p_2, p_3)}{\alpha - p_3} + K_3 = 0.$$

Taking into account the symmetry of the restrictions, the linear system to be estimated can be written as

$$X = P\beta + \epsilon,\ \epsilon \sim (0, \Sigma),$$

where X and ϵ have dimension 4, P is a 4×8 matrix and β is of dimension 8. The coefficient vector is $\beta = (\beta_0, \beta_{00}, \beta_{01}, \beta_{11}, \beta_{12}, \beta_{22}, \beta_{23}, \beta_{33})^t$. Appendix 6.2 include the available data $(X_i, P_i),\ i = 1, \ldots, n$.

We define

$$\begin{aligned} \mathcal{X} &= (X_1^t, \ldots, X_n^t)^t, \\ \mathcal{P} &= (P_1^t, \ldots, P_n^t)^t, \\ \mathcal{E} &= (\epsilon_1^t, \ldots, \epsilon_n^t)^t, \\ \mathcal{S} &= I_n \otimes \Sigma, \end{aligned}$$

where $\otimes$ denotes the Kroneker product. So, we can transform the original linear system into a regression model with the variance of the residuals different from the identity matrix:

$$\mathcal{X} = \mathcal{P}\beta + \mathcal{E},\ \mathcal{E} \sim (0, \mathcal{S}).$$

We propose an estimator of β similar to the generalized least squares estimator (GLS) based on a previous estimation of the matrix $\mathcal{S}$. First, we take the vector minimizing the sum of the squared norms of the residual, subject to the restrictions previously enumerated, as an estimator of β. Then, we compute the residuals from that estimator and Σ is estimated by the sample covariance matrix of these residuals. Finally, a second estimation of β is carried out in the same way as GLS estimator would be applied (i.e., using the estimation of $\mathcal{S}$ based on those of Σ instead of the unknown variance matrix). The restrictions of the coefficients have to be taken into account also in this second step. The algorithm used in the minimization phases is based in penalty functions. The final estimation of β is

$$\hat{\beta} = (7.12, -1.62, 1.72, -89.26, 21.86, -7.56, .000011, -.000012)^t.$$

From these values we arrive to an estimation of α equal to 4560, the lower bound for this parameter. An interesting subproduct of the system estimation is the evaluation of values θ_k^*, because equation (2) expresses the values of θ_k^* as functions of α. Then, we obtain the estimations

$$\theta_0^* = 0.71 \times 10^{-2},\ \ \theta_1^* = 0.17 \times 10^{-1},\ \text{ and }\ \theta_2^* = 0.50 \times 10^{-1}.$$

3.2 Estimating the types distribution

The sample information we have is referred to consumptions and is displayed in Appendix 6.2.4. The consumption space is divided in 21 intervals, each of them 5 cubic meters wide (except the last one). Only aggregated information for each interval is available. The column with the percentage of consumers is used as the frequency of variable q. Let $f_i,\ i = 1, \ldots, 21$, be the observed relative frequencies of q.

In order to make compatible the theoretical continuous distribution F_q with the sample information, essentially discrete, we compute the probability assigned by F_q to each of the 21 intervals of consumption for which we know the observed frequency. These theoretical probabilities $\pi_i,\ i = 1, \ldots, 21$, are

$$\pi_i = \begin{cases} F_q(5i) - F_q(5i-5) & if \quad i \leq 20 \\ 1 - F_q(5i-5) & if \quad i = 21. \end{cases} \tag{9}$$

The demand $q(\theta)$ defined in (3) implies that π_i depends on the distribution of types F_θ. We impose continuity, that is $q_k = \theta_k^*(\alpha - p_{k+1})$, $k = 0, 1, 2$, to derive from (3) following transformation:

$$q(\theta) = \begin{cases} \theta\alpha & if \quad \theta < \theta_0 \\ q_0 & if \quad \theta_0 \leq \theta \leq \theta_0^* \\ q_0 + (\theta - \theta_0^*)(\alpha - p_1) & if \quad \theta_0^* < \theta < \theta_1 \\ q_1 & if \quad \theta_1 \leq \theta \leq \theta_1^* \\ q_1 + (\theta - \theta_1^*)(\alpha - p_2) & if \quad \theta_1^* < \theta < \theta_2 \\ q_2 & if \quad \theta_2 \leq \theta \leq \theta_2^* \\ q_2 + (\theta - \theta_2^*)(\alpha - p_3) & if \quad \theta_2^* < \theta. \end{cases} \tag{10}$$

In order to relate π_i and f_i we need to propose a parameterization for F_θ. Given that observed consumptions are very concentrated in the lower values and very asymmetric, a sensible model for F_θ is the Weibull distribution with scale parameter μ and shape parameter ρ. The density function of such a distribution is

$$f_\theta(\theta; \mu, \rho) = \frac{\mu}{\rho^\mu}\theta^{\mu-1}e^{-(\theta/\rho)^\mu}$$

and its distribution function is

$$F_\theta(\theta; \mu, \rho) = 1 - e^{-(\theta/\rho)^\mu}.$$

The definition of π_i given in equation (9) and the relation between F_θ and F_q established in equation (4) leads to the following closed expression for π_i as a function of some unknown parameters: $\pi_i = \pi_i(\mu, \rho, \theta_0^*, \theta_1^*)$, $i = 1, \ldots, 21$:

$$\pi_i = \begin{cases} \exp\left\{-\left(\frac{5i}{\alpha\rho}\right)^{\mu}\right\} - \exp\left\{-\left(\frac{5(i-1)}{\alpha\rho}\right)^{\mu}\right\} & if \quad i \leq 5 \\ \exp\left\{-\left(\frac{5(i-6)}{(\alpha-p_1)\rho} + \frac{\theta_0^*}{\rho}\right)^{\mu}\right\} - \exp\left\{-\left(\frac{5(i-1)}{\alpha\rho}\right)^{\mu}\right\} & if \quad i = 6 \\ \exp\left\{-\left(\frac{5(i-6)}{(\alpha-p_1)\rho} + \frac{\theta_0^*}{\rho}\right)^{\mu}\right\} - \exp\left\{-\left(\frac{5(i-7)}{(\alpha-p_1)\rho} + \frac{\theta_0^*}{\rho}\right)^{\mu}\right\} & if \quad 7 \leq i \leq 13 \\ \exp\left\{-\left(\frac{5(i-14)}{(\alpha-p_2)\rho} + \frac{\theta_1^*}{\rho}\right)^{\mu}\right\} - \exp\left\{-\left(\frac{5(i-7)}{(\alpha-p_1)\rho} + \frac{\theta_0^*}{\rho}\right)^{\mu}\right\} & if \quad i = 14 \\ \exp\left\{-\left(\frac{5(i-14)}{(\alpha-p_2)\rho} + \frac{\theta_1^*}{\rho}\right)^{\mu}\right\} - \exp\left\{-\left(\frac{5(i-15)}{(\alpha-p_2)\rho} + \frac{\theta_1^*}{\rho}\right)^{\mu}\right\} & if \quad 15 \leq i \leq 20 \\ 1 - \exp\left\{-\left(\frac{5(i-15)}{(\alpha-p_2)\rho} + \frac{\theta_1^*}{\rho}\right)^{\mu}\right\} & if \quad i = 21. \end{cases}$$

The value of α we obtained in the previous section ($\alpha = 4560$) take part in the definition of π_i because of equation (4). Note also that π_i, does not depend on θ_2^* because the consumption related to that consumer is $q = 200$ and it belongs to the last interval of consumptions ($q > 100$). After the division of the consumptions in 21 intervals, the consumption space can be understood as a discrete space of probability with a support set of 21 points. There we have defined a parametric probability law given by the mass function $(\pi_1, \ldots, \pi_{21})$, depending on μ, ρ, θ_0^* and θ_1^*. We also have observed frequencies for these 21 points: $(f_1, \ldots, f_{21})$.

There exist several methods to estimate the unknown parameters from the observed frequencies. We use in this work a procedure that is asymptotically equivalent to maximum likelihood (possibly the most appropriate method of estimation in this context, from a theoretical point of view) but that is computationally cheaper. It consists on the minimization of the χ^2 statistic used in the goodness of fit test of the observed frequencies $(f_1, \ldots, f_{21})$ to the theoretical probabilities $(\pi_1, \ldots, \pi_{21})$:

$$T(\mu, \rho, \theta_0^*, \theta_1^*) = \sum_{i=1}^{21} \frac{(nf_i - n\pi_i)^2}{n\pi_i} = n \sum_{i=1}^{21} \frac{(f_i - \pi_i)^2}{\pi_i}.$$

So, we minimize the function

$$\Psi(\mu, \rho, \theta_0^*, \theta_1^*) = \sum_{i=1}^{21} \frac{(f_i - \pi_i(\mu, \rho, \theta_0^*, \theta_1^*))^2}{\pi_i(\mu, \rho, \theta_0^*, \theta_1^*)}$$

in the unknown parameters μ, ρ, θ_0^* and θ_1^*. Table 1 presents the results of the estimation. A comparison between the observed and fitted data is shown in Figure 3.

	θ_0^*	θ_1^*	ρ	μ
Estimated parameters	$.9487 \times 10^{-2}$	$.1840 \times 10^{-1}$	$.9236 \times 10^{-2}$	1.9012

Table 1: Estimated parameters for the Weibull distribution.

[Insert Figure 3]

From the estimated Weibull distribution we can obtain an estimation of the proportion of hidden consumers in interval k as:

$$F(\theta_k^*) - F(\theta_k), \quad k = 0, 1.$$

So, we compute a percentage of 24.26% hidden consumers in $q_0 = 30m^3$ and a negligible percentage of hidden consumers in $q_1 = 70$.

4 Optimal two-part tariff

Once the true distribution of consumers types has been estimated, we consider the effect of introducing a two-part tariff which does not reduce current revenue levels and does not increase current consumption levels. In short, the possible welfare changes comes from the reallocation of the current consumption and payments across consumers.

If we suppose that the marginal costs of water distribution are nil and that the service is currently covering costs, the optimal two-part tariff will be that which maximize the welfare of consumers, subject to the participation of all consumers, maintenance of the firm's current revenue levels and no increase in current levels of aggregated consumption. This is equivalent to maximize the expected surplus of a consumer subject to average revenue and consumption being greater that $\underline{I}$ and less than $\overline{Q}$ respectively, where $\underline{I}$ is the average bill and $\overline{Q}$ is the average consumption. Formally, the problem is expressed as

$$[P] \equiv \left\{ \begin{array}{l} \max\limits_{A,p} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{\theta}{2}(\alpha - p)^2 \right\} f(\theta) d\theta \\ \\ s.t. \quad \left\{ \begin{array}{l} \frac{(\alpha-p)}{2} - A \geq 0 \\ \\ A + p(\alpha - p) \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) \geq \underline{I} \\ \\ (\alpha - p) \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) \leq \overline{Q} \end{array} \right. \end{array} \right.$$

where $\int_p^\alpha \theta(\alpha - x)dx = \frac{\theta}{2}(\alpha - p)^2 - A$ is the net surplus of the consumer type θ individual which is faced with the $A + pq$ tariff, and $\frac{\alpha - p}{2} - A$ is the surplus of the smaller consumer defined as $1 = \underline{\theta}(\alpha - p)$.

The solution to this problem is easy. The non-existence of a marginal cost for water supply will lead to establish a null price and a fixed fee that guarantees the participation of all consumers

$$A = \underline{I} \leq \frac{\alpha}{2}.$$

However, this price policy generates unfeasibly high consumption levels. Therefore, in order to determine the optimal tariff, we saturate the capacity constraint which determines our price that induces a consumption level similar to the current one, while the fixed fee is calculated so that the firm reaches current revenue levels, that is

$$p^* = \alpha - \frac{\overline{Q}}{E(\theta)}, \quad A^* = \underline{I} - p^*\overline{Q}, \tag{11}$$

where $E(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)$.

To evaluate the optimal tariff it is necessary to estimate $E(\theta)$ previously. This can be done by two statistical methods essentially independent. The first one only use the values θ_0^* and θ_1^* obtained from the estimation of the system of equations. It is enough to take the expected value of the discrete version of random variable θ. With this method, therefore, we incorporate the information provided by the system's theoretical constraints. The second estimation, however, use the adjusted Weibull distribution and, therefore, is based on disaggregated data in various subintervals within each interval of consumption, including no information about the area of the city. With the aim of combining all the available information given by the two different sources, we take the average of the two estimations as the final estimator of $E(\theta)$[4]. That is, $\hat{E}(\theta) = 0.00349$.

Section	% Users	$\overline{x}$	$\overline{T}$
0	61.61 %	23.67	2280
1	35.33 %	43.34	3475
2	2.66 %	97.02	8521
3	0.40 %	500.30	55953
Total	**100.00 %**	$\overline{Q}$**=34.47**	$\underline{I}$**=3083**

Table 2: Users, average revenues and average consumption levels in each interval of consumption of the current tariff.

In Table 2 the average bills, the average consumption levels and the percentage of users that currently are in each interval are presented. In the last row, the average consumption $\overline{Q}$ and the average bill per consumer $\underline{I}$ are shown. Given

[4]If the estimation processes and the original data were really independent, the variance of the estimator created by averaging the previous two would be a quarter of the sum of the variances of them.

$\hat{E}(\theta) = 0.00349$ and using the date for $(\overline{Q})$ and $(\underline{I})$ we evaluate (11) and obtain $p^* = 53.2$ and $A^* = 1248$.

The introduction of the new tariff implies an increase in the fixed fee and a reduction in the marginal price for all consumers who consume more than 30 m^3. For consumers in the interval of consumption [0,30] there is a reduction of the fixed fee and an increase in the marginal price. Thus, the first thing we can state is that the former increase their consumption and the last ones reduce it. The second effect of the introduction of a two-part tariff is the change in the distribution of consumption. Table 3 presents the frequency of users in each interval of consumption with the current and new tariffs. [5] As we can see in figure 4 a two-part tariff generate a smooth distribution with less consumers consuming $30m^3$ and more in the others intervals.

Interval consumption	Current tariff users	New tariff users	q	Current tariff payment	New tariff payment
1-5	3.5	1.76	1	2280	1301.2
6-10	3.7	4.67	6	2280	1567.2
11-15	5.15	6.95	11	2280	1833.2
16-20	6.63	8.60	16	2280	2099.2
24-25	7.27	9.59	21	2280	2365.2
26-30	35.36	9.95	26	2280	2631.2
31-35	10.03	9.77	31	2596.2	2897.2
36-40	8.09	9.14	36	2947.2	3163.2
41-45	5.85	8.21	41	3298.2	3429.2
46-50	4.44	7.11	46	3649.2	3695.2
51-55	2.86	5.95	51	4678.5	3961.2
56-60	1.95	4.82	56	5096	4227.2
61-65	1.29	3.78	61	5513.5	4493.2
66-70	0.81	2.88	66	5931	4759.2
71-75	0.64	2.14	71	8301	5025.2
76-80	0.47	1.54	76	8856	5291.2
81-85	0.27	1.08	81	9411	5557.2
86-90	0.24	0.74	86	9966	5823.2
91-95	0.17	0.49	91	10521	6089.2
96-100	0.19	0.32	96	11076	6355.2
> 100	1.09	0.50	101	11631	6621.2

Table 3:Consumption and payments with the current and new tariffs.

[Insert Figure 4]

[5]The consumption distribution with the new tariff is defined as:

$$F_q(p) = F_\theta(\frac{q}{\alpha - p}).$$

with $\alpha = 4560$ and $p = 53.2$.

Table 3 presents the payments associated with both tariffs, too. The first thing we can say is that the introduction of a two part tariff reduce the payments of the consumers with levels of consumption between 1 and 20 and more than 48 cubic meters. Although some consumers pay less and others pay more with the new tariff, we cannot determine the welfare changes comparing these payments because the consumers modify their consumption. This means that in order to evaluate the welfare changes we must to calculate the individual net surplus for each tariff, the one current (C) and the new one (N), and compare them. In the case of the current tariff it is necessary to distinguish between the hidden individuals and the non-hidden ones. For the non-hidden individuals, the consumer surplus of type θ is defined as

$$CS_C^{NH}(\theta) = \int_{p_C^k}^{\alpha} \theta(\alpha - p)dp - A_C^k = \frac{\theta}{2}(\alpha - p_C^k)^2 - A_C^k$$

where p_C^k and A_C^k are the price and fixed fee for each of the intervals of consumption $k = 0, 1, 2, 3$. On the other hand, the surplus for hidden individuals, must be defined depending on the quantity

$$CS_C^H(\theta) = q_k(\alpha - \frac{1}{2\theta}q_k) - T_C^k$$

where q_C^k and T_C^k are the associated quantities and the payments for the intervals with "hidden" individuals ($k = 0, 1, 2$). Table 4 presents the values to evaluate the consumers surplus. The surplus associated to the new tariff is written for all consumers as

$$CS_N(\theta) = \frac{\theta}{2}(\alpha - p_N)^2 - A_N,$$

where $p_N = 53.2$ and $A_N = 1248$ are the prices for the new tariff.

Non hidden	p_C^k	A_C^k
0	0.0	2280
1	70.5	420
2	83.5	420
3	111.0	420
Hidden	q_C^k	T_C^k
0	30	2280
2	70	5355
3	200	17120

Table 4: Values to calculate the consumer surplus.

In this way, the increase in individual welfare derived from the change in tariff is defined as the difference of the surplus $\triangle W(\theta) = CS_N(\theta) - CS_C(\theta)$, which is written as follows for the different intervals of consumers

$$[\Delta W] \equiv \begin{cases} 1032 - 241176.88\theta & \theta \in [0, 0.006579] \\ 135768 - 101556230\theta - 450/\theta & \theta \in [0.006579, 0.009487] \\ 77817.99\theta - 828 & \theta \in [0.009487, 0.015590] \\ 101556230\theta + 2450/\theta - 315093 & \theta \in [0.015590, 0.018390] \\ 136096.98\theta - 828 & \theta \in [0.018390, 0.044670] \\ 258822.6\theta - 828 & \theta \in [0.044670, \overline{\theta}] \end{cases}$$

In Table 5 we present the welfare changes implied by the new tariff. We classify the consumers in different intervals according to whether they have improved or worsened with the new tariff.[6] Each interval of consumption is defined by the upper and lower parameters and by their associated consumption with the current tariff.[7] Likewise, we calculate the proportion of individuals for each interval, which allows us to conclude that 65.6 percent of individuals increase their welfare while the remaining 34.4% decrease it. The consumers that were previously consuming a quantity less than 20 cubic meters are better with the new tariff (they consume less but also pay less than before). Talking about the consumers that expand their consumption, only those that were located between 30 and 35 m^3 worsen (7.9%), as the improvements derived from a reduction in the marginal price do not cover the increase experimented by the fixed fee.

Type interval $[\theta_i, \theta_{i+1}] \times 10^{-4}$	Frequency	Current tariff Consumption $[q_i, q_{i+1}]$	New tariff Consumption $[q_i, q_{i+1}]$	$\triangle W$
$[0, 42.79]$	19.76	$[0, 19]$	$[0, 18.8]$	(+76)
$[42.79, 65.79]$	21.06	$[19, 30]$	$[18.8, 29]$	(-57)
$[65.79, 72.93]$	6.35	$[30, 30]$	$[29, 33]$	(-25)
$[72.93, 94.87]$	17.91	$[30, 30]$	$[33, 42]$	(+584)
$[94.87, 106.40]$	7.89	$[30, 35]$	$[43, 48]$	(-4)
$[106.40, \overline{\theta}]$	27.01	> 35	> 48	(+152)

Table 5: Welfare changes implied by the new tariff.

Finally, given that the introduction of the new tariff does not suppose an improvement for all the individuals, with the aim of finding out if the new tariff implies an efficiency improvement, it is necessary to calculate the surplus aggregated through the expression $\sum_{i=1}^{9} \triangle W_i(\theta)$ where $\triangle W_i(\cdot)$ is the increase in the welfare obtained by the individuals belonging to the i-th interval, limited by θ_i and θ_{i+1}. The total welfare is increased with the introduction of the new tariff, and the subsidies conceded to some consumers under the current tariff are eliminated.

[6] The values of the surplus for the different intervals have been evaluated by numerical integration methods using the Weibull density function estimated in section 5.

[7] To obtain the intervals we find the roots of ΔW.

5 Conclusions

In this paper we have applied a model to recover the distribution of consumer preferences from the observed consumption distribution when the current tariff induces a pooling equilibrium. To do that, we use a transformation derived from the consumers' behavior induced by the current tariff. This allows us to explain why there is a high concentration of individuals in one of the intervals of consumption.

In contrast to other studies in the literature in this work the distribution of consumers and the demand function are estimated jointly. This allows to determine empirically the relation between consumption pattern and current tariff rather than to specify it a priori. In particular, given an estimator of the reservation price, we use the observed frequencies of consumption to estimate the parameters of the distribution of consumers.

After the estimation of the "true" distribution of consumers we analyze the effects of the introduction of a new tariff. In particular, we show that there exists a two-part tariff that increases the total welfare by reallocating the current aggregate consumption, without reducing current revenues of the firm.

We find that the existence of a free of charge level of consumption when the utility must operate on a balanced budget implies a fixed fee higher than with a two part tariff. That means that the consumers with low levels of consumption are better off with the new tariff, because they pay less for their consumption. In this sense, the equity of an increasing block rate do not benefit the small consumers.

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6 Appendix

6.1 System's constraints

Differentiating each demand function with respect to prices, we have

$$\frac{\partial D_0(p_0,p_1)}{\partial p_0} = q_0 f(\theta_0^*)\frac{d\theta_0^*}{dp_0} < 0,$$

$$\frac{\partial D_k(p_{k-1},p_k,p_{k+1})}{\partial p_k} = \int_{\theta_{k-1}^*(p_k)}^{\theta_k} -\theta f(\theta)d\theta - \theta_{k-1}^*(\alpha - p_k)f(\theta_{k-1}^*)\frac{d\theta_{k-1}^*}{dp_k} + q_k f(\theta_k^*)\frac{d\theta_k^*}{dp_k} < 0,$$

$$\frac{\partial D_3(p_2,p_3)}{\partial p_3} = \int_{\theta_1^*(p_1,p_2)}^{\overline{\theta}} -\theta f(\theta)d\theta - \theta_2^*(\alpha - p_3)f(\theta_2^*)\frac{d\theta_2^*}{dp_3} < 0,$$

$k = 1, 2$. To obtain the sign of the partial derivatives we have differentiated the marginal consumers with respect to prices. So, the indifferent consumers can be implicitly defined as

$$U(q_k,\theta_k^*) - p_k q_k - M = U(\theta_k^*(\alpha - p_{k+1}),\theta_k^*) - p_{k+1}\theta_k^*(\alpha - p_{k+1}) - M.$$

Differentiating this expression with respect to prices we obtain

$$\frac{d\theta_k^*(p_k,p_{k+1})}{dp_k} = \frac{q_k}{A} < 0, \qquad \frac{d\theta_k^*(p_k,p_{k+1})}{dp_{k+1}} = -\frac{\theta_k^*(\alpha p_{k+1})}{A} > 0,$$

where $A = U'_\theta(q_k,\theta_k^*) - U'_\theta(\theta_k^*(\alpha - p_{k+1}),\theta_k^*) < 0$. Furthermore, we can write that

$$f(\theta_k^*)\frac{d\theta_k^*}{dp_k} = -\frac{\partial D_{k+1}/\partial p_k}{\theta_k^*(\alpha - p_{k+1})}, \quad k = 0,1,2,$$

$$f(\theta_k^*)\frac{d\theta_k^*}{dp_{k+1}} = \frac{\partial D_k/\partial p_{k+1}}{q_k}, \quad k = 0,1,2,$$

and, therefore, derive that for $k = 0,1,2$, $\dfrac{\partial D_{k+1}}{\partial p_k} > 0$ and $\dfrac{\partial D_k}{\partial p_{k+1}} > 0$. To prove the symmetry of the estimated parameters, we must note that $\dfrac{\partial D_k}{\partial p_{k+1}} = q_k f(\theta_k^*)\dfrac{d\theta_k^*}{dp_{k+1}}$. Differentiating the indifference condition of the marginal consumer θ_k^* with respect to prices p_k and p_{k+1} we can write that $\dfrac{d\theta_k^*}{dp_{k+1}} = \dfrac{-\theta_k^*(\alpha - p_{k+1})}{A}$ where $A = U'_\theta(q_k,\theta_k^*) - U'_\theta(\theta_k^*(\alpha - p_{k+1}),\theta_k^*)$. As $\dfrac{d\theta_k^*}{p_k} = \dfrac{q_k}{A}$, we conclude that

$$\frac{\partial D_k}{\partial p_{k+1}} = -\theta_k^*(\alpha - p_{k+1})f(\theta_k^*)\frac{d\theta_k^*}{dp_k} = \frac{\partial D_{k+1}}{\partial p_k}.$$

Thus $\beta_{01} = \beta_{10}$ $\beta_{12} = \beta_{21}$ and $\beta_{23} = \beta_{32}$. On the other hand, substituting the cross derivatives in direct derivatives, we can write

$$\frac{\partial D_1}{\partial p_1} + \delta_0\frac{\partial D_0}{\partial p_1} + \frac{1}{\delta_1}\frac{\partial D_2}{\partial p_1} = \int_{\theta_0^*(p_0,p_1)}^{\theta_1} -\theta f(\theta)d\theta = K_1, \tag{12}$$

$$\frac{\partial D_2}{\partial p_2} + \delta_1\frac{\partial D_1}{\partial p_2} + \frac{1}{\delta_2}\frac{\partial D_3}{\partial p_2} = \int_{\theta_1^*(p_1,p_2)}^{\theta_2} -\theta f(\theta)d\theta = K_2, \tag{13}$$

$$\frac{\partial D_3}{\partial p_3} + \delta_2\frac{\partial D_2}{\partial p_3} = \int_{\theta_2^*(p_2,p_3)}^{\overline{\theta}} -\theta f(\theta)d\theta = K_3, \tag{14}$$

with $\delta_k = \dfrac{\theta_k^*(\alpha - p_{k+1})}{q_k}$, $k = 0, 1, 2$, where δ_0, δ_1 and δ_2 are the changes in the consumption of the marginal individuals when they move from interval zero to one, from one to two and from two to three. Then, we can write K_j, $j = 1, 2$, using the estimated parameters, as $K_j = \beta_{j,j} + \delta_{j-1}\beta_{j-1,j} + \dfrac{1}{\delta_j}\beta_{j+1,j}$.

6.2 Data

6.2.1 Average consumption levels and effective prices in each interval and zone ($\overline{x}_k^t$ and p_k^t)

ZONE	$\overline{x}_0$	$\overline{x}_1$	$\overline{x}_2$	$\overline{x}_3$	p_0	p_1	p_2	p_3
I	22.2368	44.0353	98.5500	403.4333	55.57	63.42	67.63	91.29
II	24.3208	42.8175	96.6667	1019.2468	61.23	69.86	79.06	114.54
III	23.4708	41.7155	99.2732	366.3312	61.77	70.35	82.87	115.87
IV	22.6810	46.2906	108.1861	372.3163	61.42	69.63	80.17	106.99
V	22.8508	45.2314	101.4405	1456.2324	60.15	67.65	78.19	111.14
VI	22.2332	46.2563	94.3733	514.2123	44.22	49.01	58.06	97.21
VII	22.5133	45.6658	95.3062	305.1098	59.65	67.75	80.07	105.38

6.2.2 Drained water in each interval and zone (γ_k^t)

Interval	Zones I	II	III	IV	V	VI	VII
0	73.21	96.79	99.06	97.59	92.28	25.91	90.19
31	73.79	97.63	99.44	96.76	89.44	20.42	89.81
71	48.82	85.69	97.96	89.27	82.86	17.93	88.94
201	34.97	96.15	99.65	76.28	87.20	50.55	72.06

6.2.3 Average prices and consumptions levels

Interval (m^3)	Supply	Drain	p_k = efective price	$\overline{x}$ = average consumption
0 - 30	38 pts	24 pts	62.0 pts	23.67 m^3
31 - 70	43.5 pts	27 pts	70.5 pts	43.34 m^3
71 - 200	52.5 pts	31 pts	83.5 pts	97.02 m^3
+ 200	78 pts	38 pts	111.0 pts	500.03 m^3

6.2.4 Frequencies

Interval	% subscribers	$\sum$ % subscribers	% users	$\sum$ % users
1-5	4.59	4.59	3.50	3.50
6-10	4.67	9.26	3.70	7.19
11-15	6.09	15.34	5.15	12.34
16-20	7.29	22.63	6.63	18.97
21-25	7.38	30.00	7.27	26.25
26-30	38.02	68.02	35.36	61.61
31-35	8.40	76.42	10.03	71.64
36-40	6.39	82.81	8.09	79.73
41-45	4.56	87.37	5.85	85.58
46-50	3.29	90.67	4.44	90.02
51-55	2.27	92.94	2.86	92.88
56-60	1.68	94.62	1.95	94.83
61-65	1.13	95.75	1.29	96.12
66-70	0.81	96.56	0.81	96.94
71-75	0.60	97.16	0.64	97.58
76-80	0.47	97.64	0.47	98.05
81-85	0.34	97.97	0.27	98.32
86-90	0.29	98.26	0.24	98.56
91-95	0.21	98.47	0.17	98.73
96-100	0.18	98.65	0.19	98.92
101-200	0.16	98.81	0.13	99.06

6.3 Estimation of $F(\theta)$

	S.V.	E.V.		S.V.	E.V.
1	3.50	1.714	11	2.86	3.26
2	3.70	4.562	12	1.95	2.45
3	5.11	6.825	13	1.29	1.78
4	6.63	8.472	14	0.81	1.26
5	7.27	9.484	15	0.64	0.874
6	35.36	34.069	16	0.47	0.587
7	10.03	7.624	17	0.27	0.385
8	8.09	6.473	18	0.24	0.245
9	5.85	5.316	19	0.17	0.152
10	4.44	4.230	20	0.19	0.0927
			21	0.13	0.1255

S.V.= Sample value, E.V.= Estimated value.

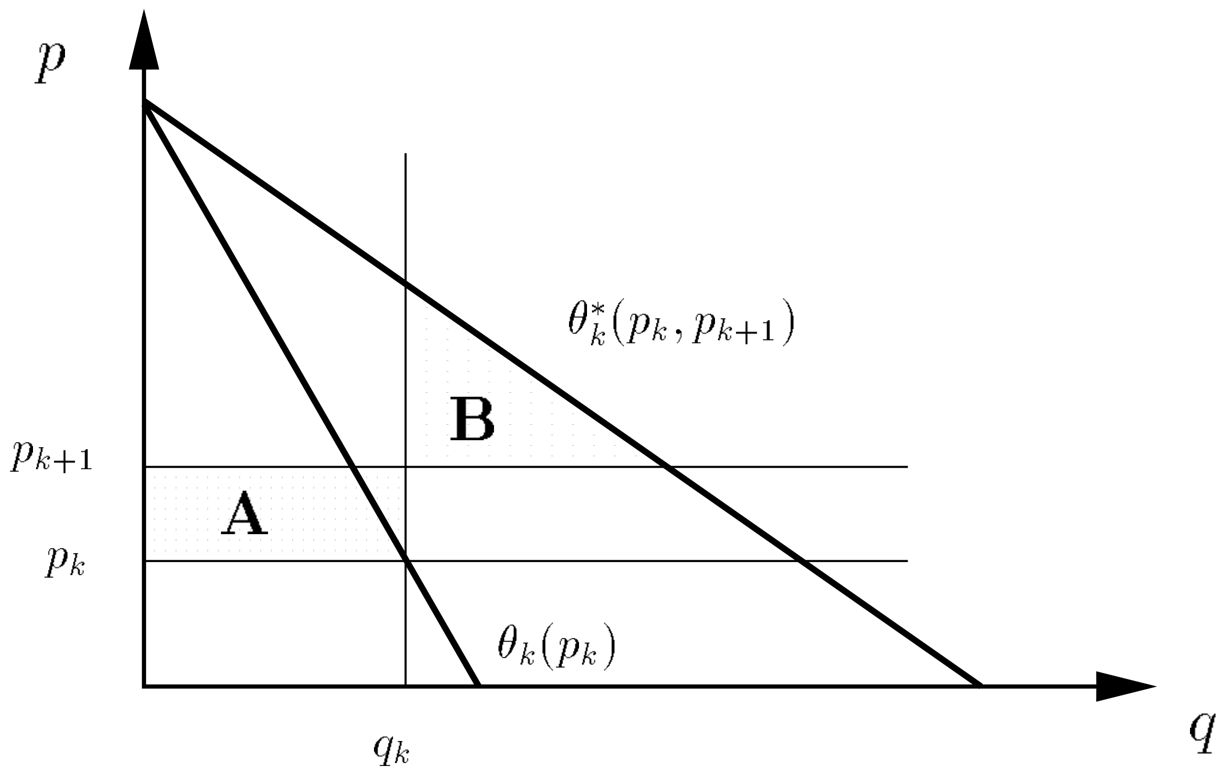

Figure 1: Consumers type $\theta \in [\theta_k, \theta_k^*]$ prefer to consume q_k.

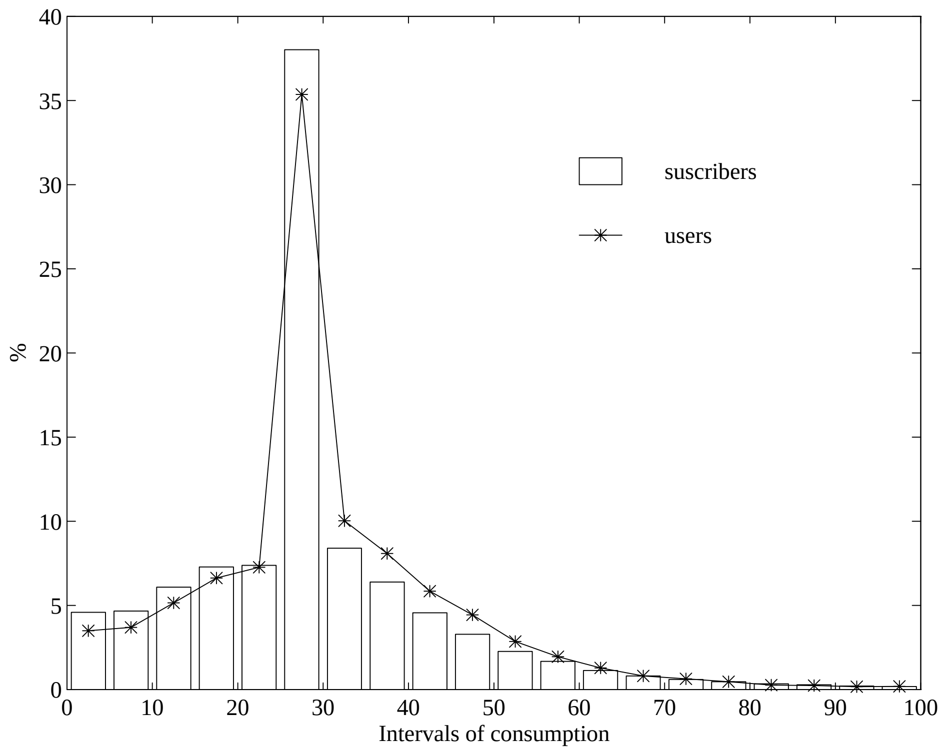

Figure 2: Frequencies of consumers.

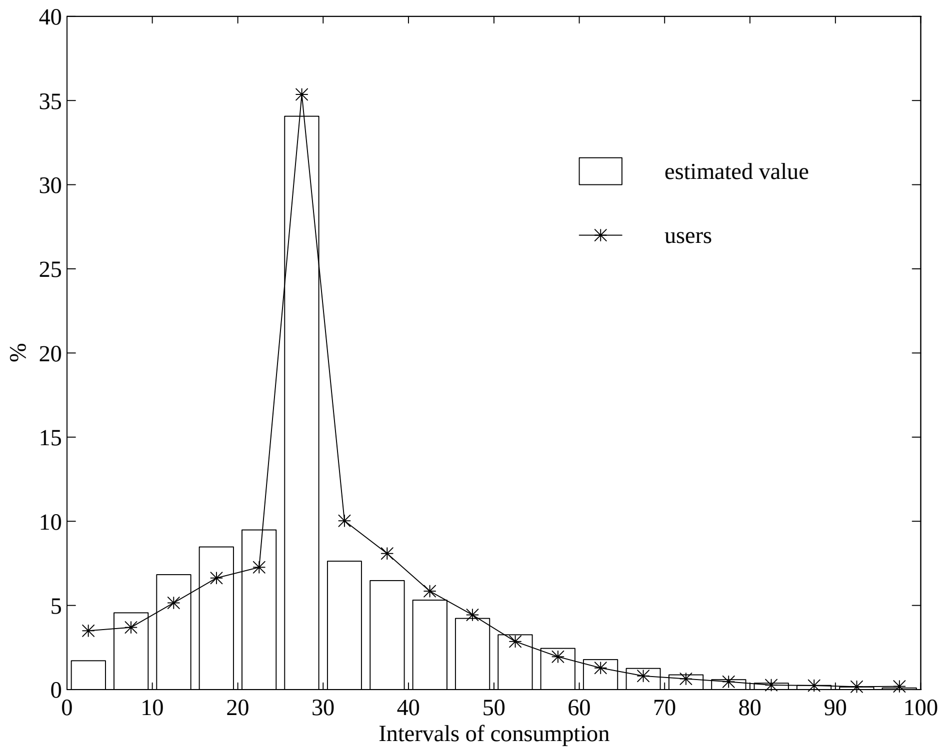

Figure 3: Observed and estimated frequencies of consumers.

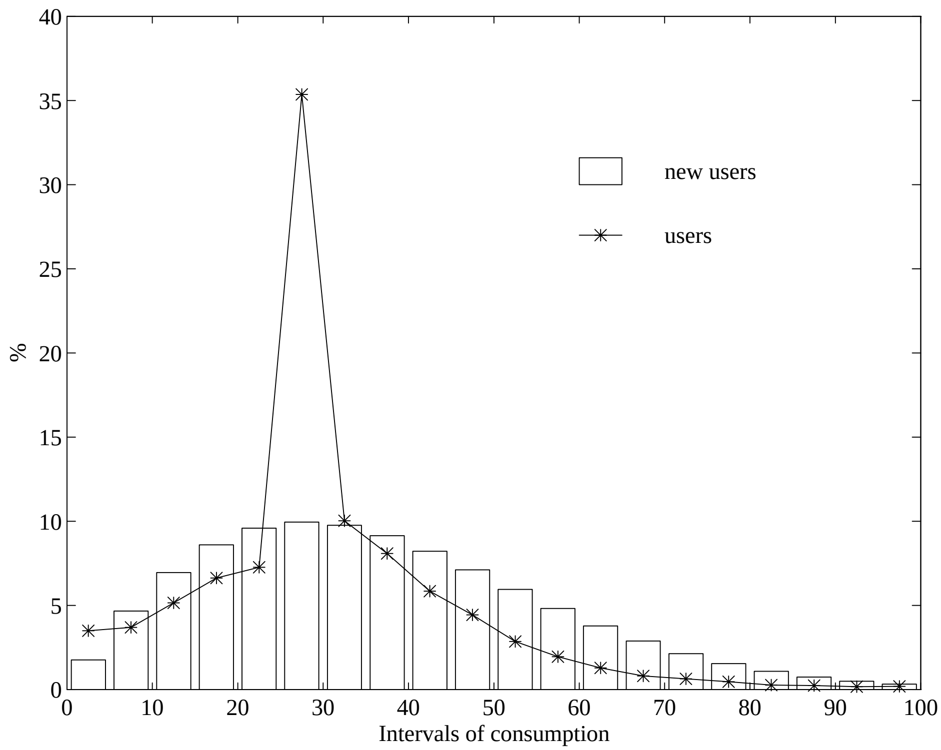

Figure 4: Changes in the distribution of consumption.