# Endogenous Price Leadership*

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**Abstract**

We consider a linear price setting duopoly game with differentiated products and determine endogenously which of the players will lead and which will follow. While the follower role is most attractive for each firm, we show that waiting is more risky for the low cost firm so that, consequently, risk dominance considerations, as in Harsanyi and Selten (1988), allow the conclusion that only the high cost firm will choose to wait. Hence, the low cost firm will emerge as the endogenous price leader.

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# 1 Introduction

Standard game theoretic models of oligopoly situations impose the order of the moves exogenously, an assumption that was already criticized in Von Stackelberg (1934), well before game theory invaded the field of industrial organization. Von Stackelberg pointed out that players have preferences over which role (leader or follower) to play in the game and he argued that a stable equilibrium would result only if the actual role assignment would be consistent with these preferences. As Von Stackelberg argued, in many situations both duopolists prefer the same role so that a stable situation does not appear to exist.

In the case of quantity competition, the typical situation is that the position of leader is most preferred and that the follower's position is least desirable, with simultaneous moves resulting in intermediate payoffs. Hence, in this situation a fight - a Stackelberg war - might arise as to which of the players will assume the leadership role. In an earlier paper (Van Damme and Hurkens (1999)) we addressed the question of which player will succeed in obtaining this most privileged position. We focused on the case of homogeneous products with linear demand and constant marginal cost, with one firm being a more efficient producer than the other. Using an endogeneous timing game introduced in Hamilton and Slutsky (1990), we showed that committing to move early is more risky for the high cost firm, hence, that risk dominance considerations (Harsanyi and Selten (1988)) imply that the efficient firm will take up the leadership position.

In the present paper we address the same question in the context of price competition in a duopoly with differentiated substitutable products, linear and symmetric demand, and constant marginal cost. Again we assume that one firm is more efficient than the other and has lower marginal cost. The question asked is which ordering of the moves will arise when this order is determined strategically by the players? Will also in this case the more efficient firm emerge as the leader in the game?

Price competition, however, is fundamentally different from quantity competition in that the leadership role is not the most preferred one. While it is indeed true that, under general conditions, a price duopolist prefers to move first to moving simultaneously, a player can benefit even more if he can move last. (See Boyer and Moreaux (1987), Dowrick (1986) and Gal-Or (1985)). The basic intuition can be easily seen when firms are identical. First of all one notices that the price of the leader $p^L$ is larger than the Nash equilibrium price $p^N$ since the leader's total profit, taking into account the rival's optimal reaction, is increasing in his price at the Nash equilibrium. Since the follower's reaction curve is flatter than the 45 degree line, the follower's price $p^F$ is smaller than $p^L$. Consequently, $\pi^F(p^L, p^F) > \pi^F(p^L, p^L) = \pi^L(p^L, p^L) > \pi^L(p^L, p^F) > \pi^L(p^N, p^N)$. (The first inequality follows since $p^F$ is on the follower's reaction curve, the second since the leader profits from a higher price of the follower, and the last since the leader could have chosen $p^N$ instead of $p^L$.) Hence, if firms are identical, each firm prefers following above leading, while any sequential order is preferred above moving simultaneously. By continuity, these preferences remain when differences between the firms are not too large[1].

As in our earlier paper, we use a model from Hamilton and Slutsky (1990) to determine which player will get which role. The model allows firms to choose a price either early or late. Choices within a period are simultaneous, but if one firm moves early and the other moves late, the latter is informed about the former's price before making its choice. Since following confers advantages, each player is tempted to move late, but the situation in which both move late is not an equilibrium: this would result in the Nash payoffs and then each player would have an incentive to move early. It follows that the game has two pure equilibria corresponding to the two possible sequential orderings of the moves and that the players have opposite preferences about these equilibria. In our view, the question of who will take up the most preferred role amounts to solving the problem of which player is willing to take the largest risk in waiting and we formally answer this question by using the risk-dominance concept from Harsanyi and Selten (1988). The conclusion is that waiting is more risky for the low cost firm, hence, the efficient firm

[1]Preferences of players may, however, be perfectly aligned when there are capacity constraints, since limited capacity reduces the follower's incentive to undercut the leader's price. Deneckere and Kovencock (1992), Furth and Kovenock (1992) and Canoy (1996) show, in a variety of circumstances, that both firms prefer the large firm to lead in this case, provided that capacities are sufficiently asymmetric.

will emerge as the price leader and the less efficient firm will take up the more favorable follower role. Relating this result to our earlier paper, we see that the identity of the leader is independent of whether prices or quantities are the strategic variables.

Before providing some intuition for our main result, we first note that one can conceive of alternative ways of selecting among the two Stackelberg equilibria. One way would be to look at which player would benefit most by moving late. Denoting player $i$'s payoff as a leader (resp. follower) by $L_i$ (resp. $F_i$), one can argue that, if

$$F_i - L_i > F_j - L_j,$$

then the equilibrium in which $i$ becomes the follower is most focal since that player has most to gain from following and, hence, that players will coordinate on this one. Alternatively, one might argue that the equilibrium in which total profits are highest is most focal, hence, that $i$ will follow if and only if

$$F_i + L_j > F_j + L_i.$$

Clearly, this latter inequality is equivalent to the first, so that both approaches would predict the same leadership pattern. Both these approaches are essentially based on an idea of collective rationality, since it is assumed that players are able to solve the coordination problem. Our approach is purely individualistic since each firm only takes into account it's own expected profit and this is why we prefer this approach. Nevertheless, it is good to point out that in this particular instance our approach produces the same outcome: the above inequalities are satisfied when $i$ is the high cost firm. Hence, all three approaches lead to the conclusion that the more efficient firm will lead.

We are now ready to provide the basic intuition for our main result. As each firm's most preferred position is to move last, it is natural to assume that each player initially expects the other to hold out. Players cannot maintain these expectations, however, as both players holding out is not a Nash equilibrium: given that the opponent holds out, each player prefers to move first. Hence, each player is forced to adjust his expectations and he will represent his uncertainty by a mixed strategy that assigns some weight to the opponent committing to a (possibly random) price and that puts the complementary weight on the opponent waiting. As the high cost firm gains more from waiting, it is more insisting on this position. To put it differently, as long as the low cost firm finds it attractive to wait, the high cost firm finds this attractive as well. Hence, given that one firm has to give in, this will be the low cost firm. The high cost firm will thus obtain the most preferred position.

Our paper thus provides a game theoretic justification for price leadership by the efficient or dominant firm. The traditional industrial organization literature has emphasized price leadership in general and leadership by the dominant firm in particular. It argues that leadership allows firms to better coordinate their prices and that it results in higher prices and lower consumer surplus, thus raising possible antitrust concerns. However, that literature is not so clear on which firm will take up the leadership role. For example, Markham (1951) in his seminal paper concluded on the one hand that "... price 'leadership' in a dominant firm market is not simply a *modus operandi* designed to circumvent price competition among rival sellers but is instead an inevitable consequence of the industry's structure," while on the other hand he stated that "... in a large number of industries which do not contain a partial monopolist, the price leader is frequently but not always the largest firm." Similarly, Scherer and Ross (1990) list as distinguishing characteristics of (barometric) price leadership "... occasional changes in the identity of the price leader (who is likely in any case to be one of the largest sellers)." We believe that the risk considerations that we stress in our paper might shed some light on these issues of price leadership in practice.

The remainder of this paper is organized as follows. The underlying duopoly game as well as the action commitment game from Hamilton and Slutsky (1990) are described in Section 2, where also the relevant notation is introduced. Section 3 describes the specifics of the risk dominance concept (Harsanyi and Selten (1988)) as it applies to this context. The main results are derived in Section 4. Section 5 shows that a shortcut, based on risk-dominance in the restricted game where each player can only choose between committing to his leader price and waiting, would have given the wrong result and argues that this is because the restricted game does not provide a faithful description of the actual risks involved. Section 6 offers a brief conclusion.

# 2 The Model

The underlying linear price setting duopoly game is as follows. There are two firms, 1 and 2. Firm $i$ produces product $i$ at a constant marginal cost $c_i \geq 0$. The goods are imperfect substitutes and the demand for good $i$ is given by

$$D_i(p_i, p_j) = \max\{1 - p_i + ap_j, 0\},$$

where $0 < a < 1$. Firms choose prices simultaneously and the profit of firm $i$ is given by $u_i(p_i, p_j) = (p_i - c_i)D_i(p_i, p_j)$. We assume that $1 > c_1 > c_2 > 0$, hence firm 2 is more efficient than firm 1. The best reply of player $j$ against the price $p_i$ of player $i$ is unique and is given by

$$b_j(p_i) = \frac{1 + ap_i + c_j}{2}. \tag{2.1}$$

The unique maximizer of the function $p_i \mapsto u_i(p_i, b_j(p_i))$ is denoted by $p_i^L$ (firm $i$'s leader price). We also write $p_j^F$ for the price that $j$ will choose as a price follower, $p_j^F = b_j(p_i^L)$, and $L_i = u_i(p_i^L, p_j^F)$ and $F_i = u_i(p_i^F, p_j^L)$. We write $(p_1^N, p_2^N)$ for the unique Nash equilibrium of the game and denote player $i$'s payoff in this equilibrium by $N_i$. For later reference we note that

$$p_i^L = \frac{2 + a + ac_j + (2 - a^2)c_i}{2(2 - a^2)} \tag{2.2$a$}$$

$$p_i^F = \frac{4 + 2a - a^2 + (4 - a^2)c_i + (2a - a^3)c_j}{4(2 - a^2)} \tag{2.2$b$}$$

$$p_i^N = \frac{2 + a + ac_j + 2c_i}{4 - a^2} \tag{2.2$c$}$$

and

$$L_i = \frac{(2 + a + ac_j + (a^2 - 2)c_i)^2}{8(2 - a^2)} \tag{2.3$a$}$$

$$F_i = \frac{(4 + 2a - a^2 + (2a - a^3)c_j + (3a^2 - 4)c_i)^2}{16(2 - a^2)^2} \tag{2.3$b$}$$

$$N_i = \frac{(2 + a + ac_j + (a^2 - 2)c_i)^2}{(4 - a^2)^2} \tag{2.3$c$}$$

One easily verifies that $p_1^L > p_2^L$ and $p_1^F > p_2^F$. It also readily follows that

$$p_i^L > p_i^F > p_i^N \qquad (i = 1, 2)$$

$$F_i > L_i > N_i. \qquad (i = 1, 2)$$

Hence, each player has an incentive to commit himself (compared to the simultaneous play equilibrium) but prefers to follow. Straightforward computations show that $F_1 - L_1 > F_2 - L_2$, hence the high cost firm benefits more from being the follower than the low cost firm. Obviously, the above inequality is equivalent to $L_2 + F_1 > L_1 + F_2$, hence total profits are larger when the efficient firm leads. The question we address in this paper is whether the players will succeed in reaching that "efficient" ordering of the moves.

To investigate which player will dare to wait when both players have the opportunity to do so, we make use of the *two-period action commitment game* that was proposed in Hamilton and Slutsky (1990). The rules are as follows. There are two periods and each player has to choose a price in exactly one of these periods. Within a period, choices are simultaneous, but, if a player does not choose to move in period 1, then in period 2 this player is informed about which action his opponent chose in period 1. This game has proper subgames at $t = 2$ and our assumptions imply that all of these have unique equilibria. We will analyze the reduced game, $g^2$, that results when these subgames are replaced by their equilibrium values. Formally, the strategy set of player $i$ in $g^2$ is $\mathbb{R}_+ \cup \{w_i\}$ and the payoff function is given by

$$\begin{aligned} u_i(p_i, p_j) &= (p_i - c_i)(1 - p_i + ap_j) && (2.4)\\ u_i(p_i, w_j) &= (p_i - c_i)(1 - p_i + a(1 + ap_i + c_j)/2) && (2.5)\\ u_i(w_i, p_j) &= (1 + ap_j - c_i)^2/4 && (2.6)\\ u_i(w_i, w_j) &= N_i && (2.7) \end{aligned}$$

Note that $u_i$ is strictly concave in $p_i$. It is easily seen that $g^2$ has three Nash equilibria in pure strategies: Either each player $i$ commits to his Nash price $p_i^N$ in the first period, or one player $i$ commits to his leader price $p_i^L$ and the other player waits till the second period. Mixed strategies represent uncertainty about whether a player will commit himself and to which price. They will play an important role below. Let $m_j$ be a mixed strategy of player $j$ in the game $g^2$. Because of the linear-quadratic specification of the game, there are only three "characteristics" of $m_j$ that are relevant to player $i$, viz. $w_j$ the probability that player $j$ waits, $\mu_j$ the average price to which $j$ commits himself given that he commits himself, and $\nu_j$, the variance of this price. Specifically, it easily follows from (2.4)-(2.7) that the expected payoff of player $i$ against a mixed strategy $m_j$ with characteristics $(w_j, \mu_j, \nu_j)$ is given by

$$\begin{aligned} u_i(p_i, m_j) &= (1-w_j)(p_i - c_i)(1 - p_i + a\mu_j) \\ &\quad + w_j(p_i - c_i)(1 - p_i + a(1 + ap_i + c_j)/2) \qquad (2.8) \\ u_i(w_i, m_j) &= (1-w_j)[a^2\nu_j/4 + (1 + a\mu_j - c_i)^2/4] \\ &\quad + w_j[(2 + a + ac_j + (a^2-2)c_i)/(4-a^2)]^2 \qquad (2.9) \end{aligned}$$

Note that the payoff function as given by (2.8) is strictly concave in $p_i$, hence, player $i$ has a unique optimal commitment price against any mixed strategy $m_j$ of player $j$. It follows that, if $m_i$ is a mixed equilibrium strategy, then $m_i$ can prescribe to mix over the periods, but not over the prices involved, hence, $m_i$ is of the form $m_i = (w_i, p_i, 0)$. We now show that the game does not admit equilibria in which both players randomize[2]. Assume that $(m_1, m_2)$ with $m_i = (w_i, p_i, 0)$ and $0 < w_i < 1$ is a mixed equilibrium of $g^2$. Because of the concavity of the payoff function and $0 < w_i < 1$, $p_i$ is in the interior of the interval spanned by $b_i(p_j)$ and $p_i^L$ for each $i$. Hence

$$\text{if } b_i(p_j) < p_i^L, \text{ then } b_i(p_j) < p_i < p_i^L \qquad (2.10)$$

$$\text{if } b_i(p_j) > p_i^L, \text{ then } p_i^L < p_i < b_i(p_j) \qquad (2.11)$$

[2]This observation was first made in Pastine and Pastine (1999). We provide a slightly different proof. We note that there are (asymmetric) equilibria in which one player randomizes. These mixed equilibria will not be considered in this paper, the reason being that we want to stick as closely as possible to the general solution procedure outlined in Harsanyi and Selten (1988), a procedure that gives precedence to pure equilibria whenever possible.

Furthermore, a necessary condition for $(m_1, m_2)$ to be an equilibrium is that no player $i$ can benefit by committing to $b_i(p_j)$ for sure. This commitment results in the same payoff as waiting does when the other player commits himself, hence, it should do no better when the opponent waits

$$u_i(b_i(p_j), w_j) \square u_i(w_i, w_j) = N_i \qquad \text{(all } i\text{)} \tag{2.12}$$

Let $[p_i^-, p_i^+]$ be the maximal interval on which $u_i(p_i, w_j) \geq N_i$. Then $p_i^- = p_i^N$ and $p_i^+ > p_i^L$. This condition thus implies that $b_i(p_j) \square p_i^N$ or $b_i(p_j) > p_i^L$ for all $i$. The first of these conditions is equivalent to $p_j \square p_j^N$. Assume $p_2 \square p_2^N$. Then $p_1 \square p_1^N$ as otherwise $p_2$ could not be a best response. Hence (by (2.10)), $p_2 > b_2(p_1)$ and $p_1 > b_1(p_2)$. The conditions $p_i \square p_i^N$ and $p_i > b_i(p_j)$ (for $i, j = 1, 2$, $i \neq j$) lead to a contradiction. Hence, assume $b_1(p_2) > p_1^L$. Then by (2.11) $p_1^N < p_1^L < p_1 < b_1(p_2)$. Since $b_2(p_1) > p_2^N$ we must have $b_2(p_1) > p_2^L$, hence $p_2^N < p_2^L < p_2 < b_2(p_1)$. But now the conditions $p_i < b_i(p_j)$ and $p_i > p_i^N$ $(i, j = 1, 2,\ i \neq j)$, lead to a contradiction.

In the remainder of this paper, mixed equilibria will not be considered (see footnote 2), but mixed strategies will play an important role. The tracing procedure that is used below to find a unique solution of the game pictures players in the situation where they are uncertain about which equilibrium will be played, and they respresent their uncertainty by a mixed strategy. Note that (2.8) and (2.9) show that uncertainty concerning the price to which $j$ will commit himself makes it more attractive for player $i$ to wait: $\nu_j$ contributes positively to (2.9) and it does not play a role in (2.8). On the other hand, increasing $w_j$ clearly increases the incentive for player $i$ to commit himself. Finally, increasing $\mu_j$ increases the incentive for player $i$ to commit himself, because of the positive effect on $i$'s demand.

# 3 Risk Dominance and the Tracing Procedure

The concept of *risk dominance* captures the intuitive idea that, when players do not know which of two equilibria should be played, they will measure the risk involved in playing each of these equilibria and they will coordinate expectations on the less risky one, i.e.

on the risk dominant equilibrium of the pair. The formal definition of risk dominance involves the *bicentric prior* and the *tracing procedure*. The bicentric prior describes the players' initial assessment about the situation. As this initial assessment need not be an equilibrium of the game, it cannot constitute the players' final view on the situation. The tracing procedure is a process that, starting from given prior beliefs of the players, gradually adjusts the players' plans and expectations until they are in equilibrium. It models the thought process of players who, by deductive personal reflection, try to figure out what to play in the situation where the initial uncertainty is represented by the given prior. Below we describe the mechanisms of the tracing procedure as well as how, according to Harsanyi and Selten (1988), the initial prior should be constructed.

## 3.1 Bicentric Prior

Let $g = (S_1, S_2, u_1, u_2)$ be a 2-person game and let $s$ and $s'$ be two equilibria of this game. We need to specify the players' initial beliefs when they are uncertain about which of these two equilibria should be played. Harsanyi and Selten (1988) argue as follows. Player $j$, being Bayesian, will assign a subjective probability $z_j$ to $i$ playing $s_i$ and he will assign the complementary probability $z'_j = 1 - z_j$ to $i$ playing $s'_i$. With these beliefs, player $j$ will play the best response against the strategy $z_j s_i + z'_j s'_i$ that he expects $i$ to play and we denote the resulting strategy of $j$ with $b_j(z_j)$.[3] Player $j$, knowing his prior $z_j$, knows which action he will play. Player $i$, however, does not know $z_j$ exactly and hence cannot predict exactly what $j$ will do. Applying the principle of insufficient reason, Harsanyi and Selten (1988) argue that $i$ will consider $z_j$ to be uniformly distributed on $[0, 1]$. Writing $Z_j$ for a uniformly distributed random variable on $[0, 1]$, player $i$ will, therefore, believe that he is facing the mixed strategy

$$m_j = b_j(Z_j) \tag{3.1}$$

[3] In general player $j$ may have multiple best replies in which case he should play all of them with equal probability. However, in our setting with strictly quasi-concave profit functions this happens with zero probability, so we may ignore multiple best replies.

and this mixed strategy $m_j$ of player $j$ is player $i$'s prior belief about $j$'s behavior in the situation at hand. Similarly, $m_i = b_i(Z_i)$, where $Z_1$ and $Z_2$ are independent, is the prior belief of player $j$, and the mixed strategy pair $m = (m_1, m_2)$ is called the *bicentric prior* associated with the pair $(s, s')$.

## 3.2 Tracing Procedure

From a mathematical point of view the tracing procedure is a mapping that maps initial beliefs into the set of equilibria of the game. In order to determine the risk dominant equilibrium we will have to apply this mapping only to the bicentric prior described above. However, in this subsection we will define the tracing procedure for any initial beliefs.

Let $m_i$ be a mixed strategy of player $i$ in $g$ $(i = 1, 2)$. The strategy $m_i$ represents the initial uncertainty of player $j$ about $i$'s behavior. For $t \in [0, 1]$ we define the game $g^{t,m} = (S_1, S_2, u_1^{t,m}, u_2^{t,m})$ in which the payoff functions are given by

$$u_i^{t,m}(s_i, s_j) = (1 - t)u_i(s_i, m_j) + tu_i(s_i, s_j). \tag{3.2}$$

Hence, for $t = 1$, this game $g^{t,m}$ coincides with the original game $g$, while for $t = 0$ we have a trivial game in which each player's payoff depends only on his own action and his own prior beliefs.[4] Write $\Gamma^m$ for the graph of the equilibrium correspondence, i.e.

$$\Gamma^m = \{(t, s) : \ t \in [0, 1], s \text{ is an equilibrium of } g^{t,m}\}.$$

It can be shown that, if $g$ is a generic *finite* game, then, for almost any prior $m$, this

[4]Loosely speaking the parameter $t$ might be thought of as time. With this interpretation, player $i$ assigns weight $1 - t$ to his prior beliefs at time $t$, while he gives weight $t$ to the reasoning process at this point in time, at time $t = 1$, when the players' actions are in equilibrium, the player fully trusts the outcome of the reasoning process.

graph $\Gamma^m$ contains a unique distinguished curve that connects the unique equilibrium $s^{0,m}$ of $g^{0,m}$ with an equilibrium $s^{1,m}$ of $g^{1,m}$. (See Schanuel et al. (1991) for details.) The equilibrium $s^{1,m}$ is called the *linear trace* of $m$. If players' initial beliefs are given by $m$ and if players' reasoning process corresponds to that as modeled by the tracing procedure, then players' expectations will converge on the equilibrium $s^{1,m}$ of $g$. Below, we will apply the tracing procedure to the *infinite* game $g^2$ that was described in the previous section. For such games, no generalizations of the Schanuel et al. (1991) results have been established yet, but as we will see in the following section, there indeed exists a unique distinguished curve in the special case analyzed here. Hence, the non-finiteness of the game $g^2$ will create no special problems.

### 3.3 Risk Dominance

Risk dominance is defined as follows. Consider two equilibria, $s$ and $s'$ of $g$. Use the construction described in subsection 3.1 to determine the bicentric prior, $m$, associated with the pair $(s, s')$. Then apply the tracing procedure of subsection 3.2 to $m$, i.e. compute the linear trace of this prior, $s^{1,m}$. We now say that $s$ *risk dominates* $s'$ if $s^{1,m} = s$. Similarly, $s'$ risk dominates $s$ if $s^{1,m} = s'$. In case the outcome of the tracing procedure is an equilibrium different from $s$ or $s'$, then neither of the equilibria risk dominates the other. Below we show that the latter situation will not occur in our 2-stage action commitment game, provided that the costs of the firms are different.

## 4 Commitment and Risk Dominance

In this section, we prove our main results. Let $g^2$ be the endogenous commitment game from Section 2. Write $S_i$ for the pure equilibrium in which player $i$ commits to his leader price in period 1, $S_i = (p_i^L, w_j)$, and write $B$ for the equilibrium in which each player commits to his Bertrand price in period 1, $B = (p_1^N, p_2^N)$. We show that both price leader equilibria risk dominate the Bertrand equilibrium and that $S_2$ risk dominates $S_1$ when $c_2 < c_1$. The first result is quite intuitive: Committing to $p_i^N$ is a weakly dominated strategy and playing a weakly dominated strategy is risky. The proof of this result is correspondingly easy.

**Proposition 1** *In $g^2$, the price leader equilibrium $S_i$ risk dominates the Nash equilibrium $B$ $(i = 1, 2)$.*

**Proof.** Without loss of generality, we just prove that $S_1$ risk dominates $B$. We first compute the bicentric prior that is relevant for this risk comparison, starting with the prior beliefs of player 1.

Let player 2 believe that 1 plays $z_2 S_{11} + (1 - z_2) B_1 = z_2 p_1^L + (1 - z_2) p_1^N$. Obviously, if $z_2 \in (0, 1)$, then the unique best response of player 2 is to wait, $b_2(z_2) = w_2$. Hence, the prior belief of player 1 is that player 2 will wait with probability 1, $m_2 = w_2$.

Next, let player 1 believe that 2 plays $z_1 S_{12} + (1 - z_1) B_2 = z_1 w_2 + (1 - z_1) p_2^N$. Obviously, waiting yields player 1 the Nash payoff $N_1$ as in (2.3c), irrespective of the value of $z_1$. When $z_1 > 0$ then committing to a price that is (slightly) above $p_1^N$ yields a strictly higher payoff, hence, the best response is to commit to a certain price $p_1(z_1)$, $b_1(z_1) = p_1(z_1)$. The reader easily verifies that $p_1(z_1)$ increases with $z_1$ and that $p_1(1) = p_1^L$. Consequently, if $m_1$ is the prior belief of player 2 then for the characteristics $(w_1, \mu_1, \nu_1)$ of $m_1$ we have: $w_1 = 0,\ \mu_1 > p_1^N,\ \nu_1 > 0$.

Now, let us turn to the tracing procedure. The starting point corresponds to the best replies against the prior. Obviously, the unique best response against $m_2$ is for player 1 to commit to $p_1^L$, while player 2's unique best response against $m_1$ is to wait. Hence, the unique equilibrium at $t = 0$ is $S_1$. Since $S_1$ is an equilibrium of the original game, it is an equilibrium for any $t \in [0, 1]$. Consequently, the distinguished curve in the graph $\Gamma^m$ is the curve $\{(t, S_1) :\ t \in [0, 1]\}$ and $S_1$ risk dominates $B$. □

We now turn to the risk comparison of the two price leader equilibria. Again we start by computing the bicentric prior based on $S_1$ and $S_2$. We show that each player's prior belief is that the other player will commit to a random price. Let player $j$ believe that $i$ commits to $p_i^L$ with probability $z$ and that $i$ waits with probability $1 - z$. Waiting yields

$$u_j(w_j, z p_i^L + (1 - z) w_i) = z F_j + (1 - z) N_j.$$

It is easily seen that committing to the follower price $p_j^F$ results in higher profits, namely the mapping $p \mapsto u_j(p, b_i(p))$ is concave and attains its maximum at $p_j^L$, and since $p_j^F \in (p_j^N, p_j^L)$, we have

$$u_j(p_j^F, b_i(p_j^F)) > u_j(p_j^N, b_i(p_j^N)) = N_j,$$

so that

$$u_j(p_j^F, zp_i^L + (1-z)w_i) > u_j(w_j, zp_i^L + (1-z)w_i).$$

Hence, it already follows that each player will believe that the opponent will commit himself to some price. To determine this price, note that committing to price $p_j$ yields

$$u_j(p_j, zp_i^L + (1-z)w_i) = (p_j - c_j)[1 - p_j + a(zp_i^L + (1-z)(1 + ap_j + c_i)/2)].$$

Given $z$, the optimal commitment price $p_j(z)$ of player $j$ must satisfy the first order condition $\partial u_j(p_j, zp_i^L + (1-z)w_i)/\partial p_j = 0$, and is, hence, given by

$$p_j(z) = \frac{(1-z)(2-a^2)p_j^L + 2zp_j^F}{2 - a^2(1-z)}. \tag{4.1}$$

Consequently, both players expect the other player to commit with probability one. Furthermore, note that $p_1(z) > p_2(z)$ for all $z \in [0, 1]$, since $p_1^L > p_2^L$ and $p_1^F > p_2^F$. This means that firm 1 expects firm 2 to commit to a low price, while firm 2 expects firm 1 to commit to a high price. From

$$p_j'(z) = \frac{2(2-a^2)(p_j^F - p_j^L)}{(2 - a^2(1-z))^2},$$

one easily verifies that $p_2'(z) < p_1'(z) < 0$ since $p_2^F - p_2^L < p_1^F - p_1^L < 0$. Hence, firm 2's price is expected to vary more than firm 1's price. (See Appendix A1 for a formal proof.) We summarize these results in Lemma 1.

**Lemma 1** *Player i's bicentric prior $m_j$ is that j will commit to a random price $p_j(z)$ with expectation $\mu_j$ and variance $\nu_j$, where $\mu_j \in \left(p_j^F, p_j^L\right)$ and $\nu_j > 0$. Moreover, we have $\mu_1 > \mu_2$ and $\nu_1 < \nu_2$.*

Now, let us turn to the tracing procedure. The starting point (the initial equilibrium) corresponds to the best reply against the prior. Since both players expect the other to commit with probability one and are uncertain about the exact price the opponent will commit to, the unique best reply for both players is to wait. As $t$ increases player $i$ attaches more and more weight (namely $t$) to the event that player $j$ will wait. At some critical point $\bar{t}_i$ it must become profitable to commit and take the leader role. We will show that the low cost firm will switch before the high cost firm will, i.e. that $\bar{t}_1 > \bar{t}_2$. The intuition is given by Lemma 1 and the equations (2.8) and (2.9): Since player 1 (the high cost firm) commits to a higher and less variable price, it is relatively more attractive for firm 2 to commit to a price. We elaborate below and relegate the formal proof to the Appendix.

Recall from Section 2 that the expected payoff of player $i$ depends only on his action and the three important characteristics of the opponent's (mixed) strategy, viz. the probability that the other player waits, the average price to which the other player commits himself (given that he commits himself), and the variance of that price. During the tracing procedure expectations about the opponent's strategy change (see section 3.2), but as long as no player switches away from waiting, only the probability that the other waits will be adjusted. The average commitment price and the variation of this price do not change. Hence, the expectation of player $i$ at time $t$, given that no one has switched yet, is given by the mixed strategy $m_j^t = (1-t)m_j + tw_j$. Identifying this mixed strategy with its important characteristics we will write $m_j^t = (t, \mu_j, \nu_j)$. The expected payoff for player $i$ from committing and waiting is given in (2.8) and (2.9), respectively. For $m^t = (t, \mu, \nu)$ define the gain from committing for $i$ as

$$g_i(m^t) = \max_{p_i} u_i(p_i, m^t) - u_i(w_i, m^t).$$

We will show that firm 2 always has a higher incentive to commit himself than firm 1, i.e. that $g_2(m_1^t) > g_1(m_2^t)$ for all $t$. Since the gain of committing is negative at $t = 0$ and positive at $t = 1$, this implies that firm 2 will switch before firm 1 does.

The formal proof is divided into three steps and is given in the Appendix. We now provide intuition for each step. In the first step we show that the gain from committing is increasing in the opponent's price. From equations (2.4) and (2.6) it follows in a straightforward manner that

$$\frac{\partial u_i(p_i, p_j)}{\partial p_j} = a(p_i - c_i)$$

and

$$\frac{\partial u_i(w_i, p_j)}{\partial p_j} = a(b_i(p_j) - c_i),$$

i.e. the marginal effect on $i$'s profit of an increase in $j$'s price is equal to the price-cost margin multiplied with the marginal increase in demand. Since $j$ will never commit to a price above $p_j^L$, $b_i(p_j) < p_i^F$. On the other hand, if firm $i$ commits himself he will (optimally) commit to a price above $p_i^F$. The effect of an increase in $p_j$ is thus larger when $i$ commits himself than when he waits.

Secondly, the gain from committing is decreasing in the variability of the price of the opponent. This is very intuitive. We know from Lemma 1 that $\nu_1 < \nu_2$ so that firm 1 is more uncertain about the price firm 2 will commit himself to. Clearly, this gives him more reason to wait and less to commit.

Finally, we show that the low cost firm has more incentive to commit than a high cost firm even if they have exactly the same expectation about the commitment price of the opponent. This follows from the fact that the high cost firm gains more from being the follower than the low cost firm, i.e. that $F_1 - L_1 > F_2 - L_2$. The above steps can now be combined to show that, with $\mu_k$ and $\nu_k$ as in Lemma 1 we get

$$g_2(t, \mu_1, \nu_1) > g_2(t, \mu_2, \nu_1) > g_2(t, \mu_2, \nu_2) > g_1(t, \mu_2, \nu_2). \tag{4.2}$$

The formal proof is in Appendix A2. The above inequalities imply that at any point in the tracing procedure player 2 gains more from committing than firm 1, and, therefore, it must be player 2 who will decide to switch first, i.e. $\bar{t}_1 > \bar{t}_2$. Thus, both players wait till $\bar{t}_2$ at which point player 2 is exactly indifferent between waiting and committing optimally (to $\tilde{p}_2(\bar{t}_2)$). The graph of the equilibrium correspondence exhibits a "vertical" segment at $t_2$. Any pair of strategies in which firm 1 waits and firm 2 mixes between waiting (with probability $w$) and committing to $\tilde{p}_2(\bar{t}_2)$ (with probability $1 - w$) is an equilibrium of $g^{\bar{t}_2,m}$ : Firm 2 is indifferent and any mixture is therefore a best reply.

Firm 1 strictly prefers to wait when $w = 1$ (since $g_1(m_2^{\bar{t}_2}) < g_2(m_1^{\bar{t}_2}) = 0$) and also when $w = 0$ (since then firm 2 commits for sure to a random price). Because of linearity (in $w$) firm 1 prefers to wait for any $w \in [0, 1]$. From $\bar{t}_2$ onward, player 2 commits with probability 1 (but changes the commitment price continuously) and player 2 waits with probability 1. Therefore, the tracing procedure ends up in an equilibrium where player 2 commits and player 1 waits, i.e. at $S_2$. This concludes the proof of

**Proposition 2** *The price leader equilibrium $S_2$ in which the low cost firm leads risk dominates the one in which the high cost firm leads.*

By combining the Propositions 1 and 2 we, therefore, obtain our main result:

**Theorem 1** *The price leader equilibrium in which the efficient firm leads and the inefficient firm follows is the risk dominant equilibrium of the endogenous price commitment game.*

Note that, if the costs of firm 1 are not much higher than the costs of firm 2, then $F_1 > L_2$, i.e. the high cost firm makes higher profits (as a price follower) in the risk dominant equilibrium than the efficient firm (as a price leader). This seems curious and counterintuitive at first sight since it could give incentives to the low cost firm to increase its cost (if he would be able to do that in a credible way). However, given the cost structure, waiting is less risky for a high cost firm than for a low cost firm, and the inefficient firm profits from its "weak" position.

# 5 Risk Dominance in the Reduced Game

It is well known that risk dominance allows a very simple characterization for $2 \times 2$ games with two Nash equilibria: the risk dominant equilibrium is that one for which the product of the deviation losses is largest. Consequently, if risk dominance could always be decided on the basis of the reduced game spanned by the two equilibria under consideration (and if the resulting relation would be transitive), then the solution could be found by straightforward computations. Unfortunately, this happy state of affairs does not prevail in general. The two concepts do not always generate the same solution and it is well-known that the Nash product of the deviation losses may be a bad description of the underlying risk situation in general. (See, Carlsson and Van Damme (1993) for a simple example.) We now show that this is also true for the game analyzed in this paper. In fact, the reduced game analysis produces exactly the opposite result from that obtained by applying the tracing procedure.

The reduced game where each player is restricted to either committing himself to his leader's price or to wait is given by Table 1

| | $p_2^L$ | $w_2$ |
|---|---|---|
| $p_1^L$ | $D_1, D_2$ | $L_1, F_2$ |
| $w_1$ | $F_1, L_2$ | $N_1, N_2$ |

**Table 1:** Reduced version of the price commitment game.

where $L_i$, $N_i$ and $F_i$ are as in (2.3) and where $D_i = u_i(p_i^L, p_j^L)$ denotes player $i$'s payoff in the case of price leader warfare.[5] It is easily verified that

$$L_i - N_i = \frac{a^4(2 + a + ac_j + (a^2 - 2)c_i)^2}{8(2 - a^2)(4 - a^2)^2} \tag{5.1}$$

$$F_j - D_j = \frac{a^4(1 + ac_i - c_j)^2}{16(a^2 - 2)^2} \tag{5.2}$$

It follows that the product of deviation losses at $S_1$ is larger than at $S_2$ if and only if

$$(c_1 - c_2)(2 + 2a + (c_1 + c_2)(a^2 - 1)) > 0 \tag{5.3}$$

which clearly holds since $c_1 > c_2$. Risk considerations based on reduced game analysis unambiguously point into the direction of the price leader equilibrium where the high cost firm leads. We see that the result based on the reduced game is the opposite of our result established in the previous section, which was based on the full commitment game.

[5]The term "warfare" is somewhat misplaced here since the corresponding payoffs are higher than the payoffs of the usual price leader, i.e. $D_i > L_i$.

Two issues arise here: First, the relevance of the $2\times 2$ game, and second, the characterization of risk dominance in $2 \times 2$ games. The result that the risk dominant equilibrium is the one at which the product of deviation losses is largest was proved by Harsanyi and Selten (1988, Lemma 5.4.4). To enable the reader a proper evaluation of our work it is convenient to reproduce their argument here. Consider the generic $2 \times 2$ game from Table 2,

| | L | R |
|---|---|---|
| T | $a_{11}, b_{11}$ | $a_{12}, b_{12}$ |
| B | $a_{21}, b_{21}$ | $a_{22}, b_{22}$ |

**Table 2:** A generic $2 \times 2$ game.

where $a_{11} > a_{21}$, $b_{11} > b_{12}$, $a_{22} > a_{12}$, and $b_{22} > b_{21}$ so that both $(T, L)$ and $(B, R)$ are pure strict Nash equilibria. Denote by $\bar{z}_j$ the probability with which player $j$ chooses his first strategy in the mixed equilibrium. Then it is easily seen that the prior belief of player $j$ (as outlined in Section 3) assigns probability $1 - \bar{z}_j$ to $i$ playing his first strategy. Assume that $\bar{z}_1 + \bar{z}_2 < 1$. Then each player's best reply against his prior is his first strategy, hence the tracing procedure for determining the risk dominant equilibrium starts at (T,L) and stays there: (T,L) risk dominates (B,R). Similarly, (B,R) is risk dominant if $\bar{z}_1 + \bar{z}_2 > 1$. Now it is easily verified that $\bar{z}_1 + \bar{z}_2 < 1$ if and only if

$$(a_{11} - a_{21})(b_{11} - b_{12}) > (b_{22} - b_{21})(a_{22} - a_{12}),$$

i.e. if the product of deviation losses at $(T, L)$ is larger than the product of deviation losses at $(B, R)$.

In order to illustrate why the reduced game analysis and the full game analysis give different solutions, let us reconsider the bicentric prior in both approaches in a numerical example with extreme values of the parameters. In particular, suppose $a = 1$, $c_1 = 1$, and $c_2 = 0$. Substituting these values into (2.2a) and (2.2b) yields $p_1^L = p_1^F = p_2^L = 2$ and $p_2^F = 3/2$. This implies that $D_1 = F_1$ so that in the reduced game of Table 1, committing to $p_1^L$ is a weakly dominant strategy for player 1. Clearly, this means that waiting is extremely risky for firm 1 and he will, therefore, be the leader in the risk dominant equilibrium. (More formally, the product of deviation losses at $S_2$ is zero while it is positive at $S_1$.)

Now reconsider the full commitment game analyzed in Section 4. If player 1 believes that 2 will commit to $p_2^L$ with probability $z$ and will wait with probability $1-z$, his best reply is to commit to $p_1(z) = 2$ for all $z$. Hence, player 2's prior is that player 1 will commit for sure to the price 2. On the other hand, player 1's prior is that player 2 will commit to some random price between 3/2 and 2. The best replies against the bicentric prior are, therefore, that player 1 waits and that player 2 commits to $p_2^F = 3/2$. During the tracing procedure the beliefs that 1 will wait and that 2 will commit to a random price are reinforced, and the linear trace must be $S_2$.

For general parameters the differences between the two approaches are not as clear-cut as with the extreme numbers used above, but the differences are still remarkable. In particular, in the full game analysis both firms believe that the other will commit for sure and, therefore, their initial best replies are to wait. In the reduced game both firms attach positive weight to the event that the other will wait. However, the high cost firm attaches a high probability to the event that the low cost firm will wait, whereas the low cost firm only assigns low probability to the event that the high cost firm will wait. The best replies are, therefore, for the inefficient firm to commit and for the efficient firm to wait. See Table 3 below.

| | Prior beliefs about | | Initial equilibrium strategies of | |
|---|---|---|---|---|
| | player 1 | player 2 | player 1 | player 2 |
| Full Game | **commit** | **commit** | **wait** | **wait** |
| Reduced Game | **likely to commit** | **likely to wait** | **commit** | **wait** |

**Table 3:** Comparing the two approaches.

We see that the two approaches differ in two respects: they produce different priors and the best replies against the priors are different. The artificial reduction of the game restricts players in their choices and forces them to do things they do not really want to do. In particular, it forces players to wait, if that is better than committing to the leader price, while we have seen that committing to the follower price always generates a higher payoff than waiting. Harsanyi and Selten (1988) emphasize that the computations should take into account all strategies that are best replies against some mixture between the two equilibrium strategies, and not only the two equilibrium strategies. To further illustrate why the $2 \times 2$ game describes the risk considerations very badly we will now consider the $3 \times 3$ game of Table 4

| | $p_2^L$ | $p_2^F$ | $w_2$ |
|---|---|---|---|
| $p_1^L$ | $D_1, D_2$ | $L_1, F_2$ | $L_1, F_2$ |
| $p_1^F$ | $F_1, L_2$ | $X_1, X_2$ | $Y_1, Z_2$ |
| $w_1$ | $F_1, L_2$ | $Z_1, Y_2$ | $N_1, N_2$ |

**Table 4:** Another reduced version of the price commitment game.

where $L_i$, $N_i$, $F_i$ and $D_i$ are as in Table 1 and $X_i = u_i(p_i^F, p_j^F)$, $Y_i = u_i(p_i^F, b_j(p_j^F))$, and $Z_i = u_i(b_i(p_j^F), p_j^F)$. In this game players are restricted to commit to the leader price, to commit to the follower price or to wait. Of course, in this game players are still quite restricted in their choices (as compared to the full game) but we will see that just allowing players to use one very natural strategy, namely committing to the follower price, already upsets the result of the $2 \times 2$ game.

Since $N_i < Y_i$, the reader easily verifies that the prior attaches positive weight only to $p_i^L$ and $p_i^F$. In fact, using the quadratic payoff structure of the game it can be established that both players attach exactly the same weight to the event that the opponent will commit to the leader price. Clearly, for both players the best reply against the initial prior is to wait, as in the analysis of the full game and as opposed to what happens in the $2 \times 2$ game. In order to compute the risk dominant equilibrium we have to determine again who will switch first as players attach more weight to the event that the other player will wait. It turns out that, if we would not allow the players to switch to the follower price (as in the $2 \times 2$ game), player 1 would switch first and become the price leader. However, in the $3 \times 3$ game players want to switch away from their waiting strategy to the safer follower price at a much earlier point in time. Each player will switch from waiting to his follower price when he becomes exactly indifferent between these strategies. Since these strategies yield the same payoff in case the other commits to his leader price, the time at which players want to switch is determined by the payoffs in the $2 \times 2$ game at the bottom right corner of the game of Table 4. Straightforward computations yield that in fact player 2 will switch first. Hence, in the game of Table 4 the equilibrium in which the low cost firm is the price leader and the high cost firm follows is risk dominant, as in the full game.[6]

To summarize, there are three objections against the shortcut analysis of the $2 \times 2$ game. First, the reduced game does not take into account all strategies that are best replies for some initial subjective beliefs. Second, this implies that the bicentric prior computed in that game is not the right one. In the full game firms are always uncertain about the commitment price of the opponent and, therefore, prefer to wait. Finally, even if we would construct the bicentric prior based on the full game, but again use the $2 \times 2$ game to determine who will switch first and become the price leader, we will get the wrong result. If firms can only commit to the leader price, the high cost firm would switch first. However, firms would switch earlier if they could use a safer strategy, like committing to the follower price. Given that opportunity, the low cost firm will switch first and become the price leader.

# 6 Conclusion

In this paper we studied the strategic choice of whether to lead or to follow in a duopoly price competition game with symmetrically horizontally differentiated products and

[6] For example, for parameters $a = 0.5$, $c_1 = 0.75$ and $c_2 = 0.25$ we find that player 1 would switch to his follower price at $t = 0.12$ and to his leader price at $t = 0.41$. For player 2 these values are, respectively, $t = 0.01$ and $t = 0.48$.

where the firms differ in their marginal costs. We analysed a model in which firms can decide to move early or late. The model has two pure equilibria corresponding to the two possible role assignments and by using the risk dominance criterion we were able to select among these. Specifically, as waiting is more risky for the efficient firm than for the firm with the higher cost, the former will act as a price leader and the latter will occupy the more preferred role. Note that this does not necessarily imply that the largest firm will lead. The efficient firm has the largest market share if and only if he charges the lowest price and whether this holds depends on the extent to which the costs differ. If the cost difference is small the efficient leader will have the higher price (hence the smaller market share) and if the difference is large it will have the lowest price (and the larger market share). So our results are in line with the empirical observation that the price leader is often, but not always, the larger firm.[7]

As compared to the alternative candidate solution, where the least efficient firm leads, the total profits in the risk dominant equilibrium are higher (since $L_2 + F_1 > L_1 + F_2$), the division of the profits is more equal ( $|L_1 - F_2| < |L_2 - F_1|$ ) and consumer surplus is lower. To see why, consider first the case where $p_1^L > p_1^F > p_2^L > p_2^F$. Since $p_2^L - p_2^F > p_1^L - p_1^F$ one sees that when we go from $S_2$ to $S_1$, the price decrease of good 2 is larger than the price increase of good 1. Since consumers buy more of good 2 than of good 1, this means that the bundle consumed under $S_2$ can be bought under $S_1$ for less money, which of course implies that consumers are better off when firm 1 is the leader. The argument for the other case where $p_1^L > p_2^L > p_1^F > p_2^F$ is similar. First note that the goods are completely symmetric so that consumers are indifferent between the situation of $S_2$ and the situation in which firm 1 charges $p_2^L$ and firm 2 charges $p_1^F$. If we compare the latter situation with $S_1$ we see that, since $p_1^F - p_2^F > p_1^L - p_2^L$, the price decrease of good 2 is larger than the price increase of good 1 so that again consumers prefer firm 1 to lead.

The conclusion that the efficient firm will move first appears to be robust. In our com-

[7] If the inefficient firm were to lead, it would certainly have the smaller market share because it sets the highest price, $p_1^L > p_2^F$.

panion paper (Van Damme and Hurkens (1999)) we derive it for the case of quantity competition, Deneckere and Kovenock (1992) obtained it for the case of capacity-constrained price competition and homogeneous goods, and Cabrales *et al.* (2000) derived the result for the case of vertical product differentiation, where firms first choose qualities and next compete in prices. This latter paper also makes use of the concept of risk dominance, but it does not derive the result analytically; instead the authors resort to numerical computations and simulations. To our knowledge, the present paper, together with its companion on quantity competition, are the first applications of the (linear) tracing procedure to games where the strategy spaces are not finite. We have seen that, although there may be some computational complexities, no new conceptual difficulties are encountered. Of course, more important than this methodological aspect is the apparant robustness result itself, which might provide the theoretical underpinning for the observed phenomenon in practise that frequently the dominant firm indeed acts as the leader (Scherer and Ross (1990)).

Note that we did not provide the solution of the endogenous timing game for the case where both firms have the same marginal costs. The reader might conjecture that in that case the Bertrand equilibrium would be selected, i.e. that firms would move simultaneously, and indeed that is correct. Clearly, if the firms are completely symmetric, none of the price leader equilibria can risk dominate the other as the solution of a symmetric game has to be symmetric. Similarly, none of the asymmetric mixed strategy equilibria cannot be the solution and since there are no symmetric mixed equilibria (as shown in Sect. 2), the solution has to be the Bertrand equilibrium. However, providing a formal direct proof is difficult. Harsanyi and Selten (1988) show that in the symmetric case the solution of the game is the linear trace of the barycentric prior $\frac{1}{2}p_i^L + \frac{1}{2}w_i$, provided the linear tracing procedure is well-defined. In Appendix A3 however, we show that the linear trace of this prior cannot be the Bertrand equilibrium. The intuition that we do not end up at the Bertrand equilibrium is simple: if the tracing path would converge there then each player would have an incentive to wait (because each firm would expect the other to commit to a random price) and that cannot be an equilibrium. It follows that the linear tracing procedure cannot be well-defined in this case and that the logarithmic tracing procedure has to be used.

We conclude by noting that the main result of this paper does not depend on the assumption that there are only two points in time when the prices can potentially be chosen. Assume that the market opens at time $t = 0$, but that firms could fix their price at any time point $t = 0, -1, -2, ..., -T$, with players being committed to a price once it has been chosen and with players being fully informed about the past history.[8] The solution may be determined by backwards induction, i.e. by applying the subgame consistency principle from Harsanyi and Selten (1988). It is common knowledge that, once the game reaches time $t = -1$ with no commitments being made, the efficient firm will commit to $p_2^L$ while the high cost firm will wait. Knowing this, at $t < -1$, both players find it in their interest to wait. The predicted outcome, hence, is not sensitive to the number of commitment periods: both firms will make their price announcements only shortly before the market opens, with the efficient firm making the announcement slightly earlier.

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[8] This game is analyzed by Robson (1990), where however it is assumed that moving early is associated with higher cost: each firm incurs additional cost $c(t)$ when it moves at time $t$, where $c(\cdot)$ is decreasing and converging to $\infty$ as $t$ tends to $-\infty$. It is easily seen that, provided $c(t) - c(t+1)$ is sufficiently small, the game only has equilibria in which players move in different periods. If both players prefer to lead then in the unique subgame perfect equilibrium that player $i$ for which $L_i - F_i$ is largest will emerge as the leader and he will commit approximately at the time where $L_j - F_j = c(t)$. If both players prefer to follow (as in the model of the present paper), the game has two subgame perfect equilibria: one player will commit at $t = -1$, while the other will wait till $t = 0$. In this case Robson (1990) cannot determine which of these equilibria will result.

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# Appendix

## A1

Let $Z \sim \text{Un}(0,1)$, $Z_i = p_i(Z)$ and $\nu_i = \text{Var}(Z_i)$. We need to prove $\nu_1 < \nu_2$. We will only use that $p_2'(z) < p_1'(z) < 0$.

$$
\begin{aligned}
\nu_1 - \nu_2 &= \int_0^1 p_1(z)^2 dz - [\int_0^1 p_1(z)dz]^2 - \int_0^1 p_2(z)^2 dz + [\int_0^1 p_2(z)dz]^2 \\
&= \int_0^1 [p_1(z)^2 - p_2(z)^2]dz - (\mu_1^2 - \mu_2^2) \\
&= \int_0^1 [(p_1(z) - p_2(z))(p_1(z) + p_2(z))]dz - (\mu_1 - \mu_2)(\mu_1 + \mu_2) \\
&= \int_0^1 [p_1(z) - p_2(z)][p_1(z) + p_2(z) - \mu_1 - \mu_2]dz \\
&= \left.(p_1(z) - p_2(z))(\int_0^z [p_1(t) + p_2(t) - \mu_1 - \mu_2]dt)\right]_0^1 \\
&\quad - \int_0^1 [p_1'(z) - p_2'(z)][\int_0^z (p_1(t) + p_2(t) - \mu_1 - \mu_2)dt]dz \\
&= 0 - \int_0^1 [p_1'(z) - p_2'(z)][\int_0^z (p_1(t) + p_2(t) - \mu_1 - \mu_2)dt]dz
\end{aligned}
$$

The first factor within the integral is positive. It suffices to show that the second factor is also nonnegative. Well, the second factor is equal to zero for $z = 0$ and for $z = 1$. The result follows once we have shown that the second factor is a concave function of $z$. The first derivative of the second factor (with respect to $z$) is

$$-\mu_1 + p_1(z) - \mu_2 + p_2(z)$$

and the second derivative is

$$p_1'(z) + p_2'(z) < 0.$$

□

## A2

We prove the three inequalities in (4.2).

(i) $g_2(t, \mu_1, \nu_1) > g_2(t, \mu_2, \nu_1)$.

*Proof.* Given expectations $m^t = (t, \mu, \nu)$, the optimal commitment price, $\tilde{p}_i(t)$ can be easily computed. The computations are almost identical to the derivation of $p_j(z)$ in (4.1), and one finds

$$\tilde{p}_i(t) = \frac{2(1-t)b_i(\mu) + t(2-a^2)p_i^L}{2-a^2t}. \tag{A.1}$$

If $\mu < p_j^L$ then $b_i(\mu) < p_i^F < p_i^L$ and it follows that $\tilde{p}_i(t) \geq b_i(\mu)$. Using the theorem of the maximum, one now easily verifies that

$$\frac{\partial g_i(m^t)}{\partial \mu} = a(1-t)(\tilde{p}_i(t) - b_i(\mu)) \geq 0. \tag{A.2}$$

Since $p_1^L > \mu_1 > \mu_2$ we have that $\tilde{p}_2(t) \geq b_2(\mu_1) \geq b_2(\mu_2)$. It follows from (A.2) that

$$g_2(t, \mu_1, \nu_1) > g_2(t, \mu_2, \nu_1).$$

□

(ii) $g_2(t, \mu_2, \nu_1) > g_2(t, \mu_2, \nu_2)$.

*Proof.* Again using the theorem of the maximum, one finds

$$\frac{\partial g_i(m^t)}{\partial \nu} = -(1-t)a^2/4 < 0. \tag{A.3}$$

Since $\nu_1 < \nu_2$, it follows from (A.3) that

$$g_2(t, \mu_2, \nu_1) > g_2(t, \mu_2, \nu_2).$$

$\square$

(iii) $g_2(t, \mu_2, \nu_2) > g_1(t, \mu_2, \nu_2)$.

*Proof.*

$$\begin{aligned}
\frac{\partial g_i(t,\mu,\nu)}{\partial c_i} &= -(1-t)(1-\tilde{p}_i(t)+a\mu) - t(1-\tilde{p}_i(t)+a(1+a\tilde{p}_i(t)+c_j)/2) \\
&\quad +(1-t)(1+a\mu-c_i)/2 - t\frac{\partial N_i}{\partial c_i} \\
&= \tilde{p}_i(t) - b_i(\mu) + t\left\{\frac{a\mu - 1 + c_i - a(1+a\tilde{p}_i(t)+c_j)}{2} - \frac{\partial N_i}{\partial c_i}\right\}
\end{aligned}$$

Taking the derivative of the right-hand side with respect to $t$ yields

$$\begin{aligned}
\frac{d}{dt}\left(\frac{\partial g_i}{\partial c_i}\right) &= (1-a^2t/2)\tilde{p}'(t) + \frac{a\mu - 1 + c_i - a(1+a\tilde{p}_i(t)+c_j)}{2} - \frac{\partial N_i}{\partial c_i} \\
&= \frac{a(-16+8a^2-2a^3-2a^4+2a^3c_i-a^5c_i-a^4c_j)}{4(4-a^2)^2} < 0
\end{aligned}$$

Hence,

$$\frac{\partial g_i}{\partial c_i}(t,\mu,\nu) \,\square\, \frac{\partial g_i}{\partial c_i}(0,\mu,\nu) = 0.$$

$$\begin{aligned}
\frac{\partial g_i(t,\mu,\nu)}{\partial c_j} &= t(\tilde{p}_i(t)-c_i)a/2 - t\frac{\partial N_i}{\partial c_j} \\
&= t(\tilde{p}_i(t)-c_i)a/2 - 2at\frac{(p_i^N - c_i)}{4-a^2} \\
&= \frac{at}{2(4-a^2)}\left\{(4-a^2)(\tilde{p}_i(t)-c_i) - 4(p_i^N - c_i)\right\} \\
&> \frac{at}{2(4-a^2)}\left\{(4-a^2)((b_i(\mu)-c_i) - 4(p_i^N - c_i)\right\} \\
&= \frac{at}{2(4-a^2)}(a^2c_i + 4(b_i(\mu) - p_i^N)) > 0
\end{aligned}$$

The gain of committing for player $i$ is decreasing in $c_i$ and increasing in $c_j$. Hence,

$$g_2(m_2|c_1,c_2) \geq g_2(m_2|c_2,c_1) = g_1(m_2|c_1,c_2) = g_1(m_2).$$

$\square$

## A3

We show that when firms have identical costs the linear trace of the barycentric prior is *not* the pure equilibrium in which firms commit themselves to the Bertrand equilibrium.

Let $m = \frac{1}{2}p^L + \frac{1}{2}w$ be the barycentric prior. Suppose that for $t$ sufficiently close to 1 the equilibrium of $g^{t,m}$ is $x(t) = (1 - w(t))p(t) + w(t)w$, i.e. each firm waits with probability $w(t)$ and commits with probability $(1 - w(t))$ to $p(t)$. Hence, at $t$ players' expectations are given by $m^t = (1-t)m + tx(t)$. Consider the derivative with respect to $t$ of the gain function $g(m^t)$. Because of the envelope theorem the effect of the (optimal) commitment price cancels out and we obtain

$$\begin{aligned}
\frac{d}{dt}g(m^t) &= \frac{\partial g(m^t)}{\partial t} + \frac{\partial g(m^t)}{\partial w}w'(t) \\
&= (p(t)-c)a\left\{-\frac{1}{2}p^L - \frac{1}{2}\frac{1+ap(t)+c}{2} + (1-w(t))p(t) + w(t)\frac{1+ap(t)+c}{2}\right. \\
&\quad \left. +tw'(t)(\frac{1+ap(t)+c}{2} - p(t))\right\} \\
&\quad +\frac{1}{2}F + \frac{1}{2}(\frac{1+ap(t)-c}{2})^2 - (1-w(t))(\frac{1+ap(t)-c}{2})^2 - w(t)N \\
&\quad -tw'(t)(N - (\frac{1+ap(t)-c}{2})^2)
\end{aligned}$$

If at $t = 1$ we would have $w(1) = 0$ and $p(1) = p^N$ then

$$\begin{aligned}
\left(\frac{d}{dt}g(m^t)\right)_{|t=1} &= (p^N - c)a(-\frac{1}{2}p^L - \frac{1}{2}p^N + p^N) + \frac{1}{2}F + \frac{1}{2}N - N \\
&= \frac{1}{2}(a(p^N - c)(p^N - p^L) + F - N) \\
&= \frac{1}{4}a(p^L - p^N)(p^F - p^N) > 0
\end{aligned}$$

Since at $t = 1$ the gain to commit is zero, this means that firms will strictly prefer to wait at $t < 1$. Hence, the outcome of the linear tracing procedure cannot be the Bertrand equilibrium. $\square$