Endogenous Business Cycles and Stabilization Policies*

Marta Aloi† Hans Jørgen Jacobsen‡ Teresa Lloyd-Braga§

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Abstract

The paper reports results on the effects of stylized stabilization policies on endogenously created fluctuations. A simple monetary model with intertemporally optimizing agents is considered. Fluctuations in output may occur due to fluctuations in labor supply which are again caused by volatile expectations which are "self fulfilling", i.e. correct given the model. It turns out that stabilization policies that are sufficiently countercyclical in the sense that government spending (on transfers or demand) depends sufficiently strongly negatively on GNP-*increases* can stabilize the economy at a monetary steady state for an arbitrarily low degree of distortion of that steady state. Such stabilization has unambiguously good welfare effects and can be achieved without features such as positive lump sum taxation or negative income taxation as part of the stabilization policy.

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† School of Economics, University of Nottingham, University Park, NG7 2RD Nottingham, UK. E-mail: marta.aloi@nottingham.ac.uk

‡ Institute of Economics and EPRU, University of Copenhagen, Studiestraede 6, DK-1455 Copenhagen K., DENMARK. E-mail:jacob@econ.ku.dk

§ Universidade Católica Portuguesa, FCEE, Palma de Cima, 1649-023 Lisboa, PORTUGAL. E-mail: tlb@fcee.ucp.pt

1 Introduction

The present paper argues that, even under perfectly competitive conditions, if endogenous output fluctuations due to self-fulfilling volatile expectations occur, there may be good and welfare based reasons for stabilization policies that involve (a certain kind of) countercyclicality in government activity. By pursuing a sufficiently countercyclical stabilization policy the government may be able to stabilize output at a monetary steady state for an arbitrarily small degree of distortion of that steady state. Such stabilization has good welfare effects. These effects can be achieved by stabilization policies that do *not* involve features such as positive lump sum taxation or negative income taxation.

We study a simple monetary dynamic model with intertemporally optimizing agents and only labor as input in production. The model can be interpreted either as an overlapping generations model or, by the argument of Woodford (1986), as a model with infinitely lived agents and cash-in-advance constraints. There is always a unique monetary steady state, but there may also be rational expectations equilibria exhibiting endogenous fluctuations, deterministic cycles or sunspot equilibria. In such equilibria output fluctuates because of variations in labor supply which are again due to self-fulfilling volatile expectations. In the considered simple model the condition for indeterminacy under laissez faire, implying the existence of endogenous fluctuations, is the one well-known from, e.g. Grandmont (1985), that the elasticity of labor supply with respect to the real wage is less than minus one half. However, the results on stabilization reported here are of relevance also for other models of endogenous competitive fluctuations where variations in the labor supply, driven by self-fulfilling variations in expectations, cause output to fluctuate.

The real spending of the model's government can be interpreted either as real lump sum transfers (to the old) or as government demand for output. If government and private demand are assumed to be perfect substitutes real transfers and government demand work in exactly the same way. The government's spending is financed either by direct proportional income taxation or by seigniorage, the latter leading to inflationary taxation. The exact mix does not matter since direct proportional and inflationary taxation work in the same way in the considered model. All the policy rules studied can therefore be interpreted as balanced-budget rules.

Government spending implies one form of taxation or the other and therefore distorts labor supply. For this reason, as far as steady state is concerned, government spending is unambiguously bad for welfare. On the other hand, due to the concavity of utility functions, if an endogenous fluctuation is the relevant dynamic equilibrium, government activity may have good welfare effects if it can help to stabilize output. In this case, however, the distortionary effect of the government's spending will still be present and should be weighted against any stabilization benefits obtained by the spending policy.

We assume that government spending is linked to the performance of the economy through policy rules which are meant to formalize stabilization principles. We axiomatize a simple class of rules according to which real government spending is always positive and depends homogeneously on the current and the past levels of GNP. The axioms exclude from the considered policy rules such exotic features as negative transfers (positive lump sum taxes), earnings subsidies (negative proportional taxes), and that government is inactive at a steady (state) GNP, but pays negative or positive transfers at varying (or different) activity levels. The rules are such that at a constant GNP, the share of government spending in GNP is given by a certain level (or taxation) parameter β, 0 □ $\beta < 1$. The larger β is the more distorted will the monetary steady state be, since a larger β means higher (direct or inflationary) taxation at a constant GNP. For varying GNP each rule involves a certain dependence of the share of government spending in GNP on GNP-increases. If this dependence is negative the rule is said to be countercyclical, and more so the stronger the dependence is. If the dependence is positive, the rule is called procyclical etc.

The main result of this paper is the demonstration that for all positive values of β, including arbitrarily small values, one can, by using a sufficiently countercyclical policy rule, obtain that the unique monetary steady state given β becomes (globally) determinate, and hence becomes the relevant dynamic equilibrium. Or in other words, one can stabilize the economy at a steady state having to accept only an arbitrarily small degree of distortion of that steady state by conducting a sufficiently countercyclical stabilization policy. Such a policy has good welfare effects: It stabilizes the economy arbitrarily close to an *efficient* steady state, and it eliminates fluctuations in output which are in themselves bad for welfare because of the concavity of utility functions. On the other hand we also show that for very low values of β, indeed very strong degrees of countercyclicality are required.

We show some side results too. If one fixes the cyclicality of the policy rule at just slightly countercyclical, then "enough government", a sufficiently large β, will create determinacy. On the other hand, if the rule is fixed to be just slightly procyclical, a sufficiently large β will create *in*determinacy. For the limiting case of an acyclical policy rule, where government spending is exactly proportional to current GNP, it turns out that large government will create determinacy or indeterminacy depending on the fundamentals of the economy.

The present paper is related to the (somewhat early) contributions Grandmont (1986), Goenka (1994), Sims (1994), and Woodford (1994), which also study the effects of fixed and supposedly realistic policy rules on endogenous fluctuations. The closest relation is to Grandmont (1986). As will be shown, the just described particular case of an acyclical policy rule is equivalent to arranging spending such that, in the absence of direct income taxation, a constant money growth rate results. Grandmont exactly studies such constant money growth rules. His main finding is that, under his assumptions on fundamentals, constant money growth rules will stabilize the economy at a monetary steady state if the money growth rate is large enough. Our results indicate that constant money growth rules are, at best, very poor stabilization instruments. First, the fact that we here include more different assumptions on fundamentals reveals that constant money growth rules, although effective in stabilizing output under some assumptions on fundamentals, are directly destabilizing under other assumptions. Second, our analysis of a broad class of parametrized policy rules reveals that the particular constant money growth rules are just at the boundary of the set of policy rules that can be output stabilizing at all, and when they are in this set, they are outperformed by countercyclical rules with respect to the welfare consequences of stabilization.

Our paper is also related to a more recent wave of papers, Guo and Lansing (1997) and (1998), Schmitt-Grohé and Uribe (1997) and (2000), Chattopadhyay (1996) and (1999), Christiano and Harrison (1999), and Guo and Harrison (1999), but our results are different. The papers of Guo and Lansing, Christiano and Harrison, and Guo and Harrison consider models in which the source of endogenous fluctuations is explicitly to be found in a productive externality and their results on stabilization depend on this feature. Here we assume no such externality. For instance, Guo and Lansing presents the interesting result that with externality based fluctuations, progressive taxation may help to create determinacy. Here we only consider proportional taxation. Further, in Guo and Lansing's work one cannot be sure that stabilization has good welfare effects, since output varying between high and low may utilize the externality better than output being at a moderate level all the time. Our emphasize is on stabilization for welfare reasons.

Christiano and Harrison (1999), the closest related of the more recent papers, do obtain stabilization on an efficient steady state by means of a policy that involves proportional tax rates that increase with employment. However, at the steady state at which the economy is stabilized the tax rate is zero, or even negative to internalize the productive externality, while the policy works through threatening with/promising lower and hence *negative tax rates* (earnings subsidies) combined with *positive lump sum taxation* at lower levels of employment etc. We have attempted to avoid such rarely seen features, or in other words, the kind of policies considered by Christiano and Harrison (1999) are not among the policy rules we consider. In an overall comparison, we avoid some less realistic features in the considered stabilization policies; we that the cost of this is that in the statement "the economy can be stabilized at an efficient steady state" the word *at* must be replaced by *arbitrarily close to.*

Other papers among the more recent are focused on the negative result that policy may create *in*determinacy. For instance Schmitt-Grohé and Uribe show how balanced budget rules may be destabilizing for sufficiently large government. Since our policy rules can be interpreted as balanced budget rules this result is closely related to the above mentioned one that with pro- or a-cyclical policy rules enough government may create indeterminacy. Our emphasize, however, is on the positive result that countercyclical policy rules may stabilize output in a good way.

One feature of the model we study is that it is simple enough to give a *one*-dimensional, first order difference equation as perfect foresight dynamic. This makes it possible to establish enough global properties to be able to use *global* determinacy as criterion for stabilization. Like Christiano and Harrison (1999), we think that in connection with *excluding* the possibility of cycles and sunspot equilibria global analysis is of importance, since local determinacy is not a sufficient condition for the non-existence of endogenous fluctuations. With two-dimensional dynamic systems it is very difficult to provide more than local analysis. This is our motivation for studying a simple model without capital.

In Section 2 we describe the basics of the economic model and the class of policy rules we consider. Section 3 derives the equilibrium dynamics, and Section 4 states the results on stabilization by the considered policy rules. Section 5 contains remarks and conclusions. Proofs are given in Appendix A. Appendix B contains a technical result that is of importance for our purposes.

2 Basics

We consider a model in discrete time with intertemporally optimizing agents. The model can be interpreted either as an overlapping generations model or as a model with infinitely lived consumers. In each period the commodities are labor, output, and money. The money prices of labor and output are $w > 0$ and $p > 0$ respectively, and labor and output markets are perfectly competitive. Subscript t is used for explicit reference to a period.

In each period a representative firm produces output $y \geq 0$ from labor input $l \geq 0$ under constant returns to scale, $y = l$.

For the overlapping generations interpretation of the model there is in each period one young and one old consumer, and a consumer is endowed with one unit of labor time in his youth. The von Neumann-Morgenstern utility function of a consumer is $u(c) + v(e)$, where $c \geq 0$ is output consumption in the consumer's old age, and $e := 1 - n \geq 0$ is leisure consumption in the youth; n is labor supply.

It is thus assumed that in the first period of a consumer's life only leisure enters utility, and in the second only consumption. As explained by Woodford (1986), this implies an equivalence to a cash-in-advance constrained economy with an infinitely-lived representative consumer, if it is assumed that this consumer's time preference rate is sufficiently large. In this interpretation the period length can be short.

We impose standard assumptions on u and v: they are continuously differentiable several times, $u'(c)$ and $v'(e)$ are strictly positive and go to infinity as c and e respectively go to zero, and $u''(c)$ and $v''(e)$ are strictly negative. We denote the Arrow-Pratt measure of relative risk aversion in u by $R(c) := -u''(c)c/u'(c) > 0$, and also define $N(n) := -v''(1-n)n/v'(1-n) > 0$. We assume that $R(0) := \lim_{c\to 0} R(c)$, and $N(0)$ both exist (are $< \infty$).

Finally, there is a government that in each period decides on a real lump sum transfer b given to the period's old consumer, and on a proportional tax rate τ, where $0 \leq \tau < 1$, by which the income of the period's young consumer is taxed. Both b and τ are taken as parametric by the consumers. For the interpretation of our model with an infinitely-lived agent, the cash-in-advance constraint should be assumed to work such that in the current period the consumer can spend last period's net of tax income plus the transfer received in the current period.

The variable b can, if positive, alternatively be interpreted as government demand for output (or labor). If it is assumed that public and private goods are perfect substitutes, so the utility function of a consumer is $v(1-n) + u(c+b)$, then the resulting dynamic model will be identical to the one in which b is a transfer. This will be demonstrated below.

Policy is conducted according to certain feedback rules linking in a systematic way the value of the real transfer, or government demand, to present and past values of the GNP. The rules are meant to formalize stabilization principles. Government spending is financed by either proportional taxation or seigniorage or a mix of both, so the government budget constraint is fulfilled. The exact financing does not matter since in the considered model direct proportional taxation and inflationary taxation have the same effects.

We confine attention to rules of the form, $b_{t+1} = b(y_{t+1}, y_t)$, and impose some further restrictions meant to express that the policy rules $b(\cdot,\cdot)$ should be realistic stabilization principles:

(i) The variable b is weakly positive in all periods. For the interpretation of b as government demand this is required. For the interpretation as a transfer there is in principle nothing wrong with negative values, but $b < 0$ means lump sum taxation (of the old) together with subsidies (to the young) proportional to income, these subsidies coming either directly or through negative inflation. Such features are seldom observed and, in particular, *variations* in lump sum taxes are *never* seen as part of stabilization policies.

(ii) At a constant GNP, the government behaves as if *it taxes GNP by a certain rate and balances the budget in each period.* That is, we require $b(y, y) = \beta y$ for

some β with $0 \,\square\, \beta < 1$. Indeed, to be a formalization of a stabilization principle the rule should dictate "neutral government behavior" at a steady GNP. For realism we let neutral behavior correspond to fixed proportional taxation and budget balance rather than, e.g. to a fixed spending that is independent of y.

(iii) When GNP varies the stabilization effort should depend on the relative *variation in GNP.* If two pairs (x, y) and (x', y') of current and past GNPs represent the same degree of relative up or down swing in economic activity, $x/y = x'/y'$, then the government stabilization effort should be relatively the same in the two situations, i.e. $b(x, y)/x = b(x', y')/x'$.

These requirements are fulfilled if and only if b is of the form $b(y_{t+1}, y_t) = \beta\phi(y_{t+1}, y_t)$, where $0 \,\square\, \beta < 1$, and ϕ is positive and homogeneous of degree one, with $\phi(1, 1) = 1$. For simplicity we will consider the specification,

$$b(y_{t+1}, y_t) = \beta y_{t+1}^{1-\alpha} y_t^{\alpha}, \tag{1}$$

where there are no a priori restrictions on the parameter α.

By virtue of mainly (i) and (ii) above we avoid to consider policy rules that prescribe zero government activity at a constant production level and negative or positive values of b otherwise. If there is government activity at all ($\beta > 0$), there is also government activity at a constant GNP.

Each policy rule of the form (1) contains a level (or resting) component given by β, and a cyclical (or reactive) component given by α. This is illustrated by the rewriting $b(y_{t+1}, y_t)/y_{t+1} = \beta(y_{t+1}/y_t)^{-\alpha}$. The level component β is spending's share in current output when output is constant, and the cyclical component $(y_{t+1}/y_t)^{-\alpha}$ is the responsiveness of this share to changes in output, $-\alpha$ being the elasticity of the spending's share with respect to the output growth factor. The larger α is, the more negative will be the reaction in spending's share to increases in output, that is, the more "countercyclical" will the rule be.

In what follows it is assumed that the policy rule $b(y_{t+1}, y_t)$ used by the government is known by the households who also have rational expectations with respect to next period's output price. Furthermore, the households are assumed to believe in the relevant policy rule.

3 Equilibrium

In (a non-trivial) equilibrium one must have $w = p$ in all periods, and that any level of production and employment is optimal for the firm.

Consider a (young) consumer who holds the expectation concerning the next period that with probability q_j the output price will be p_j and the transfer received will be b_j, where $j = 1, ..., r$. A point expectation corresponds to $r = 1$. The consumer chooses current labor supply n, ultimo money holding m, and consumption c_j in each of the r future "states", to maximize expected utility $v(1-n) + \sum_j q_j u(c_j)$, subject to the budget constraints $m = (1-\tau)wn$, and $c_j = m/p_j + b_j$ for $j = 1, ..., r$, where w and τ are the nominal wage rate and the tax rate in the period where the consumer supplis n. The optimal choices for n and c_j are uniquely given by the first order condition,

$$\frac{v'(1-n)}{(1-\tau)w} = \sum_{j=1}^{r} q_j \frac{u'(c_j)}{p_j}, \tag{2}$$

and the budget constraints,

$$c_j = (1-\tau)\frac{w}{p_j} n + b_j \text{ for } j = 1, ..., r. \tag{3}$$

In the case of a point expectation (where p and b are expected), the optimality conditions amount to $v'(1-n) = \omega u'(c)$, and $c = \omega n + b$, where $\omega := (1-\tau)w/p$. Solving for n and c gives the labor supply curve $n = n(\omega, b)$, and the future demand for produced goods $c = c(\omega, b)$. It is a consequence of our assumptions that leisure and consumption are both strict normal goods, $n'_b < 0$ and $c'_b > 0$.[1]

3.1 Temporary Equilibrium

From (3), $(1-\tau)w/p_j = (c_j - b_j)/n$. Inserting this into (2) gives, $nv'(1-n) = \sum_j q_j (c_j - b_j) u'(c_j)$. Inserting the equilibrium conditions for the labor market $n = l$, and the output market $y_j = c_j$ (the resource constraint), and using $y = l$, gives

[1] Labor supply is given by $v'(1-n) = \omega u'(\omega n + b)$. A larger b implies a lower right hand side, and to recreate equality n must fall since this both decreases the left hand, and increases the right hand, side, so $n'_b < 0$. A similar exercise on $v'(1-(c-b)/\omega) = \omega u'(c)$ shows $c'_b > 0$. For later use, we derive the elasticity of labor supply wrt. ω by log-differentiation of the first equality above,

$$\varepsilon_\omega := \frac{n'_w \omega}{n} = \frac{1 - R(\omega n + b)\frac{\omega n}{\omega n + b}}{N(n) + R(\omega n + b)\frac{\omega n}{\omega n + b}} > -1.$$

$yv'(1-y) = \sum_j q_j(y_j - b_j)u'(y_j)$. Inserting finally the policy rule $b_j = b(y_j, y)$ yields,

$$yv'(1-y) = \sum_{j=1}^{r} q_j \left[y_j - b(y_j, y)\right] u'(y_j). \tag{4}$$

This is the temporary equilibrium equation for the considered economy in terms of production levels. If the young consumer expects output in the next period to be y_j (between zero and one) with probability q_j, $j = 1, ..., r$, and knows and believes in the policy rule $b(\cdot, \cdot)$, then a y (between zero and one) is an equilibrium output of the current period if and only if it fulfils (4).

All rational expectations dynamic equilibria studied below are defined from the temporary equilibrium equation (4). The tax rates do not enter into this. Hence, for a given policy rule for spending, the rational expectations equilibrium dynamics of the considered economy is independent of how much income taxation vs. seigniorage is used in financing government spending. Proportional income taxation and inflationary taxation work in exactly the same way.

Consider the alternative interpretation of b as government demand. In this case, the consumer would maximize $v(1-n) + \sum_j q_j u(c_j + b_j)$ subject to the budget constraints $c_j = (1-\tau)\frac{w}{p_j}n$, $j = 1, ..., r$. The first order condition would be,

$$\frac{v'(1-n)}{(1-\tau)w} = \sum_{j=1}^{r} q_j \frac{u'(c_j + b_j)}{p_j}.$$

By use of the budget constraints, $(1-\tau)w/p_j = c_j/n$, one gets $nv'(1-n) = \sum_j q_j c_j u'(c_j + b_j)$. In equilibrium, $n = y$ and $y_j = c_j + b_j$, and hence $yv'(1-y) = \sum_j q_j(y_j - b_j)u'(y_j)$. Inserting a policy rule for government demand, $b_j = b(y_j, y)$, would give exactly (4). The two interpretations of b lead to the same equilibrium condition which verifies the equivalence postulated in Section 2.

3.2 Perfect Foresight Dynamics and Steady State

The economy's perfect foresight dynamics is obtained from (4) assuming that the next period's output is correctly foreseen from the current period in a deterministic sense. Inserting $y_j = y_{t+1}$ for all j, and writing current output as y_t (for y), one arrives at a first order, one-dimensional difference equation in y_t and y_{t+1},

$$y_t v'(1-y_t) = \left[y_{t+1} - b(y_{t+1}, y_t)\right] u'(y_{t+1})). \tag{5}$$

A dynamic perfect foresight equilibrium is a sequence (y_t) of production levels $0 \leq y_t < 1$, such that (5) is fulfilled for all t. A steady state is a particular case where $y_t = y$ in all periods. For all the policy rules we consider, $b(y, y) = \beta y$, and it follows from (5) that a strictly positive, or monetary, steady state production level y is given by,

$$\frac{v'(1-y)}{u'(y)} = 1 - \beta. \tag{6}$$

Since the MRS on the left hand side goes from zero to infinity as y goes from zero to one, there is for any β a unique monetary steady state $y(\beta)$, and $y(\beta) < 1$. It follows directly that $y(\beta)$ is strictly decreasing in β, and that $y(\beta)$ goes to zero as β goes to one.

If we define welfare at the monetary steady state as the common utility of all "generations", $W(\beta) := u(y(\beta)) + v(1 - y(\beta))$, then $W' = (u' - v')y'_\beta$, and from $y'_\beta < 0$ and (6), $W' < 0$ for all $\beta > 0$, and $W' = 0$ for $\beta = 0$. This proves,

Proposition A. *For all β, there is a unique monetary steady state involving production $y(\beta)$, with $0 < y(\beta) < 1$, and $y(\beta)$ is strictly decreasing in β and $y(\beta) \to 0$ as $\beta \to 1$. Welfare at the monetary steady state $W(\beta)$ is unambiguously decreasing in β, and optimal policy for steady state is $\beta = 0$.*

Proposition A implies that government activity has to be motivated by the monetary steady state not being the appropriate descriptive equilibrium. Furthermore, should endogenous fluctuations prevail (under laissez faire) and should one, by use of a policy rule belonging to the considered class, manage to stabilize the economy at the monetary steady state, then it is unambiguously to be preferred that this is done for as low a value of β as possible.[2]

The left hand side of (5) increases from zero to infinity as y_t goes from zero to one. If $\alpha \geq 0$, or $\beta = 0$, then b is (weakly) increasing in y_t, so the right hand side will, for any given $y_{t+1} > 0$, decrease weakly from a strictly positive value as y_t increases from zero. This means that for every positive y_{t+1}, there is a unique y_t between zero and one that solves (5), which thus everywhere implicitly defines

[2] It could be argued that the right welfare measure at steady state is rather $V(\beta) = u(y(\beta))/(1+\theta) + v(1 - y(\beta))$, where $\theta > 0$ is a time preference rate. In a free optimization one will then find that optimal policy for steady state is some $\beta < 0$, which, in the absence of direct taxation, is equivalent to a constant negative money growth rate, a so-called Friedman rule. If one only allows $\beta \geq 0$, then also in this case $\beta = 0$ is optimal for steady state.

y_t as a function f of y_{t+1}. From the Implicit Function Theorem, f is continuously differentiable. So, for $\alpha \geq 0$, or $\beta = 0$, the backward perfect foresight dynamic $y_t = f(y_{t+1})$ is well-defined globally. For $\alpha < 0$ and $\beta > 0$ it is not. In that case there are for y_{t+1} small enough several solutions in y_t to (5), and for y_{t+1} large enough there are none. As just shown there is, however, a unique monetary steady state $y(\beta)$, and locally around $y(\beta)$ the backward perfect foresight dynamic f is again well-defined and continuously differentiable.[3]

3.3 Rational Expectations Fluctuations

A deterministic r-cycle is a collection of r different production levels $0 < y_1, ..., y_r < 1$ in the range where f is well-defined such that $y_1 = f(y_2), ..., y_r = f(y_1)$. An r-state stationary (Markov) sunspot equilibrium, SSE, consists of r production levels $0 < y_1 \leq \cdots \leq y_r < 1$, where $y_1 < y_r$, and r^2 transition probabilities q_{ij}, $\sum_{j=1}^{r} q_{ij} = 1$ for $i = 1, ..., r$, where the matrix (q_{ij}) is irreducible, such that, whenever the young consumer expects that the output level y_j will occur with probability q_{ij} next period, $j = 1, ..., r$, then the current temporary equilibrium output level according to (4) is exactly y_i, that is,

$$y_i v'(1 - y_i) = \sum_{j=1}^{r} q_{ij} \left[y_j - b(y_j, y_i) \right] u'(y_j) \text{ for } i = 1, ..., r. \tag{7}$$

The well-known idea is that one can imagine that an irreducible Markov chain (a sunspot) on states $1, ..., r$, sending state i into state j with transition probability q_{ij}, though exogenous to the economic system, may govern its performance. If the agents know the transition probabilities and believe that in any period output must be y_i if the state is i, then output will indeed be governed by the sunspot and fluctuate accordingly, and the agents will have no reason to revise their beliefs since their expectations are probabilistically correct, i.e. rational. An r-cycle is a particular, non-stochastic r-state SSE.

Deterministic cycles and SSE are our candidates for rational expectations dynamic equilibria exhibiting endogenous fluctuations.

[3]From the Implicit Function Theorem, f is locally well-defined by (5) around steady state if the derivative of $y_t v'(1 - y_t) - [y_{t+1} - b(y_{t+1}, y_t)] \, u'(y_{t+1}))$ wrt. y_t measured at steady state is not zero. This derivative is $v'(1 - y(\beta))(1 + N(y(\beta)) + \alpha \beta u'(y(\beta))$, which, for any given β, is zero only for one particular (non-generic) negative value of α.

Our results concerning stabilization of endogenous business cycles will rely on some relationships between the perfect foresight dynamic f and the existence of cycles and sunspot equilibria. It is well-known that if f is such that an r-cycle exists then there is also a truly stochastic r-state SSE close to the cycle, see Guesnerie and Woodford (1992). It is not generally true that the existence of a SSE implies the existence of deterministic cycles, or, equivalently, that non-existence of cycles implies non-existence of SSE. For our purposes it is, however, important to establish such a connection. In Appendix B we prove a proposition stating some general conditions under which the existence of a SSE implies the existence of a 2-period cycle. The conditions are such that for the policy rules for which we show, that they eliminate all cycles through establishing global stability according to f of the monetary steady state, it can be concluded that also all SSE are eliminated.[4] By virtue of these and some other well-known results it will suffice in what follows to study the perfect foresight dynamic f. To be precise we will make use of the following standard "dynamic properties":

Indeterminacy. If f is locally well-defined around steady state and the slope of f at the steady state is below minus one or above one, then the steady state is locally stable in the *forward* direction under perfect foresight, and the steady state is said to be indeterminate. It is well known that indeterminacy implies the existence of SSE arbitrarily close to the steady state, see Guesnerie and Woodford (1992), and for the dynamics we consider, if $f'(y(\beta)) < -1$, there are also deterministic cycles. It is an "opening assumption" of ours that under laissez faire the steady state is indeterminate, $f'(y(0)) < -1$ (> 1 is not possible), and that indeterminacy indeed implies that a cycle or a sunspot equilibrium is the relevant dynamic equilibrium (if it were the steady state there would be no stabilization problem).

Determinacy. Assume that by appropriate use of one of the policy rules considered it can be obtained that the steady state $y(\beta)$ becomes *globally* stable according to f, implying that f is globally well-defined. Then there can be no deterministic cycles and, from Theorem B shown in Appendix B, for the policy rules that we find indeed can make $y(\beta)$ globally stable according to f, no SSE either. The steady

[4] The method used in Appendix B to establish that existence of a SSE implies existence of a 2-period cycle is similar to the one used by Grandmont (1986). However, the dynamics arising from our policy rules are not covered by the generality of Grandmont's result. Therefore the theorem in Appendix B generalizes Grandmont's result and it may be of independent interest.

state is then the only reasonable bounded and continuously well-defined rational expectations equilibrium, and one says that the steady state is (globally) determinate. Determinacy will be considered a sufficient condition for stabilization at steady state.

4 Stabilization

Our main result is Theorem 1 below which says that for any given positive value of β (no matter how small), one can create determinacy by choosing α large enough, or in other words, stabilization at the monetary steady state can be obtained with arbitrarily little distortion by using a sufficiently countercyclical stabilization principle. However, very high values of α are indeed required for very low values of β.

Theorem 1. *Assume $f'(y(0)) < -1$. For any $\beta > 0$, there is an $\alpha^*(\beta) > 0$, such that if a policy rule with $\alpha > \alpha^*(\beta)$ and β is used, then the steady state $y(\beta)$ is determinate and there are no cycles or stationary sunspot equilibria. On the other hand, for all sufficiently small $\beta > 0$, it is necessary for determinacy that α is greater than or equal to a certain $\alpha^{**}(\dot{\beta}) > 0$, and this $\alpha^{**}(\dot{\beta})$ goes to infinity as β goes to zero.*

Theorem 1 is about stabilization by appropriate choice of α for a given fixed β. This is the most interesting question from a welfare viewpoint. It is also of interest to investigate what can be obtained in terms of stabilization for a given α by appropriate choice of the "size of government" β, for instance because there may be limits to how countercyclical stabilization policies can be.

Proposition 1. *Assume $f'(y(0)) < -1$.*

(i) For any $\alpha > 0$, there is a $\beta^*(\alpha)$ with $0 < \beta^*(\alpha) < 1$, such that if a policy rule with α and $\beta > \beta^*(\alpha)$ is used, then the steady state $y(\beta)$ is determinate and there are no cycles or stationary sunspot equilibria.

(ii) If $\alpha = 0$ and $R(0) < 1$, there also exists a $\beta^* < 1$, such that $\beta > \beta^*$ implies determinacy of the steady state and non-existence of cycles and stationary sunspot equilibria..

Theorem 1 and Proposition 1 leave some questions open. Proposition 1 does not exclude that also for $\alpha < 0$ (or generally for $\alpha = 0$), could "enough government", a sufficiently high β, imply determinacy. Nor does Theorem 1 exclude that also negative and numerically large values of α could be stabilizing for certain (low) values of β. Proposition 2, however, rules out these possibilities. Note that it is not assumed in Proposition 2 that $f'(y(0)) < -1$. Hence Proposition 2 is about how policy can *create* indeterminacy, see also Remark 6 below.

Proposition 2. *(i) If $\beta > 0$, then $f'(y(\beta)) > 1$ for all negative and sufficiently small α; hence, the steady state $y(\beta)$ is indeterminate and stationary sunspot equilibria exist.*

(ii) If $\alpha < 0$, then $f'(y(\beta)) > 1$ for all sufficiently large β; hence, the steady state $y(\beta)$ is indeterminate and stationary sunspot equilibria exist.

(iii) If $\alpha = 0$ and $R(0) > 2 + N(0)$, then $f'(y(\beta)) < -1$ for all sufficiently large β; hence, the steady state $y(\beta)$ is indeterminate and both deterministic cycles and stationary sunspot equilibria exist.

5 Remarks

1. Welfare. The above results are strictly speaking on stabilization of output which should only be an aim for economic policy if output stabilization has good welfare implications. First note that the welfare effects are potentially good in the sense that the economy can be stabilized arbitrarily close to $y(0)$, the efficient steady state under laissez faire. This does not imply, however, that the stabilized situation Pareto dominates a fluctuating one. What speaks in favor of this latter eventuality is the concavity of utility functions (more so in the model interpretation with an infinitely lived consumer than in the overlapping generations interpretation where a concern of intergenerational equity must be added to motivate output stabilization). What speaks against it is the distortion of the steady state that the stabilization policy implies. The power of Theorem 1 is exactly that it says that stabilization at steady state can be obtained for (arbitrarily) little such distortion of the steady state by performing stabilization through a sufficiently countercyclical policy rule. The welfare implications of such stabilization should be unambiguously good.

2. Countercyclicality. The best policy rules thus entail positive values of α. Hence they are countercyclical in the sense that they imply relatively low government activity in periods up to which output has increased by a relatively large amount. Though this is not exactly countercyclicality in the usual sense of relatively low government activity when output is relatively high, such rules will, nevertheless, often appear countercyclical in the usual sense (e.g. over two-period cycles), and they certainly do have a Keynesian flavor - but not for Keynesian reasons.

3. Intuition. Why is it that the countercyclical policy rules stabilize output most effectively, and hence with the best welfare implications? The intuition is related to the intertemporal incentive effects of systematic stabilization policies. Assume that GNP increases (by a relatively large amount) from one period to the next. If this is correctly foreseen from the first period, and people know and believe in a countercyclical policy rule, then they will expect relatively low transfers during the next period. If leisure and output are normal goods (which is realistic), labor supply and output will increase in the first period, and thus the increase in output from the first to the second period will be reduced. The economy is thus stabilized. Interestingly, Benassy (1998) finds that a similar intertemporal effect is important for the stabilization of competitive fluctuations caused by exogenous shocks, and Benassy also establishes support for countercyclical policy rules.

4. Related literature. Grandmont (1986) has assumptions with the same effect as $R(0) < 1$ here and considers constant money growth rate rules which are equivalent to our rules with $\alpha = 0$.[5] One of his results is similar to our Proposition 1(ii). In view of Propositions 1 and 2, policy rules with $\alpha = 0$ are just at the boundary of the set of rules that can be stabilizing for large enough values of β, and even when they are in this set, they may well be the ones giving output stabilization in the worst possible way welfarewise, requiring the largest β.

5. A trade off. There may be limits to how countercyclical policy rules can be.

[5] The case $\alpha = 0$, gives $b(y_{t+1}, y_t) = \beta y_{t+1}$. This rule is equivalent to arranging the sequence of transfers such that with no income taxation a constant money growth rate results. To see this note that without income taxation the money stock must evolve as $M_{t+1} - M_t = p_{t+1}b_{t+1}$. The growth rate d_{t+1} of the money stock from the end of period t to the end of period $t+1$ is thus $d_{t+1} = p_{t+1}b_{t+1}/M_t \iff M_t = p_{t+1}b_{t+1}/d_{t+1}$. The second period budget constraint for the consumer reads $M_t = p_{t+1}(c_{t+1} - b_{t+1})$, where it is used that in equilibrium the amount of money held by the consumer at the end of t must be the economy's entire money stock at the end of t. By equalizing the two expressions for M_t we get $b_{t+1}/d_{t+1} = c_{t+1} - b_{t+1}$, or $b_{t+1} = (d_{t+1}/(1 + d_{t+1}))y_{t+1}$, where it was used that in equilibrium $c_{t+1} = y_{t+1}$. Hence a rule of no income taxation and constant money growth rate d is the particular case of a rule of the form (**??**) where $\alpha = 0$ and $\beta = d/(1 + d)$.

First, rules with very high values of α and correspondingly low values of β are not simple. Second, they may involve a credibility problem. At the steady state $y(\beta)$, at which the economy is stabilized, one will not see much government activity, only the low $\beta y(\beta)$. The government may have problems convincing the public that this is because fluctuations do not presently occur, and that should fluctuations occur the government would react strongly in accordance with its high α. There may thus be costs in terms of losses of simplicity and credibility of increasing α. Taking these costs into consideration our results could be read as pointing to a basic trade off between the degree of distortion and the degree of countercyclicality.

6. Policy creating indeterminacy. Inserting into the ε_ω of footnote 2, that at steady state $\omega n + b = y$, $\omega n = (1-\beta)y$, and $b = \beta y$, one gets for the real-wage elasticity of labor supply at steady state,

$$\varepsilon_\omega(\beta) = \frac{1-(1-\beta)R(y(\beta))}{N(y(\beta)) + (1-\beta)R(y(\beta))}.$$

From (10) in the proof of Theorem 1, $f'(y(0)) < -1 \iff R(y(0)) > 2 + N(y(0))$, and this implies $\varepsilon_\omega(0) < -1/2$, the well-known condition for local indeterminacy under laissez faire. Proposition 2 says that if $\alpha < 0$, or if $\alpha = 0$ and $R(0) > 2+N(0)$, a sufficient condition for indeterminacy, $f(y(\beta)) < -1$ or $f(y(\beta)) > 1$, is fulfilled for all sufficiently large β. As β goes to one, $\varepsilon_\omega(\beta)$ above goes to $1/N(0) > 0$, so for all large enough β, one has both indeterminacy, and $\varepsilon_\omega(\beta) > 0$. An inappropriate government policy may create indeterminacy, and such a policy does not have to be more peculiar than a constant money growth rate rule. Since the policy rules we have considered can be viewed as balanced budget rules, this finding is closely related to that of Schmitt-Grohe and Uribe (1997).

7. Overall conclusion. We take a modest view concerning the significance for actual stabilization policies of our model results. Insofar as fluctuations or cyclical movements in economic activity can be viewed as (at least partly) created endogenously by volatile and self-fulfilling expectations, some intertemporal effects of stabilization policies, which do not usually gain so much attention, become important. It is a logical possibility that these intertemporal effects work in such a way that policies which stabilize economic activity in a way that is good with respect to welfare involve a kind of countercyclicality in government activity that is reminiscent of what is advocated by Keynesians.

A Proofs

Proof of Theorem 1. First note that it is an assumption in Theorem 1 that $\alpha > 0$. It therefore follows from Proposition B of Appendix B, that global stability of $y(\beta)$ according to f (global determinacy), which is established in this proof and which obviously eliminates all cycles, also eliminates all SSE.

Inserting the considered specific functional form of policy rules into (5) gives,

$$y_t v'(1 - y_t) = (y_{t+1} - \beta y_{t+1}^{1-\alpha} y_t^{\alpha}) u'(y_{t+1}), \tag{8}$$

which defines $y_t = f(y_{t+1})$. For any $x > 0$, at which $f(x)$ is well-defined, the slope of f is obtained by implicit differentiation of (8) written as $f(x)v'(1 - f(x)) = [x - \beta x^{1-\alpha} f(x)^{\alpha}]\, u'(x)$. This gives,

$$f'(x) = \frac{f(x)}{x} \frac{1 - \beta(1-\alpha)(\frac{f(x)}{x})^{\alpha} - \left(1 - \beta(\frac{f(x)}{x})^{\alpha}\right) R(x)}{1 - \beta(1-\alpha)(\frac{f(x)}{x})^{\alpha} + \left(1 - \beta(\frac{f(x)}{x})^{\alpha}\right) N(f(x))}. \tag{9}$$

Measuring f' at the monetary steady state where $x = f(x) = y(\beta)$ gives,

$$f'(y(\beta)) = \frac{1 - \beta(1-\alpha) - (1-\beta)R(y(\beta))}{1 - \beta(1-\alpha) + (1-\beta)N(y(\beta))}. \tag{10}$$

When $\alpha > 0$, as assumed here, then f is globally well-defined (as explained in Section 3.2), and for any $y_{t+1} > 0$, the y_t that solves (8) is below $y_{t+1}/\beta^{1/\alpha}$. Hence, as y_{t+1} goes to zero, so must this y_t, implying $f(0) := \lim_{x\to 0} f(x) = 0$.

So, the globally well-defined backward dynamic f starts at zero, $f(0) = 0$, and stays everywhere below one, $f(x) < 1$. It may have a number of critical points $(x^c, f(x^c))$ at which $f'(x^c) = 0$ (of course, for $\beta = 0$ there must be critical points, since $f(0) = 0$, and $f'(y(0)) < -1$). In any case, f has a shape such that if all critical points are below the 45°-line (including the case where there are no critical points), i.e. fulfill $f(x^c)/x^c < 1$, then $y(\beta)$ is globally stable according to f. This is used to establish Theorem 1.

We are going to show that one can use $\alpha^*(\beta) = \frac{1-\beta}{\beta} \max_{x\in[0,1]}(R(x) - 1))$. Here $\alpha^*(\beta) > 0$, because it follows from $f'(y(0)) < -1$ and (10), that $R(y(0)) > 1$, and $0 < y(0) < 1$.

From (9), a critical point is given by $1 - \beta(1-\alpha)(\frac{f(x)}{x})^{\alpha} - \left(1 - \beta(\frac{f(x)}{x})^{\alpha}\right) R(x) = 0$. This implies that at a critical point one must have $R(x) > 1$, whenever $\alpha > 0$, and,

$$(\frac{f(x)}{x})^{\alpha} = \frac{1}{\beta} \frac{R(x) - 1}{R(x) - 1 + \alpha}. \tag{11}$$

A critical point $(x^c, f(x^c))$ is below the 45°-degree line if $f(x^c)/x^c < 1$, which has to be fulfilled if $x^c \geq 1$, since $f(x) < 1$ for all x. The denominator above is strictly positive at a critical point when $\alpha > 0$, so for $\alpha > 0$, $f(x^c)/x^c < 1$ is equivalent to,

$$\alpha > \frac{1-\beta}{\beta}(R(x^c) - 1). \tag{12}$$

Now, if $\alpha > \alpha^*(\beta)$, then in particular (12) is fulfilled for any critical point $x^c < 1$, implying that $f(x^c)/x^c < 1$. This proves the first statement of Theorem 1.

For the second statement we use that it is necessary for determinacy that -1 □ $f'(y(\beta))$ □ 1. From (10) one sees that if the denominator of $f'(y(\beta))$ is negative (which it can be for $\alpha < 0$), then $f'(y(\beta)) > 1$. So, to exclude $f'(y(\beta)) > 1$, one must set α such that the denominator is positive (for which $\alpha \geq 0$ suffices). On the other hand, for such an α, the necessary condi

o,

$$\alpha \geq \alpha^{**}(\beta) := \frac{1}{2}\frac{1-\beta}{\beta}(R(y(\beta)) - N(y(\beta)) - 2).$$

From (10), $f'(y(0)) < -1$ implies $R(y(0)) - N(y(0)) - 2 > 0$, which means that for all small enough β, the parenthesis in the expression for $\alpha^{**}(\beta)$ is positive, so an α fulfilling the inequality also fulfils $\alpha \geq 0$. Finally, as β goes to zero, the required $\alpha^{**}(\beta)$ goes to infinity because the parenthesis goes to $R(y(0)) - N(y(0)) - 2 > 0$, and $(1-\beta)/\beta$ goes to infinity. ■

Proof of Proposition 1. First note that since it is an assumption that $\alpha \geq 0$, it again follows from Appendix B, that global stability of $y(\beta)$ according to f eliminates both all cycles and all SSE.

This global stability of $y(\beta)$ is again established by sending all critical points below the 45°-line. For this to indeed imply global stability of $y(\beta)$, it is important that f is globally well-defined (which it is since $\alpha \geq 0$), and that $f(0) = 0$. For $\alpha > 0$ this follows as above. For $\alpha = 0$, (8) reads $y_t v'(1 - y_t) = (1-\beta) y_{t+1} u'(y_{t+1})$. As y_{t+1} goes to zero, so will the right hand side if and only if $R(0) < 1$.[6] Hence, if $R(0) < 1$, one still has $f(0) = 0$, whereas if $R(0) > 1$ one has $f(0) = 1$. The assumption $R(0) < 1$ in (ii) thus implies $f(0) = 0$ also when $\alpha = 0$.

[6] Note that $R(0)$ is the elasti

We will show that in (i) one can use the $\beta^*(\alpha) = \max_{x\in[0,1]} \frac{R(x)-1}{R(x)-1+\alpha}$, where one must have $0 < \beta^*(\alpha) < 1$, since, as above, $R(y(0)) > 1$, and $\alpha > 0$. From (11), since at any critical point $R(x^c) > 1$, and since $\alpha > 0$, $f(x^c)/x^c < 1$ is equivalent to,

$$\beta > \frac{R(x^c)-1}{R(x^c)-1+\alpha}, \tag{13}$$

which is fulfilled for all critical points with x^c □ 1 when $\beta > \beta^*(\alpha)$. When $x^c > 1$, one has $f(x^c)/x^c < 1$ from $f < 1$. This proves (i).

For (ii) simply note that the perfect foresight dynamic (8) for $\alpha = 0$ becomes $y_t v'(1-y_t) = (1-\beta)y_{t+1}u'(y_{t+1})$, so for β going to one the y_t that solves it must go to zero for any value of y_{t+1}. This means that $f(x)$ is pulled down arbitrarily close to the x-axis. Further, from (9), when $\alpha = 0$, a critical point is given by $R(x) = 1$ independently of β. So, as β is increased all critical points $(x^c, f(x^c))$ move downwards along the same value of x^c with $f(x^c)$ getting arbitrarily close to the x-axis, so eventually all critical points go below the 45°-line. ■

Proof of Proposition 2. *(i)* For $\beta > 0$, when α becomes negative and sufficiently large numerically, both the numerator and the denominator in (10) become negative with the numerator numerically the largest, so $f'(y(\beta)) > 1$.

(ii) When $\alpha < 0$, one sees from (10), that as β goes to one, $f'(y(\beta))$ goes to $\alpha/\alpha = 1$. B numerator and denominator become negative for a large enough β, but the numerator is numerically the largest, so $f'(y(\beta))$ goes to one fro

$\frac{}{1+N(y(\beta))}$. As β goes to one, $y(\beta)$ goes to zero (Proposition A), and hence $f'(y(\beta))$ goes to $\frac{1-R(0)}{1+N(0)}$, which is less than -1 exactly because $R(0) > 2+N(0)$. If $\lim_{\beta\to 1} f'(y(\beta)) < -1$, then from continuity also $f'(y(\beta)) < -1$ for all large enough β. Hence $y(\beta)$ is indeterminate, which suffices for the existence of SSE close to it. When f is globally well-defined and known to stay below a

■

B

transition probabilities, such that (14) is fulfilled, then there are also y', y'' with $0 < y' < y'' < 1$, such that $y' = f(y'')$ and $y'' = f(y')$. That is, if there is a stationary Markov sunspot equilibrium SSE, then there is also a two-period cycle, or, if there is no two-period cycle, then there is no SSE either.

Proof.[7] One can safely assume that all transition probabilities fulfill $q_{ij} > 0$.[8] For each $i = 1, ..., r$ define,

$$y_i^{\min} := \arg \min_{j \in \{1,...,r\}} v_2(y_i, y_j),$$

$$y_i^{\max} := \arg \max_{j \in \{1,...,r\}} v_2(y_i, y_j).$$

Since from (14), each $v_1(y_i)$ is an average of the r values of $v_2(y_i, y_j)$, $j = 1, ..., r$, one must have $v_2(y_i, y_i^{\min}) \leq v_1(y_i) \leq v_2(y_i, y_i^{\max})$ for $i = 1, ..., r$. In particular for $i = 1$ and r,

$$v_2(y_1, y_1^{\min}) \leq v_1(y_1) \leq v_2(y_1, y_1^{\max}),$$

$$v_2(y_r, y_r^{\min}) \leq v_1(y_r) \leq v_2(y_r, y_r^{\max}).$$

Since v_2 is decreasing in its first argument we have: $v_2(y_r, y_r^{\max}) \leq v_2(y_1, y_r^{\max}) \leq v_2(y_1, y_1^{\max})$, and $v_2(y_1, y_1^{\min}) \geq v_2(y_r, y_1^{\min}) \geq v_2(y_r, y_r^{\min})$. So, now using that $v_1(y_i)$ is strictly increasing in y_i, we get,

$$v_2(y_r, y_r^{\min}) \leq v_2(y_1, y_1^{\min}) \leq v_1(y_1) < v_1(y_r) \leq v_2(y_r, y_r^{\max}) \leq v_2(y_1, y_1^{\max}).$$

Part of this is $v_2(y_1, y_1^{\min}) < v_2(y_1, y_1^{\max})$, and since all transition probabilities q_{1j} are strictly positive, one gets $v_1(y_1) > v_2(y_1, y_1^{\min})$. Similarly, $v_1(y_r) < v_2(y_r, y_r^{\max})$. We have thus established,

$$v_2(y_1, y_1^{\min}) < v_1(y_1) \leq v_1(y_2) \leq \cdots \leq v_1(y_r) < v_2(y_r, y_r^{\max}). \tag{15}$$

For one i, one has $y_i = y_1^{\min}$, and hence $v_1(y_1^{\min}) > v_2(y_1, y_1^{\min}) \geq v_2(y_1^{\min}, y_1^{\min})$, where the latter follows since v_2 is decreasing in its first argument. Hence, $v_1(y_1^{\min}) >$

[7] This proof extends the result of Grandmont (1986) from the case where v_2 is independent of y_i, to the case where v_2 is weakly decreasing in y_i.

[8] We appeal here to known results. For dynamic systems as considered here, if there is a deterministic cycle, that is, a completely non-stochastic SSE where for each i, only one q_{ij} is greater than zero (equal to one), then there is also a fully stochastic SSE where all q_{ij} are strictly positive. By the same reasoning, if there is an SSE where for each i, some, but not all, q_{ij} are strictly positive, then there is also a fully stochastic SSE, cf. Guesnerie and Woodford (1992).

$v_2(y_1^{\min}, y_1^{\min})$, but this implies that $f(y_1^{\min}) < y_1^{\min}$. (Remember that $f(y_1^{\min})$ is the solution in z to $v_1(z) = v_2(z, y_1^{\min})$. For $z = y_1^{\min}$, one gets "strictly larger than". The solution is then to be found strictly below $y_1^{\min}$, since v_1 is strictly increasing, and v_2 is decreasing, in z). Similarly, for one i, one must have $y_i = y_r^{\max}$, so $v_1(y_r^{\max}) < v_2(y_r, y_r^{\max}) \square v_2(y_r^{\max}, y_r^{\max})$, implying $f(y_r^{\max}) > y_r^{\max}$. So, we have both $f(y_1^{\min}) < y_1^{\min}$ and $f(y_r^{\max}) > y_r^{\max}$. This implies, of course, that $y_1^{\min} \neq y_r^{\max}$, but also that,

$$y_r^{\max} < y_1^{\min}.$$

Otherwise one would have $f(y_1^{\min}) < y_1^{\min} < y_r^{\max} < f(y_r^{\max})$, which from the continuity and $f < 1$ parts of Assumption 2 would imply the existence of a monetary steady state strictly between $y_1^{\min}$ and $y_r^{\max}$, *and* one strictly above $y_r^{\max}$, contradicting the uniqueness of monetary steady state part of Assumption 2.

Also from (15), one has directly that $v_1(y_1) > v_2(y_1, y_1^{\min})$, which implies $f(y_1^{\min}) < y_1$ (by the same reasoning as above), and similarly $v_1(y_r) < v_2(y_r, y_r^{\max})$, implying $f(y_r^{\max}) > y_r$. Since also $y_1 \square y_r^{\max}$, and $y_1^{\min} \square y_r$, one has,

$$f(y_1^{\min}) < y_r^{\max} \text{ and } y_1^{\min} < f(y_r^{\max}).$$

Combining the two last displayed inequalities gives,

$$f(y_1^{\min}) < y_r^{\max} < \ y_1^{\min} < f(y_r^{\max}).$$

Given that f is continuous and stays below the "ceiling" one, this suffices for the existence of a two period cycle: Note that the obtained inequality states that f has a negative slope below minus one over an interval around the steady state, not necessarily infinitesimally close to it. However, the kind of non-local negative slope below minus one obtained suffices from a standard argument. If one constructs the mirror image of f around the 45°-line then this has, under the obtained condition and Assumption 2, to intersect f itself at two points y' and y'' different from the steady state. These y' and y'' define a two-period cycle. ■

References

Benassy, J.-P. (1998): "On the Optimality of Activist Policies with a Less Informed Government", mimeo.

Chattopadhyay (1996): "...", *Economic Theory* **8**, pp. 122-135.

Cattopadhyay (1999): "...", *Macroeconomic Dynamics* **3**, pp. 463-481.

Christiano, L. and S. G. Harrison (1999): "Chaos, Sunspots, and Automatic Stabilizers", *Journal of Monetary Economics* **44**, pp. 3-31.

Goenka, A. (1994): "Fiscal Rules and Extrinsic Uncertainty", *Economic Theory* **4**, pp. 401-416.

Grandmont, J.-M. (1985): "Endogenous Competitive Business Cycles", *Econometrica* **53**, pp. 995-1045.

Grandmont, J.-M. (1986): "Stabilizing Competitive Business Cycles", *Journal of Economic Theory* **40**, pp. 57-76.

Guesnerie, R. and M. Woodford (1992): "Endogenous Fluctuations", in J.-J. Laffont (ed.), *Advances in Economic Theory: Proceedings of the Sixth World Congress*, vol. II, Cambridge University Press.

Guo and Harrison (1999): "..."

Guo, J.-T. and K. J. Lansing (1997): "Fiscal Policy, Increasing Returns, and Endogenous Fluctuations", *mimeo*.

Guo, J.-T. and K. J. Lansing (1998): "Indeterminacy and Stabilization Policy", *Journal of Economic Theory* **82**, pp. 481-490.

Jacobsen, H. J. (2000): "Endogenous Markups and Endogenous Fluctuations", *mimeo*, Institute of Economics, University of Copnhagen.

Schmitt-Grohe, S. and M. Uribe (1997): "Balanced-Budget Rules, Distortionary Taxes, and Aggregate Stability", *Journal of Political Economy* **105**, pp. 976-1000.

Schmitt-Grohé and Uribe (2000): "..."

Sims, C. A. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy", *Economic Theory* **4**, pp. 381-399.

Woodford, M. (1986): "Stationary Sunspot Equilibria in a Finance Constrained Economy", *Journal of Economic Theory* **40**, pp. 128-137.

Woodford, M. (1994): "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy", *Economic Theory* **4**, pp. 345-380.