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Equilibrium Heterogeneous-Agent Models as Measurement Tools: some Monte Carlo Evidence

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Abstract

This paper discusses a series of Monte Carlo experiments designed to evaluate the empirical properties of heterogeneous-agent macroeconomic models in the presence of sampling variability. The calibration procedure leads to the welfare analysis being conducted with the wrong parameters. The ability of the calibrated model to correctly predict the welfare changes induced by a set of policy experiments is assessed. The results show that, for the economy and the policy reforms under analysis, the model always predict the right sign of the welfare effects. Quantitatively, the maximum errors made in evaluating a policy change are very small for some reforms (in the order of 0.05 percentage points), but bigger for others (in the order of 0.5 pp). Finally, having access to better data, in terms of larger samples, does lead to sizable increases in the precision of the welfare effects estimates.

JEL Classification Codes: C15, C54, C68, D52.

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http://qed.econ.queensu.ca/pub/faculty/cozzi/Webpage/

1 Introduction

Fact: quantitative macroeconomics is a burgeoning field. In this field, Heterogenous-Agent (HA) models have become a fruitful approach to conduct modern research in macroeconomics, as discussed in Krusell and Smith (2006) and Heathcote, Storesletten and Violante (2009). The need to go beyond the assumption of the representative agent for several important questions, Carroll (2000), the availability of cheap computational power, the increased access to both cross-sectional and panel data, together with a wider exposure of the profession to both numerical and recursive methods are among the reasons why macroeconomics research is performed more often with the aid of a computer and with household level heterogeneity.

The typical project starts by posing a well defined economic question, develops a model firmly grounded on microfoundations (specifically tailored to address the question at hand, particularly for the role of heterogeneity), and numerically solves the model by implementing a calibration strategy. Often the aim is to use the theory to quantify a variable that cannot be directly measured for, say, lack of data (e.g. the extent of frictions in a market, as in Krusell, Mukoyama, Rogerson, and Sahin (2010), or the nature of unobservable shocks, as in Storesletten, Telmer and Yaron (2004) and in Guvenen (2007)). Another objective is to perform counterfactual analysis, by computing the welfare effects of a policy change together with its distributional impacts in an ex-ante policy evaluation, or its optimal scheme, as in Alvarez and Veracierto (2001) and in Conesa, Kitao and Krueger (2009).

Ultimately, quantitative stochastic macroeconomic models are measurement devices. Important and influential contributions relying on HA quantitative models have found, for example, that the observed changes in the US wage structure led on average to welfare gains, Heathcote, Storesletten and Violante (2010), that a theory of uninsured income risk accounts for the US earnings and wealth inequality almost exactly, Castaneda, Diaz-Gimenez, and Rios-Rull (2003), that skill-biased technological change accounts for the evolution of wage inequality in the US, Heckman, Lochner and Taber (1998), that the size of precautionary savings is modest, Aiyagari (1994), just like the welfare costs of Business Cycles, Imrohoroglu (1989).[1]

These models are designed to quantify some variables of interest, being functions of a set of empirical facts: the features of the data they are asked to replicate in the calibration stage. However, little is known about their empirical properties, in particular their reliability as tools to perform welfare analysis. As noted in the past in the Real Business Cycle literature, when the objective of the computational experiment is empirical in nature, the calibration methodology is not free of potential

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[1] This list is by no means exhaustive: it is just a subset of the several important contributions in the HA macroeconomic literature. In particular, since the analysis will be in a steady-state, models with aggregate uncertainty are omitted. For a comprehensive survey see Heathcote, Storesletten and Violante (2009).

pitfalls.[2] On the one hand, relying on external estimates to pin down some of the parameters leads to identification issues, whenever the assumptions of the empirical methodology estimating those parameters are not consistent with the ones of the macroeconomic model. On the other hand, sampling variability can be an obstacle that could cast doubts on some of the findings. More precisely, in face of sampling variability, it is not known how much the object of measurement is going to depend on the tight requirement of asking the model to match exactly selected features of the data.[3]

This paper is going to deal with the latter aspect in a class of equilibrium HA economies.[4] In order to shed some light on the empirical properties of these models I am going to propose and perform a Monte Carlo experiment. First, I am going to specify a simple HA economy. This is a version of the Huggett (1993) and Aiyagari (1994) economies, appropriately modified to allow for some interesting public policies and for a clear-cut calibration of key parameters. More in detail, the economy is going to be a production economy with an endogenous asset distribution, where a government collects taxes to finance both an unemployment benefit scheme and public expenditure.

Agents are ex-ante identical, while they differ ex-post, due to different realizations of an exogenous employment opportunity shock. This is a one-shock economy, where the sequence of shocks is independently distributed among the agents, and follows a two-state Markov chain. This is an important aspect, because the probabilities of the exogenous stochastic process do not depend on other parameters, and are uniquely identified: data on the unemployment rate and the unemployment duration are sufficient to pin them down.

A baseline parameterization of the stochastic process, consistent with US and European labor market outcomes, will be considered as the DGP. I will simulate the economy and I will draw 500 samples of different sizes, namely $1,000$, $4,000$, $16,000$, and $64,000$ individuals.[5] For each replication, a sample with different realizations of the employment opportunity shocks is drawn. This is going to lead to non-degenerate distributions of two key moments: the unemployment rate and the unemployment duration.

With these data in hand, I am going to follow Huggett (1993) strategy to calibrate the model economy, solve it, compute the relevant endogenous variables, together with welfare measures under both the current policy regime and under a policy change. The procedure is going to deliver sampling

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[2] See Gregory and Smith (1990), Canova (1995), Hoover (1995), and Hansen and Heckman (1996), among others.

[3] These are among the reasons why carefully designed calibration exercises tend to provide some robustness checks, by perturbing the benchmark parameters.

[4] The general idea is not new: almost two decades ago some authors, Gregory and Smith (1991), Watson (1993), and Canova (1994), developed a set of methodologies to assess the empirical properties of RBC models.

[5] These artificial datasets are meant to mimic the size of the US datasets used more often by macroeconomists to estimate/calibrate the stochastic processes driving the uncertainty in the model economy, such as the CPS, the NLSY, and the PSID.

distributions of the calibrated parameters and, more importantly, of the endogenous variables.

The contributions of this paper are several. First, to the best of my knowledge, this is the first Monte Carlo experiment dealing with equilibrium HA models. With this methodology it is possible to assess the small sample properties of these models as measurement devices. This allows to gauge some of their features as tools to conduct empirical analysis. Second, by varying the sample size it is possible to see how the behavior of the relevant statistics changes, namely if there is a quick convergence to the true values. Finally, ranges of estimates for the proposed policy changes are going to be provided.

For the policy changes under analysis, I will be able to answer questions such as: How big is the largest mistake due to sampling variability that a quantitative macroeconomist can incur into when evaluating a policy change with these models? Is there a tendency for the distribution of welfare effects to become substantially more concentrated when the researcher has access to more precise information (i.e. larger datasets, which lead to better calibrations)?

The most important finding is that, irrespective of the postulated DGP, the specific policy change being examined, and the sample size in the Monte Carlo simulation, the calibration exercise never fails in assessing the sign of the welfare change. In the presence of several policy changes, the answer from the calibrated models has always proven to be of the right sign. This is true both for the welfare enhancing cases and for the welfare reducing ones.

As for the magnitude of the welfare effects, the results are less straightforward. Quantitatively, when calibrating from the largest sample, the maximum errors made in evaluating a policy change are relatively small for some reforms (in the order of $\pm 3\%$ of the true value), but bigger for others (in the order of $\pm 10\%$ of the true value). Unfortunately, there does not seem to be a powerful rule-of-thumb suggesting when we are going to be in presence of small mistakes and when of large ones. These errors obviously get larger when considering smaller samples (in the worst cases considered here they get above $\pm 20\%$ of the true value).

Moving from $1,000$ to $64,000$ sampled individuals does lead to a decrease in the dispersion of the welfare effects. The model and the implementation of the welfare comparisons are consistent by construction: with knowledge of the fixed parameters, a law of large numbers applies, there is a unique stationary distribution, and a unique equilibrium. Hence, if we could sample an infinite number of agents for a long enough period of time, the calibration would deliver unique values for the free parameters, and the results would collapse to the population ones, recovering the DGP. The Monte Carlo experiments show that the distributions of welfare effects become more and more concentrated around the true values. However, the speed of convergence is faster in some experiments than in others. Working with the wrong parameters triggers an endogenous response of the model, due to its

GE nature, leading to changing aggregates, prices, distributional features and welfare measures. In some experiments, this prevents the welfare effects to converge quickly to their true values.

The rest of the paper is organized as follows. Section 2 briefly presents the theoretical model. Section 3 is devoted to the description of the Monte Carlo experiment. Section 4 provides the main results, while Section 5 concludes. Some appendices are also included: they discuss in more detail the model and the numerical methods used. They also present additional results and a set of robustness exercises.

2 The HA Economy

Time is discrete. The economy is populated by a measure one of infinitely lived ex-ante identical agents.[6]

2.1 Preferences

Agents' preferences are assumed to be represented by a time separable utility function $U(.)$. Agents' utility is defined over stochastic consumption sequences $\{c_t\}_{t=0}^{\infty}$: their aim is to choose how much to consume (c_t), and how much to save in an interest bearing asset (a_{t+1}) in each period of their lives, in order to maximize their objective function. The agents' problem can be defined as:

$$\max_{\{c_t,a_{t+1}\}_{t=0}^{\infty}} E_0 U(c_0, c_1, ...) = \max_{\{c_t,a_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where E_0 represents the expectation operator over the employment opportunity shocks $s \in S = \{e, u\}$. $\beta \in (0,1)$ is the subjective discount factor. I assume that $u(c_t) = \frac{c_t^{1-\sigma}-1}{1-\sigma}$, that is the per period utility function is strictly increasing, strictly concave, satisfies the Inada conditions, and has a CRRA$= \sigma$. Notice that there is no direct disutility from work, hence labour supply is fixed.[7]

2.2 Endowments

Agents can be employed (e), or unemployed (u). If employed, they earn a wage w and pay proportional taxes (τ_u, τ_a, τ_w) to finance the unemployment benefit scheme and the public expenditure G. If

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[6] In the interest of space, I am going to present just a sketch of the model. For more details see Appendix A, Huggett (1993), Aiyagari (1994), and Rios-Rull (1999).

[7] The analysis will focus on steady states, hence from now on we drop the time subscripts. Hereafter, the prime symbol $'$ denotes future variables.

unemployed, they collect unemployment benefits, which are specified as a constant replacement rate ϕ of the going wage, and pay proportional taxes (τ_a, τ_w).

The stochastic employment opportunities follow a two state first order Markov process. The transition function of the employment opportunity state is represented by the matrix $\Pi(s', s) = [\pi(i,j)]$, where each element $\pi(i,j)$ is defined as $\pi(i,j) = \Pr\{s_{t+1} = i | s_t = j\}$, $i, j = \{e,\ u\}$. Finally, every agent is endowed with exogenous efficiency units denoted as ε, normalized to 1.

2.3 Technology

The production side of the model is modeled as a constant returns to scale technology of the Cobb-Douglas form, which relies on aggregate capital K and labor L to produce the final output Y.

$$Y = F(K, L) = K^{\alpha} L^{1-\alpha}.$$

Capital depreciates at the exogenous rate δ and firms hire capital and labor every period from competitive markets. The first order conditions of the firm give the expressions for the net real return to capital r and the wage rate w:

$$r = \alpha \left(\frac{L}{K}\right)^{1-\alpha} - \delta, \tag{1}$$

$$w = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}. \tag{2}$$

Notice that the marginal productivity of labor is always positive, hence all the people with a favorable employment shock are going to be employed. It follows that in the steady-state:

$$L = \frac{\pi(e', u)}{1 - \pi(e', e) + \pi(e', u)}.$$

2.4 Government

The role of the government in this economy is twofold.

On the one side it runs the Unemployment insurance (UI) benefits scheme, by taxing the labor income of the employed workers at rate τ_u and subsidising the unemployed workers at the replacement rate ϕ. ϕ is a policy parameter exogenously given, while τ_u is set residually to ensure a self-financing scheme.

On the other side, the government can carry out some public expenditure G. In order to finance these purchases, both capital (τ_a) and labor (τ_w) taxes are levied. G and τ_a are going to be policy parameters set exogenously, while labor taxes are set residually to guarantee a balanced budget.

2.5 Other market arrangements

The final good market is competitive, and firms hire capital and labor every period from competitive markets. Capital is supplied by rental firms that borrow from workers at the risk-free rate r and invest in physical capital.

There are no state-contingent markets to insure against the unemployment risk, but workers can self-insure by saving into the risk-free asset. The agents also face an exogenous borrowing limit, denoted as $b \geq 0$.

2.6 Discussion

Why the focus on this specific HA economy?

1. At its heart it represents the core of many richer HA economies, sharing with them similar intertemporal trade-offs, insurance motives, and distortions.
2. It is simple enough to be efficiently solved on a computer with an extremely high numerical precision in a matter of minutes, allowing for a Monte Carlo experiment to be feasible.
3. It has a limited number of parameters, allowing for a neat calibration procedure.
4. The simple stochastic structure allows to solve numerically for the endogenous distributions without relying on simulations of large samples of agents, which can lead to confounding effects in the welfare effects computation.[8]

3 Design of the Monte Carlo Experiment

The Monte Carlo experiment consists of two steps.

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[8] As shown in Michaelides and Ng (2000) in a Monte Carlo comparison of simulation estimators, exploiting the numerical approximation of the invariant distribution rather than simulating samples of agents provides substantial efficiency gains.

In the first step, I postulate the stochastic process for our economy and parameterize it to match US data. I then solve the model economy and compute the welfare effects arising from a set of policy experiments. Coming from the DGP, these represent the true welfare effects.

In the second step, relying on the true stochastic process, I simulate 500 samples of different sizes from it. This procedure generates a sequence of moments computed from the simulated samples. I then re-calibrate the model's parameters according to the simulated moments. With the new calibration, I implement the same policy changes considered in the first step and compute the welfare effects induced by the new policy regime. These will (most likely) differ from the true ones, because of the underlying parameterization being different from the DGP. A crucial aspect, related to the reliability of calibrated HA equilibrium models as empirical devices, will be how disperse the welfare effects estimates are going to be, and by how much they will differ from the true ones.

I repeat these experiments four times (with samples of different sizes, to consider the effect of having more data, namely of size $n = 1,000$, $4,000$, $16,000$, and $64,000$), and perform some robustness checks (with different fixed parameters, to check if these affect the results).

3.1 Parameterization

3.1.1 DGP

The baseline DGP parameterization targets an unemployment rate of 9.4% and an average unemployment duration of 33.0 weeks. These are recent figures (as of December 2010) for the US labor market and are broadly consistent with many European experiences.

In order to properly capture the labor market dynamics, I need to work with a short time period: in the benchmark case, one model period corresponds to six months. In this case, $\pi(e', e)_{DGP} = .9182$, and $\pi(e', u)_{DGP} = .7879$.[9]

3.1.2 Monte Carlo Simulations and Calibration

The HA economy under study has seven independent parameters: $\pi(e', u), \pi(e', e), b, \sigma, \beta, \alpha$, and δ.

These parameters are divided into two categories. The first two parameters, $\pi(e', u)$ and $\pi(e', e)$, are "simulated", meaning that they are going to be recalibrated at each replication of the experiment,

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[9] A previous version of the paper considered also a second DGP parameterization targeting long-run averages for the US. For this alternative case the unemployment rate was 5.9%, the average unemployment duration was 19.0 weeks, and the model was solved with a period being a quarter. The results were qualitatively very similar, and are not worth reporting. The robustness checks with respect to two parameters, β and δ, were done mainly with this alternative DGP specification, which implied different values for them, because the model was asked to match the same targets at the annual frequency.

on the basis of the specific results obtained in the sample simulation of the DGP. The five remaining parameters are going to be kept fixed. However, several robustness checks with respect to their values are performed in order to understand if changing these parameters affects the findings.

Notice that I limit the attention to calibrating only two parameters while fixing the other ones. With the information from the simulated samples the parameters I focus on are exactly and uniquely identified and do not need the model solution to be assigned a value. The calibration procedure would be substantially more complex if I were to allow for richer simulations (i.e. more moments) and less fixed parameters (which would need to be calibrated in equilibrium, increasing the computational time by orders of magnitude, requiring the use of super-computers).[10]

3.1.3 "Simulated" Parameters

The parameters driving the uncertainty in the economy are going to be assigned many different values, one for each replication of the simulated sample. The two independent probabilities representing the Markov chain for the employment opportunities, $\pi\left(e',e\right)$ and $\pi\left(e',u\right)$, are going to be re-calibrated for every sample drawn in iteration m of the Monte Carlo, according to the following formulas:

$$\pi\left(e',u\right)_m = \frac{1}{U\ duration_m} \tag{3}$$

$$\pi\left(e',e\right)_m = 1 - \pi\left(e',u\right)_m * \left(\frac{U\ rate_m}{1 - U\ rate_m}\right) \tag{4}$$

where "$U\ duration_m$" stands for the average unemployment duration computed in the simulated sample m, while "$U\ rate_m$" stands for the corresponding simulated unemployment rate.

[Figures 1 and 2 about here]

Figures (1) and (2) show the distribution of the two parameters resulting from the calibration procedure. Figure (1) presents the non-parametric kernel estimate of four densities (one for each sample size in the simulation stage) of the job finding probability $\pi\left(e',u\right)$. These are computed relying on equation (3). Figure (2) presents a similar graph for the job retention probability $\pi\left(e',e\right)$, computed relying on equation (4). As expected, sampling more individuals from the true stochastic process leads to more precise estimates of both the unemployment rate and the unemployment duration. These, in

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[10] Although conceptually feasible, matching the simulated labor shares, investment rates, and capital-output ratios would require reliable data on the value of asset holdings, a type of information which is usually hard to get in real household level panel data. At the same time, this alternative procedure would lead to several parameters changing at once, taking several hours per replication, and making the interpretation of the results less transparent.

turn, lead to more concentrated distributions of the job finding and job retention probabilities. The rate of convergence is $\sqrt{n}$, which visually can be seen by moving from one panel to the next, with the estimated densities doubling their maximum value.

[Table 1 about here]

Table (1) provides a set of descriptive statistics for the two Markov chain probabilities. It is worth stressing that the parameters' range used to solve the model and compute the related welfare effects is quite large. As for $\pi(e', e)$, with only $1,000$ observations at our disposal this parameter ranges from 0.8545 to 0.9583. This is even more so for the other parameter in the Markov chain, the job finding probability $\pi(e', u)$. In the $1,000$ observations case this parameter ranges from 0.5137 to 1.0. As a consequence, the amount of idiosyncratic uncertainty in the economy varies substantially, getting less disperse with larger samples. With $64,000$ artificial observations $\pi(e', e)$ ranges from 0.9113 to 0.9244, while $\pi(e', u)$ ranges from 0.7525 to 0.8355. Notice how the median values are always very close to the DGP, irrespective of the sample size.

3.1.4 Fixed Parameters

The concavity of the utility function is pinned down by the CRRA coefficient σ, which is set to 1.5, a common value in the literature. In the robustness checks I will set σ to 1.0 (i.e. *log* utility) and 2.0, instead.

The borrowing limit b is set to three different values, all strictly lower than the natural borrowing limit. In the benchmark economy $b = 4$. This means that the agents can borrow up to (almost) two model period average income. With this limit, 8.8% of the households are in debt, a value consistent with the SCF data, reported for example by Cagetti and De Nardi (2008). Other experiments rely on $b = 0$ and $b = 2$.

The capital depreciation rate is set to replicate an investment/output ratio of approximately 22.5%. This is achieved with $\delta = 0.04$. With a Cobb-Douglas production function the capital share is captured by the parameter $\alpha = 0.34$, which matches the capital share of income. The rate of time preference β is calibrated to get an equilibrium interest rate of approximately 4% on an annual basis, obtained when $\beta = 0.97979589$.

The complete parameterization of the model is reported in Table (2).

[Table 2 about here]

3.1.5 Policy Parameters

Differently from the parameters above, ϕ, τ_a, and G are policy parameters. They are pinned down by the institutional features of the economy they are meant to represent (its labor market and fiscal policies, in particular). Their values are reported in Table (3), and will be explained in detail in the policy experiments description.

[Table 3 about here]

3.2 Model Solution and Policy Changes

For each set of calibrated parameters the equilibrium of the model is computed twice. The first time under the current policy regime, i.e. for specific values of the triplet $\{\tau_a, G, \phi\}$, the second time under the counterfactual economy, i.e. after a policy change.[11]

The policy changes that I consider fall into two broad categories. The first experiment is going to deal with a reduction in the generosity of the UI scheme. This is meant to highlight the insurance properties of this policy and the endogenous response of precautionary savings. The second experiment is going to deal with a reduction in capital income taxation. This is meant to highlight the distortionary effects of capital taxation and the endogenous response of savings.[12]

It is important to stress that this paper does not deal primarily with the optimality of the UI program, as in Hansen and Imrohoroglu (1992), or the actual desirability of changing the current capital tax code, as in Ventura (1999). It is mainly targeted at assessing the empirical properties of the underlying quantitave macroeconomic HA model.

Moreover, notice that in the numerical solution of the model there is no sampling variability (due for example to Monte Carlo integration). The only sources of error are the numerical errors induced by the discretization of the state space and by the convergence criteria, which are kept as small as

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[11] The welfare effects are going to compare two different steady-states. I consider both the percentage change in aggregate welfare, and a consumption equivalent welfare measure, while I don't consider transitional dynamics. See equations (11) and (12) in Appendix A.

[12] In the experiments, I consider both a big change in capital taxation and in the UI benefit. An alternative, and perhaps more informative, exercise could have been to consider their elimination. However, in the model I am preventing agents to react to such changes through relevant margins. Agents cannot change their labor supply (taxes on labor increase as a consequence of the decreased capital taxes), and they can't put more effort while searching for a job (in the UI experiment). Although admittedly important, these extensions would increase substantially both the computational burden and the likelihood of the algorithm failing to converge for some calibrations. These complications could make the Monte Carlo experiment quite intractable.

the computational burden makes possible. It follows that, across replications, the computed equilibria vary only because of the different calibrated parameters. Differently, the change in equilibria would partially reflect the simulation error: aggregate quantities would vary randomly, leading to an induced endogenous response of the model and an additional source of error for the welfare effects.[13]

3.3 Experiment 1 - UI

For this experiment I assume that the only public policy in place in the economy is the UI. Unexpectedly, the government implements a cut in the replacement rate $\phi : \phi = 0.5$ in the benchmark economy and $\phi^{new} = 0.25$ in the counterfactual one.

The economics of this policy change are very simple. With a lower ϕ, agents are less insured against their idiosyncratic shocks. A priori we cannot say if the policy reform is going to lead to a welfare gain or to a welfare loss. Income during an unemployment spell decreases, making it more difficult for agents to achieve consumption smoothing. However, quantitatively, this reform leads to an aggregate welfare increase. This is due to the ability of agents to self-insure by accumulating more assets: capital supply increases, because of the increased precautionary savings. At the same time employed agents pay less taxes, and are going to consume part of these resources. Overall, this leads unemployed agents to be worse off in the new steady-state, while employed ones are better off.

The unemployment rate in the economy is kept constant, hence the capital demand schedule does not change and we are just moving along the curve. A higher capital supply is going to trigger a decrease in the interest rate clearing the asset market, leading to: a) a larger capital stock, b) higher wages and a lower decrease in the UI benefits, c) cheaper borrowing, d) a lower return on savings. The last two effects imply that some agents could start substituting away from saving and consuming more, with some borrowing more. However, in the aggregate, these shifts are dominated by the increased precautionary saving motive.

Overall, together with the larger relative size of employed agents, this explains why there is a welfare gain when the unemployment benefits are reduced.

3.3.1 Case 1a)

For this case $G = \tau_a = \tau_w = 0$, $\phi = 0.5$ and $\phi^{new} = 0.25$.[14]

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[13] This is why I don't consider a stochastic process for wages, as in Aiyagari (1994). The approximation procedures typically used to solve that model introduce a source of error that I can dispense of with this alternative and simpler specification. For more details on the numerical solution, see Appendices B and C.

[14] Taxes react to the new regime, and their new equilibrium value decreases to $\tau_u = \frac{\phi^{new}(1-L)}{L}$. See equation (7) in appendix A.

The true welfare effect for this case is quantified in a 0.556% increase of the average steady-state utility. Employed agents enjoy a 0.667% welfare increase, while unemployed ones suffer a 0.528% welfare loss. The corresponding figures for the consumption equivalent welfare measure are a 1.104% increase for the overall economy, a 1.320% increase for employed agents, and a 1.064% decrease for the unemployed ones.

3.3.2 Case 1b)

In case 1b I consider the same set-up as in case 1a. The only difference stems from a tighter borrowing limit: b is decreased from $b = 4$ to $b = 0$. Agents in this economy cannot borrow at all, hence it becomes more difficult for them to smooth consumption.

Irrespective of the more stringent borrowing limit, the true average welfare effect for this case does not change much compared to the previous one. It is now quantified in a 0.370% increase of the average steady-state utility. A slightly lower improvement in welfare is obtained because of the lower impact of precautionary savings, and the greater utility loss of the unemployed. Due to the stricter b, agents in this economy are saving more than in case 1a: the same change in ϕ leads to a response of savings which is proportionally lower. The capital stock does not increase as much as in case 1a, implying a lower increase in welfare. Employed agents enjoy a 0.478% welfare increase, while unemployed ones suffer a 0.699% welfare loss. Due to their inability to borrow, an income cut proves to be even more detrimental for these agents. The corresponding figures for the consumption equivalent welfare measure are a 0.735% increase for the overall economy, a 0.950% increase for employed agents, and 1.413% decrease for the unemployed.

3.3.3 Welfare Effects Sampling Distributions

Figures (3) and (6) plot the non-parametric kernel density estimates of the economy-wide welfare effects obtained in the four Monte Carlo experiments (with different sample sizes) for the cases 1a and 1b.

[Figures (3) and (6) about here]

A few features of the plots are worth stressing. The distributions are almost centered around the true values. Moreover, when moving from the top left panel ($n = 1,000$) to the bottom right one ($n = 64,000$), the dispersion of the welfare effect estimates gets smaller. The more people are sampled, the tighter the calibration, and the less imprecise the welfare effects estimates. However, the rate of convergence is not close to the rate of convergence of the parameters, and seems to be happening very slowly for case 1b. The endogenous response of the model, when working with parameters different from the true ones, seems to be preventing a rapid convergence of the welfare effects. When the true welfare effects are quantitatively very small, as in case 1b, even small differences in the computed equilibria involve a non-negligible impact on the welfare effects.

The welfare analysis can be broken down for two subgroups, the employed and the unemployed agents. The related plots for case 1a are shown in Figures (4) and (5). Compared to the overall economy-wide welfare effects it is possible to appreciate some differences. The welfare effects of the unemployed can be very imprecise when working with small samples, but they show a rate of convergence which is very fast and closer in magnitude to the parameters' one.[15] As for the employed agents, the same observations seem to hold, but less strongly. The errors made with very limited sample sizes are less extreme, but they tend to improve at a lower rate.

[Figures (4) and (5) about here]

Finally, for these economies, studying the welfare effects directly in terms of the average steady state utility change vis-à-vis a consumption based welfare measure turns out to be immaterial. When considering the latter, there is just a level effect, with the shape of the distributions and the convergence behavior being identical.[16]

3.4 Experiment 2 - Capital Taxation and Public Expenditure

The second experiment deals with a change in the tax code, starting from a very distortionary capital income taxation τ_a. An analogous policy reform is studied in a similar economy by Imrohoroglu (1998).

There is an exogenously given public expenditure G, which needs to be financed by collecting taxes on capital and labor income. G and τ_a are set by a policy maker, while τ_w is set residually to ensure a balanced budget.

In the initial policy regime capital income is heavily taxed. This leads agents to save little, as the after-tax rate of return is low. On the other hand, labor supply is inelastic, so taxing labor does not imply any distortions. Agents cannot reduce their labor supply and do not accumulate human capital while on the job. Unexpectedly, the government implements a budget neutral tax cut: $\tau_a = 0.3$ in

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[15] This finding provides evidence that the slow convergence of the welfare effects for the employed agents and for the overall economy is a genuine result. The numerical error could have been behind those behaviors.

[16] For example, compare Figure (3) to Figure (9) in Appendix D.

the benchmark economy and $\tau_a^{new} = 0.2$ in the counterfactual one. This simple case implies that, for a given level of publc expenditure, lower capital taxes are going to increase the incentives to save, leading to a larger capital stock, and higher wages. Overall this explains why there is a welfare gain from reducing the capital tax, even though a priori the policy reform could lead to a welfare loss (labor income is going to be taxed more, hence unemployment spells might become less attractive).

Finally, notice that for these experiments the UI scheme is kept at its benchmark specification, and the borrowing limit is set to $b = 0$, to avoid capital income taxes turning into subsidies for agents in debt.

3.4.1 Case 2a)

For this case $\phi = 0.5$, $\tau_a = 0.3, \tau_a^{new} = 0.2$ and $G = \overline{G} = 0.42$. This value of G matches the Public Expenditure/Output ratio in the initial steady-state, which is approximately 20%.

For a given (and constant) public expenditure, a decrease in capital taxes τ_a mechanically leads to an increase in labor taxes τ_w.[17] However, labor supply is rigid in this economy and we are getting closer to the principles of Ramsey taxation: higher taxes should be set for the goods with lower elasticities of substitution. Notice also that the decrease in distortionary taxation has a GE effect as well: a higher return on capital increases savings and investment, leading to a higher output and a lower interest rate.

It follows from these considerations that in the counterfactual economy the government share of output is no longer going to be at a constant value of 20%, and in the final steady-state it will account for a lower share.

As expected, the true welfare effect for this case is sizable, and it is quantified in a 8.415% increase of the average steady-state utility. Unlike in the first policy change, both the employed and unemployed agents enjoy a welfare gain, equal to 8.363% and to 8.939% respectively. The increased capital supply raises output, wages, and the unemployment benefits, which more than compensate the increased taxation of labor earnings. The corresponding figures for the consumption equivalent welfare measure are stunning: the increases are 14.921% for the overall economy, a 14.840% increase for employed agents, and 15.737% for the unemployed ones.

3.4.2 Case 2b)

For this case $\phi = 0.5$, $\tau_a = 0.3, \tau_a^{new} = 0.2$ and $\frac{G}{Y} = \overline{\frac{G}{Y}} = 0.2$.

Differently from case 2a, now the government share of output $\left(\frac{G}{Y}\right)$ in the counterfactual economy is kept always constant at 20%. This means that, after the decrease in the capital tax, output will

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[17] See equation (8) in Appendix A.

increase, and so will the public expenditure. Compared to case 2a, there are now less resources left for both private consumption and investment. This explains why the welfare gains turn out to be substantially lower.

The true welfare effect for this case is quantified in a 3.645% increase of the average steady-state utility, with the increase for the employed being 3.619%, and for the unemployed being 3.909%. The consumption based welfare measures are now 6.910%, 6.864% and 7.383%.

3.4.3 Welfare Effects Sampling Distributions

Figures (7) and (8) plot the non-parametric kernel densities of the welfare effects obtained in the four Monte Carlo experiments for the cases 2a and 2b.

[Figures (7), and (8) about here]

The tax experiments differ from the UI ones in some aspects. First, the economy-wide welfare effects of the capital tax reforms are converging at a fast rate to their true value, in case 2a in particular. Second, for this policy reform breaking down the welfare analysis into the employed and the unemployed agents does not show any difference from the patterns of the economy-wide welfare gains. They both track the overall welfare effect closely, being just a shift to the right or to the left of the economy-wide distributions. Third, both groups of workers always enjoy a welfare gain.

The top left panels of the two figures (referring to $n = 1,000$) show that a small sample and an extreme calibration can lead to relatively large mistakes in the assessment of aggregate welfare. Differently, the bottom right panels (referring to $n = 64,000$) show that the welfare effect distributions are very concentrated around their true value.

4 Estimating Welfare Effects with a Calibrated HA Model

What happens to the estimated welfare effects obtained from the Monte Carlo experiments? In general, working with a calibrated model with parameters that differ from the true ones leads to errors that many would consider tolerable. Table (4) reports several statistics of the economy-wide welfare effects computed in each experiment.[18]

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[18] Tables (5) and (6) in Appendix D provide the same statistics for the employed and the unemployed agents. The main comments apply also to these group specific welfare effects.

[Table 4 about here]

The calibrated HA model always got the right sign of the welfare effects. Irrespective of the policy experiment considered, the alternative calibrations of the fixed parameters, and the data abundance, the quantified welfare effects were always of the same sign as the true ones.

Another positive, but perhaps not surprising, aspect is that the range of the welfare effects decreases monotonically with the sample size. For example, for case 1a, it moves from $[0.453, 0.629]$ in the least precise case to $[0.508, 0.594]$ in the most precise one. Both the upper bound and the lower bound of the welfare effects support are shrinking considerably with the sample size.

An implication of this behavior is that the maximum error that we could incur into when evaluating the welfare effects of a policy change decreases with the quantity of the information at our disposal. In the first experiment the maximum error never exceeds 0.103 percentage points, and (in absolute value) it ranges between 6.83% and 18.52% of the true value. In experiment 1b, the maximum error never exceeds 0.062 percentage points, and it ranges between 9.73% and 16.76% of the true value. The capital tax experiments show larger errors, because the underlying true values are substiantially higher. The maximum error never exceeds 3.253 percentage points for case 2a, and 0.681 percentage points for case 2b. The range of the maximum error is between 3.29% and 38.66% of the true value for the former, and between 3.79% and 18.68% for the latter.

The means and medians of each Monte Carlo experiment are remarkably close to the corresponding true values. If we consider the model as an estimator for the welfare effects, it displays a small sample bias that does not seem to have a regular pattern with the sample size. Hence, the average of the welfare effects sampling distribution does not coincide with the true effect. The bias, though, is quantitatively negligible for all possible experiments. This result seems to suggest that a well designed sensitivity analysis can go a long way in quantifying a reliable estimate of an ex-ante policy evaluation.

Very similar considerations apply for all the cases considered. This implies that, irrespective of the size and sign of the true welfare effects, a quantitatively negligible small sample bias is going to be always present, and that more information makes large mistakes to become less and less likely.

The main differences in the experiments under analysis consist of the rate at which the welfare effects distributions converge to their true values. In cases 1a and 1b the sampling distributions show a very slow rate of convergence, unlike cases 2a and 2b, which seems to be collapsing to their true values at quite a fast rate. The welfare effects sampling distributions do get more concentrated around the true value, but the speed of convergence is often far from what is observed for the Markov Chain probabilities.

5 Discussion and Conclusions

Fifteen years ago, in their survey on computational experiments in macroeconomics, Hansen and Heckman (1996) argued that equilibrium HA models represented a potential solution to some of the problems inherent in the calibration methodology. In this paper I performed a Monte Carlo experiment for a class of HA economies. The general idea is to evaluate some of the properties of equilibrium HA models as tools to perform empirical research. More precisely, I focused on the misspecifications of the model due to sampling variability in the moments that the calibration methodology aims at matching.

This paper does not claim to have provided the final word on the matter. However, as a first assessment, the findings seem to be encouraging: 1) in the simulations, for all possible parameter configurations and policy changes, the quantified measure of the welfare change is always of the right sign; 2) for many parameter configurations and policy changes the distribution of welfare effects are quite concentrated around the true value, leading to what many would consider tolerable errors when evaluating the effects of a policy change.

On a less bright note, in some experiments, the welfare effects were less concentrated, leading to a maximum error as big as 38.6% of the true value. However, mistakes as extreme as this one seemed to be rare, with the $[5\%, 15\%]$ interval being the most common range, even with only $1,000$ observations.

For some experiments, the distribution of welfare effects shows a fast convergence to the true value when increasing the sample size (hence the quality of the calibration). While for other experiments the rate of convergence appeared to be slow.

Rather than fully specifying the DGP and creating artificial samples from it, I could have considered real data from, say, the CPS for one or several years, bootstrap the sample, calibrate and solve the model. Although the statistical foundations of bootstrapping are different, I believe that I would have obtained similar results, at least qualitatively. The advantage of the fully specified DGP approach is that it provides a natural benchmark comparison: by design, I know how far each replication is from the true model, and from the true welfare effects. With a bootstrapping experiment I would still be able to compute, for example, bounds for the welfare gains. However, I would not be able to assess how far these computations are from the true value, whenever I rely on parameters that are not the true ones. That is, I would not be able to tell how the endogenous response of the model to parameter mispecifications "confounds" the estimates of the welfare gains.

It would be interesting to evaluate the empirical properties of this class of HA models both with boostrapping methodologies, and with full-fledged structural estimation ones. I leave these extensions and modifications for future work.

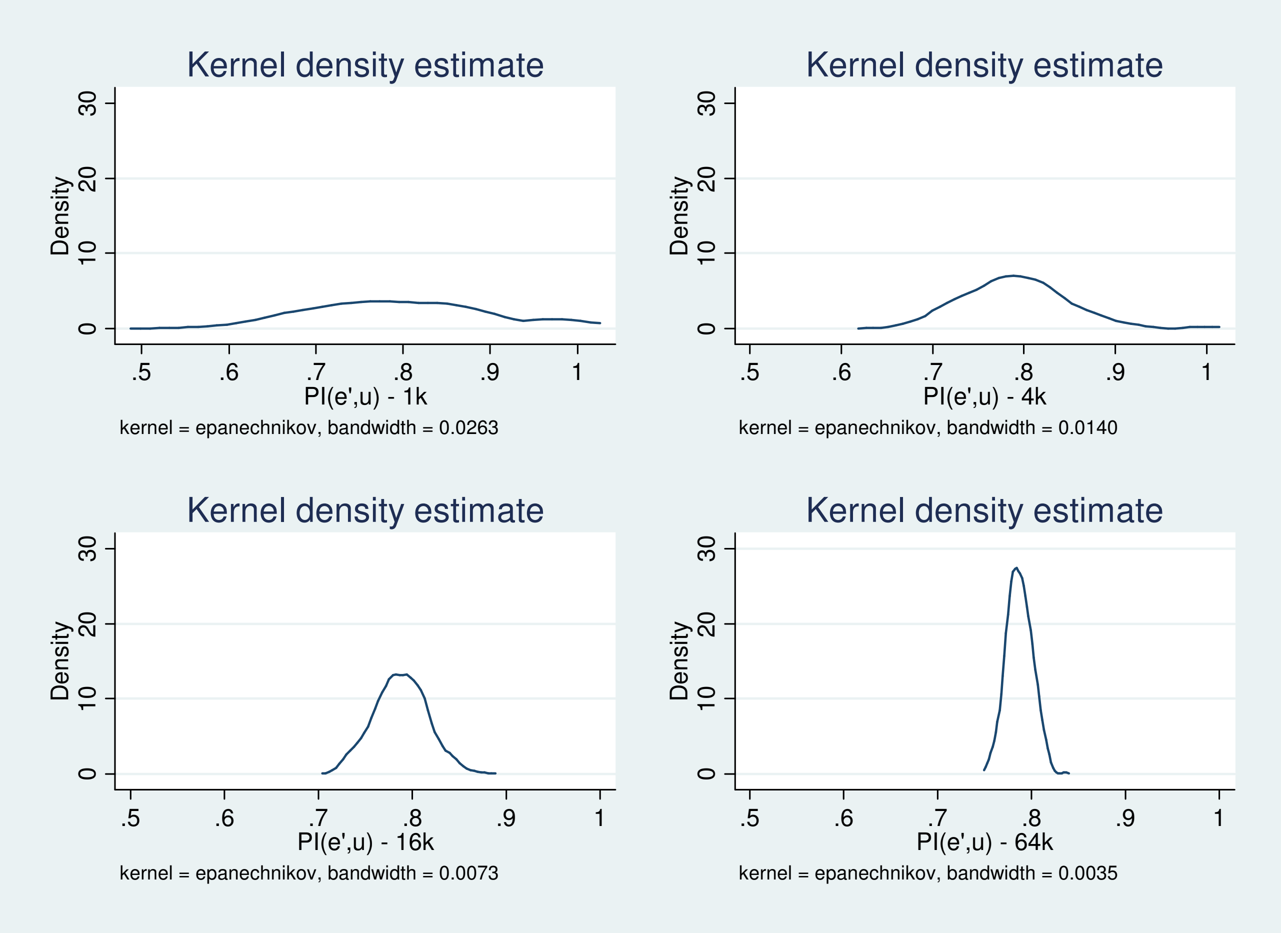

Figure 1: Calibration - "Simulated" Parameters: $\pi(e\prime, u)$

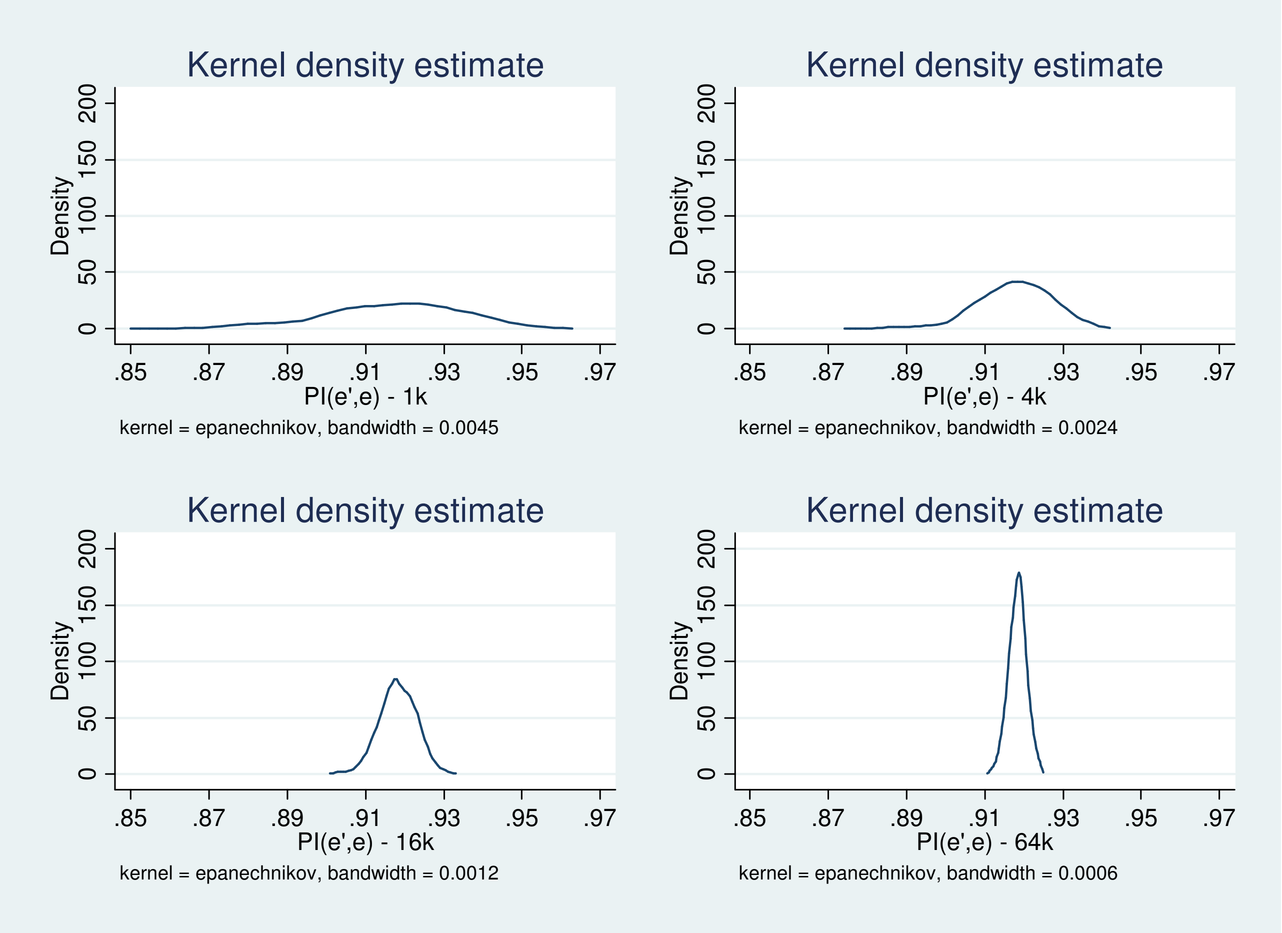

Figure 2: Calibration - "Simulated" Parameters: $\pi(e\prime, e)$

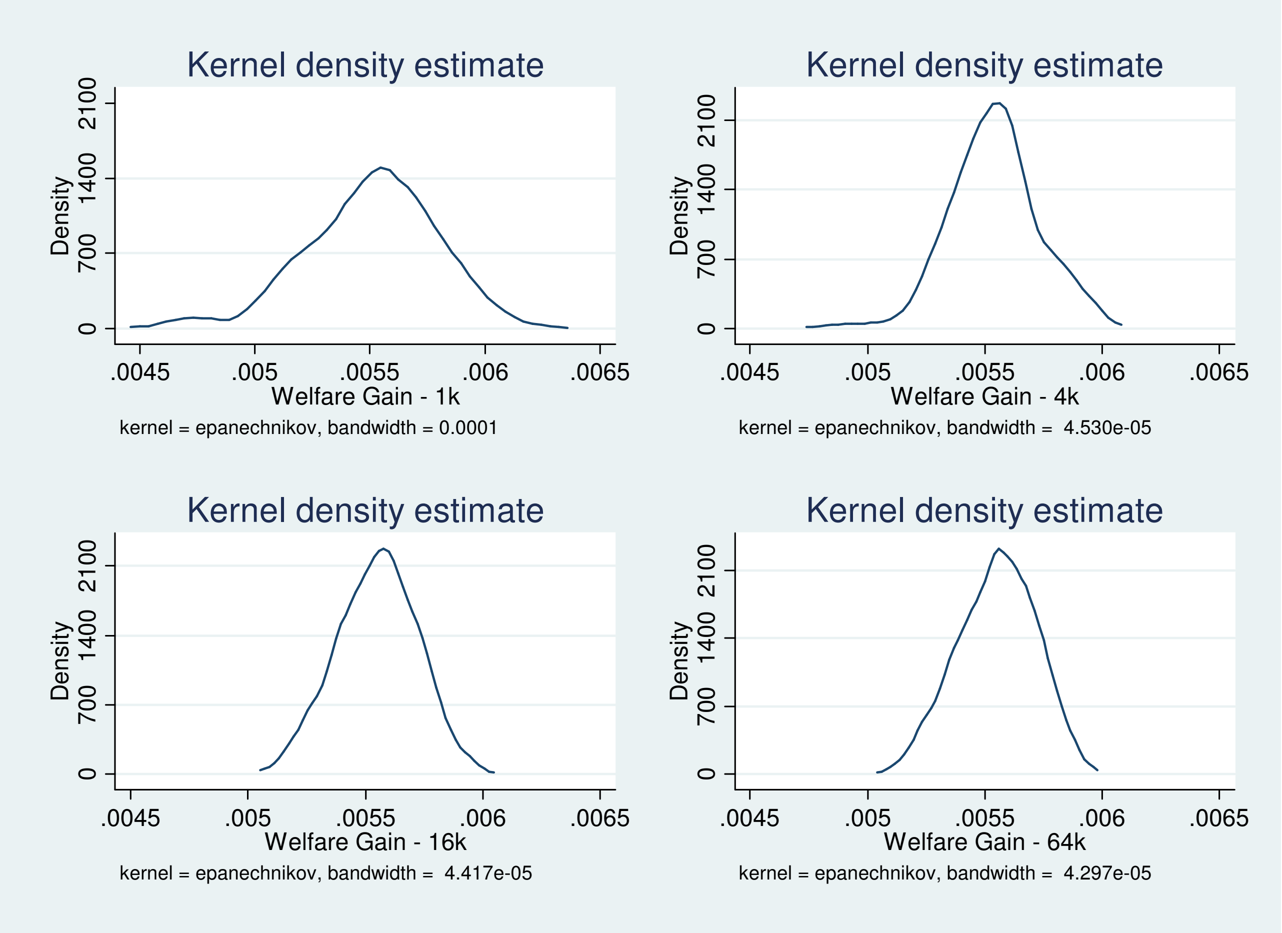

Figure 3: Welfare Gains, Experiment 1a $- b = 4$

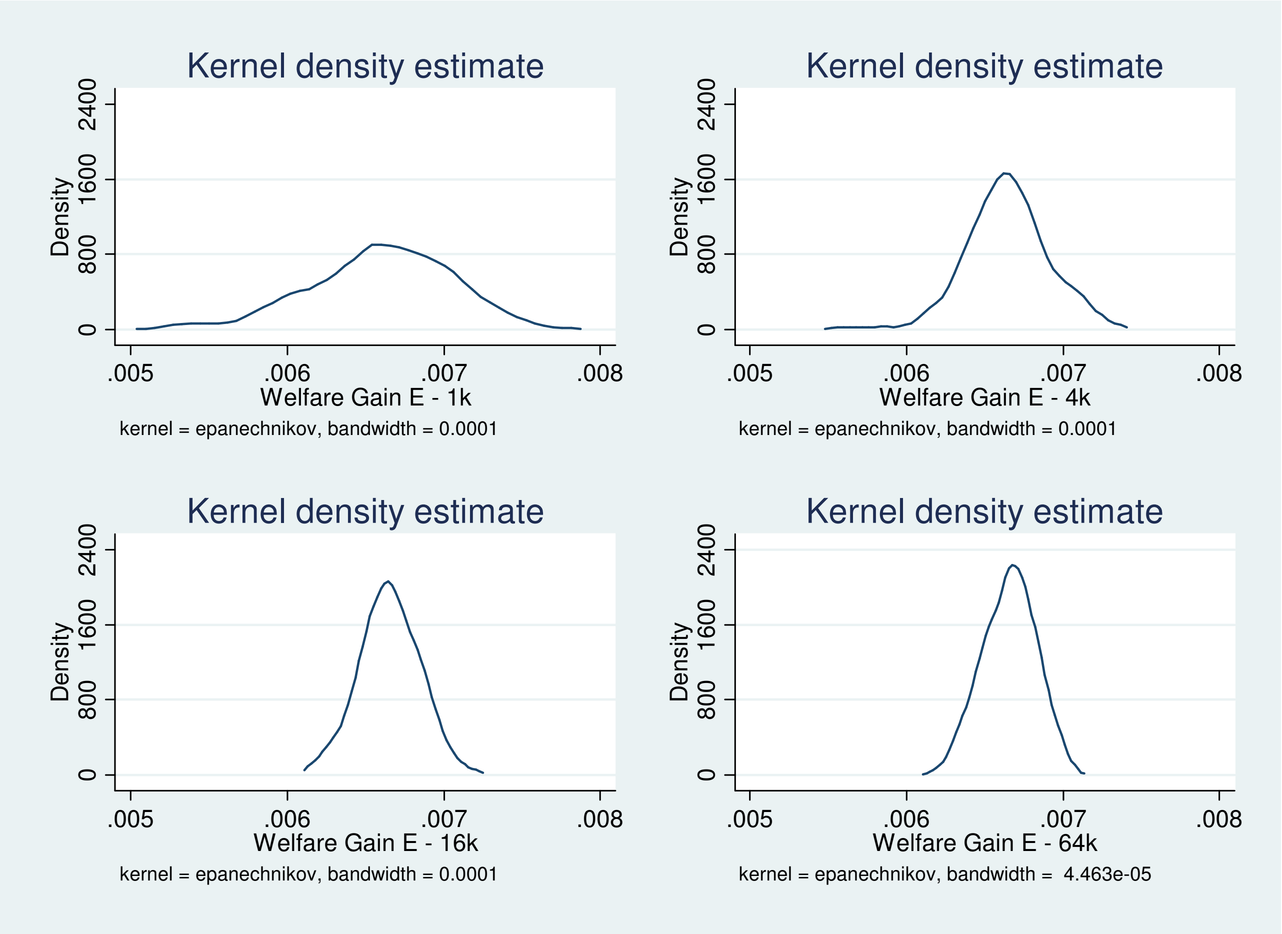

Figure 4: Welfare Gains Employed, Experiment 1a $- b = 4$

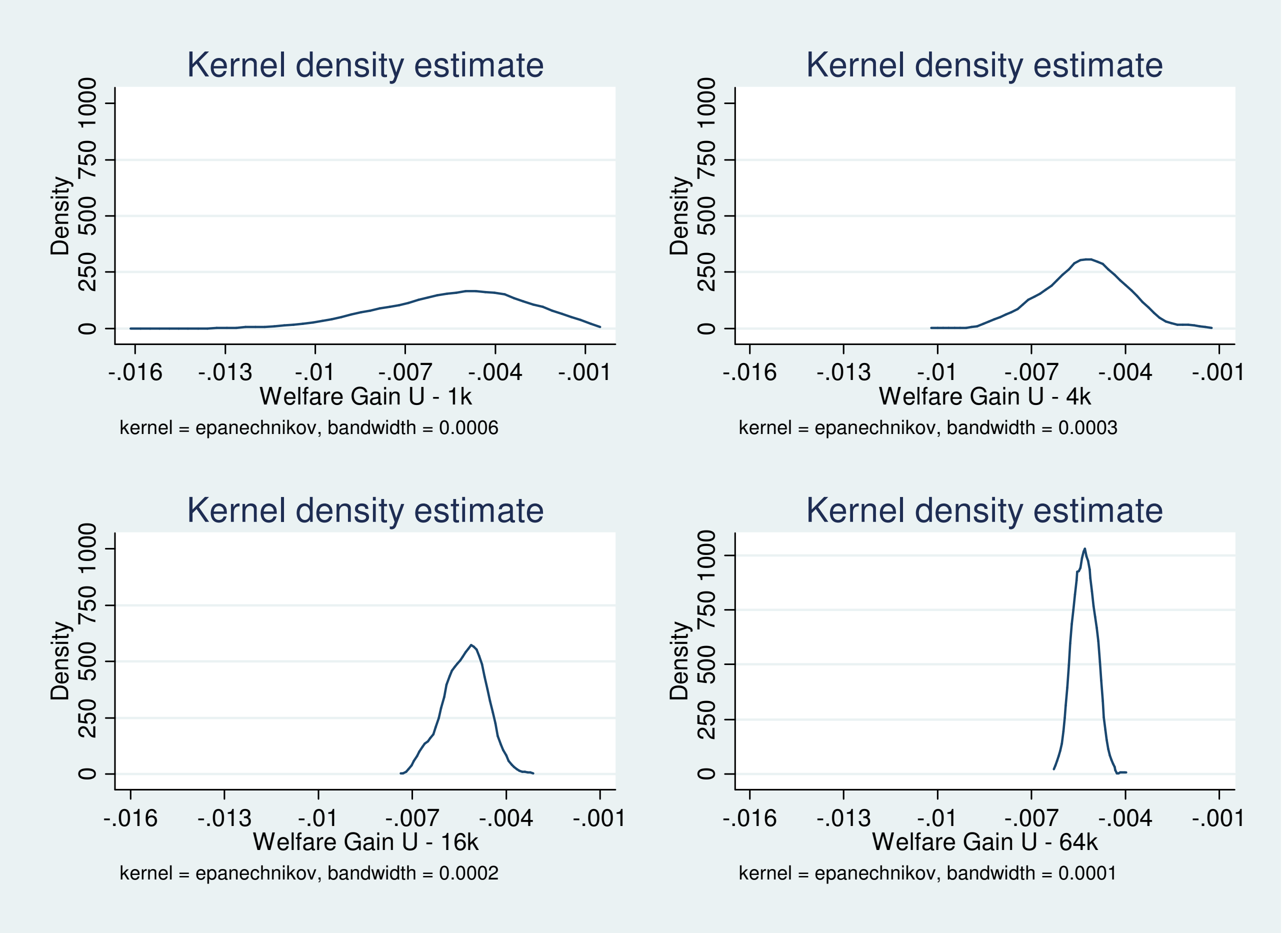

Figure 5: Welfare Gains Unemployed, Experiment 1a $- b = 4$

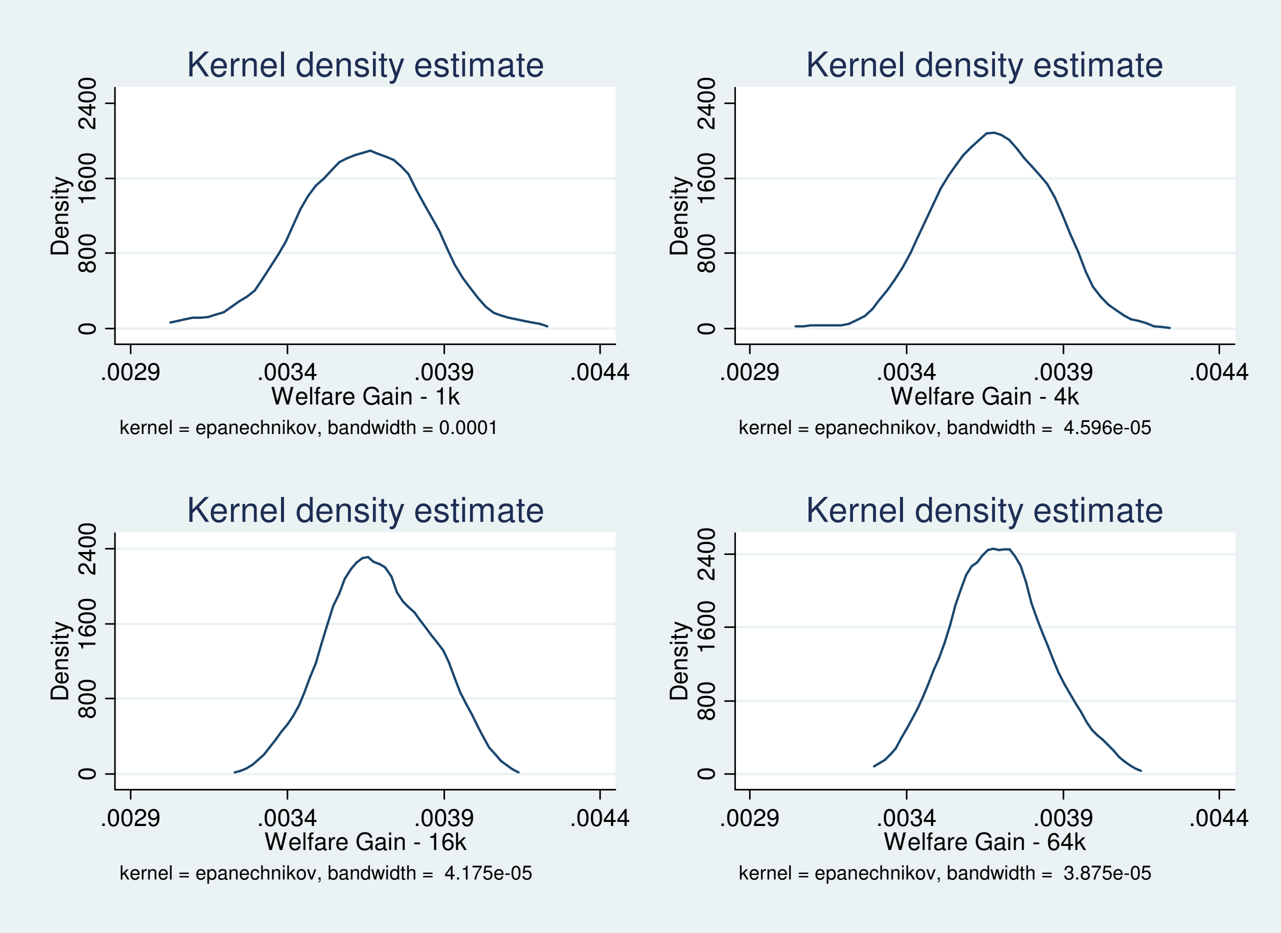

Figure 6: Welfare Gains, Experiment 1a $- b = 0$

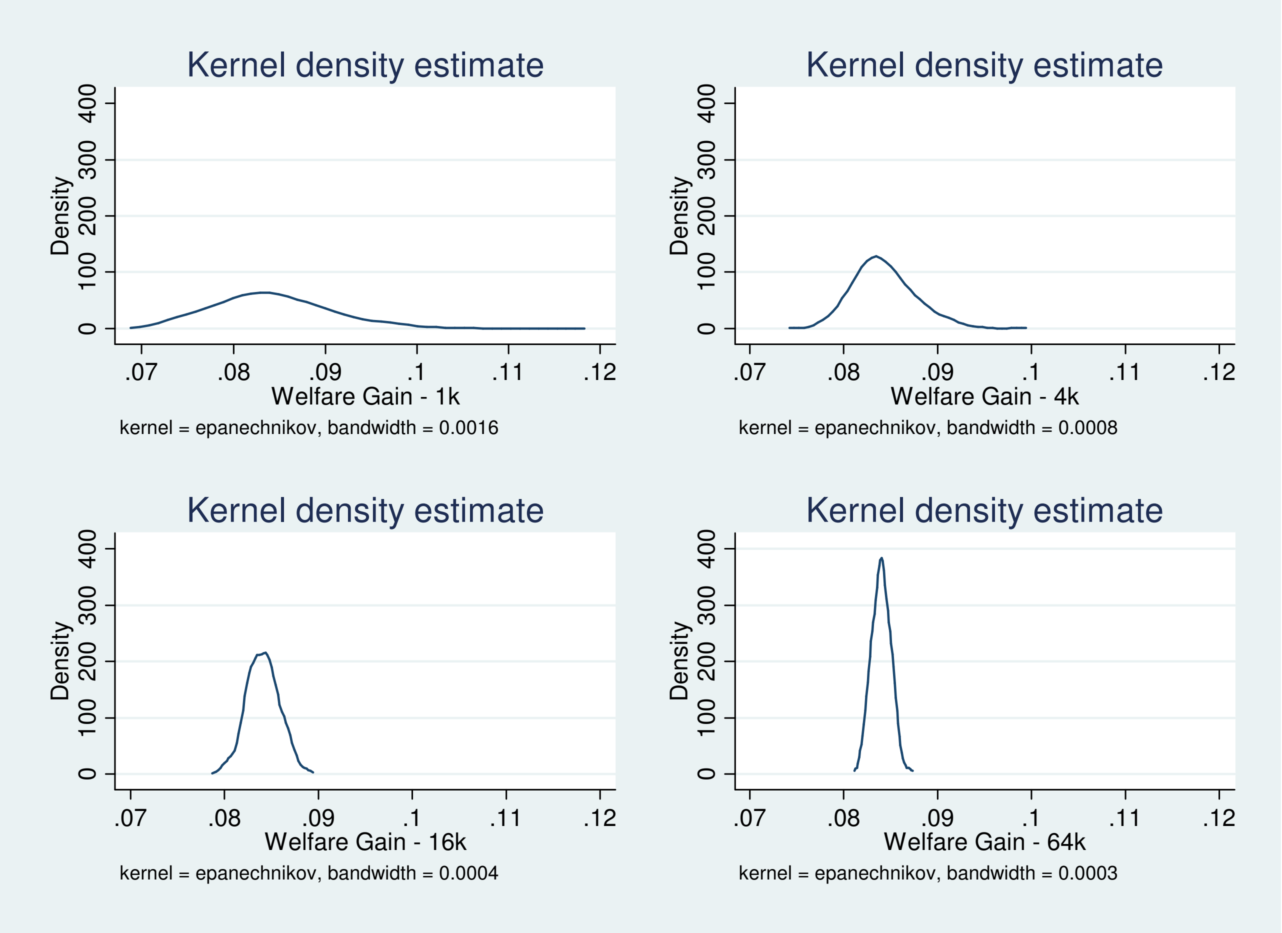

Figure 7: Welfare Gains, Experiment 2a

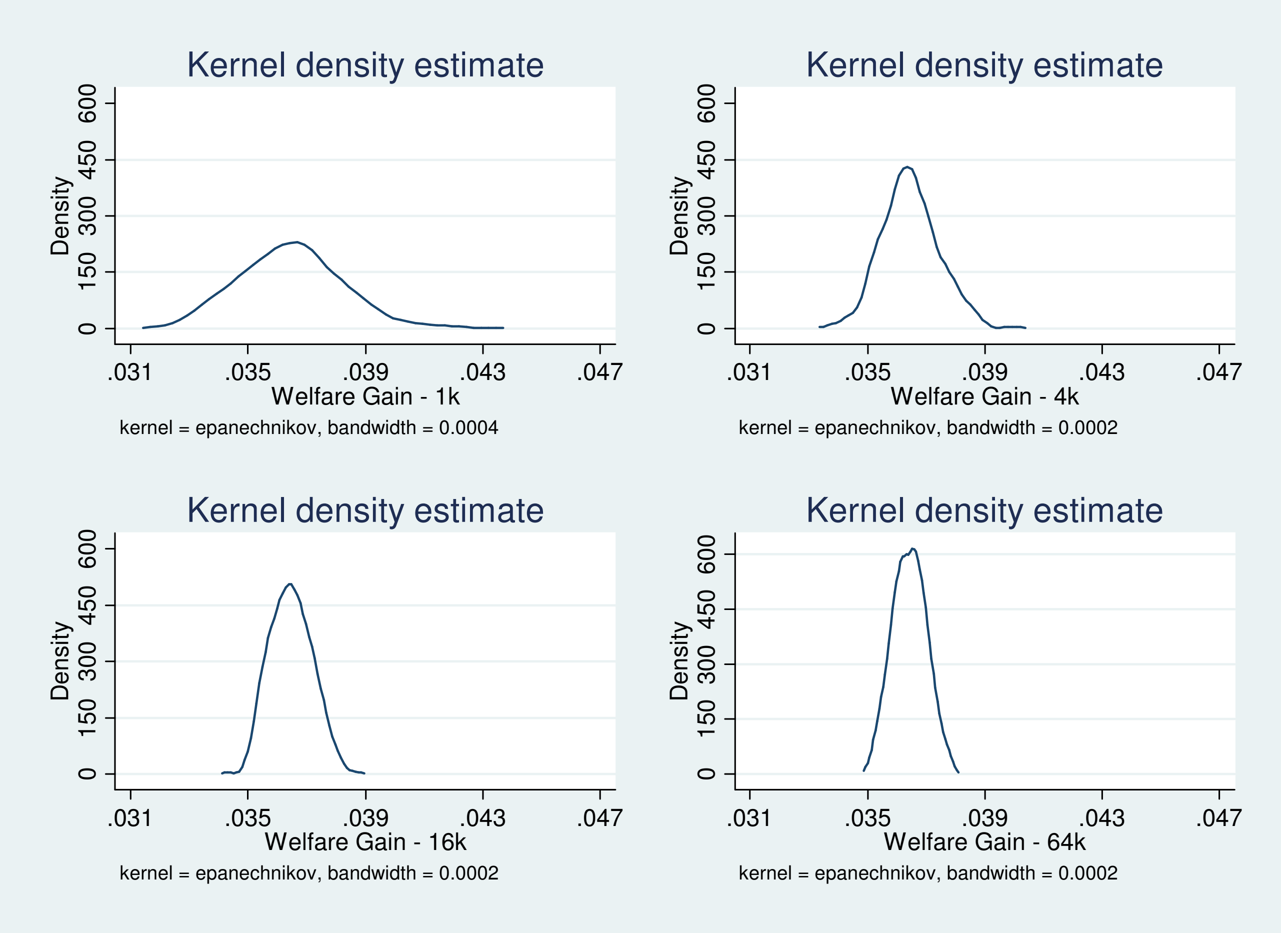

Figure 8: Welfare Gains, Experiment 2b

Sample Size	*Min*	*Max*	*Mean*	*Med*	*C.v.*
$\pi\,(e', e)$ (DGP=.9182)					
1k	.8545	.9583	.9169	.9181	.0189
4k	.8764	.9396	.9176	.9182	.0104
16k	.9022	.9319	.9183	.9183	.0051
64k	.9113	.9244	.9184	.9185	.0025
$\pi\,(e', u)$ (DGP=.7879)					
1k	.5137	1	.7968	.7897	.1271
4k	.6329	1	.7909	.7892	.0730
16k	.7109	.8810	.7874	.7879	.0361
64k	.7525	.8355	.7870	.7865	.0173

Table 1: Calibration - "Simulated" Parameters: Descriptive Statistics

Parameter	*Value*	*Target*
Model Period	*Half year*	
σ - *CRRA*	1.5 $\{1.0, 2.0\}$	*Micro Estimates*
b - *Borrowing limit*	4 $\{2, 0\}$	*Two* $\{$*one, zero*$\}$ *period income*
δ - *Capital depreciation rate*	0.04	*Investment share of output* $\approx 22.5\%$
α - *Capital share*	0.34	*Capital share of output* $= 34\%$
β - *Rate of time preference*	0.97979	*Interest rate* $\approx$ *4% (annual)*

Table 2: Calibration - Fixed Parameters

Parameter	*Value*	*Policy*
ϕ	0.5	*UI replacement rate*
$\overline{G}$	0.42	*Government Expenditure (20% of initial S.S. Output)*
$(\overline{G/Y})$	0.2	*Government Expenditure (Fixed at 20% of S.S. Output)*
τ_a	0.3	*Capital income tax*

Table 3: Policy Parameters

Welfare Change (%)	*Min*	*Max*	*Mean*	*Med*	*C.v.*
*Experiment 1a (true=*0.556*)*					
1k	0.453	0.629	0.551	0.552	0.052
4k	0.479	0.604	0.554	0.554	0.035
16k	0.509	0.600	0.555	0.556	0.031
64k	0.508	0.594	0.555	0.556	0.030
*Experiment 1b (true=*0.370*)*					
1k	0.308	0.418	0.364	0.364	0.054
4k	0.310	0.419	0.368	0.368	0.048
16k	0.328	0.410	0.369	0.368	0.044
64k	0.334	0.410	0.369	0.369	0.042
*Experiment 2a (true=*8.415*)*					
1k	7.043	11.668	8.426	8.364	0.076
4k	7.497	9.862	8.425	8.390	0.039
16k	7.910	8.894	8.407	8.403	0.021
64k	8.138	8.704	8.397	8.396	0.012
*Experiment 2b (true=*3.645*)*					
1k	3.185	4.326	3.645	3.645	0.049
4k	3.360	4.013	3.646	3.640	0.027
16k	3.429	3.877	3.647	3.644	0.020
64k	3.507	3.796	3.644	3.643	0.016

Table 4: Monte Carlo Results - Welfare Effects

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Appendix A - The Model and its Recursive Representation

6 Stationary Equilibrium

First the problem of employed and unemployed workers is defined. The individual state variables are the employment status $s \in \mathcal{S} = \{e, u\}$, and asset holdings $a \in \mathcal{A} = [-b, \overline{a}]$.[19] The stationary distribution of employed agents is denoted by $\mu_e(a)$ whereas the distribution of unemployed agents is $\mu_u(a)$.

6.1 Problem of the agents

In this Section first the problem of the agents in their recursive representation is defined, then I provide a formal definition of the equilibrium concept used in this model, the recursive competitive equilibrium.

6.1.1 Problem of the unemployed workers

The value function of an unemployed agent whose current asset holdings are equal to a is denoted with $V_u(a)$. The problem of these agents can be represented as follows:

$$V_u(a) = \max_{c,a'} \{u(c) + \beta [\pi(u', u) V_u(a') + (1 - \pi(u', u)) V_e(a')]\} \tag{5}$$

$$s.t.$$

$$c + a' = [1 + (1 - \tau_a) r] a + (1 - \tau_w) \phi w$$

$$a_0 \; given, \quad c \geq 0, \quad a' > -b$$

Unemployed agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period they can still be unemployed, with probability $\pi(u', u)$, or they can find a job and enjoy an employment spell, with probability $1 - \pi(u', u)$.

While unemployed these workers receive an unemployment benefit equal to ϕw. The unemployment benefit consists of the replacement rate ϕ (a policy parameter) of the wage w an employed worker is

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[19] A formal argument proving that $\overline{a} < \infty$ appears for a similar economy in Huggett (1993).

receiving. These agents pay both proportional labor and capital income taxes, τ_a and τ_w, respectively. Finally, they are subject to an exogenous borrowing constraint, $b \geq 0$.

6.1.2 Problem of the employed workers

The recursive representation of the problem of an employed worker is as follows:

$$V_e(a) = \max_{c,a'} \{u(c) + \beta [\pi(u', e) V_u(a') + (1 - \pi(u', e)) V_e(a')]\} \tag{6}$$

$$s.t.$$

$$c + a' = [1 + (1 - \tau_a) r] a + (1 - \tau_w - \tau_u) w$$

$$a_0 \textit{ given}, \quad c \geq 0, \quad a' > -b$$

Employed agents enjoy utility from consumption and face some uncertain events in the future. In the next period they can still be employed, or they can experience a job separation and begin an unemployment spell. Finally, notice that these agents pay labor and capital income taxes, together with τ_u, which stands for an additional proportional labor income tax paid by the agents currently employed to finance the unemployment benefit scheme.

6.2 Recursive Stationary Equilibrium

Definition 1 *For given policies $\{\tau_a, G, \phi\}$ a recursive stationary equilibrium is a set of decision rules $\{c_e(a), c_u(a), a'_e(a), a'_u(a)\}$, value functions $\{V_e(a), V_u(a)\}$, prices $\{r, w\}$, proportional taxes $\{\tau_u, \tau_w\}$ and a set of stationary distributions $\{\mu_e(a), \mu_u(a)\}$ such that:*

- Given relative prices $\{r, w\}$, proportional taxes $\{\tau_a, \tau_u, \tau_w\}$ and unemployment benefits ϕw, the individual policy functions $\{c_e(a), c_u(a), a'_e(a), a'_u(a)\}$ solve the household problems (5)-(6), and $\{V_e(a), V_u(a)\}$ are the associated value functions.
- Given relative prices $\{r, w\}$, K/L solves the firm's problem and satisfies (1)-(2).
- The labor market is in flow equilibrium, that is the measure of people becoming unemployed is identical to the measure of people finding a job

$$\int_{\mathcal{A}} \pi(u', e) \, d\mu_e(a) = \pi(u', e) L = \pi(e', u)(1 - L) = \int_{\mathcal{A}} \pi(e', u) \, d\mu_u(a)$$

- The asset market clears

$$K = \int_{\mathcal{A}\times\mathcal{S}} a'_s(a) d\mu_s(a)$$

- The goods market clears

$$F(K, L) = C + I + G = \int_{\mathcal{A}\times\mathcal{S}} c_s(a) d\mu_s(a) + \delta K + G$$

- The unemployment benefit scheme is self-financing, i.e. the proportional tax τ_u satisfies

$$\tau_u = \frac{\int_{\mathcal{A}} \phi w d\mu_u(a)}{\int_{\mathcal{A}} w d\mu_e(a)} = \frac{\phi(1-L)}{L} \tag{7}$$

- The government's budget is balanced, that is tax revenues from (capital and labor) income taxation are equal to government purchases G

$$G = \int_{\mathcal{A}\times\mathcal{S}} \tau_a r a d\mu_s(a) + \int_{\mathcal{A}} \tau_w w d\mu_e(a) + \int_{\mathcal{A}} \tau_w \phi w d\mu_u(a)$$

- It can be shown that, given G and τ_a, the proportional tax τ_w satisfies

$$\tau_w = \left(\frac{1}{1-\alpha}\right)\left(\frac{L}{L+\phi(1-L)}\right)\left[\frac{G}{Y} - \tau_a\left(\frac{\alpha r}{r+\delta}\right)\right] \tag{8}$$

$$\text{with } \frac{G}{Y} = \begin{cases} \text{either } \left(\frac{\overline{G}}{Y}\right), \overline{G} = \text{constant, case } \mathit{2a} \\ \text{or } \overline{\left(\frac{G}{Y}\right)}, \overline{\left(\frac{G}{Y}\right)} = \text{constant, case } \mathit{2b} \end{cases}$$

- The stationary distributions $\{\mu_e(a), \mu_u(a)\}$ satisfy

$$\mu_e(a') = \int_{a:a'_u(a)=a'} \pi(e', u) d\mu_u(a) + \int_{a:a'_e(a)=a'} \pi(e', e) d\mu_e(a) \tag{9}$$

$$\mu_u(a') = \int_{a:a'_u(a)=a'} \pi(u', u) d\mu_u(a) + \int_{a:a'_e(a)=a'} \pi(u', e) d\mu_e(a) \tag{10}$$

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by the above two equations (9)-(10).

- The welfare measures W, W_e and W_u are utilitarian, i.e. they weight agents' utilities by their mass in the steady-state

$$W = \int_{\mathcal{A}\times\mathcal{S}} V_s(a) d\mu_s(a),\ W_e = \int_{\mathcal{A}} V_e(a) d\mu_e(a),\ W_u = \int_{\mathcal{A}} V_u(a) d\mu_u(a) \tag{11}$$

- The consumption based welfare measure $\widehat{\rho}$ is the percentage increase in consumption in all states of the world that makes welfare in the counterfactual economy $W^1(\rho)$ equal to welfare in the baseline one W^0

$$\begin{aligned}
W^0 &= W^1(\rho) \\
\int_{\mathcal{A}\times\mathcal{S}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c^0(.)_t\right]^{1-\sigma}-1}{1-\sigma} d\mu_s^0(a) &= \int_{\mathcal{A}\times\mathcal{S}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[(1+\rho)\, c^1(.)_t\right]^{1-\sigma}-1}{1-\sigma} d\mu_s^1(a) \\
\widehat{\rho} &= \left(\frac{W^1}{W^0}\right)^{\frac{1}{1-\sigma}} - 1 \qquad (12)
\end{aligned}$$

Appendix B - Computation

- All codes solving the economies and simulating samples of agents were written in the FORTRAN 95 language, relying on the Intel Fortran Compiler, build 11.1.048 (with the IMSL library). They were compiled selecting the O3 option (maximize speed), and without automatic parallelization. They were run on a 64-bit PC platform, running Windows 7 Professional Edition, with an Intel i7-870 Quad Core 2.93 Ghz processor.

- The 500 Monte Carlo replications take up to 100 hours to complete. Notice that $1,000$ equilibria have to be computed, and typically from 10 to 14 iterations on the interest rate are needed to find each equilibrium.

- The sample simulations are performed outside the numerical solution of the model. I simulate for 500 times and for $3,000$ periods (to ensure stationarity) a sample of individuals from the true stochastic process. These simulations take from 5 minutes (with $1,000$ individuals) to 5 hours (with $64,000$ ones). For a given sample size, in order to avoid sampling variability affecting the results, I rely on the same sample realizations to parameterize and solve all models.

- In the actual solution of the model I need to discretize the continuous state variable a (the employment status is already discrete). I rely on an unevenly spaced grid, with the distance between two consecutive points increasing geometrically. This is done to allow for a high precision of the policy rules at low values of a, where the change in curvature is more pronounced.

- The model is solved with a time iteration procedure on the set of euler equations. In order to allow for very good approximations of the policy functions, I use $2,001$ grid points on the asset space, the lowest value being the borrowing constraint b and the highest one being a value $a_{\max} > \overline{a}$ high enough for the saving functions to cut the 45 degree line ($a_{\max} = 2,200$ for the half year model). Notice that I do not restrict agents' asset holdings to belong to a discrete set. As for the approximation method, I rely on a linear approximation scheme for the saving and consumption functions, for values of a falling outside the grid.

- A collocation method is implemented. I look for the policy functions such that the residuals of the Euler equations are (close to) zero at the collocation points (the asset grid points). For all possible combinations of state variables I need to solve a non linear equation. A time iteration scheme is applied to get the policy functions, i.e. the first order conditions with respect to a' and the envelope condition deliver a set of euler equations, whose unknowns are the policy functions: $a'_e(a)$, and $a'_u(a)$. I start from a set of guesses, $a'_e(a)_0$, and $a'_u(a)_0$, and keep on iterating until a fixed point is reached, i.e. until two successive iterations satisfy:

$$\underset{a}{Sup}\,|a'_e(a)_{n+1} - a'_e(a)_n| < 10^{-8} \text{ and } \underset{a}{Sup}\,|a'_u(a)_{n+1} - a'_u(a)_n| < 10^{-8}$$

Typically, around 900 iterations are needed to reach a fixed point.

- The stationary distributions are computed relying on their definitions (9)-(10). I start from a set of guesses, $\mu_e(a)_0$, and $\mu_u(a)_0$ and keep on iterating until convergence, i.e. until two successive iterations satisfy:

  $$\underset{a}{Sup}\,|\mu_e(a)_{n+1} - \mu_e(a)_n| < 10^{-10} \text{ and } \underset{a}{Sup}\,|\mu_u(a)_{n+1} - \mu_u(a)_n| < 10^{-10}$$

  Between $10,000$ and $140,000$ iterations are needed to reach a fixed point. Between grid-points, I use a linear approximation scheme.

- The asset market is in equilibrium when the current guess for the interest rate r_0 achieves a capital excess demand which is less than 0.1% of the market size. In turn, this implies that the excess demand in the final good market is always less than 0.1% of the market size. This tolerance level could seem a minor aspect of the analysis. However, it is quite important. It was found that, for the policies with a quantitatively small true welfare effect, relying on a loose criterion convergence was preventing the distribution of welfare effects to display convergence when increasing the sample size.

- The value functions are computed relying on the Bellman equations (5)-(6). I start from a set of guesses, $V_e(a)_0$, and $V_u(a)_0$ and by using the optimal consumption and saving functions already obtained I keep on iterating until convergence, i.e. until two successive iterations satisfy:

  $$\underset{a}{Sup}\,|V_e(a)_{n+1} - V_e(a)_n| < 10^{-10} \text{ and } \underset{a}{Sup}\,|V_u(a)_{n+1} - V_u(a)_n| < 10^{-10}$$

  Typically, around $1,150$ iterations are needed to reach a fixed point.

- The welfare measures W, W_e and W_u are just the numerical integral of the value functions, integrated with respect to the steady state distributions. For more details, see Rios-Rull (1999).

- All convergence criteria are quite strict, and more stringent than what is normally used for these models. However, they are needed in order to avoid numerical errors systematically biasing the computation of the equilibria and their associated welfare measures. Some experimentation showed that, for example, asset grids with less than $1,001$ points lead to welfare effects that differed in a non trivial way from the true ones, when these were less than 1%. The same comment applies to virtually all other convergence criteria. The required level of precision, however, drastically impacts the computational time.

Appendix C - Monte Carlo Algorithm

The computational procedure used to solve the Monte Carlo experiments can be represented by the following algorithm:

1. Simulate 500 times a sample of individuals of given size n and compute two statistics: the average unemployment rate and the average unemployment duration; store them.
2. Read the two simulated moments and match them exactly with the markov chain probabilities.
3. Generate a discrete grid over the asset space $[-b, ..., a_{\max}]$.
4. Start the loop for the benchmark economy.
5. Get the employment level L.
6. Set the capital tax τ_a.
7. Guess the interest rate r_0.
8. Get the capital demand K_0, wages w_0, and unemployment benefits ϕw_0.
9. Get the equilibrium taxes τ_u, τ_w.
10. Get the saving functions $a'_e(a), a'_u(a)$.
11. Get the stationary distributions $\mu_e(a), \mu_u(a)$.
12. Get the aggregate capital supply.
13. Check asset market clearing; Get r_1.
14. Update r'_0 (with a bi-section method).
15. Iterate until asset market clearing.
16. Get the consumption functions $c'_e(a), c'_u(a)$.
17. Check final good market clearing.
18. Compute the equilibrium value functions $V_e(a), V_u(a)$ and the ex-ante welfare W, W_e and W_u.
19. Start the loop for the counterfactual economy (i.e. under the new policy regime) and repeat steps $5 - 18$.
20. Save the output and repeat from step 2 for 500 times.

Appendix D - Additional Results and Figures

Welfare Change Employed (%)	*Min*	*Max*	*Mean*	*Med*	*C.v.*
Experiment 1a (true=0.667*)*					
1k	0.515	0.776	0.660	0.662	0.068
4k	0.554	0.735	0.664	0.664	0.040
16k	0.616	0.720	0.665	0.665	0.029
64k	0.615	0.709	0.666	0.666	0.026
Experiment 1b (true=0.478*)*					
1k	0.372	0.558	0.472	0.475	0.070
4k	0.385	0.548	0.477	0.477	0.049
16k	0.429	0.526	0.478	0.478	0.036
64k	0.439	0.526	0.478	0.478	0.033
Experiment 2a (true=8.363*)*					
1k	6.998	11.597	8.375	8.315	0.077
4k	7.451	9.812	8.374	8.340	0.039
16k	7.858	8.839	8.356	8.349	0.021
64k	8.086	8.651	8.346	8.344	0.012
Experiment 2b (true=3.619*)*					
1k	3.169	4.298	3.620	3.619	0.049
4k	3.334	3.981	3.620	3.611	0.027
16k	3.406	3.852	3.621	3.618	0.020
64k	3.478	3.770	3.618	3.617	0.016

Table 5: Monte Carlo Results - Welfare Effects for the Employed

Welfare Change Unemployed (%)	*Min*	*Max*	*Mean*	*Med*	*C.v.*
Experiment 1a (true=-0.528)					
1k	-1.555	-0.111	-0.542	-0.527	0.083
4k	-0.987	-0.159	-0.533	-0.525	0.050
16k	-0.721	-0.332	-0.533	-0.528	0.036
64k	-0.619	-0.408	-0.532	-0.531	0.032
Experiment 1b (true=-0.699)					
1k	-1.746	-0.294	-0.715	-0.695	0.342
4k	-1.156	-0.317	-0.704	-0.696	0.192
16k	-0.903	-0.493	-0.703	-0.699	0.100
64k	-0.801	-0.607	-0.702	-0.702	0.052
Experiment 2a (true=8.939)					
1k	7.609	12.178	8.950	8.893	0.070
4k	8.060	10.279	8.950	8.917	0.036
16k	8.464	9.419	8.932	8.932	0.019
64k	8.662	9.222	8.921	8.920	0.011
Experiment 2b (true=3.909)					
1k	3.395	4.526	3.911	3.916	0.044
4k	3.640	4.295	3.911	3.906	0.024
16k	3.681	4.117	3.911	3.910	0.018
64k	3.755	4.060	3.908	3.909	0.015

Table 6: Monte Carlo Results - Welfare Effects for the Unemployed

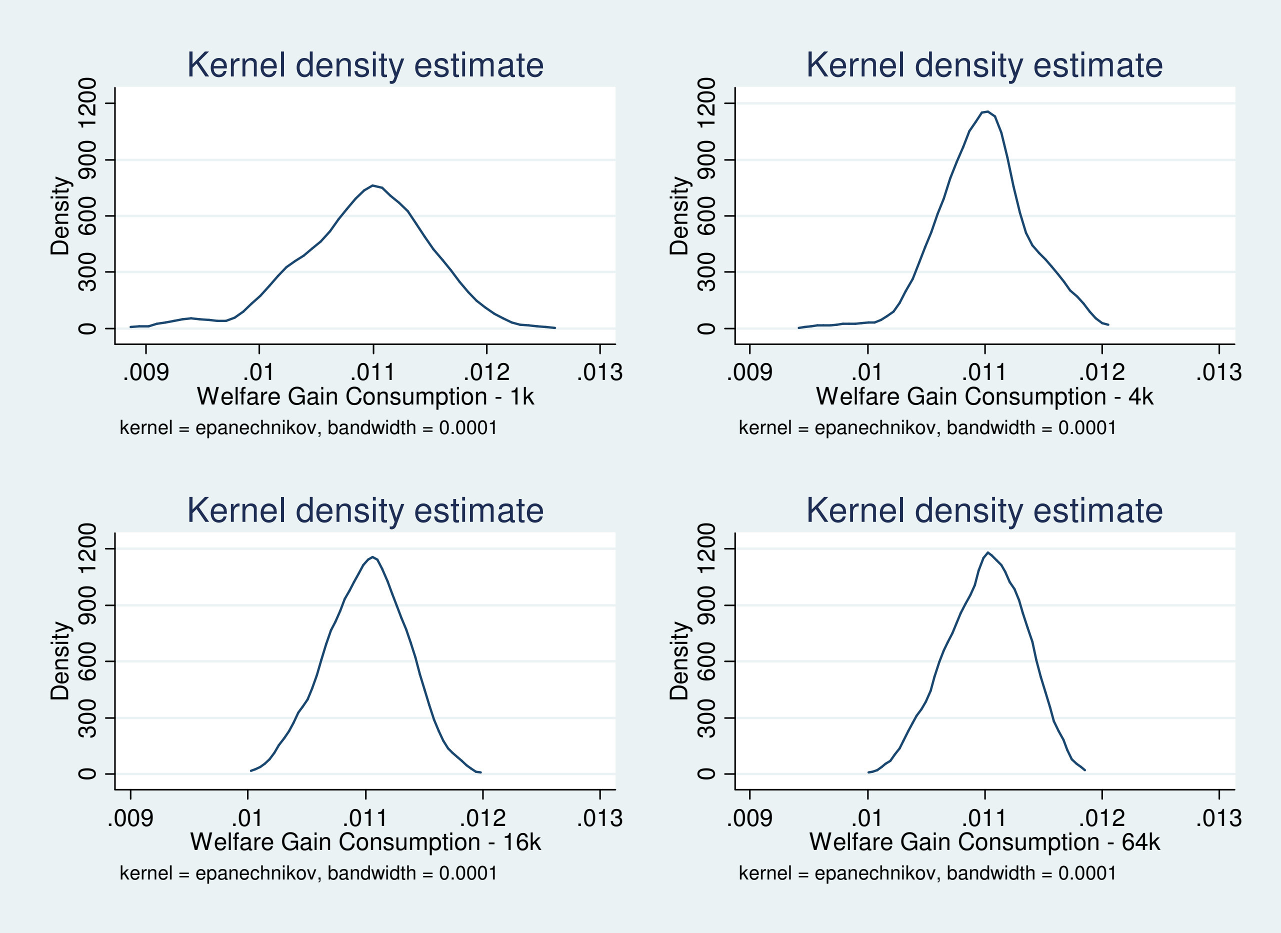

Figure 9: Welfare Gains Consumption, Experiment 1a $- b = 4$

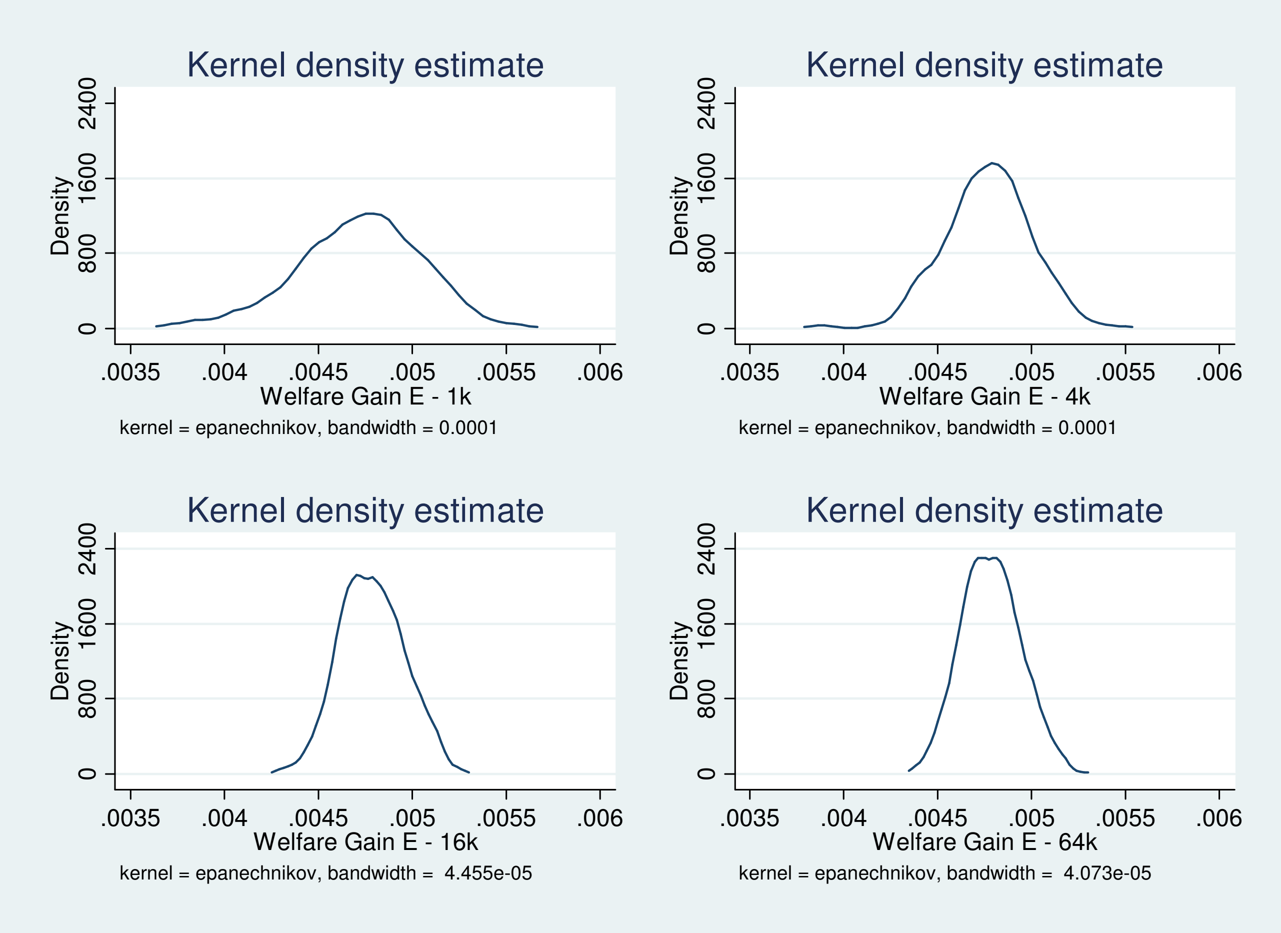

Figure 10: Welfare Gains Employed, Experiment 1b $- b = 0$

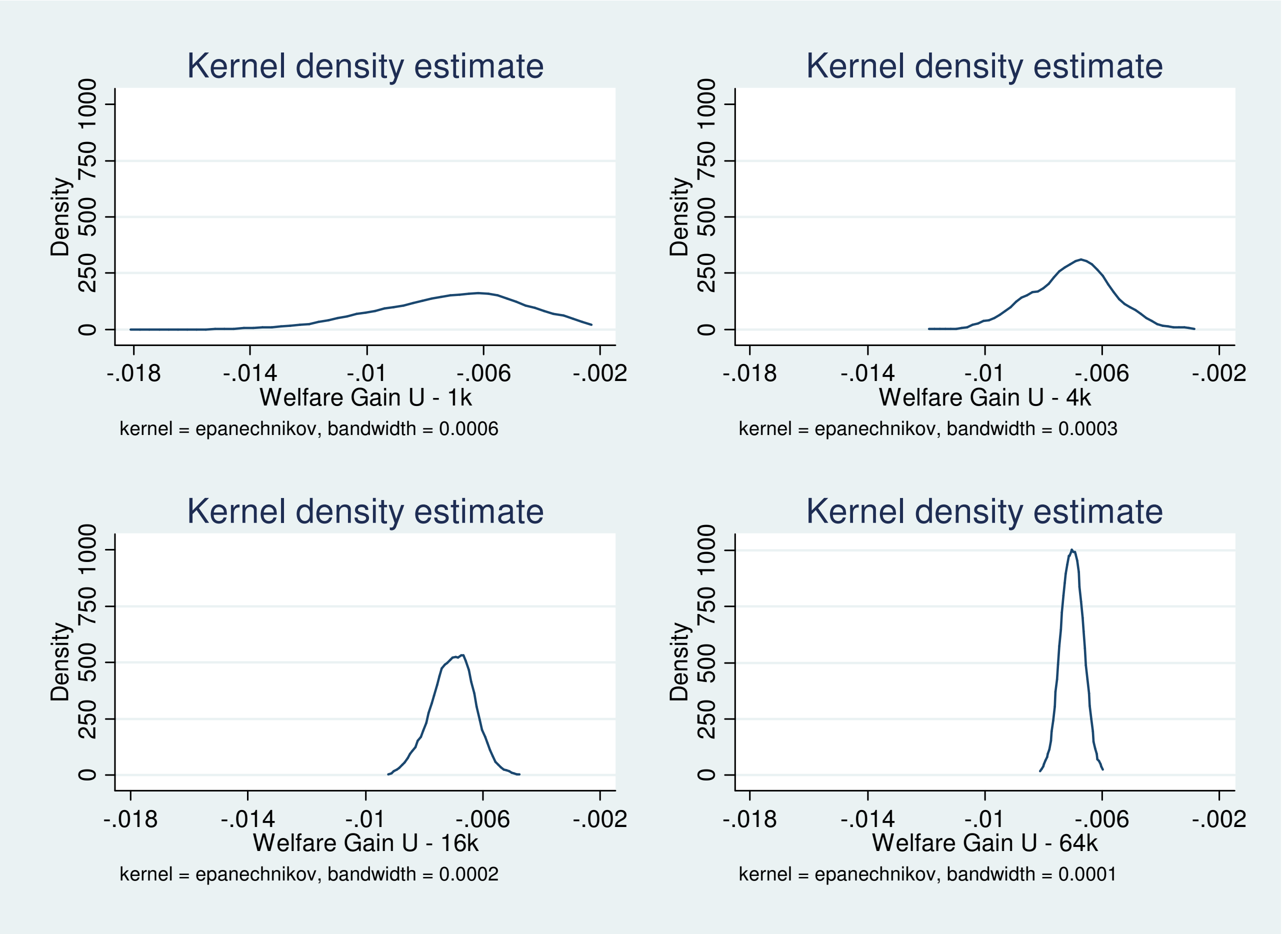

Figure 11: Welfare Gains Unemployed, Experiment 1b $-$ $b = 0$

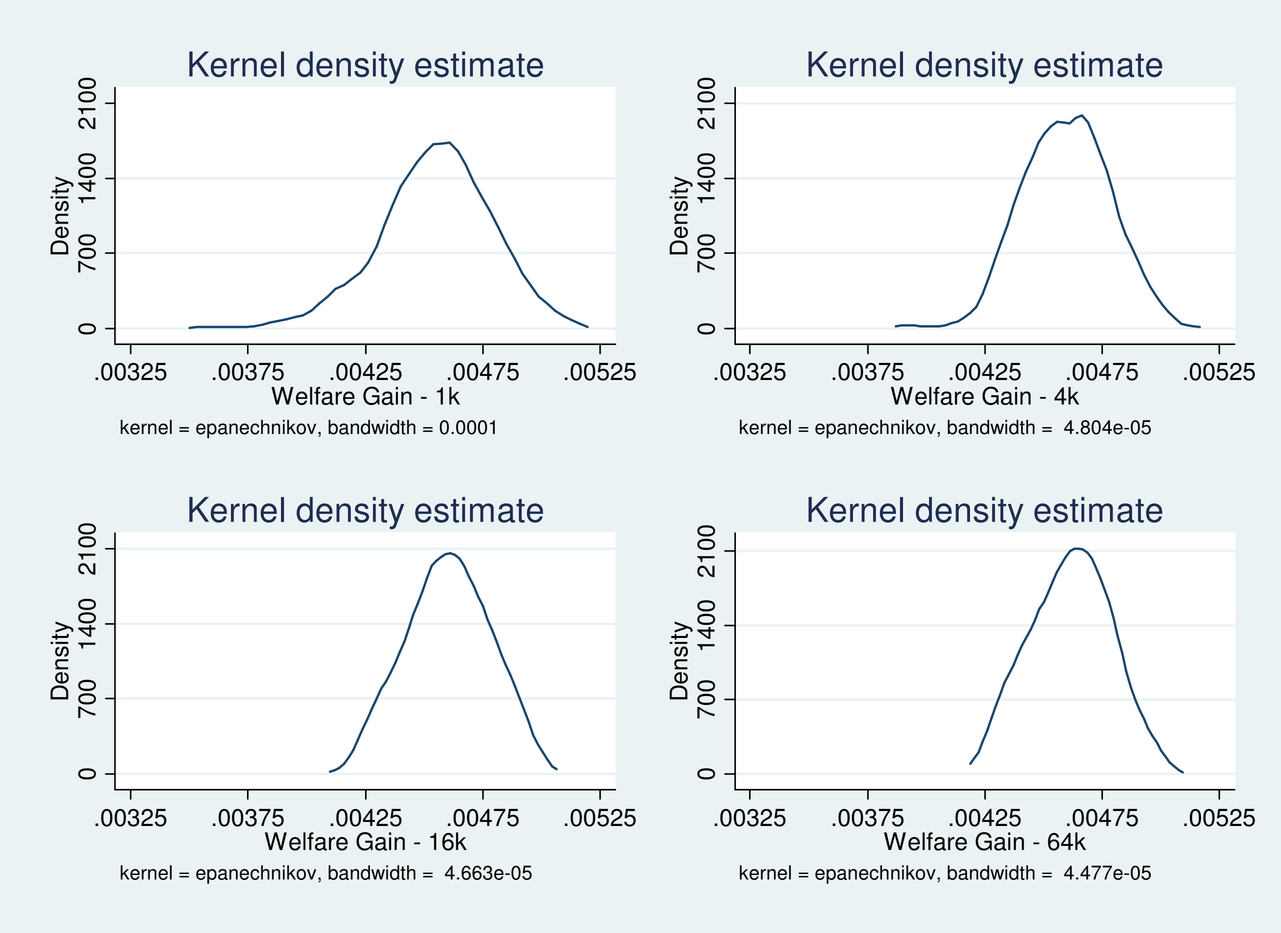

Figure 12: Welfare Gains, Experiment $1 - b = 2$

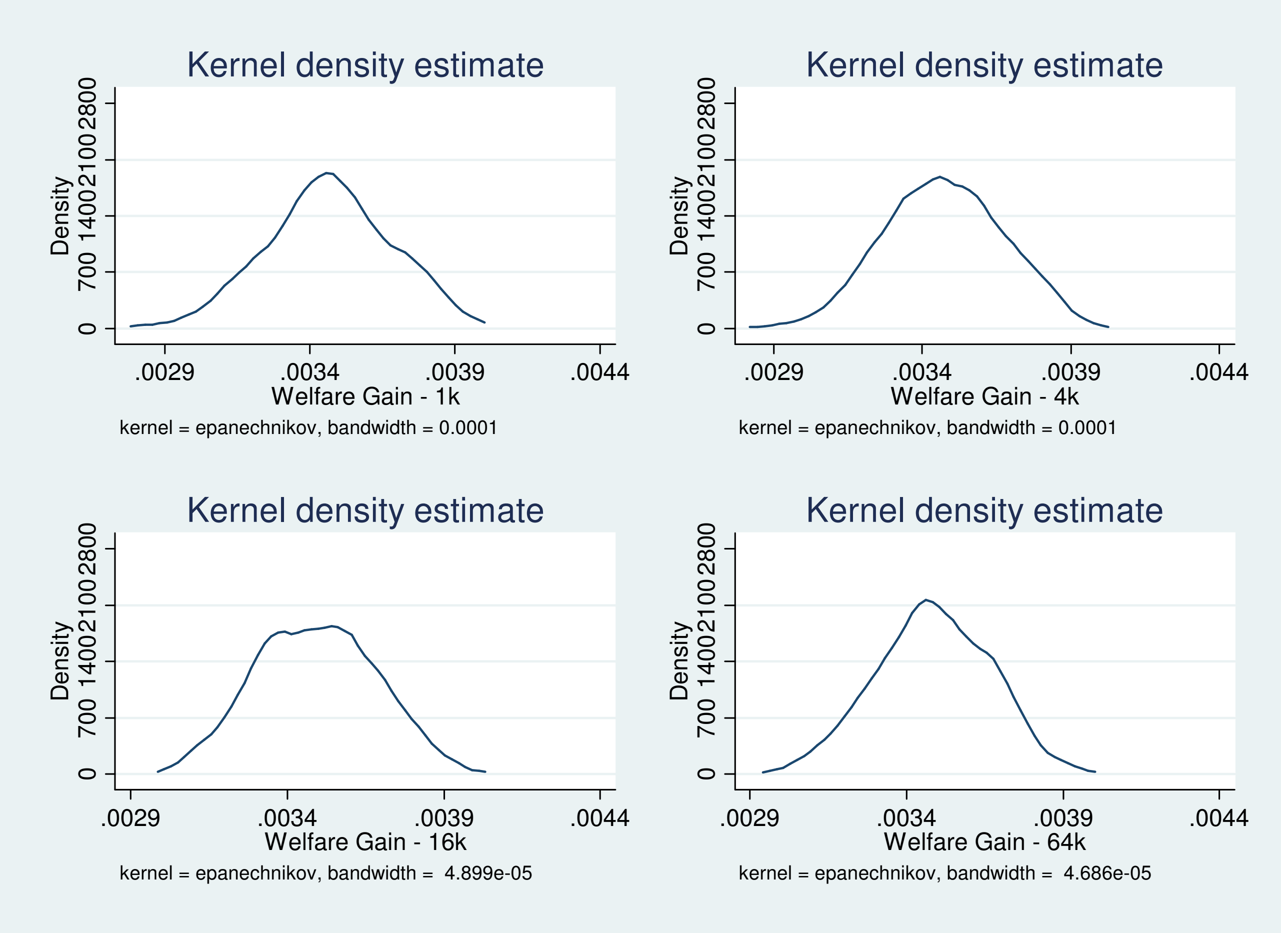

Figure 13: Welfare Gains, Experiment $1 - b = 2$ and $\sigma = 1$

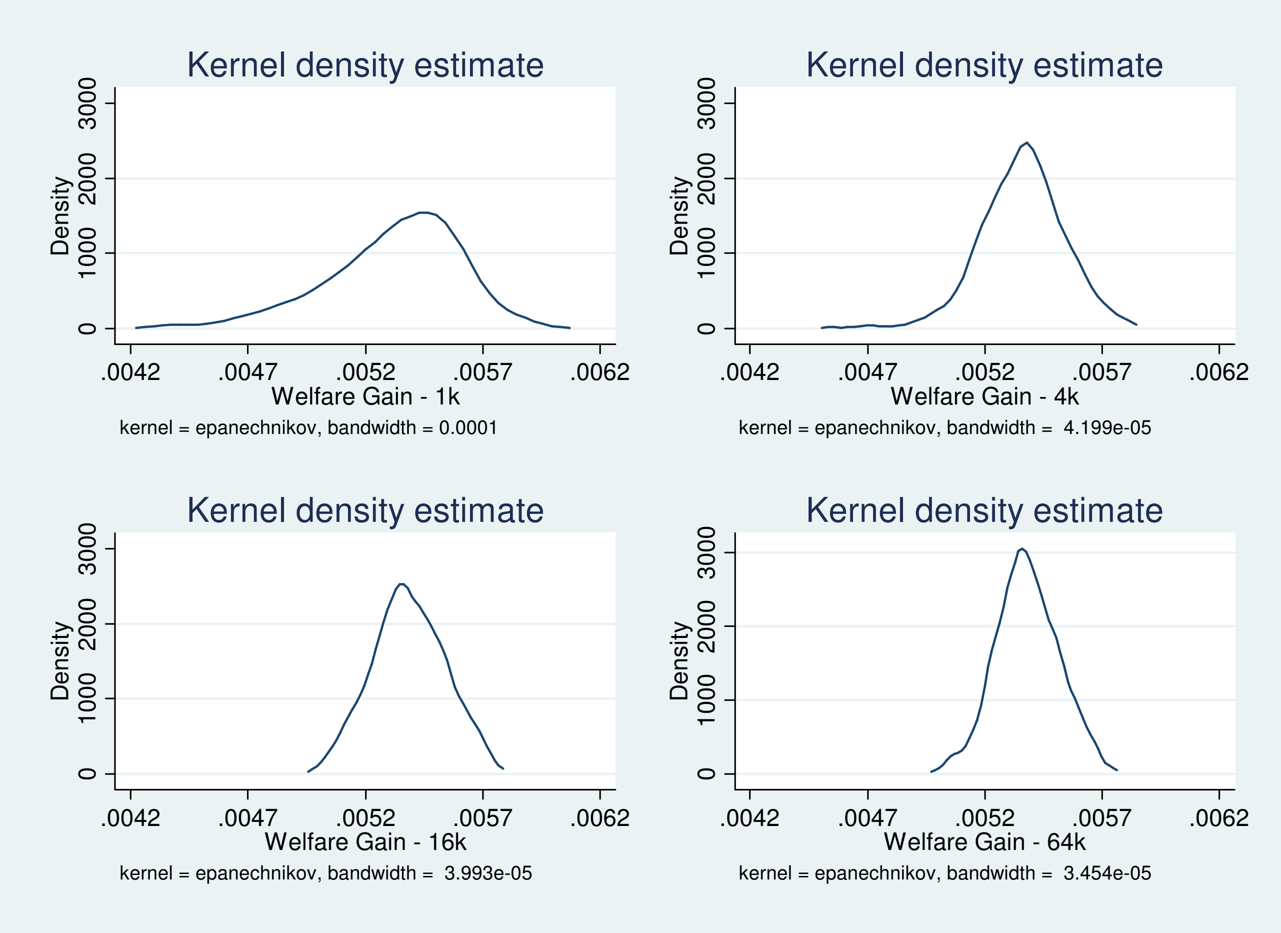

Figure 14: Welfare Gains, Experiment $1 - b = 2$ and $\sigma = 2$

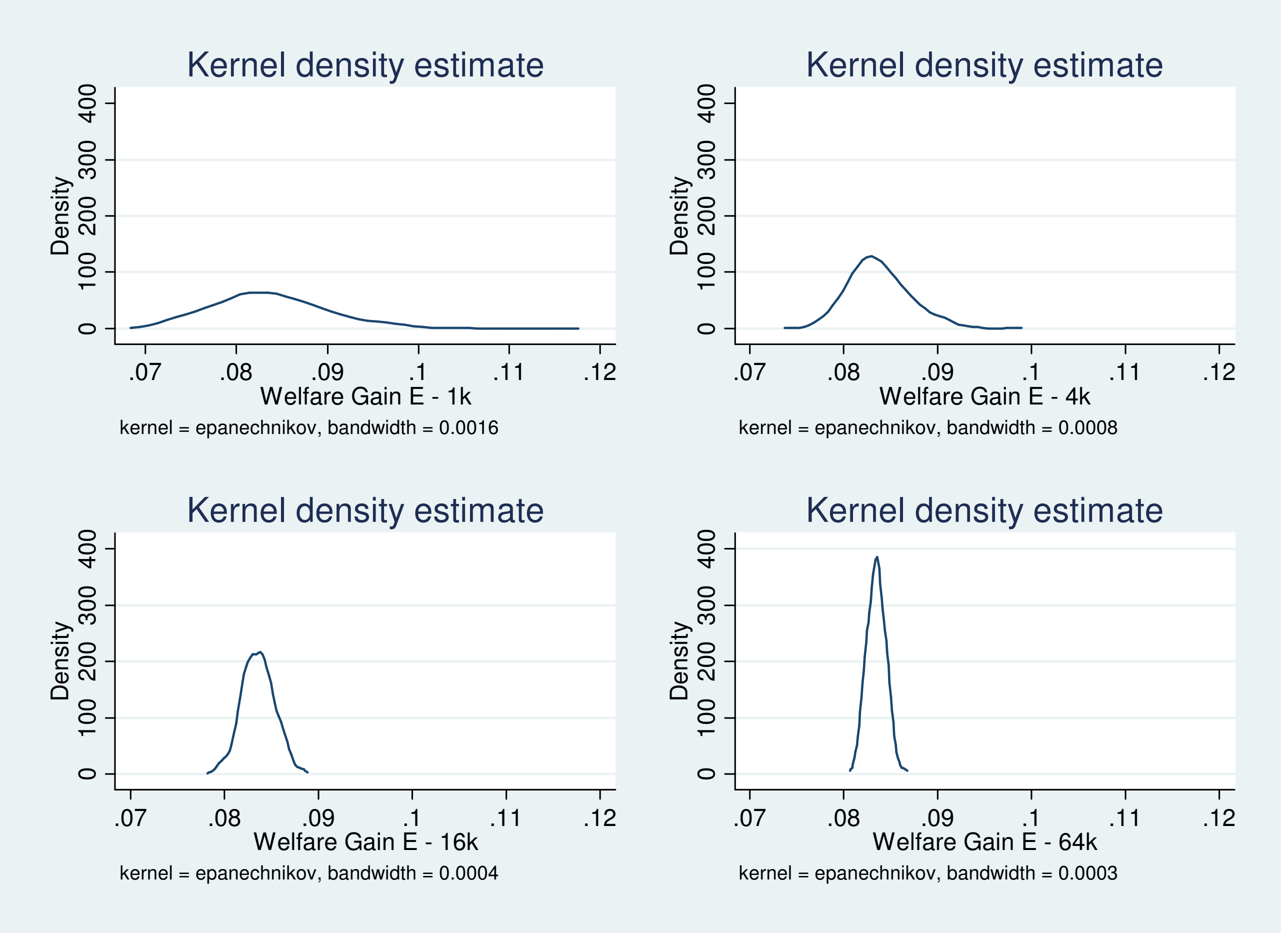

Figure 15: Welfare Gains Employed, Experiment 2a

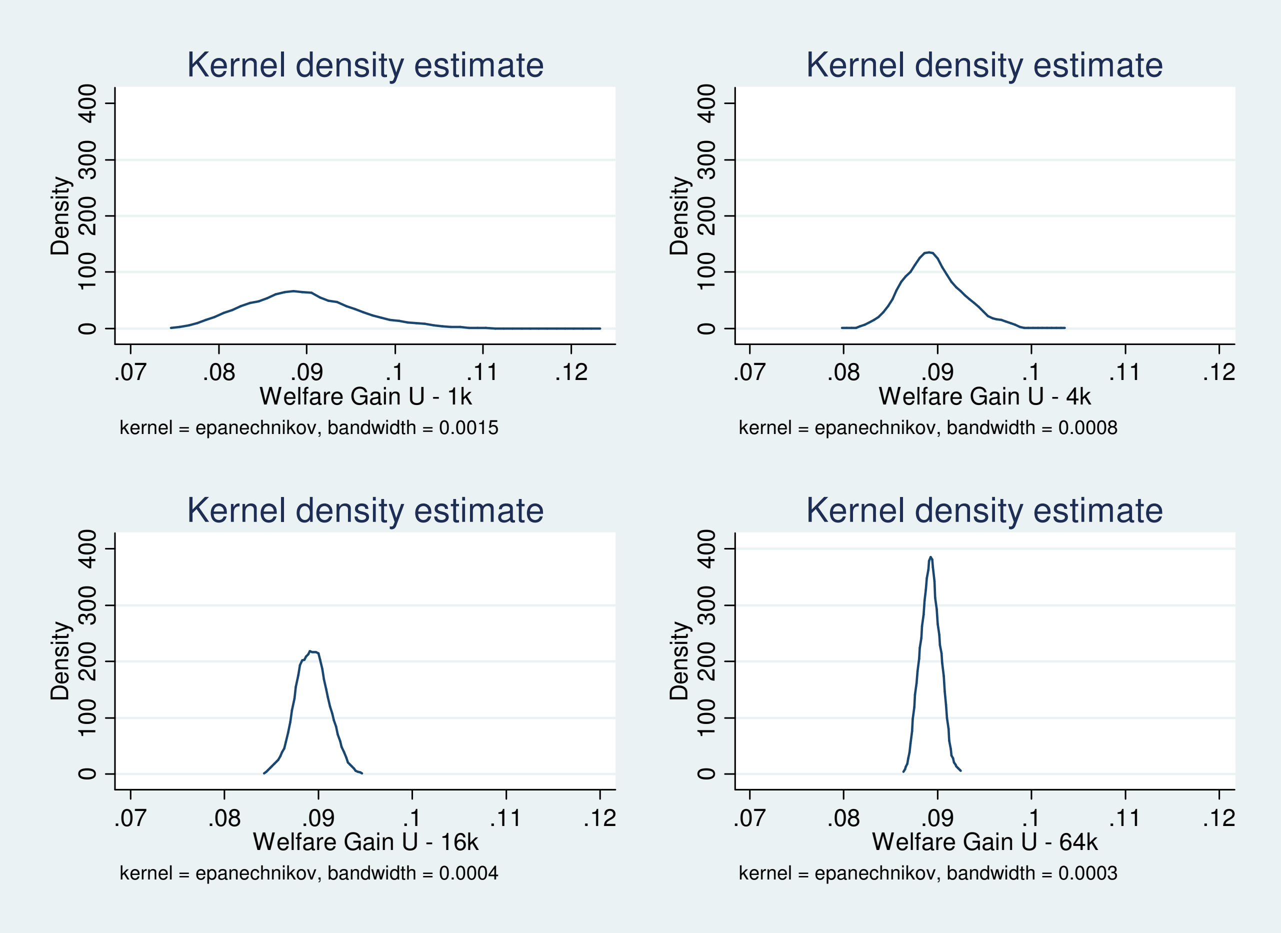

Figure 16: Welfare Gains Unemployed, Experiment 2a

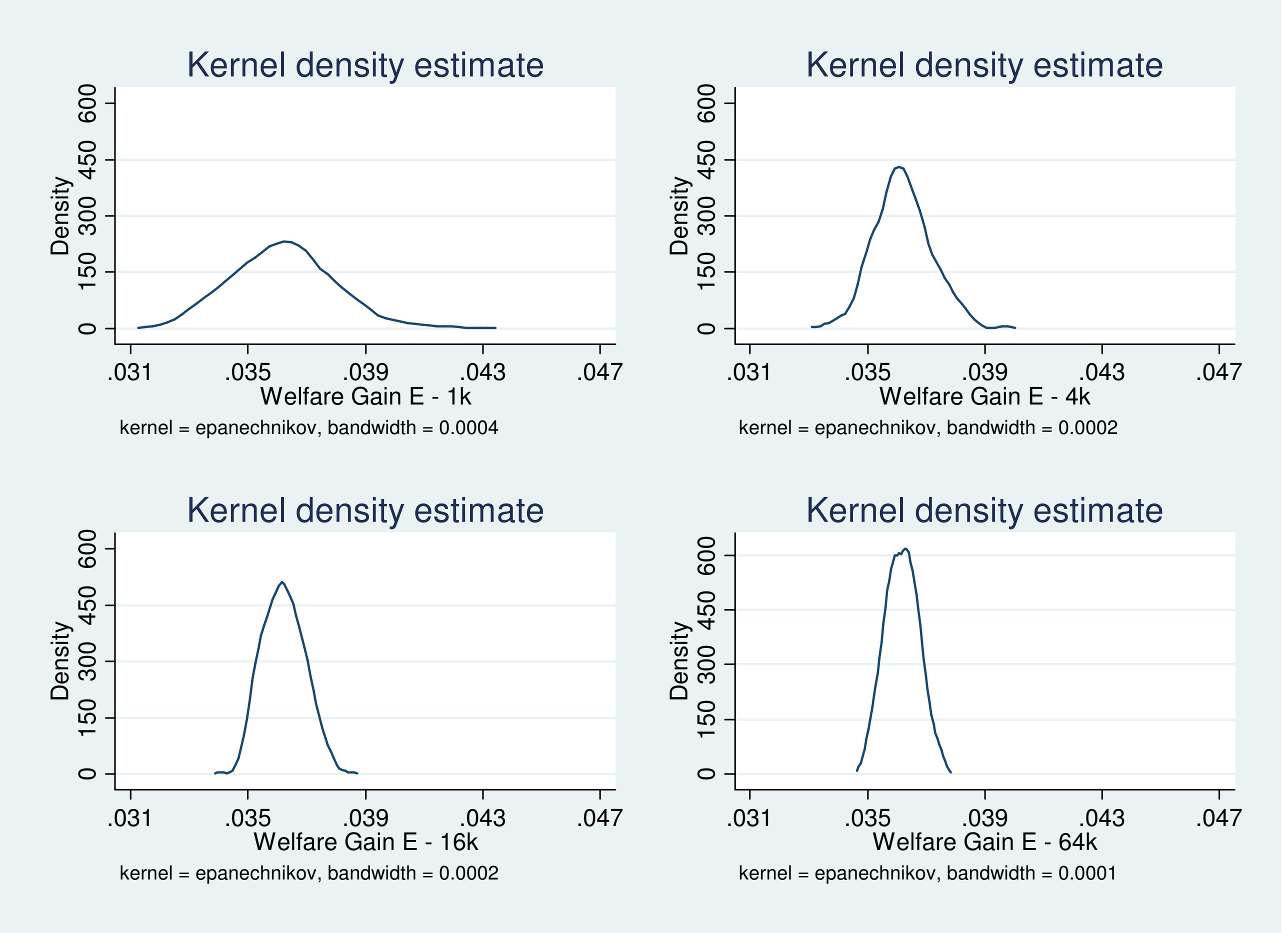

Figure 17: Welfare Gains Employed, Experiment 2b

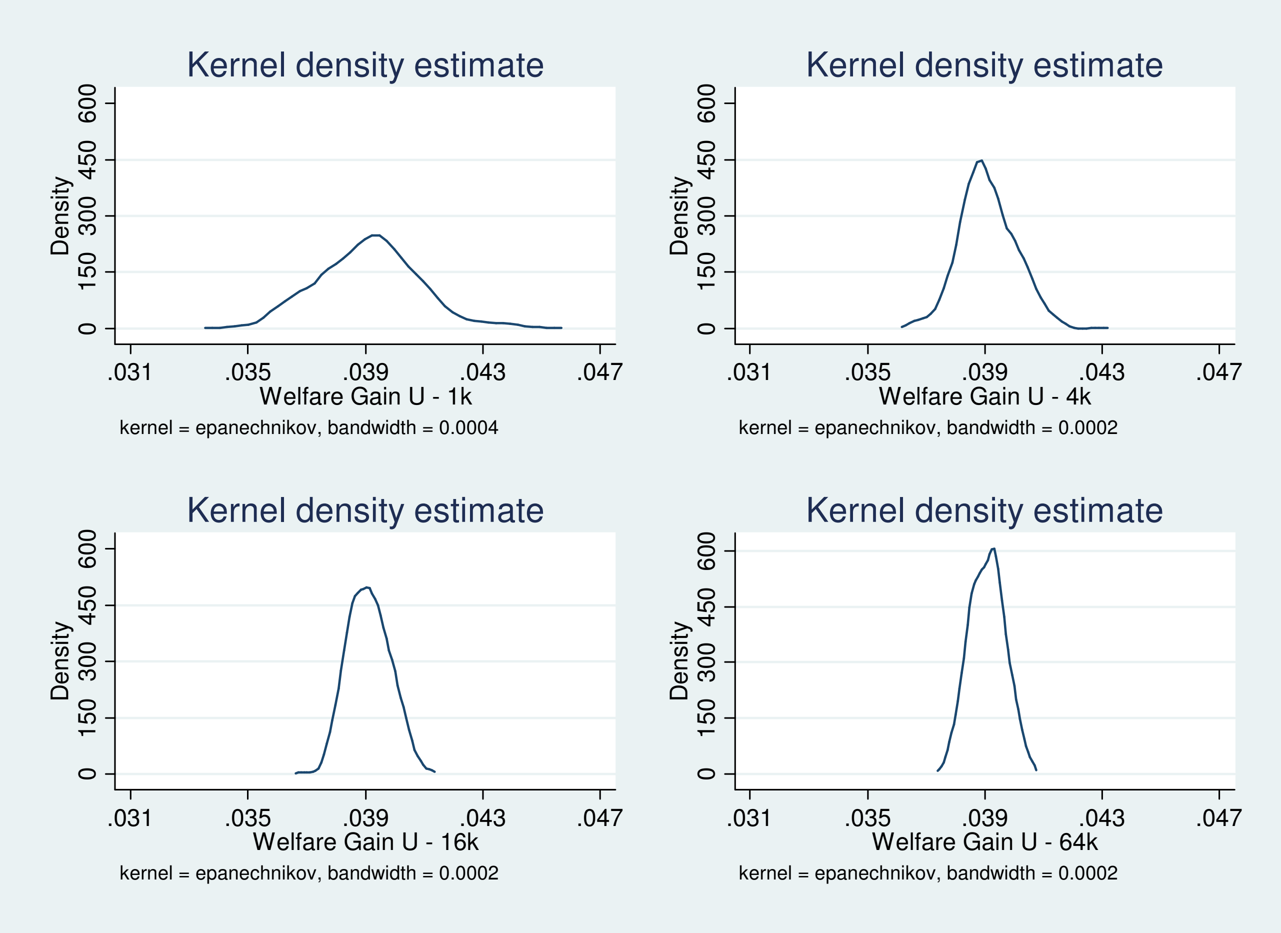

Figure 18: Welfare Gains Unemployed, Experiment 2b