School of Economics and Management
TECHNICAL UNIVERSITY OF LISBON

Department of Economics

António Afonso & João Tovar Jalles

A Longer-run Perspective on Fiscal Sustainability

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A Longer-run Perspective on Fiscal Sustainability*

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António Afonso [#] [$] and João Tovar Jalles [+]

Abstract

This paper investigates the sustainability of fiscal policy in a set of 19 countries by taking a longer-run secular perspective over the period 1880-2009. Via a systematic analysis of the stationarity properties of the first-differenced level of government debt, and disentangling the components of the debt series using Structural Time Series Models, we are able to conclude that the solvency condition would be satisfied in mostly all cases since non-stationarity can be rejected, and, therefore, longer-run fiscal sustainability cannot be rejected (Japan and Spain can be exceptions). The same would be true for the panel sample analysis.

JEL: C23, E62, H62
Keywords: fiscal sustainability, government debt, unit roots, breaks, structural time series models

* The opinions expressed herein are those of the authors and do not necessarily reflect those of the ECB or the Eurosystem.

[#] European Central Bank, Directorate General Economics, Kaiserstraße 29, D-60311 Frankfurt am Main, Germany. email: antonio.afonso@ecb.europa.eu.
[$] ISEG/UTL - Technical University of Lisbon, Department of Economics; UECE – Research Unit on Complexity and Economics. R. Miguel Lupi 20, 1249-078 Lisbon, Portugal. UECE is supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal). email: aafonso@iseg.utl.pt.
[+] University of Cambridge, Faculty of Economics, Sidgwick Avenue, Cambridge CB3 9DD, United Kingdom, email: jodstj2@cam.ac.uk.

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Non-technical summary

The importance of maintaining sustainable fiscal policies has become paramount in the aftermath of the economic and financial crisis in 2008/09. Indeed, maintaining a stable long-term relationship between government expenditures and revenues is one of the key requirements for a stable macroeconomic environment and a sustainable economy. Such is also the challenge of several developed economies, notably in the European Union.

Therefore, the main purpose of this paper is to investigate the sustainability of fiscal policy in a set of 19 countries by taking a longer-run secular perspective covering the period 1880-2009. In addition, in our empirical approach we perform a systematic analysis of the stationarity properties of the first-differenced level of government debt. This approach will provide us with an indirect test on the solvency of public finances in these countries.

Consequently, this paper adds to the existing literature by applying a plethora of different (and recent) unit-root tests to 19 countries using consistent public finance data from one single source that put together the longest government debt database to date. It also tests for the existence of structural breaks during the time sample of each country. By means of Structural Time Series Models allowing for stochastic unobserved components and taking advantage of the long time series we have available, we decompose them into cycle, trend and irregular components, and analyse the evolution and behaviour of the corresponding trend growth over the period under scrutiny.

Via a systematic analysis of the stationarity properties of the first-differenced level of government debt, we are able to conclude that the solvency condition would be satisfied in mostly all cases since non-stationarity can be rejected, and, therefore, longer-run fiscal sustainability cannot (Japan and Spain can be exceptions). In addtion, the implications of the global financial crisis for government debt in 2008 and 2009 are also picked up in the analysis.

Interestingly, the results of our paper may be considered as more “pleasant” over the very long run from a policy-maker’s point of view, than some previously existing fiscal sustainability analysis where the time spans were much shorter. Still, and even if that may be the case for the existing explicit government liabilities, one needs to bear in mind that any policy measures in that area are probably more than needed to tackle the burden of incoming implicit liabilities in most countries. On the other hand, is also not possible to reject the hypothesis of sustainability of public finances in the context of this panel sample, which is nevertheless in line with other shorter time span panel analysis of fiscal sustainability.

1. Introduction

The importance of maintaining sustainable fiscal policies has become paramount in the aftermath of the economic and financial crisis in 2008/09. Indeed, maintaining a stable long-term relationship between government expenditures and revenues is one of the key requirements for a stable macroeconomic environment and a sustainable economy. Such is also the challenge of several developed economies, notably in the European Union.

Fiscal sustainability is a useful criterion to evaluate whether or not fiscal policy is on a right long-term track.[1] Theoretically, sustainable fiscal policies would need to prevail without any modification in the existing policy stance, or, in other words, if the intertemporal government budget constraint holds in present value terms. Conversely, if sound fiscal policies are absent, economic policies at both macro and microeconomic levels will become unmanageable and will require policy changes. On the other hand, fiscal sustainability is challenged when the debt-to-GDP ratio reaches an excessive value, and government revenues are not enough to keep on financing the new issuance of government debt.

The empirical assessments of fiscal sustainability, stemming from the intertemporal government budget constraint, usually test for the existence unit roots in government debt and budget deficit series, and/or for cointegration between government revenues and expenditures. Such strand of analysis deals with explicit government liabilities.[2] The empirical studies tend to focus mostly looking on the US and on European cases (see, Hamilton and Flavin, 1986; Hakkio and Rush, 1991; Trehan and Walsh, 1991; MacDonald, 1992; Ahmed and Rogers, 1995; Quintos, 1995; Makrydakis et al., 1999; Feve and Henin, 2000; Martin, 2000; Bravo and Silvestre, 2002; Hatemi-J, 2002; Afonso, 2005; Mendoza and Ostry, 2007; Arghyrou and Luintel, 2007, to name a few).

Since fiscal sustainability needs to be tackled at the country level, a country assessment is naturally necessary. On the other hand, a panel analysis of the sustainability of public finances is also relevant, notably in the case of the European Union (EU). In this context, recent panel analysis of fiscal sustainability that has been carried out for the EU, which points to the solvency of government public finances when considering the EU15, suggesting that fiscal policy may not have been sustainable for several countries, although it may have been less unsustainable for some countries (Denmark, Finland, Luxembourg, and the Netherlands).[3]

[1] Analysis on sustainability has focused on both the univariate properties on debt (e.g. Hamilton and Flavin, 1986) and the long-run relationship between revenues and expenditures (e.g. Hakkio and Rush, 1991).
[2] See Afonso (2005) for an overview.
[3] See Afonso and Rault (2010) who assess the sustainability of public finances using 2nd generation unit root tests and panel cointegration

Still, usually the time spans used, either in country specific analysis or in a panel set up, tend to go far back as only the 1970s, which can limit the full assessment of long-run fiscal sustainability.

Therefore, the main purpose of this paper is to investigate the sustainability of fiscal policy in a set of 19 countries by taking a longer-run secular perspective covering the period 1880-2009. In addition, in our empirical approach we perform a systematic analysis of the stationarity properties of the first-differenced level of government debt. This approach will provide us with an indirect test on the solvency of public finances in these countries.

Consequently, this paper adds to the existing literature by applying a plethora of different (and recent) unit-root tests to 19 countries using consistent public finance data from one single source, Abbas et al. (2010) who put together the longest government debt database to date. It also tests for the existence of structural breaks during the time sample of each country. By means of Structural Time Series Models allowing for stochastic unobserved components and taking advantage of the long time series we have available, we decompose them into cycle, trend and irregular components, and analyse the evolution and behaviour of the corresponding trend growth over the period under scrutiny.

Essentially, our results show that the solvency condition would be satisfied in mostly all cases since non-stationarity can be rejected. Therefore, longer-run fiscal sustainability cannot be rejected (Japan and Spain can be exceptions), and the same would be true for the panel sample analysis.

The structure of the paper is as follows. Section 2 briefly reviews the underlying theoretical framework, which is the basis for the empirical analysis. Section 3 presents the empirical and econometric methodology. Section 4 discusses our results and findings. Section 5 concludes.

2. Theoretical Framework

If fiscal developments turn out to be unsustainable, fiscal policy has to guarantee that the future primary balances are consistent with the intertemporal government budget constraint.[4] The hypothesis of fiscal policy sustainability is related to the condition that the trajectory of the main macroeconomic variables is not affected by the choice between the issuance of government debt or the increase in taxation. Under such conditions, it would not be crucial how the deficits are financed, implying also the assumption of the Ricardian Equivalence hypothesis.[5]

[4] For instance, Cuddington (1997) and Hénin (1997) discuss this topic.

[5] Afonso (2008) provides an analyis of Ricardian fiscal behaviour notably for the EU.

One can start with the government budget constraint to derive the present value of the budget constraint (PVBC). Therefore, the flow budget constraint is written as

$$G_t + (1 + r_t)B_{t-1} = R_t + B_t , \tag{1}$$

where G is the government expenditures, excluding interest payments, R is the government revenues, B is the government debt and r is the real interest rate.[6]

Rewriting equation (1) for the subsequent periods, and solving recursively leads to the intertemporal budget constraint:

$$B_t = \sum_{s=1}^{\infty} \frac{R_{t+s} - G_{t+s}}{\prod_{j=1}^{s}(1 + r_{t+j})} + \lim_{s\to\infty} \prod_{j=1}^{s} \frac{B_{t+s}}{(1 + r_{t+j})} . \tag{2}$$

When the second term from the right-hand side of equation (2) is zero, the present value of the existing stock of government debt will be identical to the present value of future primary surpluses. For empirical purposes it is useful to make several algebraic modifications to equation (1). Assuming that the real interest rate is stationary, with mean r, and defining

$$E_t = G_t + (r_t - r)B_{t-1} , \tag{3}$$

it is possible to obtain the following so-called PVBC:

$$B_{t-1} = \sum_{s=0}^{\infty} \frac{1}{(1+r)^{s+1}} (R_{t+s} - E_{t+s}) + \lim_{s\to\infty} \frac{B_{t+s}}{(1+r)^{s+1}} . \tag{4}$$

A sustainable fiscal policy should ensure that the present value of the stock of government debt, the second term of the right hand side of (4), goes to zero in infinity, constraining the debt to grow no faster than the real interest rate. In other words, it implies imposing the absence of Ponzi games and the fulfilment of the intertemporal budget constraint. When facing this transversality condition, the government will have to achieve future primary surpluses whose present value adds up to the current value of the stock of government debt. In other words, government debt in real terms cannot increase indefinitely at a growth rate that is higher than real interest rate.[7]

A common practice in the literature, among the set of methods to evaluate fiscal policy sustainability, is to investigate past fiscal data to see if government debt follows a stationary process.[8] Recalling the PVBC, equation (4), it is possible to have two complementary definitions of sustainability or empirical testing:

i) The value of current government debt equals the sum of future primary budget surpluses:

[6] Sometimes in the literature the real interest rate is assumed stationary, but this is a much more difficult assumption for the nominal interest rate.

[7] See McCallum (1984) and Joines (1991).

[8] See Hamilton and Flavin (1986), Trehan and Walsh (1991), and Hakkio and Rush (1991).

$$B_{t-1} = \sum_{s=0}^{\infty} \frac{1}{(1+r)^{s+1}} (R_{t+s} - E_{t+s}), \quad (5)$$

ii) The present value of government debt must approach zero in infinity:

$$\lim_{s\to\infty} \frac{B_{t+s}}{(1+r)^{s+1}} = 0. \quad (6)$$

Therefore, and in order to test empirically the absence of Ponzi games, one can test the stationarity of the first difference of the stock of government debt (ΔB_t).

3. Empirical Methodology

3.1 Unit Roots and Structural Breaks

Before disentangling the different components of our government debt series using Structural Time Series Models (STM), one should formally test the possibility of lasting shocks in our variables of interest. It is important to be aware of the possibility of a spurious break phenomenon. Whenever a series is non-stationary, with a unit-root, one or more breaks may be erroneously suggested by the data even if it is stable over time. This problem was first raised from a graphic perspective by Hendry and Neale (1991). Therefore, our structural time series analysis should be complemented by (ex-ante) appropriate statistical hypothesis tests.

Hence, in order to narrow down the number of suitable structural time series models for the 19 countries, some statistics have been computed, which provide additional information in relation to the main characteristics of the different components of the variable.[9] In relation to the trend of the variables, unit root tests can provide a valuable insight into the presence of either a deterministic or stochastic secular component in the government debt series. In this context, in addition to standard Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) unit root tests – for purposes of robustness and completeness[10] – we also conduct the four tests (M-tests) proposed by Ng and Perron (2001) (NP) based on modified information criteria (MIC): the modified Phillips-Perron test MZ_α; the modified Sargan-Bhargava test (MSB); the modified point optimal test MP_T; and the modified Phillips-Perron MZ_T. These tests improve the PP-tests both with regard to size distortions and power.

In addition, we resort to unit root tests allowing for breaks, notably the Zivot-Andrews (1992) (ZA) one. This endogenous structural break test is a sequential test which utilizes the

[9] The countries are: Argentina, Austria, Belgium, Brazil, Denmark, France, Germany, Greece, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Russian Federation, Spain, Sweden, the United Kingdom, and the United States

[10] Moreover, this test is especially appropriate under certain dynamic data structure, and when their random components are not white noise.

full sample and uses a different dummy variable for each possible break date. The break date is selected where the t-statistic from the ADF test of unit root is at a minimum (most negative). Consequently a break date will be chosen where the evidence is least favourable for the unit root null.[11] We complement this with the modified ADF test proposed by Vogelsang and Perron (1998) (VP) also allowing for one endogenously determined break. Finally, we also use the two-break unit root test described by Clemente, Montanes and Reyes (1998) (CMR).[12] This latter test tests the null of unit root against the break-stationary alternative hypothesis and provides us supplementary insights vis-à-vis conventional unit root tests that do not account for any break in the data.

For the unit root tests that allow for one or two endogenously determined breaks it is assumed that the shift can be modelled by a dummy variable $DU_t = 0$ for $t \leq TB$ and for $t > TB$, where TB is the shift date (time break). In the time series literature, two generating mechanisms of shifts are distinguished, additive outlier (AO) and innovational outlier (IO) models. The former results in an abrupt shift in the level, whereas the latter allows for a smooth shift from the initial level to a new level. Although both results are reported, we will mainly discuss tests constructed for AO models. As discussed in Vogelsang and Perron (1998), who consider an unknown shift date situation, the AO framework may be preferable to the IO statistics, even if the Data Generating Process (DGP) is an IO process.

However, it is important to recognize some important drawbacks in both previous unit root tests, particularly, the ZA and VP tests. In particular, with relation to the VP test, it has been shown that the critical values are substantially smaller in the *I(0)* case than in the *I(1)* case (therefore, suggesting that the test is conservative in the *I(0)* case). The solution was then to devise a procedure that would have the same limit distribution in both cases. This was first attempted by Vogelsang (2001) but simulations provided support for the lack of power in the *I(1)* case. Perron and Yabu (2009) (PY) were more successful on this endeavour by proposing a new test for structural changes in the trend function of the time series without any prior knowledge of whether the noise component was stationary or integrated and making use of Andrews and Ploberger's (1994) exponential functional and Roy and Fuller's (2001) finite

[11] The critical values in Zivot and Andrews (1992) are different from the critical values in Perron (1989). The difference is due to that the selecting of the time of the break is treated as the outcome of an estimation procedure, rather than predetermined exogenously.

[12] For more detailed discussion of these tests that allow for endogenously determined breaks, the reader should refer to the original references.

sample correction procedure. This newer test has better properties in terms of size and power.[13]

3.2 Unobserved Components and Structural Time Series Models

We further inspect the properties of our time series by applying univariate versions of a STM[14] with unobserved components developed by Harvey (1989), Harvey and Shephard (1993) and Harvey and Scott (1994). In choosing this methodology, we have also considered other trend-cycle filters. For example, the Hodrick-Prescott (HP) filter, which has been employed widely in the recent business cycle literature, is not considered appropriate here as it explicitly neglects low frequency long swings by assumption. Moreover, Harvey and Jaeger (1993), and Cogley and Nason (1995), show that the HP filter may create spurious cycles and other distortions.[15]

3.3 Panel Unit Roots

Given the notoriously low power of individual country-by-country tests for unit roots and cointegration, it may be preferable to pool the time series of interest together and conduct panel analysis. We implement three different types of panel unit root tests: two first generation tests, namely the Im et al. (2003) test (IPS); the Maddala and Wu (1999) test (MW) and one second generation test – the Pesaran (2007) CIPS test. The latter test is associated with the fact that 1st generation tests do not account for cross-sectional dependence of the contemporaneous error terms, and not considering it may cause substantial size distortions in panel unit root tests (O'Connell, 1998 and Pesaran, 2007). In our context, the fact that country specific government debt puts pressure on the overall available savings may indeed, together with possible spillover effects and correlated movements in long-term government bond yields, constitute another argument for allowing for cross-section dependences.

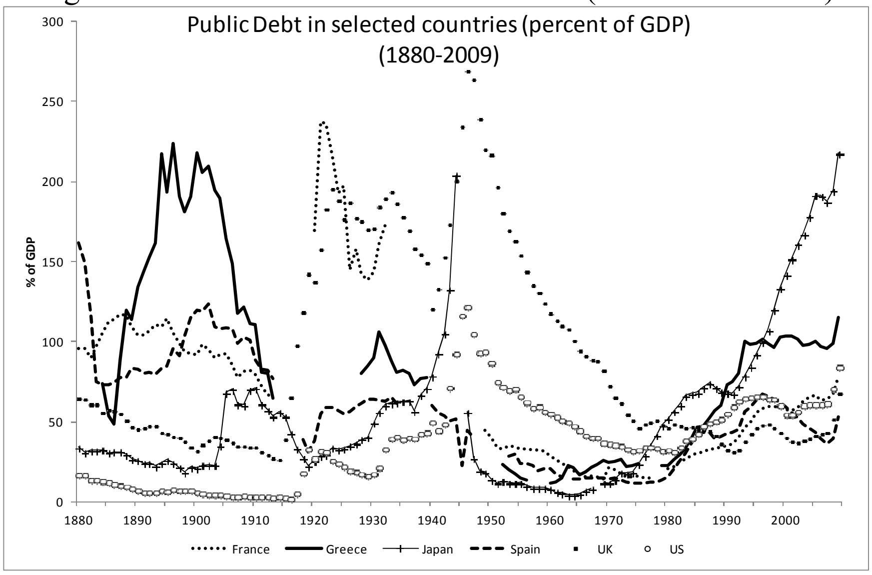
4. Empirical Analysis

Stylised Facts

[13] We thank Pierre Perron and Tomoyoshi Yabu for providing their GAUSS code.
[14] A brief exposition on these type of models and estimation procedures is presented in the appendix.
[15] The last authors show that under certain conditions the HP trend-cycle decomposition is similar to a smooth trend structural model. The HP filter constraints $\sigma_\eta^2 = 0$ and a "a priori" constraint of the signal-to-noise ratio, λ to 6.25 for annual data. When λ=0 the HP filter returns the original series without smoothing. As λ→∞, it returns an OLS trend for interior points in the sample, but distortions occur near the endpoints. However, in a structural time series model the signal-to-noise ratio is estimated and not imposed ex-ante.

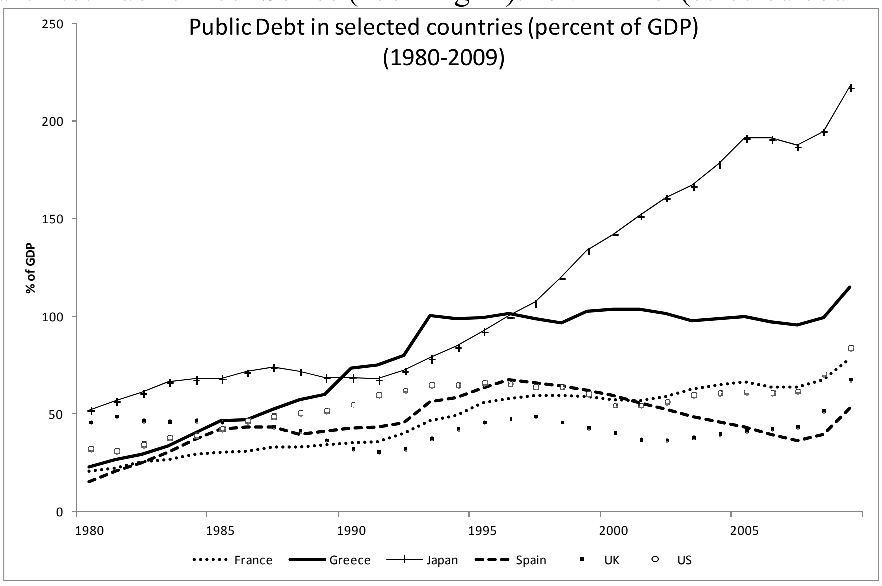
A brief characterization of the government debt series for the countries under scrutiny is appropriate before performing the empirical testing on the fiscal sustainability hypothesis. In fact, the consequences of choosing different fiscal policies may be exemplified by looking at the public paths of selected countries, as depicted in Figure 1a. It is clear from this chart that government debt-to-GDP ratios peaked around the two World Wars in the 20th century and then again after the 1970s till the end of the 1990s – with the exception of Japan where debt kept on rising. Government debt restarted an increasing trend with the 2008 economic crisis and the continuous worsening state of public finances in most advanced and emerging economies (Figure 1b).

Figure 1.a: Public Debt Series: 1880-2009 (selected countries)



Source: Abbas et al. (2010).

Figure 1.b: Public Debt Series (zooming-in): 1980-2009 (selected countries)



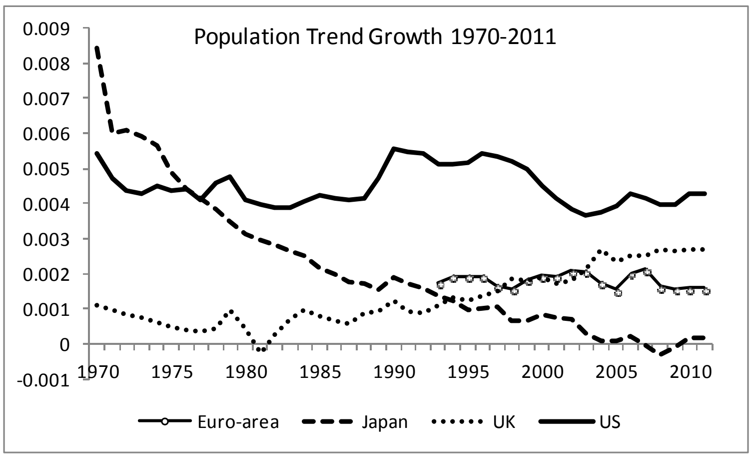
Source: Abbas et al. (2010).

For instance, government debt increased in Italy from an average of 51.8% of GDP in the 1970s to an average of 112.3% in the 2000s (Table 1). Nevertheless, Italy reduced its debt level in 3.8 percentage points (pp) relative to the average figure in 1880s. On the other hand, Japan's debt increased by about 143.5 pp between the 1880s and the 2000s, followed by Belgium, the US, Sweden and Argentina (see last column in Table 1). In the case of Greece, Italy and Japan government debt has surpassed 100% of GDP, an average value that was kept during the 2000s. In the cases of Belgium and Italy, their high debt service payments induced substantial budget deficits despite primary surpluses. A reversal of that general trend is noticeable only at the end of the 1990s, as several "more indebted" European countries tried to fulfil or at least come closer to the Maastricht criteria (much of that effort was afterwards reversed, notably in the context of the 2008-2009 economic and financial crisis). All in all, the main conclusion is that the burden of government debt has increased over time in almost every country under scrutiny.

[Table 1]

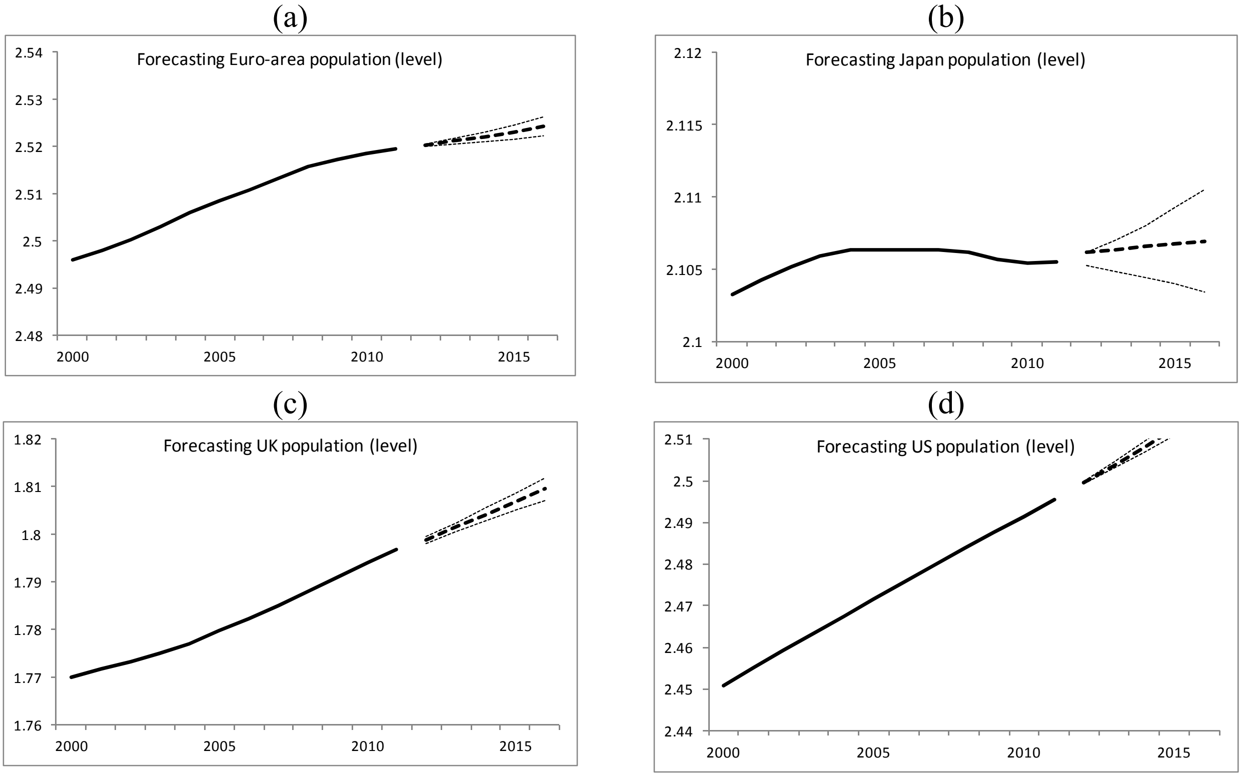
It is nevertheless important to be aware that the main driver for (potential) lack of fiscal sustainability in the near future (particularly in the context of the government debt increase following the last economic and financial crisis) will be population growth (see Figure 2a. and 2.b), combined with generous pay-as-you-go social security systems in advanced economies.

Figure 2.a: Trend Growth of Population: 1970-2011 (selected countries)



Note: Authors' calculations.

Figure 2.b: Forecasting Population (level) with STM: 2000-2016 (selected countries)



Note: Authors' calculations based on fitted univariate STM applied to the log of total population and recursively forecasted using the Kalman filter up to 5 years ahead and using information from 2000 till 2011.

Since this population shift towards older societies is an entirely new phenomenon, it cannot be considered in terms of the econometric analysis based exclusively on past data, notably regarding explicit liabilities. Indeed, implicit pension liabilities will impinge on future borrowing requirements, and can carry additional sustainability issues. Figure 2.b suggests that population growth will put extra pressure on Euro-area, UK and US budgets (given computed projections), less so in the case of Japan (also attested by declining, and close to zero, population trend growth).

Country Unit Root Analysis

Before presenting the results of unit roots tests, it should be mentioned that the fact that our series are in ratio to GDP does not rule them out being integrated processes (see, Ahmed and Yoo, 1989). Hence, we focus on fiscal policy sustainability for each of the 19 countries by means of several unit root tests in an attempt to validate the sufficient sustainability condition using the stock of government debt. Table 2 shows the stationarity tests results for the first difference of the debt ratio for the period 1880-2009.

[Table 2]

The results for the ADF and PP test (considering both a constant and a time trend) allow the rejection of the null of a unit root in all countries but Japan. Therefore, the series of the first difference of government debt might be I(0) over the very long-run and the solvency condition would be satisfied in those cases since non-stationarity can be rejected, and, therefore, longer-run fiscal sustainability cannot. The Ng and Perron (2001) tests give us similar conclusions (apart from the case of Spain). One should also note that contrary to several other studies on fiscal sustainability, which have to rely on a small number of observations, accuracy problems of unit-root tests with small samples do not apply in our case.

The previous set of results assumes that there is no structural break in the government debt series. However, this might not be the case in some countries – for example, in periods of war or important economic downturns. In the presence of structural changes in the trend function, ADF and PP tests that do not take into account the break in the series have low power, and are biased toward the non-rejection of a unit root. Therefore, in Table 2 we also report the identified structural breaks, with many occurrences taking place in the 1st half of the 21st century. Interestingly, we also see structural breaks in European countries when the so-called expansionary fiscal consolidations took place, notably in Sweden, in 1991 and in 1996, and in Denmark in the periods 1982-1983.[16]

Consequently, the longer historical time span proves to be crucial to assess fiscal sustainability. Indeed, while most existing empirical analysis conclude for absence of fiscal sustainability for many countries, such studies usually have a much more limited data time span, starting essentially in the 1970s.

Cyclical Behaviour

In order to evaluate the possibility of the presence of a cyclical component in the government debt series, some descriptive statistics (not shown) such as the correlogram and the power spectrum can provide useful information. If sometimes the correlogram shows only small individual autocorrelations, not providing enough strong evidence of the presence of cyclical movement in the series (despite that some cycle evidence may be buried with noise), then a much clearer message emerges from the examination of the spectrum.

Based on the information gathered by conducting unit root tests and the descriptive statistics employed to evaluate the presence of cyclical movements in the debt series, a likely

[16] Several studies detect in such periods expansionary fiscal consolidations episodes (see, for instance, Afonso, 2010).

specification for the trend and the cyclical components of a structural time series model for the different data can be estimated. Table 3 shows main diagnostic and goodness-of-fit statistics for a basic structural model.[17] All these models assume the presence of a trend, one cycle and an irregular component.[18]

[Table 3]

The diagnostics are generally satisfactory and the estimation of alternative models, such as a linear trend model, a smooth trend model or even a random walk with drift, yielded poorer statistics relative to the selected Basic Structural Model.

The estimated variances for the hyper-parameters together with the period (in years) of the cycle are presented in Table 4.

[Table 4]

A first comment goes to the period of the cycle which varies between 7.03 years for Greece and 23.71 years for Portugal. Secondly, none of the models show a q-ratio[19] associated with the irregular component exactly equal to 1 (with the exception of Greece), meaning that all the variation in those time series is explained by the trend, cycles and interventions (whenever present). Ideally we would like σ_{ε}^{2} to be as small as possible, and let the other components explain most of the model's variation.

Given this model for each country, the graphs of the trend growth resulting from the STM for the government debt series are displayed in Figure 3 for a subset of countries (for reasons of parsimony other graphs are available upon request).

Increases in government debt are naturally associated with times of increased military expenditure, hence notably during the two World Wars. A spike is also visible around the 1970s with the oil price shocks and government's efforts to keep their energy supplies even at increasing costs. More recently, since the early 2000s both the UK and the US to a greater extent and Spain and Greece to a lesser extent experienced a sudden increase in their debt trend growth rates; less so in the case of France, whereas in Japan such phenomenon began

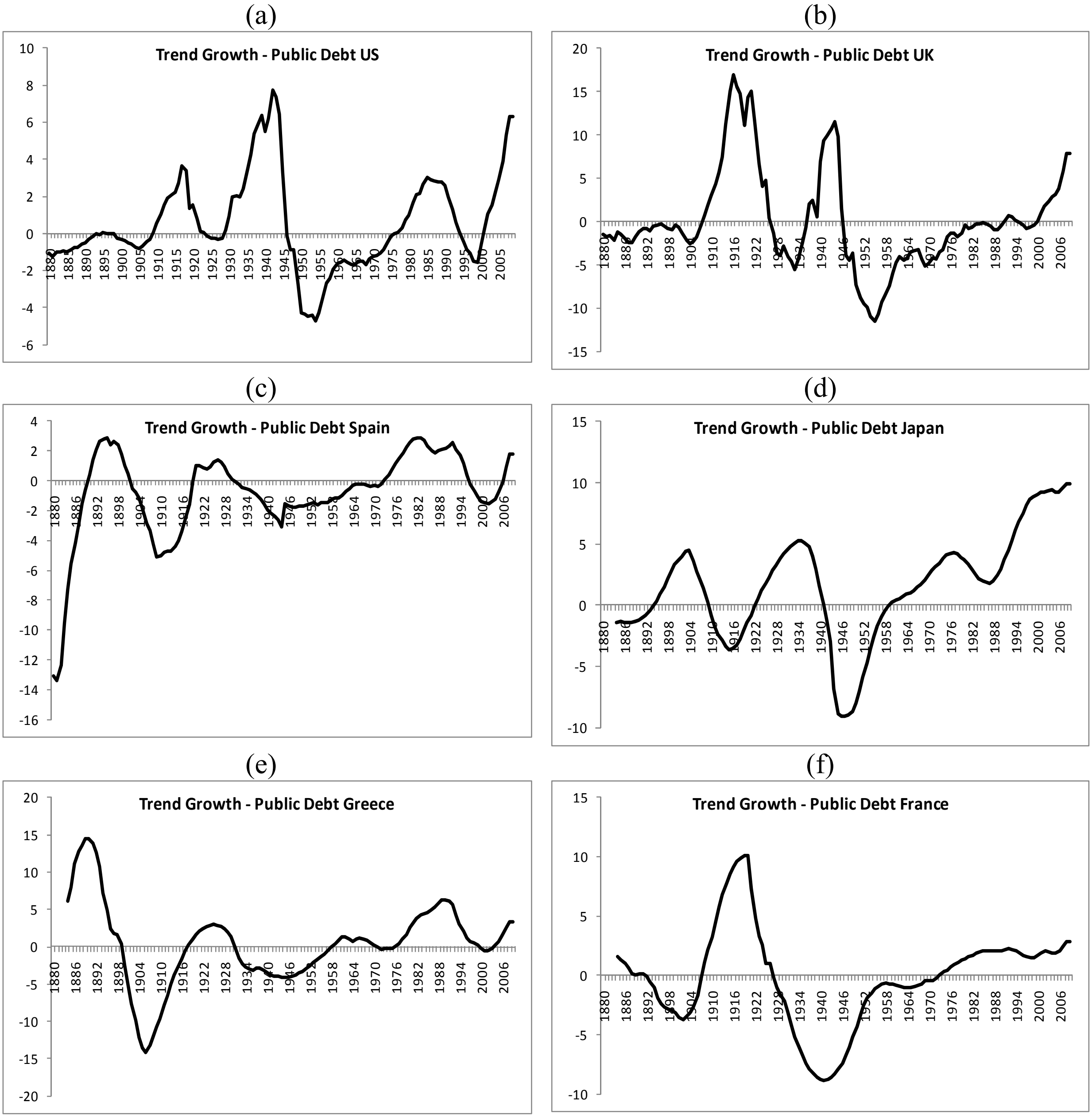
[17] Diagnostic checking tests are conducted by computing the Box-Ljung Q(n) statistic for serial correlation and a simple test for heteroscedasticity H(n,m). The Prediction Error Variance (PEV), the coefficient of determination (Rd^2) and the information criteria (Akaike Information Criteria, AIC; and Bayesian Information Criteria, BIC) proved goodness-of-fit measures. Table 2 also presents information related to the Log-Likelihood.

[18] Other models have been estimated but yield less satisfactory diagnostics, based on information criteria assessment. These are available upon request and consisted of: i) a statistical specification that assumes that the trend component follows a random walk with drift, with a deterministic slope; ii) a second which introduces a somewhat smoother trend with a deterministic level and a stochastic slope; iii) finally, we have estimated local linear trend model, which stipulates the level and slope to be stochastic.

[19] The q-ratios are the ratios of each variance to the largest one. By observing it we can examine which of the components is the most volatile (in relative terms) in explaining the model's variation.

back in the early 1990s following their economic (asset price) bubble and financial crisis from 1986-1991 which continued up to the current period.

Figure 3: Trend Growth of Public Debt Series: 1880-2009 (selected countries)

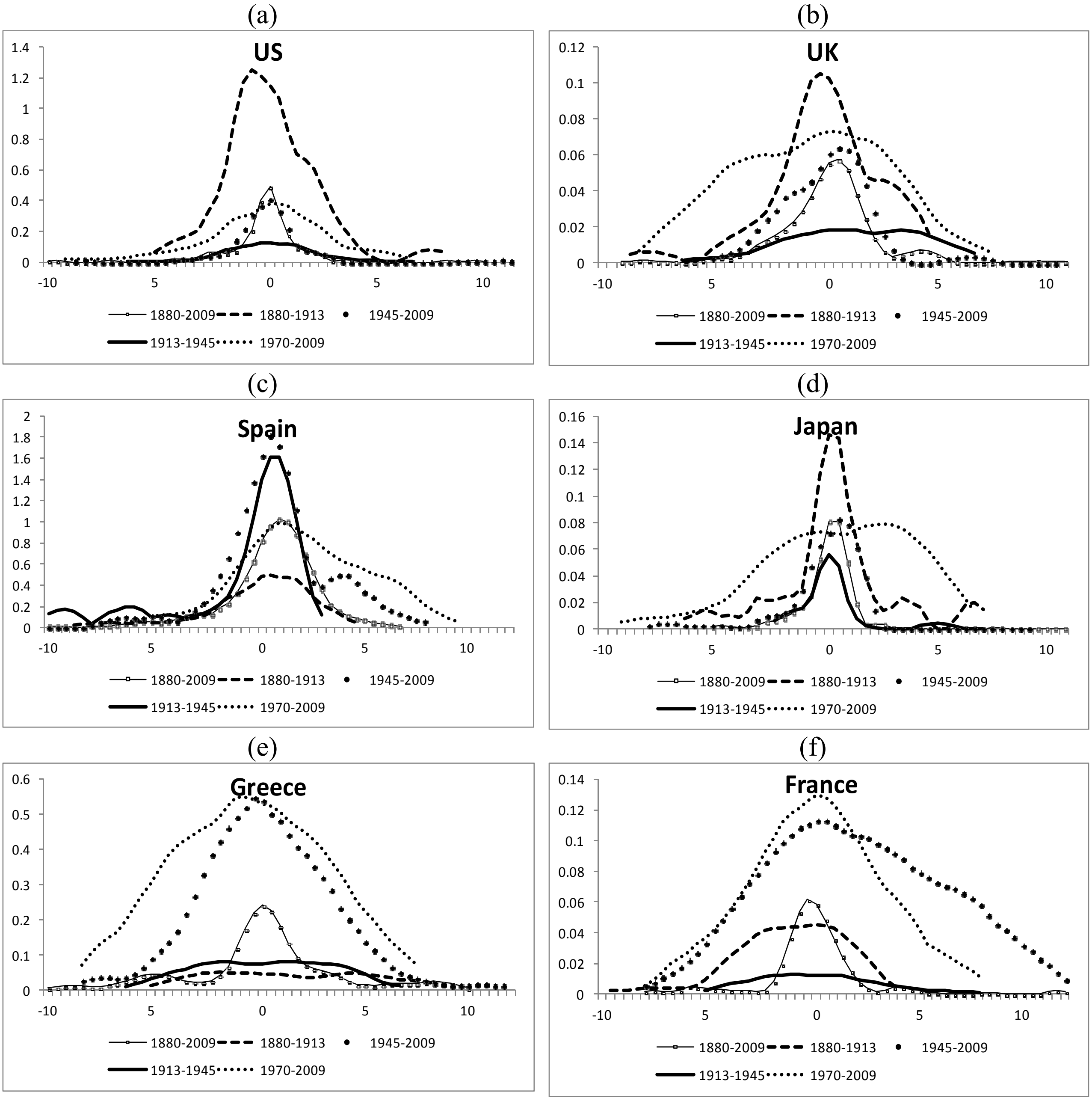


Note: Authors' calculations.

The government debt series depicted above are characterized by cycles which show great volatility during the first half of the 20th century and then again around the period associated with the oil price shocks. Particularly over the 1950s and 1960s (the so-called "Golden Age" of economic growth), and then after the entry in the EU of many European countries, these cyclical variations are much reduced. Given the important cyclical variation of the short cycles and the clear identification of major peaks and troughs the contrast is evident in Figure

4, which presents the Kernel density estimates[20] of the (STM) cyclical deviation of government debt over the full span of the period 1880-2009 and the sub-periods 1910-1973, 1973-1999, 1926-1973, 1913-1945, 1945-2009 (according to Angus Maddison's growth phases split). It is clear that any period is more stable than the inter-war years.

Figure 4: Short cycles: Kernel Density Estimates of Public Debt Series: 1880-2009 (selected countries)



Note: Authors' calculations.

[20] These estimates were generated with an Epanechnikov kernel and a band-width proportional to the sample size raised to the power -2.

Panel Unit Root Analysis

A panel analysis can also be considered to take advantage of the increased number of observations. In addition, the nature of the interactions and dependencies that generally exist, over time and across the individual units in the panel, can then also be taken into account. For instance, and as already mentioned, cross-country dependence can also be envisaged notably via some level of integration of financial markets, while spillover effects in government bond markets and interest rates co-movements are to be expected.

Therefore, we take advantage of the long cross-section of time series available and conduct first and second general panel unit root tests. The results of such analysis are displayed in Tables 5a and 5b. The conclusions go in the same direction as the ones reached for the individual country unit-root tests, that is, for the first-differenced debt at lags 0-2 both first and second generation panel unit root tests reject the null that all country series contain a nonstationary process. Therefore, is not possible to reject the hypothesis of sustainability of public finances in the context of this panel sample.

[Tables 5a-5b]

"Debt-trackers"

Finally, as an additional illustration, and following Matheson's (2011)[21] approach, we constructed the so-called "debt-trackers" using our long series for the same selection of countries. The heat map in Figure 5 displays information about government debt for selected countries: the US, UK, Japan, Spain, France and Greece. The trends used in the heat map are computed by means of univariate STM estimated using the Kalman filter. The colors are based on the behavior of the smoothed series relative to trend: a yellow color indicates growth below trend and moderating; red and pink indicate growth of debt above trend at increasing and decreasing rates, respectively; the darkest shade of green represents contraction of debt, with the lightest green indicating a debt growth below trend and moderating.

Therefore, the heat map in Figure 5 clearly shows the implications of the global financial crisis for government debt in 2008 and 2009. The effects of the crisis were seen

[21] The author has employed a Dynamic Factor Model (DFM) to estimate growth indicators. In his case the DFM is particularly useful, because it can utilize a large number of economic time series (such as PMIs, consumer and business confidence, retail sales, industrial production, exports, imports, exchange rates, interest rates, equity prices, credit conditions, employment, wages, PPIs, CPIs and inflation expectations) in a timely fashion and it has been shown to produce reliable short-term forecasts. He also employs a multi-step procedure for cleaning the data of outliers and missing observations (by estimating the factors and parameters using principal components and OLS, and then re-estimating them using the Kalman filter).

across the represented countries (to a less extent in Japan, which had a very high debt level to start with).

[Figure 5]

5. Conclusion

In this paper we have revisisited the issue of fiscal policy sustainability in a set of 19 countries by taking a longer-run secular perspective covering the period 1880-2009. In our empirical assessment we also performed a systematic analysis of the stationarity properties of the first-differenced level of government debt.

Via a systematic analysis of the stationarity properties of the first-differenced level of government debt, we are able to conclude that the solvency condition would be satisfied in mostly all cases since non-stationarity can be rejected, and, therefore, longer-run fiscal sustainability cannot (Japan and Spain can be exceptions). In addtion, the implications of the global financial crisis for government debt in 2008 and 2009 are also picked up in the analysis.

Interestingly, the results of our paper may be considered as more “pleasant” over the very long run from a policy-maker’s point of view, than some previously existing fiscal sustainability analysis where the time spans were much shorter. Still, and even if that may be the case for the existing explicit government liabilities, one needs to bear in mind that any policy measures in that area are probably more than needed to tackle the burden of incoming implicit liabilities in most countries.

On the other hand, it is also not possible to reject the hypothesis of sustainability of public finances in the context of this longer time span panel sample, which is nevertheless in line with other shorter time span panel analysis of fiscal sustainability.

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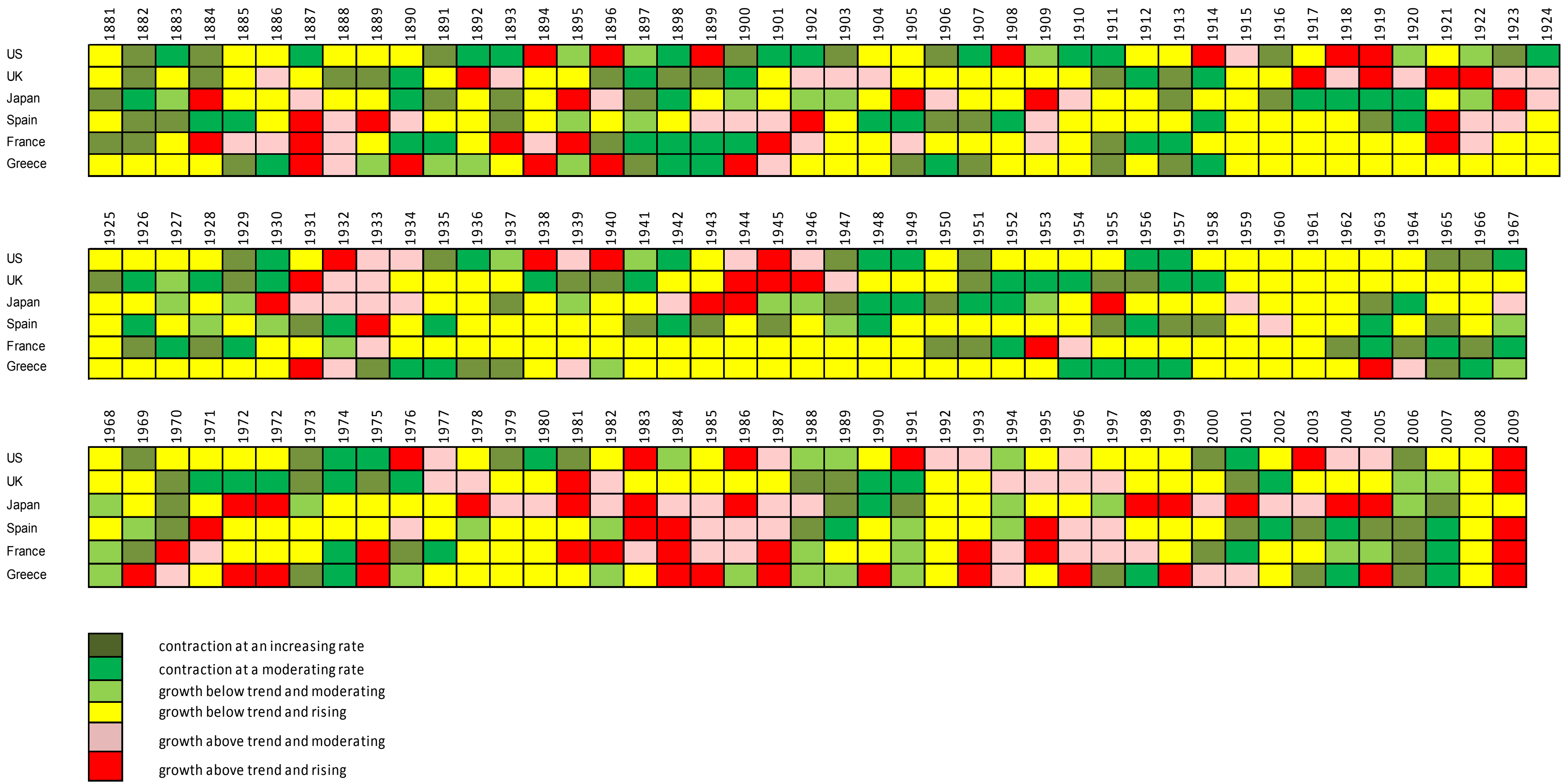
Figure 5: Debt-Trackers (selected countries) 1881-2009



Note: Authors' calculations. The debt trackers are constructed using a large time span of yearly debt-to-GDP data. The trackers are estimated and forecasted at the annual frequency. The classifications represented in the table are based on the behaviour of a backward 2-year-moving average. The most recent estimates implicitly include forecasts.

TABLE 1. Public Debt (% of GDP), decade averages: 1880-2009

country	1880s	1890s	1900s	1910s	1920s	1930s	1940s	1950s	1960s	1970s	1980s	1990s	2000s	Diff. (p.p.)	ranking
Argentina	50.8	87.9	44.0	27.3		43.7	51.3	29.4	15.3	13.8	46.1	39.5	83.9	33.1	
growth rel. prev.		*23.8*	*-30.1*	*-20.7*			*7.0*	*-24.1*	*-28.6*	*-4.4*	*52.5*	*-6.8*	*32.8*		5
Austria	78.5	85.1	71.6	64.1	19.9	32.3	27.9	14.0	15.9	21.5	47.0	63.0	64.8	-13.7	12
growth rel. prev.		*3.5*	*-7.5*	*-4.8*	*-50.8*	*21.0*	*-6.4*	*-29.8*	*5.4*	*13.1*	*33.9*	*12.8*	*1.2*		
Belgium	40.0	47.7	48.3	45.6	107.6	81.9	123.3	66.9	58.4	44.8	92.7	124.1	97.6	57.6	2
growth rel. prev.		*7.7*	*0.5*	*-2.4*	*37.2*	*-11.8*	*17.8*	*-26.6*	*-5.9*	*-11.5*	*31.6*	*12.7*	*-10.5*		
Brazil	104.6	70.8	54.3	36.9	25.0	32.5	25.2	9.3		31.1	53.6	60.4	70.2	-34.4	14
growth rel. prev.		*-16.9*	*-11.5*	*-16.8*	*-16.9*	*11.4*	*-11.1*	*-43.4*			*23.6*	*5.2*	*6.5*		
Denmark	23.0	18.5	16.3	14.6	19.6	20.1	10.9	28.0	16.4	14.8	54.2	65.1	50.8	27.8	8
growth rel. prev.		*-9.4*	*-5.6*	*-4.8*	*12.9*	*1.1*	*-26.4*	*40.8*	*-23.3*	*-4.4*	*56.4*	*8.0*	*-10.8*		
France	105.2	102.8	87.7	90.9	179.6	159.2	42.2	33.4	20.0	17.4	29.2	50.4	63.5	-41.6	16
growth rel. prev.		*-1.0*	*-6.9*	*1.6*	*29.6*	*-5.2*	*-57.7*	*-10.2*	*-22.1*	*-6.0*	*22.4*	*23.7*	*10.0*		
Germany	35.3	42.8	39.7	39.9	10.2	20.3	17.8	18.7	19.4	23.3	39.2	52.1	64.5	29.2	7
growth rel. prev.		*8.4*	*-3.3*	*0.3*	*-59.3*	*30.0*	*-5.9*	*2.3*	*1.4*	*7.9*	*22.7*	*12.3*	*9.3*		
Greece	90.3	182.4	162.9	84.2	84.9	85.2		16.9	19.4	23.7	44.5	93.6	101.3	11.0	9
growth rel. prev.		*30.6*	*-4.9*	*-28.7*	*0.4*	*0.1*			*5.9*	*8.8*	*27.3*	*32.3*	*3.4*		
Italy	111.4	116.7	97.9	94.6	121.3	82.8	61.7	32.1	31.8	51.8	77.9	112.3	107.6	-3.8	10
growth rel. prev.		*2.0*	*-7.7*	*-1.5*	*10.8*	*-16.6*	*-12.8*	*-28.4*	*-0.5*	*21.2*	*17.7*	*15.9*	*-1.9*		
Japan	30.7	23.3	48.0	45.9	35.0	62.8	75.0	11.0	7.3	28.3	66.1	97.1	174.2	143.5	1
growth rel. prev.		*-12.0*	*31.4*	*-2.0*	*-11.7*	*25.4*	*7.7*	*-83.5*	*-17.4*	*58.5*	*36.9*	*16.7*	*25.4*		
Netherlands	89.2	90.1	67.7	50.6	49.1	63.5	218.0	76.7	71.1	56.4	79.8	86.3	54.2	-35.0	15
growth rel. prev.		*0.4*	*-12.4*	*-12.7*	*-1.3*	*11.2*	*53.6*	*-45.4*	*-3.3*	*-10.1*	*15.1*	*3.4*	*-20.2*		
New Zealand	108.6	126.5	115.1	116.2	149.0	181.5	134.1	82.7	62.1	47.9	61.6	50.8	24.4	-84.3	19
growth rel. prev.		*6.6*	*-4.1*	*0.4*	*10.8*	*8.6*	*-13.1*	*-21.0*	*-12.5*	*-11.2*	*10.9*	*-8.4*	*-31.9*		
Norway	16.2	20.0	27.1	17.8	30.2	33.8	42.3	26.7	25.1	30.2	39.3	40.7	47.3	31.1	6
growth rel. prev.		*9.1*	*13.2*	*-18.2*	*22.8*	*5.0*	*9.7*	*-20.0*	*-2.7*	*8.1*	*11.4*	*1.6*	*6.5*		
Portugal	84.2	99.9	78.0	73.2			31.3	24.3	38.7	30.7	50.7	56.5	60.1	-24.1	13
growth rel. prev.		*7.5*	*-10.7*	*-2.8*				*-11.1*	*20.3*	*-10.1*	*21.8*	*4.7*	*2.6*		
Russia	78.7	71.8	57.9	49.8								72.7	30.9	-47.8	18
growth rel. prev.		*-4.0*	*-9.3*	*-6.5*									*-37.2*		
Spain	94.9	92.4	107.9	62.5	57.1	62.6	47.2	23.6	15.6	12.9	34.5	57.1	48.7	-46.2	17
growth rel. prev.		*-1.1*	*6.7*	*-23.7*	*-4.0*	*4.1*	*-12.3*	*-30.2*	*-18.0*	*-8.2*	*42.8*	*21.9*	*-6.9*		
Sweden	18.1	17.0	15.8	15.6	18.9	23.5	39.9	30.4	28.9	31.3	60.1	73.2	52.8	34.6	4
growth rel. prev.		*-3.0*	*-2.9*	*-0.6*	*8.3*	*9.4*	*23.0*	*-11.8*	*-2.2*	*3.4*	*28.4*	*8.5*	*-14.2*		
UK	56.7	41.9	36.9	68.5	174.8	169.0	202.5	157.7	97.5	54.5	44.0	41.1	44.4	-12.4	11
growth rel. prev.		*-13.1*	*-5.5*	*26.9*	*40.7*	*-1.5*	*7.9*	*-10.9*	*-20.9*	*-25.3*	*-9.3*	*-3.0*	*3.3*		
US	12.5	7.2	4.6	9.9	23.3	37.3	84.4	66.9	45.5	34.1	43.3	62.4	62.7	50.2	3
growth rel. prev.		*-24.1*	*-18.9*	*33.0*	*37.2*	*20.3*	*35.5*	*-10.1*	*-16.7*	*-12.5*	*10.4*	*15.9*	*0.1*		

Note: each column shows the decade-by-decade average public debt level in percentage of GDP. “growth rel. prev.” Means the growth rate (%) relative to the previous decade. “Diff” computes the difference between 2000s and 1980s. “p.p.” mean percentage points. The last column, “ranking”, orders the countries according to the next-to-last column, “Diff”.

TABLE 2. Unit Root Tests and Structural Breaks: First-Differenced Public Debt 1880-2009

Countries	ADF	PP	NP				ZA	VP(AO)	VP(IO)	CMR(AO)	CMR(IO)	PY2009
			MZa	MZt	MSB	MPT						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Argentina	-5.29***	-9.35***	-74.29***	-6.09***	0.08***	0.32***	1906***	2000**	2000**	1999, 2002	1892, 2000**	1961***
Austria	-6.65***	-6.65***	-34.06***	-4.11***	0.12***	0.75***	1926***	1912	1913	1912, 1922	1913, 1923**	1912***
Belgium	-6.00***	-5.73***	-17.05***	-2.87***	0.16**	1.60**	1944***	1939**	1940**	1944, 1946	1938, 1942**	1962***
Brazil	-10.48***	-11.17***	-13.21**	-2.57**	0.19***	1.85***	1990***	1986	1987**	1891, 1986	1892, 1987**	1946*
Denmark	-5.30***	-5.13***	-31.84***	-3.98***	0.12***	0.77***	1977***	1980	1983**	1978, 1982	1975, 1984**	1959***
France	-13.02***	-12.17***	-50.17***	-4.94***	0.09***	0.65***	1923***	1919	1920**	1919, 1947	1920, 1948	1925***
Germany	-7.20***	-7.26***	-20.51***	-3.15***	0.15***	1.37***	1926***	1923	1924**	1912, 1923	1913, 1924**	1956***
Greece	-4.77***	-9.22***	-11.22**	-2.25**	0.20***	2.61***	1901***	1892	1893**	1892, 1909	1895, 1912**	1920***
Italy	-8.38***	-8.35***	-7.47*	-1.92*	0.25*	3.29*	1921***	1917**	1918**	1917, 1944	1918, 1945**	1944***
Japan	0.09	-2.75*	-16.49***	-2.67***	0.16**	2.19**	1945***	1942**	1943**	1942, 1948	1943, 1948**	1945***
Netherlands	-6.39***	-6.40***	-45.28***	-4.78***	0.10***	0.56***	1947***	1944	1945	1944, 1958**	1945, 1959**	1930***
New Zealand	-7.49***	-6.27***	-83.71***	-6.45***	0.07***	0.30***	1934***	1935	1931**	1935, 1939	1931, 1939	1949*
Norway	-7.66***	-7.37***	-49.91***	-4.98***	0.09***	0.50***	1948***	1948**	1949**	1991, 1999	1992, 1998**	1987***
Portugal	-7.06***	-7.06***	-36.45***	-4.19***	0.11***	0.88***	1915***	1911**	1912	1900, 1911	1890, 1912**	1911***
Russian Federation	-4.82***	-4.89***	-13.04**	-2.55**	0.19***	1.87**	1913***	1990**	1991**	1990, 1996	1991, 1998**	1925***
Spain	-7.74***	-7.67***	-0.27	-0.12	0.47	16.97	1900***	1943**	1944**	1912, 1943	1913, 1944**	1964***
Sweden	-4.51***	-4.65***	-29.45***	-3.79***	0.12***	0.95***	1977***	1990**	1991**	1978, 1996**	1989, 1996**	1954***
United Kingdom	-5.18***	-5.24***	-37.95***	-4.28***	0.11***	0.85***	1947***	1943**	1945**	1942, 1944	1939, 1945	1917***
United States	-6.33***	-4.98***	-72.10***	-5.87***	0.08***	0.61***	1947***	1943	1944**	1939, 1943	1941, 1944**	1941***

Note: ADF critical values: -4.028, -3.445, -3.145 for 1, 5 and 10% levels respectively. For the Ng-Perron test (NP), none of the test statistics are significant at the usual levels. The critical values are taken from Ng and Perron (2001), table 1 and the autoregressive truncation lag (zero) has been selected using the modified AIC. The ZA test statistic reported is the minimum Dickey-Fuller statistic calculated across all possible breaks in the series, when both a break in the intercept and the time trend is allowed for. The year in parenthesis denotes the year when this minimum DF statistic is obtained. The 1% critical value is -5.57 and the 5% critical value is -5.08. As for the VP test, "AO" means addictive outlier and "IO" means innovational outlier and critical values are taken from Perron and Vogelsang (1992), in particular, -3.56 (AO) and -4.27 (IO) for 5% level. As for CMR the 5% critical value is -5.49 (both AO and IO), also taken from Perron and Vogelsang (1992). In column 10 we run the Perron-Yabu (PY) unit root test. For the structural-break type tests only dates are presented and when applicable, a statistically significant symbol is added. The null in the non-break type tests is of unit root. The null in the break-type tests is of unit root against the break stationary alternative hypothesis.

TABLE 3. Structural Time Series Models: Diagnostic and Goodness-of-fit statistics, Public Debt 1880-2009

Basic Structural Model	Log-Li.	P.E.V.	H(h)	Q(p,q)	Rd^2	AIC	BIC
Argentina	-250.925	212.744	4.451	4.991	0.084	5.407	5.475
Austria	-124.965	9.519	0.479	14.743	0.623	2.299	2.365
Belgium	-240.111	74.728	0.907	39.514	0.774	4.36	4.426
Brazil	-212.365	104.901	2.201	11.802	0.122	4.700	4.768
Denmark	-108.627	7.160	8.321	12.979	0.558	2.016	2.083
France	-241.475	108.976	0.031	15.016	0.708	4.738	4.806
Germany	-92.118	5.394	0.607	9.0637	0.684	1.732	1.800
Greece	-239.807	152.784	0.091	24.729	0.257	5.076	5.142
Italy	-267.251	71.390	0.124	24.548	0.075	4.315	4.383
Japan	-316.344	170.517	0.476	21.820	0.371	5.185	5.254
Netherlands	-174.364	24.073	1.004	16.062	0.934	3.228	3.296
New Zealand	-270.969	78.792	0.267	45.993	0.134	4.414	4.482
Norway	-155.037	14.621	5.319	15.314	0.387	2.730	2.797
Portugal	-136.207	18.390	0.407	22.023	0.533	2.958	3.024
Russian Federation	-99.498	61.374	2.404	17.903	0.679	4.165	4.232
Spain	-231.959	54.259	0.153	6.3130	0.238	4.039	4.106
Sweden	-122.354	9.176	15.77	20.662	0.519	2.262	2.329
United Kingdom	-271.397	68.033	0.222	22.660	0.348	4.266	4.332
United States	-195.735	20.820	0.577	20.936	0.238	3.082	3.148

Notes: The Log-Li is the Log-likelihood statistic. The P.E.V. statistic is the prediction error variance, which is equivalent to the variance of the disturbance term in the reduced form; the Rd^2 statistic is a modified coefficient of determination, which is obtained by using the sum of squares of the first differences about their mean, rather than the observations themselves, to compute the coefficient; H(n,m) is a test for heteroscedasticity, distributed as an F(n,m) under the null of no heteroscedasticity; Q(n) statistic is the Ljung-Box Portmanteau test for serial correlation based on n lags; AIC is the Akaike Information Criteria and BIC is the Bayesian Information Criteria.

TABLE 4. Structural Time Series Models: Estimated variances of disturbances, Public Debt 1880-2009

country	σ_{η}^{2}	σ_{ζ}^{2}	σ_{c1}^{2}	σ_{ε}^{2}	**Period 1**
Argentina	43.45	0.00	94.05	25.23	19.87
	(0.46)	(0.00)	(1.00)	(0.26)	
Austria	5.85	0.60	0.26	0.00	7.15
	(1.00)	(0.10)	(0.04)	(0.00)	
Belgium	0.00	27.79	10.12	2.23	9.65
	(0.00)	(1.00)	(0.36)	(0.08)	
Brazil	0.00	0.08	84.37	0.00	19.43
	(0.00)	(0.00)	(1.00)	(0.00)	
Denmark	3.38	0.81	0.66	0.00	15.13
	(1.00)	(0.24)	(0.19)	(0.00)	
France	0.00	4.12	26.18	18.06	15.16
	(0.00)	(0.16)	(1.00)	(0.69)	
Germany	3.12	0.46	0.14	0.00	7.46
	(1.00)	(0.15)	(0.04)	(0.00)	
Greece	16.53	6.99	0.74	49.33	7.03
	(0.34)	(0.14)	(0.02)	(1.00)	
Italy	58.00	0.06	4.10	0.00	11.09
	(1.00)	(0.00)	(0.07)	(0.00)	
Japan	0.00	4.60	84.30	0.00	11.60
	(0.00)	(0.05)	(1.00)	(0.00)	
Netherlands	4.86	15.69	0.00	0.00	12.45
	(0.31)	(1.00)	(0.00)	(0.00)	
New Zealand	0.00	2.18	39.99	0.00	13.20
	(0.00)	(0.05)	(1.00)	(0.00)	
Norway	4.34	0.00	7.14	0.00	16.39
	(0.61)	(0.00)	(1.00)	(0.00)	
Portugal	18.03	0.02	0.00	0.00	23.71
	(1.00)	(0.00)	(0.00)	(0.00)	
Russian Federation	0.00	0.52	28.75	0.00	9.06
	(0.00)	(0.02)	(1.00)	(0.00)	
Spain	31.14	3.03	1.06	2.75	11.29
	(1.00)	(0.10)	(0.03)	(0.09)	
Sweden	0.00	0.32	4.41	0.00	13.65
	(0.00)	(0.07)	(1.00)	(0.00)	
United Kingdom	0.00	12.30	24.38	0.00	13.92
	(0.00)	(0.50)	(1.00)	(0.00)	
United States	0.00	2.48	7.67	0.00	11.81
	(0.00)	(0.32)	(1.00)	(0.00)	

Note: σ_{η}^{2} , σ_{ζ}^{2} , σ_{ε}^{2} , σ_{c1}^{2} are variances of the stochastic components of the level, slope, irregular and cycle, respectively. Period 1 refers to the number of years of the cycle. In parenthesis we present the q-ratios. See main text for details.

Table 5.a First Generation Panel Unit Root Tests

Im, Pesaran and Shin (2003) Panel Unit Root Test (IPS) (a)

Full	**debt**
in levels	
lags	*[t-bar]*
1.47	*0.04*

Maddala and Wu (1999) Panel Unit Root Test (MW) (b)

Full	**debt**	
lags	p_{λ}	*(p)*
in levels		
0	18.63	0.99
1	37.90	0.47
2	40.19	0.37
in first differences		
0	712.35	0.00
1	485.40	0.00
2	317.90	0.00

Notes: The debt series are in percent of GDP. (a) We report the average of the country-specific "ideal" lag-augmentation (via AIC). We report the t-bar statistic, constructed as $t-bar=(1/N)\sum_{i}t_{i}$ (t_{i} are country ADF t-statistics). Under the null of all country series containing a nonstationary process this statistic has a non-standard distribution: the critical values are -1.73 for 5%, -1.69 for 10% significance level – distribution is approximately *t*. We indicate the cases where the null is rejected with **. (b) We report the MW statistic constructed as $p_{\lambda}=-2\sum_{i}\log(p_{i})$ (p_{i} are country ADF statistic p-values) for different lag-augmentations. Under the null of all country series containing a nonstationary process this statistic is distributed $\chi^{2}(2N)$. We further report the p-values for each of the MW tests.

Table 5.b: Second Generation Panel Unit Root Tests

Pesaran (2007) Panel Unit Root Test (CIPS)

Full	**debt**	
lags	p_{λ}	*(p)*
in levels		
0	3.00	0.99
1	0.94	0.82
2	1.21	0.88
in first differences		
0	-19.04	0.00
1	-16.43	0.00
2	-11.90	0.00

Notes: The debt series are in percent of GDP. Null hypothesis of non-stationarity. We further report the p-values for each of the CIPS tests.

Appendix: Time Series and Signal Extraction

Univariate Structural Time Series Model

Structural time series models are the appropriate practice for signal extraction, as they admit each of the unobserved components to have a stochastic nature. That is, the components describing the evolution of a given time series (trend, seasonality, cycles and irregular) have been traditionally modelled as deterministic; however, with sufficiently long series it is reasonable to consider that these components evolve randomly over time. This flexibility is recognized by structural models in the sense that they are no more than regression models in which explanatory variables are function of time, and parameters change over time (Harvey, 1989). The most simple example is one in which the observations fluctuate around an average level which is kept constant over time. If these fluctuations are stationary, in the sense that some values move around the average level in the short-run but they will tend and converge to that level in the long-run; and if one assumes that they are not correlated, we have the following formulation:

$$y_t = \mu_t + \varepsilon_t \tag{A1}$$

where ε_t is a white noise process with constant variance $\sigma^2{}_\varepsilon$. This is a model with an irregular component ε_t and a level component μ_t, which is fixed or deterministic. It is rare for an economic time series to be described by such a model.

The previous formulation can be extended as to allow the level of the series to change over time, giving place to a model in which the level at each moment is a function of the previous level plus a random element. This model can be described as:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t \\ \mu_t &= \mu_{t-1} + \eta_t, t = 1,...,n \end{aligned} \tag{A2}$$

where η_t is a white noise process with variance $\sigma^2{}_\eta$. This model has a random disturbance term around the underlying level which fluctuates without any particular direction – random walk with noise. The model may be used to represent a series' behaviour without seasonality or cycles, whose average level changes over time (stochastic level) but without having a systematic increasing or decreasing trend. When $\sigma^2{}_\eta = 0$ we end up with the first simplest model above in which the level is deterministic. If the irregular component variance is zero, $\sigma^2{}_\varepsilon = 0$, but the level variance is different from zero, then the series is a pure random walk.

Now, if we add a trend to the elements described above for the level component, then we have:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t \\ \mu_t &= \mu_{t-1} + \beta + \eta_t, t = 1,...,n \end{aligned} \tag{A3}$$

where β is a constant that measures the average growth rate of the series, that is, the slope of the trend. In this model, the level changes randomly over time, however the average growth rate, β, is constant. If one wants to make the series' dynamics even more flexible, allowing such growth rate to fluctuate over time, we have:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t \\ \mu_t &= \mu_{t-1} + \beta_t + \eta_t \\ \beta_t &= \beta_{t.1} + \xi_t, t = 1,...,n \end{aligned} \tag{A4}$$

where ξ_t is a white noise disturbance with variance $\sigma^2{}_\xi$. The disturbance term ξ_t gives to the slope a stochastic character. This model is known as local linear trend model and it represents the behaviour a time series without seasonality or cycles, whose underlying growth rate changes over time.

Theoretically, interesting cases of the previous model are when $\sigma^2{}_{\xi} = 0$, i.e., the series has slope, however with a constant average growth rate over time. If additionally, $\beta_t = 0$, then the series has no slope and the model transforms into a random walk with noise. Finally, it is possible to keep the slope's stochastic nature if $\sigma^2{}_{\xi}$ and simultaneously, assume that $\sigma^2{}_{\eta} = 0$, i.e., the level is deterministic. Below we present the formulation of this particular case. This fixed level model with stochastic trend is labelled smooth trend model:

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_t \\ \beta_t &= \beta_{t.1} + \xi_t, t = 1,...,n \end{aligned} \qquad \text{(A5)}$$

Variances of the random disturbances which affect the distinct components of the series are called hyperparameters. When these variances are different from zero it means that the associated component is stochastic.

Finally, in economic time series it is important to distinguish between a long-run trend and cyclical movements. The univariate structural model allows us to estimate several cycles. A stochastic representation of the cycle[22] is given by:

$$\begin{aligned} \psi_t &= \cos(\lambda_c \psi_{t-1}) + \sin(\lambda_c \psi^*{}_{t-1}) + \kappa_t \\ \psi^*{}_t &= -\sin(\lambda_c \psi_{t-1}) + \cos(\lambda_c \psi^*{}_{t-1}) + \kappa^*{}_t \end{aligned} \qquad \text{(A6)}$$

where κ_t and $\kappa^*{}_t$ are white noise disturbances not mutually correlated or with any other disturbance in the model, and they share a common variance $\sigma^2{}_{\kappa}$; the parameter λ_c is the frequency of the cycle measured in radians, i.e., it measures the number of time the cycle repeats itself over a period equal to 2π.

Outliers and Structural breaks

An outlier can be captured by a dummy explanatory variable in the measurement equation, known as an impulse intervention variable, which takes the value one at the time of the outlier and zero elsewhere.

A structural break in the level can be modelled by a step intervention variable in the measurement equation which is zero before the originating event and one on the event and after. Alternatively, it can be modelled by a dummy explanatory variable in the corresponding transition equation which takes the value one at the time of the structural break in the level and zero elsewhere.

A structural break in the slope can be modelled by a staircase intervention in the measurement equation which is a trend variable taking the values 1, 2, 3,..., starting in the period of the break. Alternatively, it can be modelled by a dummy variable in the corresponding transition equation, which takes the value one at the time of the structural break in the slope and zero elsewhere.

It is helpful to note that the level and slope breaks can be viewed in terms of impulse interventions applied to the level and slope equations of the model defined above. The structural framework also suggests that it may sometimes be more natural to think of an outlier as an unusually large value for the irregular disturbance. This leads to the notion of a level shift arising from an unusually large value of the level disturbance while a slope break can be thought of as a large disturbance to the slope component. Thus, interventions can be seen as fixed or random effects, however, the random effects approach is more flexible.

Viewing intervention effects as random is consistent with the representation of a stochastic trend in the STM model discussed above. For the detection of structural breaks, de Jong and Penzer (1998) showed that all that is required is to have the model set up in state space form in such a way that the level shift can be introduced by a pulse intervention somewhere in the transition equation.

[22] Sometimes, a damping factor, ρ, can be included, giving more flexibility to the cyclical component. If $|\rho|<1$ the cycle is stationary.