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Can Markov-regime switching models improve power price forecasts ? Evidence for German daily power prices

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Abstract

Nonlinear autoregressive Markov regime-switching models are intuitive and frequently proposed time series approaches for the modelling of electricity spot prices. In this paper such models are compared to an ordinary linear autoregressive model with regard to their forecast performance. The study is carried out using German daily spot prices from the European Energy Exchange in Leipzig. Four nonlinear models are used for the forecast study. The results of the study suggest that Markov regime-switching models provide better forecasts than linear models.

Keywords: Electricity spot prices, Markov regime-switching, forecasting.

JEL Classification: Q40, L94, C22.

1 Introduction

Until recently, the electricity sector has been a vertically integrated industry and prices have been fixed by regulators. The rapidly progressing deregulation is now leading to gradually privatised electricity markets. Increasingly large volumes are traded in these markets OTC as well as at electricity exchanges. Moreover, generated electricity must always match electricity consumption exactly. However, electricity demand depends to a great extent on weather conditions and is therefore stochastic. In order to ensure the reliable operation of the system, the so-called reserve capacity has to be maintained. In case of unforeseen high demand, for example, this reserve capacity is then exploited to balance electricity consumption and electricity generation. The suppliers of electricity can either make use of their own reserve capacities or buy additional electricity. The decision whether to buy electricity at an exchange, for example, strongly depends on the expected market price for electricity. Time series models capture the salient characteristics of electricity prices very well. Therefore, in order to support the make-or-buy decision, time series models with the aim to obtain reliable forecasts are of crucial importance.

Electricity prices, referred to as power prices in the remainder of the paper, exhibit stylized facts which differ from those of other traded commodities and financial securities. Electricity cannot be stored and as a result enormous price fluctuations, reflected by high volatility in electricity markets, are observed. Furthermore, several seasonality cycles, mean reversion and spikes are typical of power prices. Spikes are usually explained either by unexpected outages of large power plants or unpredicted changes of weather conditions. The literature on power prices is still limited. Important initial articles are those of Knittel and Roberts (2001) and of Lucia and Schwartz (2002). Knittel and Roberts (2001) evaluate the forecast performance of several univariate models using Californian power prices. Lucia and Schwartz (2002) present analytic formulas for the pricing of power derivatives. In addition, they take seasonality and mean reversion into account. Escribano, Peña and Villaplana (2002) suggest a very general jump model approach. They incorporate mean reversion, spikes and GARCH in their approach for the modelling of power prices. Moreover, Cuaresma, Hlouskova, Kossmeier and Obersteiner (2004) carry out a forecast study with several linear univariate time series models. They use data from the EEX in Germany. More recently, Angeles Carnero, Koopman and Ooms (2003) provide empirical evidence of periodic heteroskedastic RegARFIMA models. They apply them to four different markets. Furthermore, Burger, Klar, Müller and Schindlmayer (2004) derive a spot market model for hourly power prices at the EEX. They base their model on economic fundamentals of power prices in combination with a Seasonal-ARIMA approach. Nonlinear Markov regime-switching approaches in the spirit of Hamilton (1989) have been suggested and successfully applied for instance by Ethier and Mount (1998), Huisman and Mahieu (2003) and De Jong and Huisman

(2003). Basic idea behind these approaches is to model spikes as a separate regime.

The focus of this paper is on the forecasting performance of ordinary linear autoregressive models compared to nonlinear autoregressive Markov regime-switching models. For the sake of simplicity, we assume the autoregressive part of each model to consist merely of an AR(1) process. We consider models of de Jong and Huisman (2003), Ethier and Mount (1998) and, in addition, modified versions of both approaches. Modified models are to a great extent motivated by the fact that the original models only roughly distinguish between weekend days and holidays on one hand and working days on the other. However, distinguishing between different types of days provides a better fit and forecasting performance. The model proposed by de Jong and Huisman (2003) deviates from the pure Hamilton (1989) framework because the autoregressive part of the model is assumed to be regime- dependent, too. Therefore, a pure approach in terms of the Hamilton (1989) framework of Ethier and Mount (1998) is additionally incorporated in the study. The jump model is not considered in the study. There are mainly two reasons for this. On one hand, there is no elaborate forecasting methodology for these models and, on the other hand, jump models, to some extent, are a special case of Markov regime-switching models. So, whenever the more general model provides reasonable estimates, it captures more of the structure in the data and is therefore preferable.
For the empirical forecast study we estimate each considered model for a subsample of the given historical data. Then we carry out forecasts up to 100 steps ahead for observations held back at the estimation stage. In the following step, we augment the subsample which we use for estimation by one observation and carry out forecasting again. The results of the study indicate that Markov regime-switching models provide better forecasts, in particular, with respect to long run forecasts.
The remainder of the paper is organised as follows. In Section 2 we present the data, we use. Moreover, we show some descriptive statistics and define a deterministic model component of the logged power price. In Section 3 we introduce the considered stochastic models. Furthermore, in Section 4 we present and discuss results of the forecast comparison study. Section 5 concludes the paper and gives hints for further research.

2 Data and Descriptive Statistics

The EEX is the largest national power exchange in Europe where volumes up to nearly one fifth of the average daily electricity consumption of about 1350 000 MWh in Germany are traded. EEX wholesale electricity prices for 24 hours of the following day are determined through an auction. These day-ahead prices are typically referred to as spot prices. Besides hourly prices, so-called baseload and peakload prices are traded. The exchange EEX defines baseload prices as an equally weighted average of the 24 individual hourly prices, while peakload prices are determined by the equally weighted average of prices from 9 am to 8 pm. In this paper, we use data including baseload and peakload price series which range from June 16$^{\text{th}}$ 2000 to July 28$^{\text{th}}$ 2004. Figure 1 shows the baseload series that exhibits typical features of power prices like mean reversion and spikes. Power prices usually rather seem to follow a lognormal than a normal distribution. Therefore, most authors e.g. Burger, Klar, Müller and Schindlmayer (2004), de Jong and Huisman (2003), Escribano, Peña and Villaplana (2002) prefer working with the log of power prices instead of the original price series. In this paper, we follow their approach. Furthermore, Figure 1 shows the Q-Q plots of baseload against a normal distribution and a lognormal distribution, respectively. According to e.g. de Jong and Huisman (2003) and Escribano, Peña and Villaplana (2002) logged power prices $\log(\mathrm{P}_t)$ will be assumed to consist of two parts, a deterministic part denoted by f_t and a stochastic part X_t,

$$\log(P_t) = f_t + X_t \,. \tag{1}$$

Since the goal of this paper is to compare the forecasting properties of some stochastic models, we model the deterministic part of logged power prices as simply as possible but, on the other hand, still realisticly. Figure 2 shows the weekly seasonality. In order to take into account the weekly seasonality, weekend dummys for saturdays and sundays as well as a dummy for holidays are included. Moreover, since the range of the data covers more than four years, we include a deterministic trend and a sinusodal term to consider yearly seasonality.
The deterministic part of the logged power price f_t is specified as

$$f_t = \beta_1 \cdot dummy_{sat} + \beta_2 \cdot dummy_{sun} + \beta_3 \cdot dummy_h + \beta_4 \cdot t + \gamma_1 \cdot \sin\left((\gamma_2 + t) \cdot \frac{2\pi}{365} \right) .$$

We perform estimation and forecasting in `Eviews 5.0`. We use the implemented BHHH or Marquardt algorithm for numerical optimization.

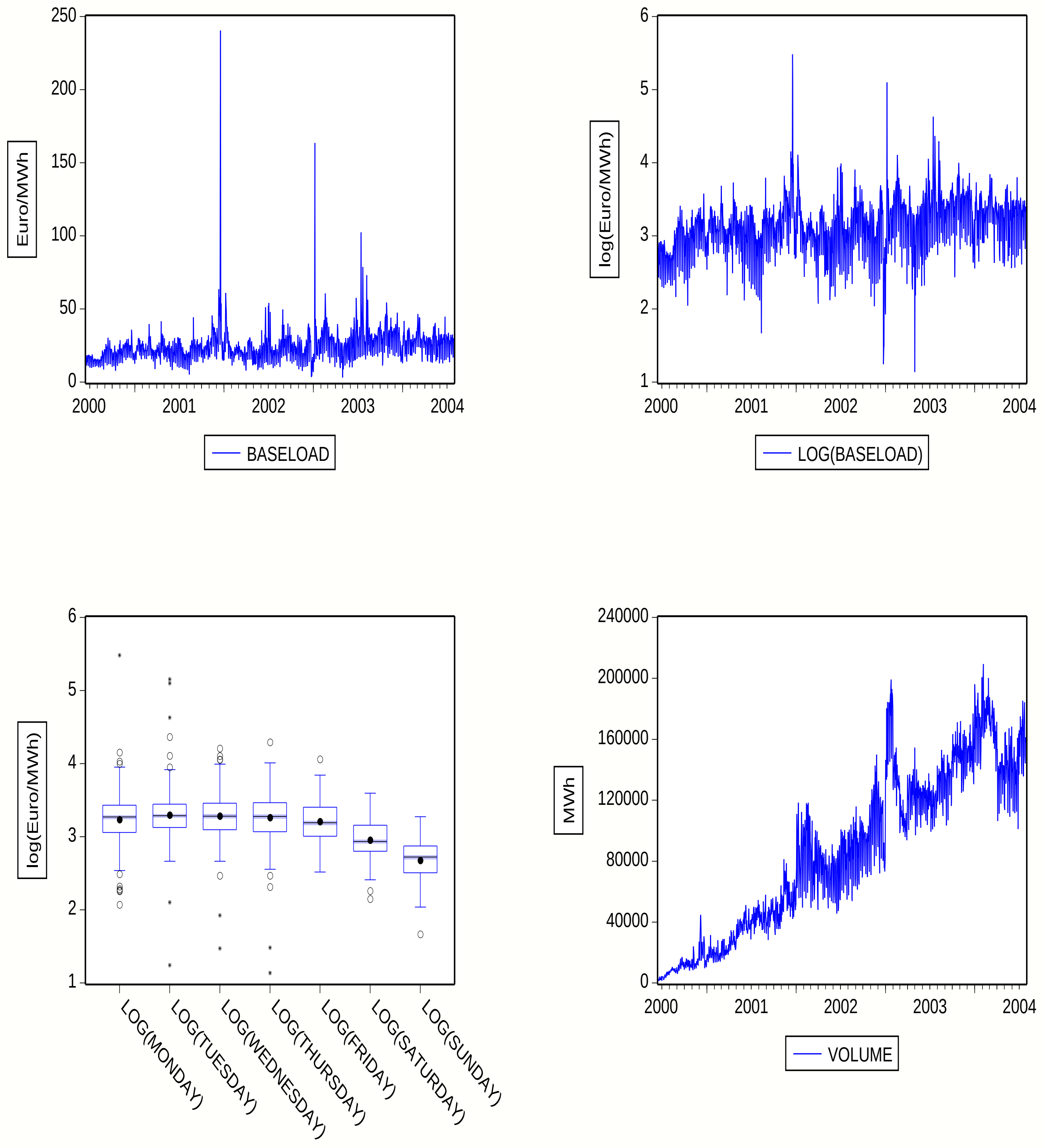

Figure 1: Plots for the baseload power prices, the log(baseload) logged power prices and the traded volume at the EEX.

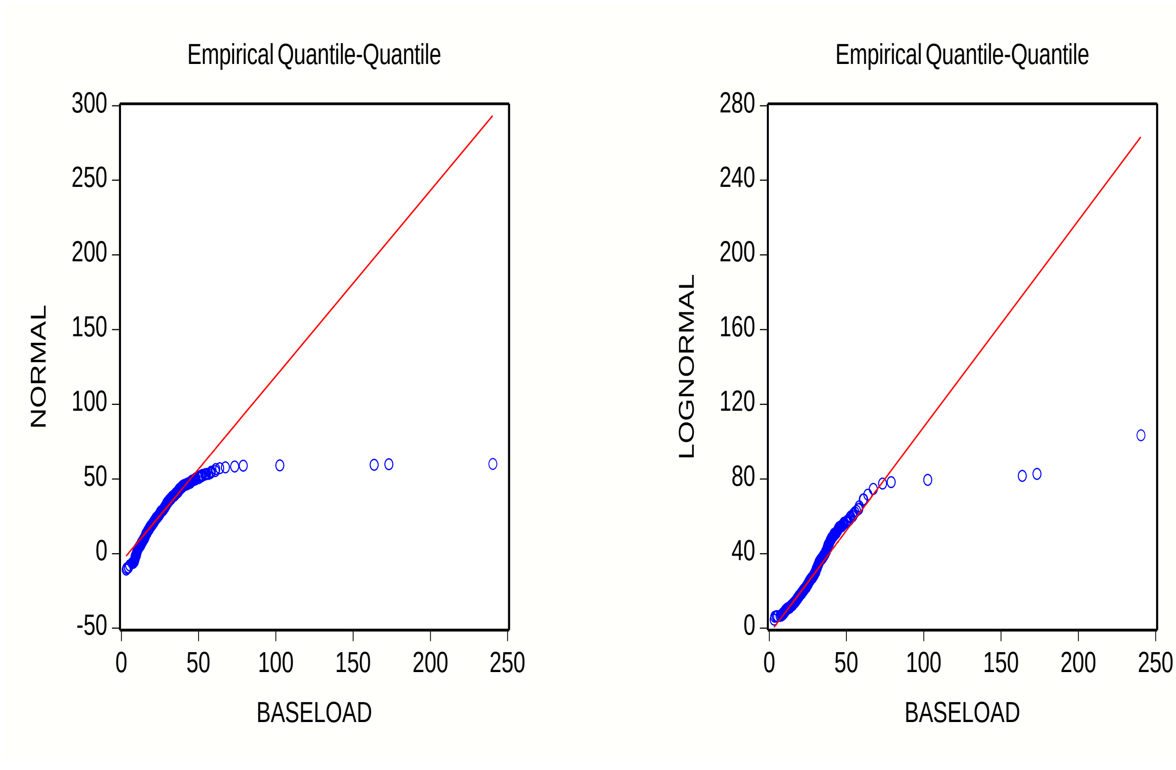

Figure 2: Q-Q plots for the baseload and log(baseload) power prices at the EEX.

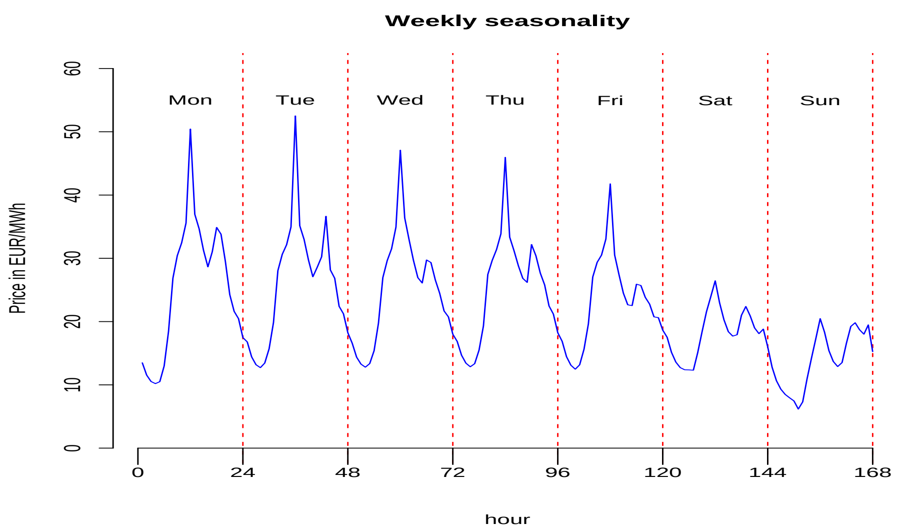

Figure 3: Weekly seasonality of baseload power prices at the EEX.

3 Stochastic Models for Power Prices

In this section, models for the stochastic part of power prices are outlined and discussed with a special focus on their predictive power.

3.1 Model I: AR(1) Process with Drift

A main stylized fact of power prices is mean reversion. This behavior of power prices can be modelled by an AR(1)-process. The mean, to which the process reverts, is μ_M.
Model I:

$$X_t = \alpha \cdot \mu_M + (1-\alpha) \cdot X_{t-1} + u_t \ , \qquad u_t \sim \mathcal{N}(0, \sigma^2). \tag{2}$$

We carry out the h-step ahead forecast the usual way based on the conditional expectation $E[X_{T+h}|\psi_T]$, where ψ_T denotes the information set at time T. Generally, there are two ways of carrying out forecasts. Both versions ($\log(P^f_{T+h,1})$, $\log(P^f_{T+h,2})$) are depicted below. Moreover, it should be noted that these forecasts procedures yield different forecasts for the logged power prices $\log(P^f_{T+h})$ in the presence of deterministic components. To clarify why this is so, the stochastic part X_t is replaced by $\log(P_t) - f_t$ according to equation (2),

$$\begin{aligned} 1) \quad X^f_{T+h,1} &= \log(P^f_{T+h,1}) - f^f_{T+h,1} \\ &= \alpha \cdot \mu_M + (1-\alpha) \cdot (\log(P_{T+h-1}) - f_{T+h-1}) \, , \end{aligned}$$

or

$$\begin{aligned} 2) \quad X^f_{T+h,2} &= \log(P^f_{T+h,2}) - f^f_{T+h,2} \\ &= \mu_M \cdot (1-(1-\alpha)^h) + (1-\alpha)^h \cdot (\log(P_T) - f_T) \, . \end{aligned}$$

The empirical results suggest that the recursive proceeding provides better forecasts than the second based on the forecast origin.

3.2 Model II : De Jong and Huisman (2003)

The basic idea behind this model is to distinguish between two independent regimes. In particular, one regime which is best described as the normal or stable regime, while the second independent regime serves to model spikes. As outlined in the introductory section, the peculiarity of this model is that the autoregressive part is presumed to prevail in the stable regime only.

Model II:

$$\begin{aligned} X_{M,t} &= X_{M,t-1} + \alpha \cdot (\mu_M - X_{M,t-1}\ ) + u_{M,t} && \text{stable regime,} \\ X_{S,t} &= \mu_S + u_{S,t} && \text{spike regime,} \end{aligned}$$

with $u_{M,t} \sim \mathcal{N}(0, \sigma^2)$, $u_{S,t} \sim \mathcal{N}(0, \sigma_S^2)$ $\quad (*)$

and transition matrix

	$S_{t-1} = M$	$S_{t-1} = S$
$S_t = M$	q	$1-p$
$S_t = S$	$1-q$	p

$(**)$.

Next we explain how the logarithmic likelihood can be constructed. Let $\ln L = \sum_{t=1}^{T} \ln f(X_t|\psi_{t-1})$ be the logarithmic likelihood and S_t denote the regime parameter S_t taking M when power prices are in the stable regime and S else. The conditional density is expressed as follows:

$$\begin{aligned} f(X_t|\psi_{t-1}) &= f(X_t, S_t = M|\psi_{t-1}) + f(X_t, S_t = S|\psi_{t-1}) \\ &= f(X_t|S_t = M, \psi_{t-1}) \cdot f(S_t = M|\psi_{t-1}) + \\ &\quad\ f(X_t|S_t = S, \psi_{t-1}) \cdot f(S_t = S|\psi_{t-1}) \end{aligned}$$

Moreover, the density $f(S_t = i|\psi_{t-1})$,$i \in \{M, S\}$, has to be determined. It holds,

$$f(S_t = j|\psi_{t-1}) = f(S_t = j, S_{t-1} = M|\psi_{t-1}) + f(S_t = j, S_{t-1} = S|\psi_{t-1}), \quad j \in \{M, S\},$$

$$f(S_t = j|S_{t-1} = M) \cdot f(S_{t-1} = M|\psi_{t-1}) + f(S_t = j|S_{t-1} = S) \cdot f(S_{t-1} = S|\psi_{t-1}).$$

The densities $f(S_t = j|S_{t-1} = i)$ are the one-step transition probabilities.

Densities of type $f(S_{t-1} = j|\psi_{t-1})$ are recursively calculated.
Due to $\psi_{t-1} = \{\psi_{t-2}, X_{t-1}\}$ it holds

$$f(S_{t-1} = j|\psi_{t-1}) = f(S_{t-1} = j|\psi_{t-2}, X_{t-1}) = \frac{f(S_{t-1} = j, X_{t-1}|\psi_{t-2})}{f(X_{t-1}|\psi_{t-2})}$$

$$= \frac{f(X_{t-1}|S_{t-1} = j, \psi_{t-2}) \cdot f(S_{t-1} = j|\psi_{t-2})}{f(X_{t-1}|S_{t-1} = M, \psi_{t-2}) \cdot f(S_{t-1} = M|\psi_{t-2}) + f(X_{t-1}|S_{t-1} = S, \psi_{t-2}) \cdot f(S_{t-1} = S|\psi_{t-2})}$$

The key problem with regard to this conditional density is the determination of $X_{M,t-1}$ because the last spot price originating from the stable regime is not known. De Jong and Huisman (2003) propose a modification of this conditional density before they use it to maximize the logarithmic likelihood. In order to take into account the mentioned problem, they replace the part of the conditional density that represents the stable regime $f(X_t|S_t = M, \psi_{t-1})$ by an approximation,

$$f(X_t|S_t = M, \psi_{t-1}) \approx \sum_{i=1}^{K} Prob[\lambda(t-i) = M \wedge \lambda(t-j, j < i) \neq M] \cdot f(X_t|S_t = M, \psi_{t-i})$$

K denotes how far we maximally go back to find the last spot price originating from the stable regime.

$$f(X_t|S_t = M, \psi_{t-i}) \approx \frac{1}{Var[X_{M,t}|\psi_{t-i}] \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{(X_{M,t} - E[X_{M,t}|\psi_{t-i}])^2}{2 \cdot Var[X_{M,t}|\psi_{t-i}]}\right)$$

with

$$\begin{aligned} E[X_{M,t}|\psi_{t-1}] &= \alpha \cdot \mu_M + (1-\alpha) \cdot E[X_{M,t-1}|\psi_{t-2}] \\ Var[X_{M,t}|\psi_{t-1}] &= (1 + (1-\alpha)^2) \cdot Var[X_{M,t-1}|\psi_{t-2}] \end{aligned}$$

Applying these equations for the conditional expectations and variances yields :

$$E[X_{M,t}|\psi_{t-i}] = (1-\alpha)^i \cdot X_{M,t-i} + \mu_M \cdot (1 - (1-\alpha)^i)$$

$$Var[X_{M,t}|\psi_{t-i}] = \sigma_M^2 \cdot \frac{(1-\alpha)^{2 \cdot i} - 1}{(1-\alpha)^2 - 1}$$

The expression $Prob[\lambda(t-i) = M \wedge \lambda(t-j, j < i) \neq M]$ is the probability of the logged spot price X_{t-i} to be the last logged spot price before X_t originating from the stable regime. X_{t-j} with $j < i$, whereas, are supposed to be spikes. The remaining problem is to determine the right K. De Jong and Huisman (2003) propose $K = 5$. In this paper, $K = 5$ is considered as sufficient, too.

3.3 Model IIb : Two Regime Model with a Modified Spike Regime

The original model proposed by de Jong and Huisman (2003) assumes that deviations from the stable regime are independent of the type of the day. However, it seems more convenient to distinguish between working days on one hand and weekends and holidays on the other. Very low demand is typical of weekends and holidays. Therefore, upward directed spikes are rather not to expect, whereas we can detect downward directed deviations from the stable regime in data. In order to take into account different types of days, in this modified model high spikes and low spikes are distinguished.
Practically, we decompose spikes by declaring an indicator function $\mathbf{1_H}$ which takes the value zero on holidays, weekend days, and two days before and after a holiday. All remaining days are candidates for high spikes only, so in these

cases the indicator function takes value 1. The decomposition fits to observed German data. However, weekends and holidays are days of low demand not only in Germany, so this decomposition might be suitable for other countries, too.

Model IIb :

$$\begin{aligned} X_{M,t} &= X_{M,t-1} + \alpha \cdot (\mu_M - X_{M,t-1}) + u_{M,t} && \text{stable regime,} \\ X_{S,t} &= \mathbf{1_H} \cdot (\mu_{S,H} + u_{S,H,t}) + (\mathbf{1} - \mathbf{1_H}) \cdot (\mu_{S,L} + u_{S,L,t}) && \text{spike regime,} \end{aligned}$$

with $u_{M,t} \sim \mathcal{N}(0, \sigma^2)$, $u_{S,H,t} \sim \mathcal{N}(0, \sigma^2_{S,H})$, $u_{S,L,t} \sim \mathcal{N}(0, \sigma^2_{S,L})$ $\quad (***)$

and transition matrix

	$S_{t-1} = M$	$S_{t-1} = S$
$S_t = M$	q	$1-p$
$S_t = S$	$1-q$	p

$(****)$.

Although the de Jong and Huisman (2003) model apparently goes beyond the popular Hamilton methodology, nevertheless, this model still fits into the theoretical Hamilton framework. Consequently, the forecasting methodology, we use, is based on Hamilton (1989). How to apply this methodology is described below.

Let $\xi_{(T|\psi_T)}$ be the vector of posterior densities at time T,

$$\xi_{(T|\psi_T)} = \begin{pmatrix} \dfrac{f(X_T, S_T = M|\psi_{T-1})}{f(X_T|\psi_{T-1})} \\ \\ \dfrac{f(X_T, S_T = S|\psi_{T-1})}{f(X_T|\psi_{T-1})} \end{pmatrix}.$$

Moreover let P be the transition matrix,

$$P = \begin{pmatrix} q & 1-p \\ 1-q & p \end{pmatrix}.$$

The h-step ahead forecasts for the posterior probabilities are computed as follows

$$\xi^f_{T+h} = P \cdot \xi_{(T+h-1|\psi_T)}.$$

V_{T+h} is defined as the vector containing the conditional expectations $E[X_{S_{T+h},T+h}|\psi_T]$ for each regime. Then in case of Model II holds

$$V_{T+h} = \begin{pmatrix} E[X_{M,T+h}|\psi_T] \\ E[X_{S,T+h}|\psi_T] \end{pmatrix},$$

with $E[X_{S,T+h}|\psi_T] = \mu_S$. Furthermore, in the case of Model IIb holds

$$V_{T+h} = \begin{pmatrix} E[X_{M,T+h}|\psi_T] \\ E[X_{S,T+h}|\psi_T] \end{pmatrix}$$

with $E[X_{S,T+h}|\psi_T] = \mu_{S,H} \cdot \mathbf{1_H} + \mu_{S,L} \cdot (1 - \mathbf{1_H})$. Finally the forecast results as

$$X^f_{T+h} = V^T_{T+h} \cdot \xi^f_{T+h}.$$

In this framework the two regimes are assumed to be independent. Therefore, the forecast for the stable regime is

$$E[X_{M,T+h}|\psi_T] = \mu_M \cdot (1 - (1-\alpha)^h) + (1-\alpha)^h \cdot E[X_{M,T}|\psi_T].$$

Again, the problem remains to determine the last spot price originating from the stable regime. We refer to the ideas of de Jong and Huisman (2003). Therefore, a possible solution is to approximate $E[X_{M,T}|\psi_T]$ by

$$E[X_{M,T}|\psi_T] \approx \sum_{i=0}^{K-1} Prob[\lambda(T-i) = M \wedge \lambda(T-j, j<i) \neq M] \cdot E[X_{M,T}|\psi_{T-i}].$$

The probabilities $Prob[\lambda(T-i) = M \wedge \lambda(T-j, j<i) \neq M]$ can be very easily computed, since posterior probabilities are given, and

$$E[X_{M,T}|\psi_{T-i}] = (1-\alpha)^i \cdot X_{M,T-i} + \mu_M \cdot (1 - (1-\alpha)^i).$$

The results of the empirical forecast study, however, support to take the ordinary value of X_T as forecast origin rather than the conditional expectation described above. Moreover, one should bear in mind that taking X_T as forecast origin, when X_T is indeed a spike, means to make a mistake. This kind of mistake is of minor importance because spikes rarely occur in the given data. Power price series which exhibit more spikes might require a modification as outlined above.

3.4 Model III: Ethier and Mount (1998)

Recalling the introductory statements, a pure approach according to Hamilton is, additionally, considered in this subsection. Contrary to the basic model proposed by Hamilton (1989) and according to Ethier and Mount (1998) heteroscedasticity is now assumed. Similar to the de Jong and Huisman (2003) model, variances of disturbances are presumed to be regime-dependent, too. We consider two models, Model III and a modification denoted as Model IIIb.

Model III:

$$X_{\{S_t=j\},t} = \mu_{\{S_t=j\}} + (1-\alpha)\cdot\left(X_{\{S_{t-1}=i\},t-1} - \mu_{\{S_{t-1}=i\}}\right) + u_{\{S_t=j\},t}, \qquad j,i \in \{M,S\}.$$

Model IIIb :

$$\begin{aligned} X_{\{S_t=j\},t} &= \mu_{\{S_t=j\}} + (1-\alpha)\cdot\left(X_{\{S_{t-1}=i\},t-1} - \mu_{\{S_{t-1}=i\}}\right) + u_{\{S_t=j\},t}, \qquad j,i \in \{M,S\}, \\ \mu_S &= \mathbf{1_H}\cdot(\mu_{S,H}) + (\mathbf{1}-\mathbf{1_H})\cdot(\mu_{S,L}), \\ u_{S,t} &= \mathbf{1_H}\cdot(u_{S,H,t}) + (\mathbf{1}-\mathbf{1_H})\cdot(u_{S,L,t}), \end{aligned}$$

Furthermore, we define the disturbances and the transition matrix as in $(*)$ and $(**)$ for Model III and as in $(***)$ and $(****)$ for Model IIIb.
We carry out forecasting in the same way as in Models II and IIb with the matrix

$$P = \begin{pmatrix} q & q & 0 & 0 \\ 0 & 0 & 1-p & 1-p \\ 1-q & 1-q & 0 & 0 \\ 0 & 0 & p & p \end{pmatrix}$$

and the matrix of posterior densities

$$\xi_{(T|\psi_T)} = \begin{pmatrix} \dfrac{f(X_T, S_T = M, S_{T-1} = M|\psi_{T-1})}{f(X_T|\psi_{T-1})} \\ \\ \dfrac{f(X_T, S_T = M, S_{T-1} = S|\psi_{T-1})}{f(X_T|\psi_{T-1})} \\ \\ \dfrac{f(X_T, S_T = S, S_{T-1} = M|\psi_{T-1})}{f(X_T|\psi_{T-1})} \\ \\ \dfrac{f(X_T, S_T = S, S_{T-1} = S|\psi_{T-1})}{f(X_T|\psi_{T-1})} \end{pmatrix}$$

We compute the h-step ahead forecasts for the posterior densities as follows

$$\xi^f_{T+h} = P\cdot\xi_{(T+h-1|\psi_T)}.$$

Let V_{T+h} again be the vector that contains the conditional expectations $E[X_{S_{T+h},T+h}|\psi_T]$ for each regime. Then, according to Clements and Krolzig (1998), the following recursion holds

$$X^f_{T+h} = V^T_{T+h}\cdot\xi^f_{T+h},$$

$$X^f_{T+h} = \mu_{T+h|T} + (1-\alpha) \cdot (X_{T+h-1} - \mu_{T+h-1|T}),$$

and

$$\mu_{T+h|T} = \sum_j \mu_j \cdot Prob(S_{T+h} = j|X_T), \qquad j \in \{M, S\}.$$

In case of Model IIIb, μ_S is replaced by $\mathbf{1_H} \cdot \mu_{S,H} + (1 - \mathbf{1_H}) \cdot \mu_{S,L}$.
The forecast for Model III can also be written as

$$X^f_{T+h} = \sum_j \mu_j \cdot Prob(S_{T+h} = j|X_T) + (1-\alpha)^h \cdot \left(X_T - \sum_j \mu_j \cdot Prob(S_T = j|X_T) \right),$$

and again in model IIIb μ_S is replaced by $\mathbf{1_H} \cdot \mu_{S,H} + (1 - \mathbf{1_H}) \cdot \mu_{S,L}$.
As $h \to \infty$ the posterior probability to be in regime j, $Prob(S_{T+h} = j|X_T)$, converges to the unconditional probability to be in regime j since the Markov chain is assumed to be ergodic. This also holds in the de Jong and Huisman (2003) framework,

$$\lim_{h\to\infty} Prob(S_{T+h} = M|X_T) = \frac{1-p}{2-p-q},$$

$$\lim_{h\to\infty} Prob(S_{T+h} = S|X_T) = \frac{1-q}{2-p-q}.$$

Remarks

Instead of the modification proposed in this paper, a three regime approach, modelling low spikes as a separate regime, might seem appropriate. Empirical evidence, however, supports the modification proposed in this paper. A three regime approach is much harder to estimate and yields poor results in terms of fit compared to the modified Models IIb and IIIb.
The modification of the likelihood proposed by de Jong and Huisman (2003) is necessary because posterior probabilities $f(S_{t-1} = M|\psi_{t-1})$ and $f(S_{t-1} = S|\psi_{t-1})$ are not 1 or 0 but probabilities in between. The consequence is that the regime-dependent parameters are not estimated independently from parameters of the other regime and, by this, are biased. The modification proposed by de Jong and Huisman reduces the impact of spikes on the stable regime parameters and vice versa. Moreover, it seems that the modification of de Jong and Huisman is not easily applicable to more sophisticated models because computing conditional expectations and conditional variances is very cumbersome. Being aware of this problem, the study has been restricted to the simple specification of the autoregressive process. Further research is needed to develop an approach similar to that of de Jong and Huisman which is applicable to more sophisticated models.

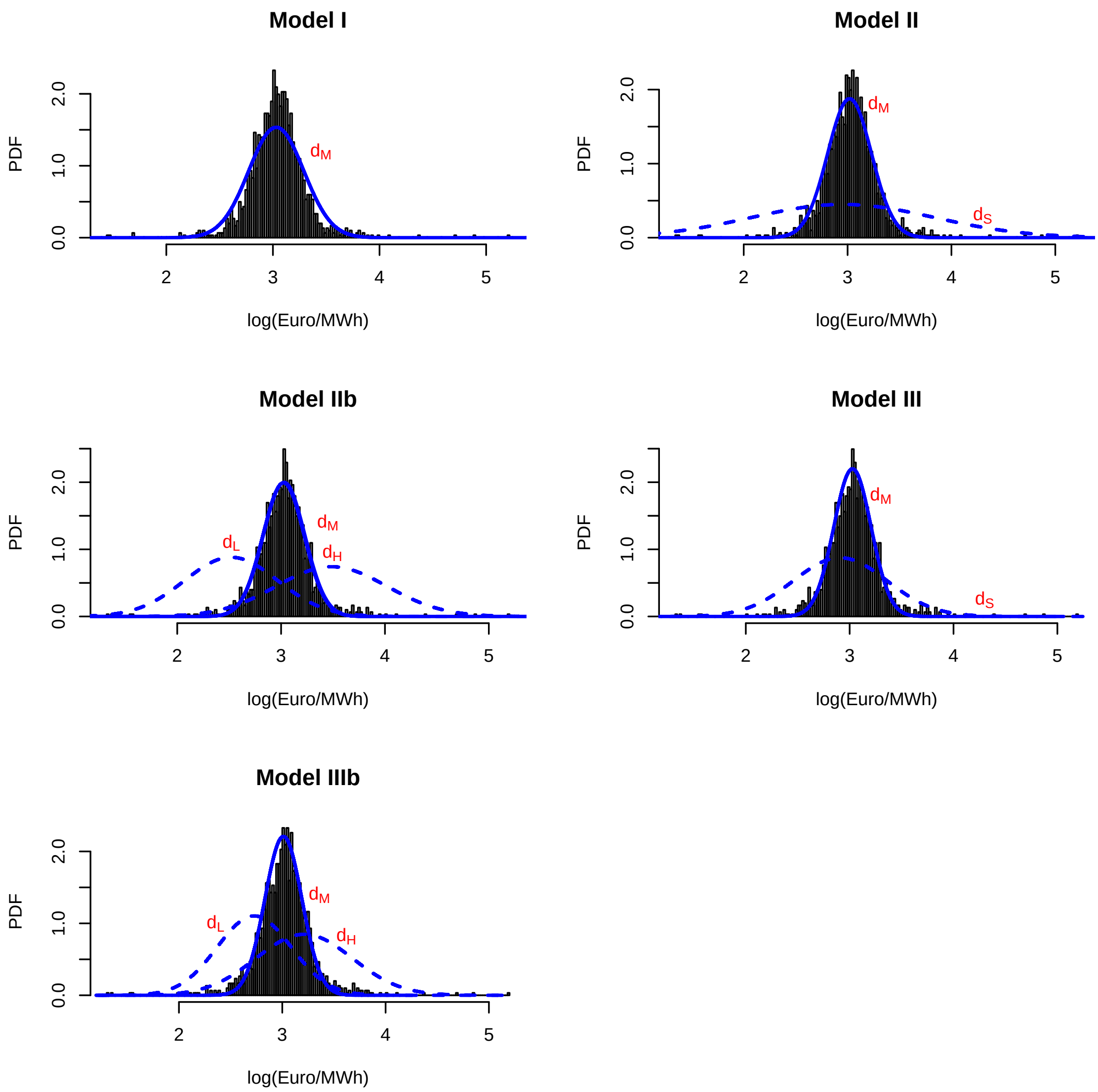

Figure 4: Histograms for log(baseload) from which the deterministic effects have been removed are plotted together with the estimated normal densities for all considered models.

4 A Forecast Comparison Study

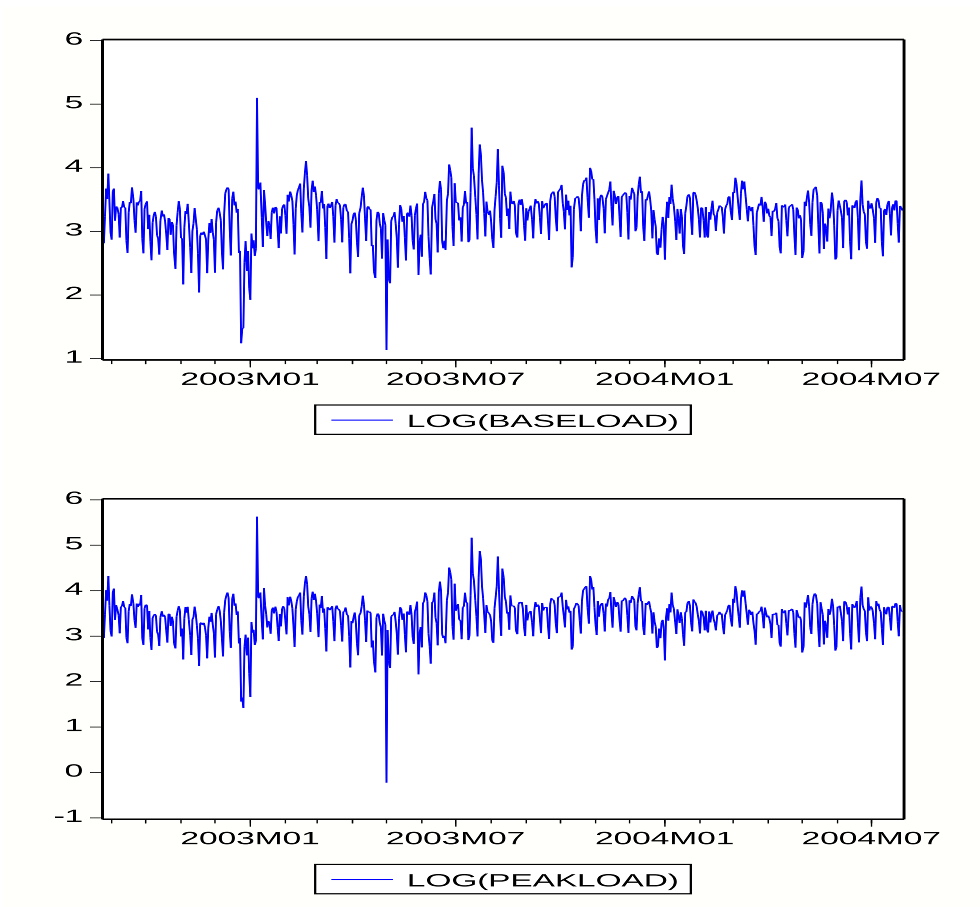

Figure 5: Forecast horizon : 25^{th} August 2002 to 28^{th} July 2004.

In this forecast comparison study, we carry out and evaluate ex ante forecasts in terms of RMSE and MAE. All given information available at time T is exploited and, by this, we use all known electricity prices up to T to estimate the parameter values. This proceeding is reasonable since electricity prices exhibit strong seasonality and autocorrelation that are estimated the better the more data is available. The forecasting procedure is close to that of Cuaresma,Hlouskova,Kossmeier and Obersteiner (2004) applied to hourly prices and is described below.
The given dataset is divided into an in-sample period which includes observations from
6/16/2000 : 8/24/2002 at the beginning. Moreover, the out-of-sample period

ranges from 8/25/2002:7/28/2004. The forecasting experiment is designed as follows. We use in-sample data to estimate the parameters of the model of interest. We, then, make out-of-sample make forecasts up to 100 steps ahead and evaluate them. The in-sample period is then enlarged by one observation and again forecasts for the out-of-sample period are made and evaluated. We repeat this procedure 604 times. This forecasting study has been carried out using logged baseload and logged peakload prices, respectively. Furthermore, the used measures have been computed for each h- step ahead forecast with $h \in \{1, 2, \ldots, 100\}$. P_t denotes the actual observed price at time t, while P_t^f refers to the predicted price at time t. The measures used for comparison are

$$RMSE = \sqrt{\frac{1}{k} \cdot \sum_{i=1}^{k} \left(\log(P_t) - \log(P_t^f) \right)^2},$$

$$MAE = \frac{1}{k} \cdot \sum_{i=1}^{k} \left| \log(P_t) - \log(P_t^f) \right|.$$

All of the presented five models are considered in the comparison study.

4.1 Short Discussion of Results

First of all, we want to stress that prices at the edge of the forecasting sample do not have the same weight on the outcome of the study as prices which lie rather in the middle. This aspect must be taken into consideration when the results of the empirical study are gauged. One consequence is that nonlinear models clearly outperform the linear model in terms of long run forecasting (30-80 steps ahead). Periods with larger sized spikes are settled rather in the middle while stable periods prevail at the beginning and at the end of the whole forecasting sample 8/25/2002-7/28/2004. In order to scrutinize the outcome of the study, the linear autoregressive model performs well in terms of very short run forecasts, in particular, one up to two steps ahead. For both measures, we cannot observe any clear difference from the remaining models. With respect to the long run ability, however, nonlinear models outperform the linear model which is partly due to the position of spikes in the time series. Another reason is the improved estimation compared to the linear model. Estimation of important parameters for forecasting like deterministic components and μ_M is less influenced by spikes. For short run forecasting better estimates of α are of interest, too. Moreover, modified Models IIb and IIIb outperform their basic counterparts II and III with respect to the RMSE. This is a consequence of the modification of the spike regime, since the direction of spikes is better predicted by the modified models. Finally, Model IIIb provides better long run forecasts than model IIb, while in terms of short run forecasting, the opposite is true. Forecasts for the stable regime have to be based on the forecast origin X_T in Models II and IIb. A recursive proceeding, like in

the pure Hamilton framework, is prohibited due to the assumption of independent regimes. However, there is empirical evidence that in the presence of deterministic components, a recursive proceeding provides better forecasts. Another problem arises if X_T is indeed a spike but treated as originating from the stable regime. Obviously, forecasts for the stable regime based on a spike are biased. Short run forecasts of Model IIb are better than those of Model IIIb. Therefore, forecasting the stable regime based on X_T, when X_T is indeed a spike, is of minor importance. The recursive proceeding in forecasting is the advantage of Model IIIb compared to Model IIb. Additionally, we have only used the stable regime of Models II and IIb to make forecasts. These forecasts are denoted by II-stable and IIb-stable in Figures 5 and 6. For Model II, we obtain better forecasts if we renounce to exploit the whole nonlinear methodology. However, this holds unless the modification proposed in this paper is implemented.
Furthermore, outcomes with respect to the two proposed measures are different. In terms of MAE, models which perform best with respect to the RMSE are often nearly or indeed outperformed by their unmodified counterparts. Moreover, the performance of forecasts if we only use the stable regime is remarkably good and sometimes even best with respect to the MAE. To understand these results, it is necessary to bear in mind that deviations due to outliers have much more impact and are more penalized by the RMSE than by the MAE. Therefore, the advantage of modified models compared to the unmodified models is of minor importance with respect to the MAE.

5 Conclusion

Markov regime-switching models are frequently discussed in the literature that deals with electricity spot prices. In this paper, the focus goes beyond the mere estimation and takes account of the forecast ability of nonlinear models. Furthermore, modified versions of these models are also considered within the study. The key question in the paper is whether the nonlinear approach provides better forecasts than an ordinary linear autoregressive specification.
The obtained results of the forecast study suggest that there is a benefit of taking the nonlinear model at least for long run forecasting. In the case of the de Jong and Huisman (2003) framework, we already obtain better forecasts compared to the linear model when forecasting is carried out with the stable regime only.
To discuss the practical implication of this study, we recall the underlying practical problem of electricity suppliers:Electricity suppliers have to take volume risk due to weather into account. They can either maintain own reserve capacities or buy additional electricity. This decision depends on the expected price for additional electricity which has to be predicted. Nonlinear models provide better forecasts than linear models. Therefore, suppliers can reduce the risk of a wrong decision due to a bad forecast of the expected price. Provided that the other

market participants base their forecasts on worse models, a better model offers arbitrage opportunities. The suppliers are buyers as well as sellers of electricity. Since baseload and peakload are determined through an auction, a better forecast of the expected price helps to improve the bidding strategies of sellers and buyers. For example, the sellers of electricity can reduce the risk of demanding too high as well as too small bid prices for the volume that they offer.

Besides the day-ahead spot market, monthly futures and options based on monthly futures are traded at the EEX. These financial instruments are used to hedge price risk, since power prices are subject to high volatility. The good long run forecasts of nonlinear models suggest that nonlinear models can also be useful for forecasting of future spot prices. Therefore, nonlinear models can help to value electricity derivatives.

This study has been restricted to a very simple specification for the deterministic as well as for the stochastic component. However, several studies like Angeles Carnero, Koopman and Ooms (2003), Burger, Klar, Müller and Schindlmayer (2004) and Cuaresma,Hlouskova,Kossmeier and Obersteiner (2004) indicate that more sophisticated time series specifications are needed. Therefore in future research, specifications that incorporate time varying parameters should be considered.

Furthermore, the distribution of spikes among days of the week suggests that time varying transition probabilities should be included. Moreover, it could be an asset to include explanatory variables associated with weather or economic fundamentals of electricity in a good model. Finally, models for hourly prices should be conceived, since hourly prices and so-called blocks of hourly prices are traded at the EEX power exchange.

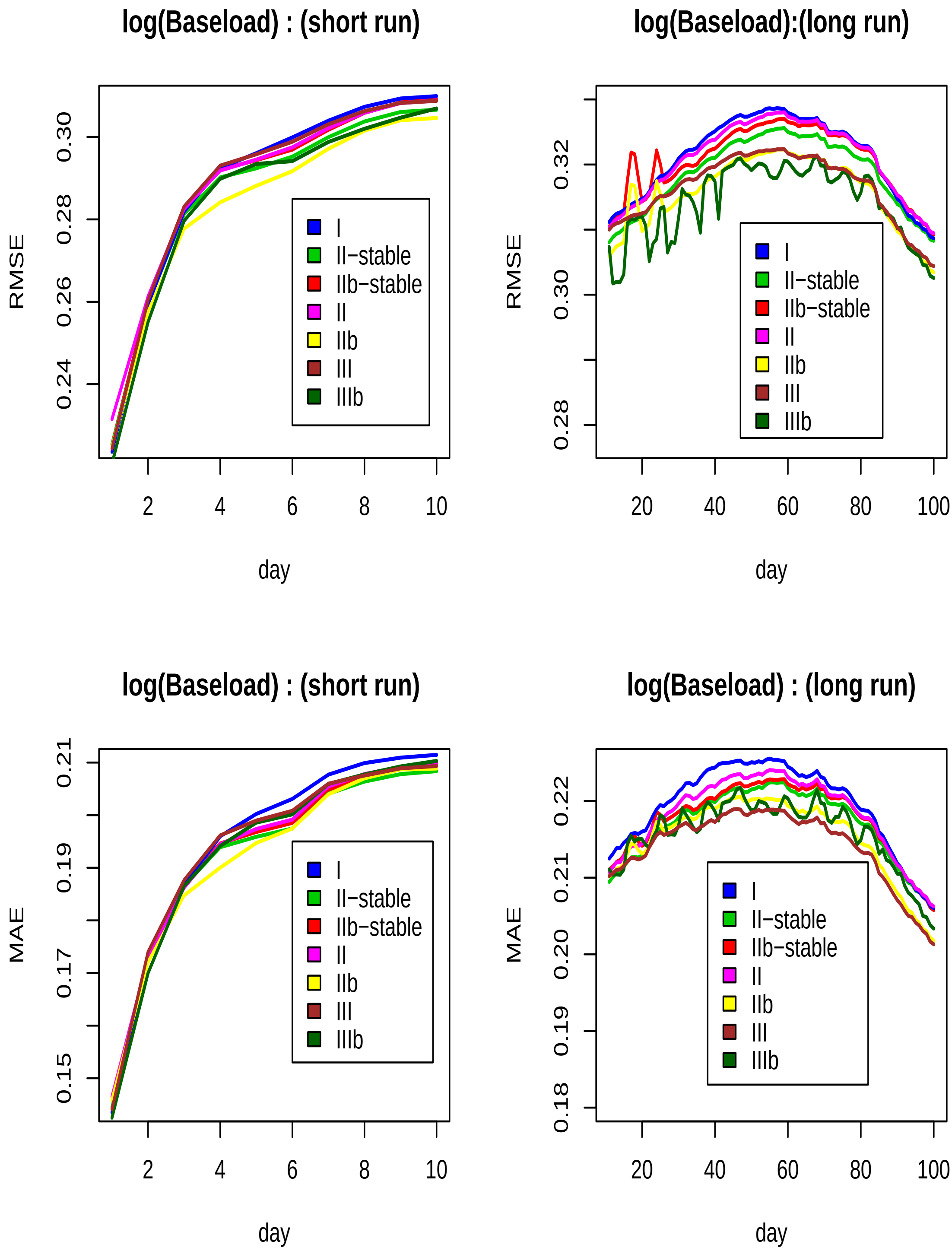

Figure 6: Results of all models for the log(baseload) time series.

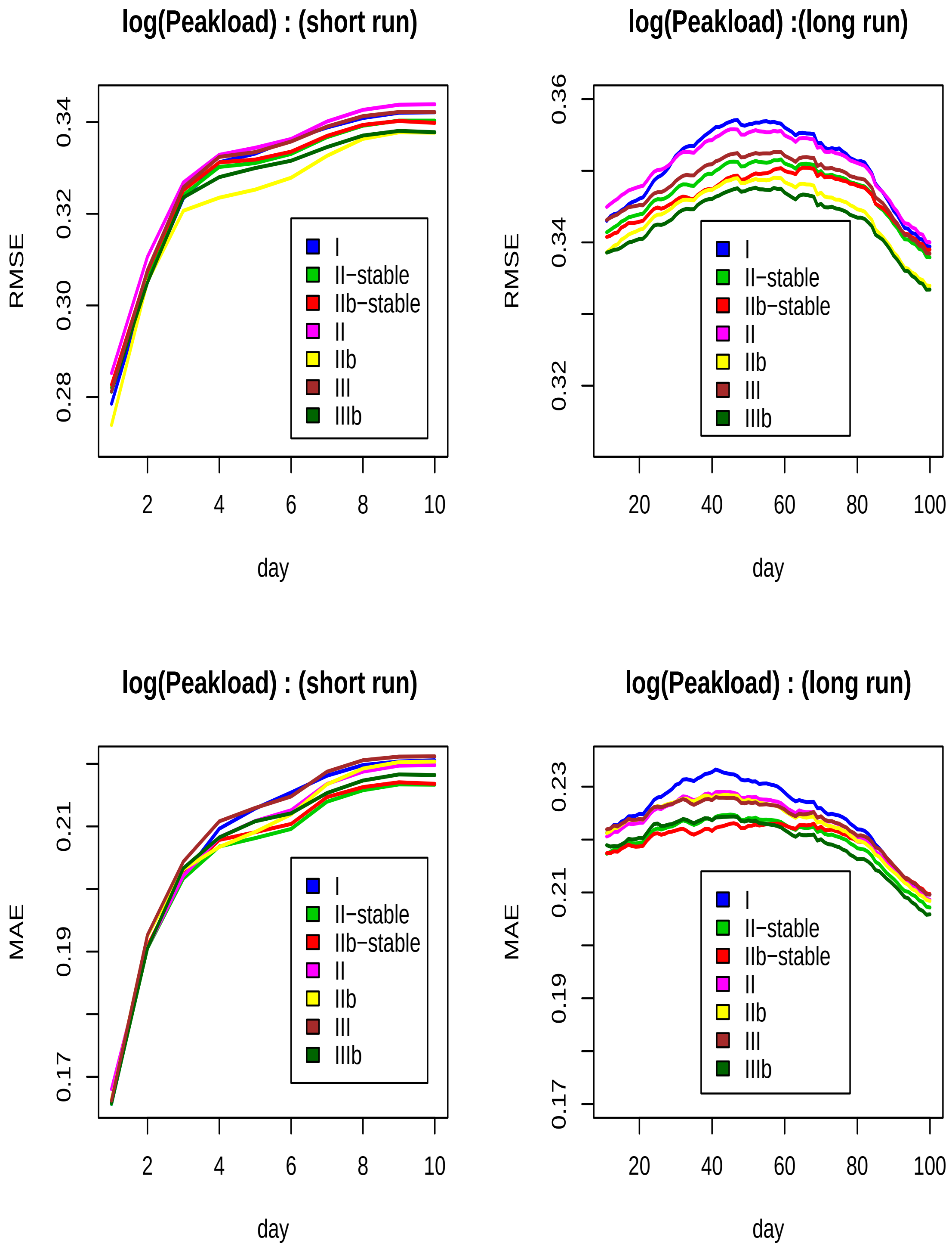

Figure 7: Results of all models for the log(peakload) time series.

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