The University of Adelaide
School of Economics

Research Paper No. 2011-23
May 2011

Variety Matters

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March 25, 2011

Abstract

Countercyclical markups are a key transmission mechanism in many endogenous business cycle models. Yet, recent findings suggest that aggregate markups in the US are procyclical. The current model adresses this issue. It extends Galí's (1994) composition of aggregate demand model by endogenous entry and exit of firms and by product variety effects. Endogenous business cycles emerge with procyclical markups that are within empirically plausible ranges.

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1 Introduction

Beginning with Benhabib and Farmer (1994), Farmer and Guo (1994) and Galí (1994) to name just three, a large body of work now exists in which a standard real business cycle model is modified to generate indeterminate equilibria driven by sunspot shocks. The two central mechanisms that produce this indeterminacy result are increasing returns to scale technologies and countercyclical markups. While the existence of scale economies has undergone much empirical testing, empirical work on the aggregate markup's behavior over the business cycle remains less evolved. Moreover, Nekarda and Ramey (2010) suggest that markups in the US are procyclical or acyclical. Hence, the Nekarda-Ramey findings put doubt on the plausibility of endogenous business cycle models that build on countercyclical markups.[1] The current paper addresses this issue by laying out an artificial economy that is susceptible to endogenous business cycles even if markups are procyclical.

Specifically, we investigate the roles of taste for variety and the composition of aggregate demand in a general equilibrium economy with endogenous entries and exits of monopolistically competitive firms.[2] These firms supply differentiated intermediate goods that are used in the production of final consumption goods and investment goods. The technologies of assembling the two final goods differ, hence, the composition of demand affects the degree of market power of the monopolistic firms. At this stage, the model resembles Schmitt-Grohé (1997) and Galí (1994). However, we are able to show that endogenous net business formation eliminates the existence of sunspot equilibria. For these equilibria to re-emerge, we introduce taste for variety. Variety effects connote the idea that a greater number of differentiated products enhances efficiency.[3] A rise in product variety then leads to a fall in

[1] The finding is not uncontroversial and Rotemberg and Woodford (1999), for example, claim that markups are countercyclical.

[2] The procyclicality of firm entry and its implications for the business cycle has been discussed by Chatterjee and Cooper (1993), Devereux, Head and Lapham (1996), and more recently by Jaimovich (2007) who also offers empirical support.

[3] Our formulation of the variety effect (aka love of variety, variety gains, increasing returns to variety, returns to specialization) follows Benassy (1996) as it allows to separate the effect from the elasticity of substitution.

aggregate prices (relative to the price of intermediate goods) or equivalently to an endogenous rise in the efficiency wedge, which can be interpreted as a rise in productivity. If the taste for variety is sufficiently large (in a well defined way) then economic fluctuations can be driven by self-fulfilling beliefs. Moreover, we show that these belief shocks generate artificial business cycles that resemble empirically observed fluctuations. Returning to Nekarda and Ramey's (2010) findings, artificial markups can be either procyclical or countercyclical. In fact, we identify situations where procyclical markups (in the empirically plausible range) make indeterminacy easier to obtain.

Several studies affirm the presence of variety effects. This empirical evidence includes Funke and Ruhwedel (2001) who report a positive relationship between an index of product variety and per capita income. Feenstra and Kee (2008) and Feenstra (2010) find similar effects in the trade context. Ardelean (2009) estimates consumer's love for variety and suggests that variety matters for both imported and domestically produced goods. Drescher, Thiele and Weiss (2008) present evidence on consumers' preferences for variety in food consumption. While these authors aver the presence of variety effects, we cannot pan a general agreement on point estimates. Hence, we do not calibrate our artificial economy to such an estimate, rather, our strategy is to ferret out indeterminacy situations that require this effect to be as small as possible.

The economic mechanism that concocts the indeterminacy is easily understood in terms of the equilibrium wage-hour locus: expectations about the future turn out to be self-fulfilling if this locus is upwardly sloping. Technologically increasing returns are not the reason for the positive slope. Instead, a combination of taste for variety and changes of the markup delivers this result. With a constant number of firms, a countercyclical markup leads to a procyclical efficiency wedge and thus can give rise to an upwardly sloping wage-hour locus. In this case, a countercyclical efficiency wedge due a procyclical markup cannot give rise to indeterminacy. However, in the presence of positive variety effects, endogenous firm entry and exit implies that an increase in the number of firms leads to an increase in productivity. Then a procyclical markup, despite itself lowering the efficiency wedge, can induce greater firm entry. Alternatively, a countercyclical markup improves the efficiency wedge, but has a negative impact on firm entry. In reduced form, these work like increasing returns at the aggregate level, and hence, the wage-hour locus can become upwardly sloping.

Related work includes Benhabib and Farmer (1994), Farmer and Guo (1994) and Wen (1998) who show that technological increasing returns lead to sunspot equilibria in real business cycle models. However, these authors rely on declining marginal costs, which does not receive empirical support.[4] Schmitt-Grohé (1997), Weder (2000), and Jaimovich (2007) lay out models that generate indeterminacy through countercyclical markups.[5] Seegmuller (2007) and Chang, Hung and Huang (2011) show that taste for variety can be the source of sunspot equilibria. Seegmuller (2007) argues this in an overlapping generations model with constant markups. Markups are also constant in Chang et al. (2011) who additionally assume production externalities, i.e. declining marginal costs. To our knowledge, Galí's (1994) composition of demand model is the only model in which indeterminacy can result due to procyclical markups. However, his model then entails a countercyclical investment share and requires markups that are outside of the empirically plausible range.

The rest of this paper proceeds as follows. Section 2 lays out the model. Section 3 analyzes the local dynamics. Variable capital utilization is introduced in Section 4. Section 5 checks robustness regarding the formulation of the variety effect. Simulations are presented in Section 6. Section 7 concludes.

2 Model

The artificial economy is based on the composition of aggregate demand model laid out by Schmitt-Grohé (1997) and originally put forth by Galí (1994). Each intermediate good firm produces a differentiated intermediate

[4] For example, see Basu and Fernald (1997).

[5] See also Dos Santos Ferreira and Dufourt (2006) as well as Dos Santos Ferreira and Lloyd-Braga (2008).

good and acts as a monopolist competitor for this good. These goods are bought by a final sector that welds them together into two different final goods. One final good is consumed, while the other increases the capital stock. Monopolistic competitors cannot price-discriminate between the consumption and investment related demands, hence, the composition of demand affects their market power. Our model differs from Schmitt-Grohé (1997) and Galí (1994) in two important ways. First, the number of intermediate good varieties, N_t, is endogenously determined each period: free entry takes place up to the point where the zero profit condition holds. Empirically, zero or close to zero pure profits seem to be a reasonable assumption. Second, we allow for variety effects. These effects imply that net business formation induces productivity increases. Unlike other work on indeterminacy, including Chang et al. (2011), the only source of technological increasing returns are fixed costs to operate the firms. Time evolves continuously. We will begin with the presentation of the economy's technology.

2.1 Firms

The final goods sector is perfectly competitive and produces the final consumption good, C_t and the final investment good, X_t. The production functions linking the final outputs to intermediate goods are

$$C_t = N_t^{1+\omega} \left(\frac{1}{N_t} \int_0^{N_t} y_{i,c,t}^{\sigma} di \right)^{1/\sigma} \qquad \omega \geq 0, \sigma \in (0,1)$$

and

$$X_t = N_t^{1+\tau} \left(\frac{1}{N_t} \int_0^{N_t} y_{i,x,t}^{\eta} di \right)^{1/\eta} \qquad \tau \geq 0, \eta \in (0,1)$$

where $y_{i,c,t}$ ($y_{i,x,t}$) stands for the amount of the unique intermediate good i used in manufacturing consumption (investment) goods. The specific functional form implies constant elasticities of substitution between intermediate goods equal to $1/(1-\sigma)$ and $1/(1-\eta)$. Parameters ω and τ govern the strength of the variety effects, which are absent when $\omega = 0$ and $\tau = 0$. Our formulation follows Benassy (1996): the variety effect is independent of the elasticity of substitution parameters σ and η. Intermediate good producers are not able to price discriminate regardless of whether their goods will be used in the production of consumption or investment goods, thus, they charge the identical price $p_{i,t}$ to both demands (see also Galí, 1994). Then, the conditional demand for intermediate good i to be used in the production of the consumption good is

$$y_{i,c,t} = \left(\frac{p_{i,t}}{P_{c,t}}\right)^{1/(\sigma-1)} N_t^{\frac{\sigma(1/\sigma-1-\omega)}{\sigma-1}} C_t \tag{1}$$

with the price index

$$P_{c,t} = N_t^{(1-\sigma)/\sigma-\omega} \left(\int_0^{N_t} p_{i,t}^{\sigma/(\sigma-1)} di\right)^{(\sigma-1)/\sigma}.$$

Similarly, investment demand becomes

$$y_{i,x,t} = \left(\frac{p_{i,t}}{P_{x,t}}\right)^{1/(\eta-1)} N_t^{\frac{\eta(1/\eta-1-\tau)}{\eta-1}} X_t \tag{2}$$

and

$$P_{x,t} = N_t^{(1-\eta)/\eta-\tau} \left(\int_0^{N_t} p_{i,t}^{\eta/(\eta-1)} di\right)^{(\eta-1)/\eta}.$$

Intermediate goods are produced using capital, $k_{i,t}$, and labor, $h_{i,t}$, both supplied on perfectly competitive factor markets. Each firm i produces according to the production function

$$y_{i,t} = k_{i,t}^{\alpha} h_{i,t}^{1-\alpha} - \phi \qquad 0 < \alpha < 1, \phi > 0 \tag{3}$$

where ϕ stands for fixed overhead costs. The presence of ϕ implies internal increasing returns to scale. Each monopolist faces demands (1) and (2) and sets the profit maximizing price $p_{i,t}$ such that the markup, $\mu_{i,t}$, equals

$$\mu_{i,t} = \frac{\frac{1}{\sigma-1} y_{i,c,t} + \frac{1}{\eta-1} y_{i,x,t}}{\frac{\sigma}{\sigma-1} y_{i,c,t} + \frac{\eta}{\eta-1} y_{i,x,t}}.$$

The implicit demands for labor and capital are

$$\frac{\mu_{i,t}}{p_{i,t}} w_t = (1-\alpha) \frac{k_{i,t}^{\alpha} h_{i,t}^{1-\alpha}}{h_{i,t}} \tag{4}$$

and

$$\frac{\mu_{i,t}}{p_{i,t}} r_t = \alpha \frac{k_{i,t}^{\alpha} h_{i.t}^{1-\alpha}}{k_{i,t}} \tag{5}$$

where w_t is the wage and r_t is the rental rate earned by agents for their labor and capital services. Free entry into the intermediate goods sector leads to zero profits (net of fixed costs) for each active firm in every period. Entry and exit decisions are static and simply depend on the current period's profits.

We restrict our analysis to a symmetric equilibrium where all monopolists produce the same amount and charge the same price. Therefore, $y_{i,c,t}+y_{i,x,t} = y_{i,t} = y_t$, $k_{i,t} = k_t$, $h_{i,t} = h_t$, $\mu_{i,t} = \mu_t$, $p_{i,t} = p_t$ and aggregate capital and hours are $K_t = N_t k_t$ and $H_t = N_t h_t$. When choosing the consumption good as the numeraire, we find

$$p_t = N_t^{\omega}$$

and the relative price of the investment good, P_t, becomes

$$P_{x,t} = N_t^{\omega-\tau} \equiv P_t.$$

P_t moves with the number of firms if the variety effects in final goods differ. If $\omega = \tau$, the relative price remains constant. Using (3), (4), and (5) with the zero profit condition leads to

$$y_t = \frac{\phi}{\mu_t - 1}$$

and to aggregate output

$$Y_t = \frac{P_t N_t^{\tau}}{\mu_t} K_t^{\alpha} H_t^{1-\alpha}.$$

The efficiency wedge, $P_t N_t^{\tau}/\mu_t$, is therefore a positive function of the relative price and the number of firms and it is negatively related to the markup. The number of firms moves positively with aggregate output and the markup:

$$N_t = \left(\frac{Y_t}{P_t} \frac{\mu_t - 1}{\phi} \right)^{1/(1+\tau)}.$$

Lastly, we define s_t as the share of the value of investment in aggregate output, that is

$$s_t \equiv \frac{P_t X_t}{Y_t} = 1 - \frac{C_t}{Y_t}.$$

Then the optimal markup can be rewritten as a function of this share

$$\mu_t = \frac{\frac{1}{1-\sigma}(1 - s_t) + \frac{1}{1-\eta} s_t}{\frac{1}{1-\sigma}(1 - s_t) + \frac{1}{1-\eta} s_t - 1}.$$

The price elasticity of demand is given by $\frac{1}{\sigma-1}(1-s_t)+\frac{1}{\eta-1}s_t$. Note that when $\sigma=\eta$ the markup is constant. If $\sigma>\eta$ the markup is procyclical to s_t, then a shift in demand from consumption to investment means that each monopolist faces a more inelastic demand curve and this leads to a rise in the markup. We restrict the markup elasticity, $\varepsilon_\mu \equiv (\partial\mu/\partial s)(s/\mu)$, to permissible values via $\mu>1$ and $\sigma,\eta\in(0,1)$. Some algebra restricts ε_μ to

$$\frac{1-\mu}{\mu}<\varepsilon_\mu<\frac{\mu-1}{\mu}\frac{s}{1-s}$$

where μ and s are the steady state values of the markup and the investment share.[6]

2.2 Agents

The representative agent derives lifetime utility from the function

$$U=\int_0^\infty e^{-\rho t}u(C_t,H_t)dt \qquad\qquad \rho>0$$

where ρ denotes the subjective discount rate. Period utility takes on the functional form

$$u(C_t,H_t)=\ln C_t-\upsilon H_t \qquad\qquad \upsilon>0.$$

Labor is indivisible. This assumption keeps solutions analytically tractable and it allows us to compare our results to existing work on sunspot equilibria.[7] The agent owns the capital stock and sells labor and capital services. He owns all firms and receives any (potential) profits, Π_t, generated by them. Then, the representative agent's budget is constrained by

$$w_tH_t+r_tK_t+\Pi_t\geq P_tX_t+C_t.$$

Capital accumulation follows

$$\dot{K}_t=X_t-\delta K_t \qquad\qquad 0<\delta<1.$$

[6] Under indeterminacy and in the absence of changes to fundamentals, when $\varepsilon_\mu>0$ ($\varepsilon_\mu<0$), the markup is also procyclical (countercyclical) with respect to output. See Appendix 8.1.

[7] Yet, extensions are straightforward.

Here, time derivatives are denoted by dots and δ stands for the constant rate of physical depreciation of the capital stock. Optimality implies

$$\frac{P_t}{C_t} = \lambda_t \tag{6}$$

$$\upsilon = \frac{w_t}{C_t} \tag{7}$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = \delta + \rho - \frac{1}{C_t}\frac{r_t}{\lambda_t} \tag{8}$$

where λ is the current value multiplier. Equations (6) and (7) describe the agent's leisure-consumption trade-off, while (8) is the intertemporal Euler equation. In addition the usual transversality condition holds.

3 Dynamics

Next we analyze the local dynamical property of the artificial economy. In particular, we take a log-linear approximation to the equilibrium conditions. The dynamical system boils down to

$$\begin{bmatrix} \dot{K}_t/K_t \\ \dot{\lambda}_t/\lambda_t \end{bmatrix} = \mathbf{J} \begin{bmatrix} \hat{K}_t \\ \hat{\lambda}_t \end{bmatrix}.$$

Hatted variables denote percent deviations from their steady-state values and $\mathbf{J}$ is the 2×2 Jacobian matrix of partial derivatives. Note that λ_t is a non-predetermined variable and that K_t is predetermined. Hence, indeterminacy requires that the two roots of $\mathbf{J}$ to be negative, or simply Det$\mathbf{J} > \mathbf{0} >Tr\mathbf{J}$.

3.1 Constant markup

We first consider the case where the markup is constant, meaning that $\mu = 1/\eta = 1/\sigma$ and $\varepsilon_\mu = 0$. Then, the determinant of $\mathbf{J}$ is given by

$$\frac{(1-\alpha(1+\tau))(\rho+\delta)(\delta(1-\alpha)+\rho)}{\alpha[\tau(1-\alpha)-\alpha)]}$$

and the trace of $\mathbf{J}$ equals

$$\frac{\tau\delta + \alpha\rho(1+\tau)}{\alpha - \tau(1-\alpha)}.$$

Note that the markup, μ, as well as the variety effect in the assembling of consumption goods, ω, are completely absent here, hence, they have no effect on the stability of the steady state.[8] Indeterminacy is driven by the variety effect in investment alone. Since indeterminacy requires a positive determinant and a negative trace, it must be that $\tau(1-\alpha) > \alpha$.[9]

Proposition 1 *If $\varepsilon_\mu = 0$, then indeterminacy arises if $\tau > \dfrac{\alpha}{1-\alpha} > 0$.*

In the absence of a variety effect, $\tau = 0$, Det$\mathbf{J} = -(1-\alpha)(\rho+\delta)(\delta(1-\alpha)+\rho)\alpha^2 < 0$ and Tr$\mathbf{J}$ $=\rho$, thus, the possibility of indeterminacy disappears.

Lemma 1 *If $\varepsilon_\mu = 0$ and $\tau = 0$, then indeterminacy cannot arise.*

The condition for indeterminacy is similar to Benhabib and Farmer (1994) and rests on an upwardly sloping wage-hour locus now generated by the variety effect: the minimum returns to variety equal $\alpha/(1-\alpha)$ or numerically, 0.428 (with $\alpha = 0.3$). Figures 1 and 2 show this case along their $\varepsilon_\mu = 0$-axis. Note that the firm's internal increasing returns to scale equal μ which can be pushed towards unity without affecting the indeterminacy condition.[10] Thus, unlike Benhabib and Farmer (1994) and others, indeterminacy is possible in the absence of decreasing marginal costs and at essentially zero increasing returns to scale at the firm level. Given the very limited evidence of significant returns to scale (e.g. Basu and Fernald, 1994), taste for variety offers a potentially more plausible mechanism to generate sunspot equilibria.[11] Next, we allow indeterminacy to arise for smaller values of τ.

[8] The second component of this result is reminiscent of Harrison and Weder's (2002) findings for two-sector models; the increasing returns originating in the consumption goods sector are irrelevant for the stability of the steady state.

[9] We restrict $\alpha(1+\tau) < 1$ to rule out endogenous growth.

[10] In fact, market power has no effect on dynamics. This is due to the instantaneous adjustment to zero profits (also see Kim, 2004).

[11] This being said, existing empirical work on variety effects is sparse and we do not have good point estimates yet, and therefore we abstain for this reason to make any suggestion that the values used here are reasonable.

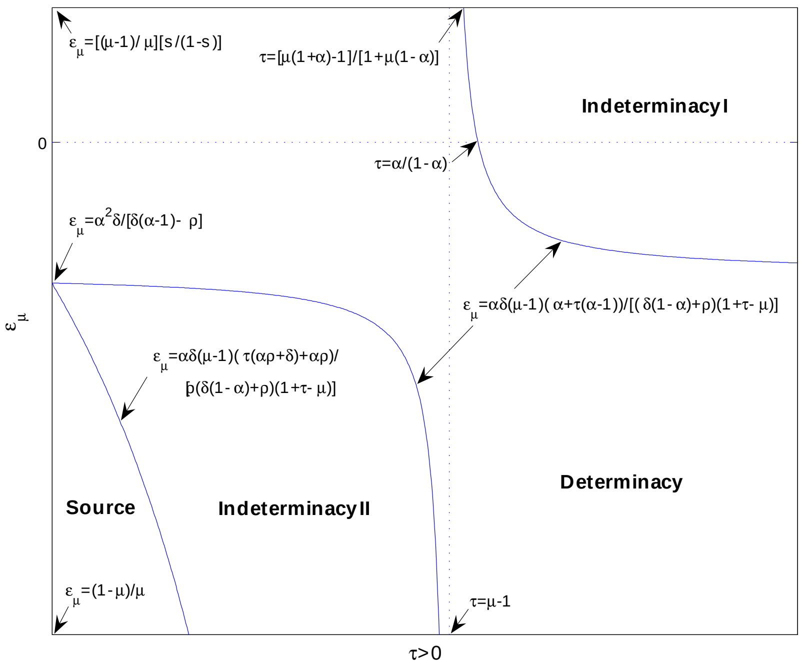

Figure 1: $\mu < 1 + \alpha/(1-\alpha)$.

3.2 Variable Markup

We now consider variable markups, that is $\varepsilon_\mu \neq 0$. Then, the determinant of $\mathbf{J}$ equals

$$\frac{\delta(\mu-1)(\delta(1-\alpha)+\rho)(\delta+\rho)(\alpha(1+\tau)-1)}{\alpha^2\delta(\mu-1)(1+\tau)+\alpha\delta[\tau(1-\mu)+\varepsilon_\mu(1+\tau-\mu)]-\varepsilon_\mu(\delta+\rho)(1+\tau-\mu)}$$

and the matrix's trace is given by

$$\frac{\rho\alpha^2\delta(\mu-1)(1+\tau)+\alpha\delta[\tau\delta(\mu-1)+\rho\varepsilon_\mu(1+\tau-\mu)]-\rho\varepsilon_\mu(\delta+\rho)(1+\tau-\mu)}{\alpha^2\delta(\mu-1)(1+\tau)+\alpha\delta[\tau(1-\mu)+\varepsilon_\mu(1+\tau-\mu)]-\varepsilon_\mu(\delta+\rho)(1+\tau-\mu)}.$$

Again, the variety effect in consumption, ω, does not appear in these expressions. This suggests that relative price movements have no effect on the occurrence of sunspot equilibria, rather, the indeterminacy mechanism must come from variety effects and markup variations. Indeterminacy cannot emerge if there is no variety effect in investment, i.e. $\tau = 0$: a positive determinant, which occurs if $\varepsilon_\mu < \alpha^2\delta/(\delta(\alpha-1)-\rho) < 0$ will go in hand with

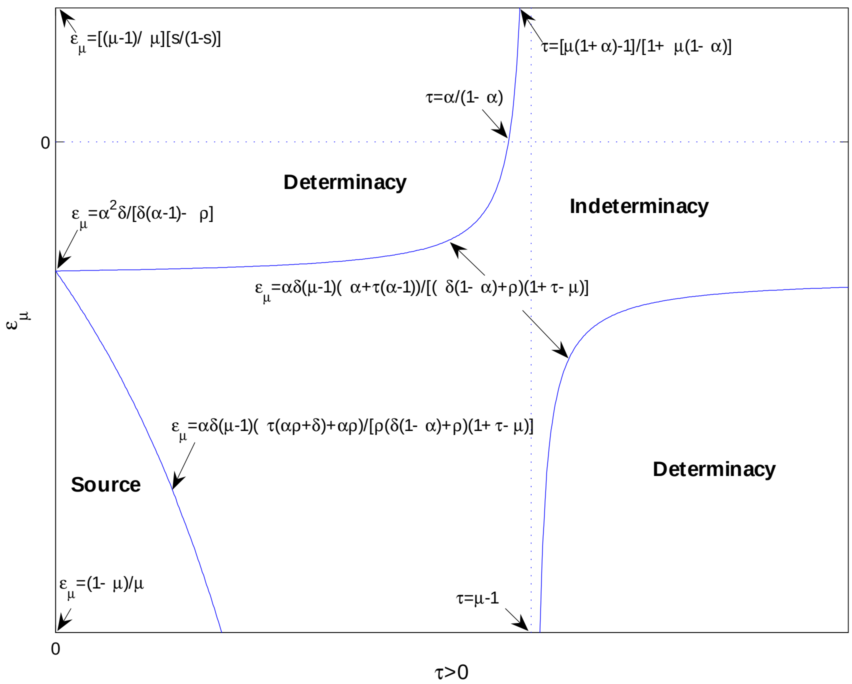

Figure 2: $\mu > 1 + \alpha/(1 - \alpha)$.

Tr**J** $=\rho$. Phrased alternatively, net business formation eliminates sunspot equilibria in Schmitt-Grohé (1997).

Lemma 2 *If* $\varepsilon_\mu \neq 0$ *and* $\tau = 0$*, then indeterminacy cannot arise.*

We plot the stability zones in Figure 1 and 2. In both Figures, the function that divides the indeterminacy and determinacy regions is given by $\varepsilon_\mu = \alpha\delta(\mu-1)(\alpha+\tau(\alpha-1))/[(\delta(1-\alpha)+\rho)(1+\tau-\mu)]$. This borderline is discontinuous at $\tau = \mu - 1$ and this results in two separate indeterminacy regions if $\mu < 1 + \alpha/(1-\alpha)$. That is, the regions are separate when the discontinuity occurs at lower variety values than Proposition 1's critical level of τ. This situation is plotted in Figure 1, where we denote the two regions Indeterminacy I and II. The two regions are merged in Figure 2. There are several important results to this. First, market power now affects indeterminacy. Second, sunspot equilibria arise for procyclical and countercyclical markups – this stands in contrast to Schmitt-Grohé (1997), Jaimovich (2007) and others who require countercyclical markups. The result is noteworthy in light of Nekarda and Ramey's (2010) claim on US markup dynamics. Third, for $\mu < 1 + \alpha/(1-\alpha)$, the determinacy-indeterminacy borderline is downwardly sloping. For indeterminacy region I, this suggests that the minimum τ from $\varepsilon_\mu = 0$ can be reduced by increasing ε_μ, and in a sense, a more procyclical markup makes indeterminacy easier to obtain. To move from determinacy into region II, the (negative) elasticity of the markup has to be increased. In the empirically less appealing case of $\mu > 1 + \alpha/(1-\alpha)$, the borderline is upwardly sloping. Here, a larger procyclical ε_μ then requires a stronger variety effect.

Proposition 2 *Indeterminacy appears for procyclical and countercyclical markups.*

The economic mechanism that creates the continuum of solutions in our model is easily understood in reference of the equilibrium wage-hour locus and an elastic labor supply curve. For example, upon optimistic expectations about the future, the agent anticipates higher prospective income. Today's consumption expenditures will rise. As a consequence, the labor supply curve shifts inwards. If the wage-hour locus slopes upwardly, employment and investment will jump up today. The future capital stock, output and consumption will be high and the initial optimistic expectations are self-fulfilled.

There are essentially two ways to generate an upwardly sloping wage-hour locus: variety effects and variable markups. This thought can be organized via the log-linearized wage-hour locus[12]

$$\hat{w}_t = (1+\tau)\alpha \hat{K}_t + \left[\tau(1-\alpha) - \alpha + \varepsilon_\mu \frac{(1+\tau-\mu)(\delta(1-\alpha)+\rho)}{\alpha\delta(\mu-1)}\right] \hat{H}_t.$$

Since labor is indivisible, indeterminacy requires the term in front of $\hat{H}_t$ to be positive. When is this the case? If $\varepsilon_\mu = 0$, then $\tau(1-\alpha) - \alpha > 0$ guarantees indeterminacy as in Proposition 1. If $\varepsilon_\mu(1+\tau-\mu) > 0$, then τ may be lowered; the third term in squared brackets is positive and works in the same direction as the variety effect. Therefore, a procyclical (countercyclical) markup endogenously expands the efficiency wedge if $\tau > \mu - 1$ ($\tau < \mu - 1$). The former is possible because a higher markup induces net firm entry. If the variety effect is sufficiently large, this entry will dominate the contractionary clout of a procyclical markup and, in effect, will work like an improvement of the efficiency wedge. If $\tau < \mu - 1$, as in region II, indeterminacy arises even at very low values of τ. Here, a countercyclical markup leads to a procyclical efficiency wedge. Next, we show how to decrease minimum τ even further.

4 Capital utilization

Section 1 mentioned papers report empirical evidence of variety effects. However, there seems to be no general agreement on point estimates, hence, our strategy is to make the size of the effect as small as possible. For countercyclical markups, variety effects can be close to zero. For indeterminacy to arise with a procyclical markup, we need larger magnitudes. To see this, we calibrate the model as in Farmer and Guo (1994) and Wen (1998): $\alpha = 0.3$, $\rho = 0.01$, $\delta = 0.025$. Then, indeterminacy then requires $\tau > 0.176$ in the extreme case where μ is close to unity (and hence ε_μ is close to zero). This value

[12] To be precise, here we set $\tau = \omega$.

can be significantly reduced by introducing endogenous capital utilization. To do this, we amend the model such that an intermediate good producer i operates the production technology

$$y_{i,t} = (u_t k_{i,t})^{\alpha} h_{i,t}^{1-\alpha} - \phi$$

and aggregate capital accumulation follows

$$\dot{K}_t = X_t - \delta_t K_t = X_t - \frac{1}{\theta} u_t^{\theta} K_t \qquad \theta > 1$$

where u_t stands for the intensity of capital utilization set by the capital stock's owners. The rate of depreciation, δ_t, is an increasing function of the utilization rate. Figures 3 and 4 show numerical indeterminacy regions; the qualitative pattern parallels the constant utilization model and the source of sunspot equilibria remains an upwardly sloped wage-hour locus.[13] The constant markup case, $\varepsilon_{\mu} = 0$, delivers indeterminacy if $\tau > \alpha/(1-\alpha+\delta/\rho) = 0.094$. Hence, indeterminacy is independent of μ and ω; it is driven by the variety effect in the investment technology only. When the markup is variable, $\varepsilon_{\mu} \neq 0$, market power again affects indeterminacy and sunspot equilibria arise for procyclical and countercyclical markups. In Figure 3, the steady state markup equals 1.05, which is also the size of firm level scale economies. At this level of market power, there are two regions of indeterminacy and for slightly procyclical markups, e.g. the upper limit $\varepsilon_{\mu} = 0.013$, the variety effect can be as low as 0.072. Figure 4 assumes $\mu = 1.10$ which is of magnitude that is commonly assumed in New Keynesian models. Here, the two indeterminacy regions are merged; analogous to the constant utilization model, this occurs if $\mu > 1+\alpha/(1-\alpha+\delta/\rho)$. Overall, with the addition of variable capital utilization we have shown that indeterminacy does not require implausibly high levels of market power and that the size of the required variety effects can be significantly lowered. Before addressing the business cycle dynamics, we present an alternative formulation of the variety effect.

[13] We provide analytical results in Appendix 8.2.

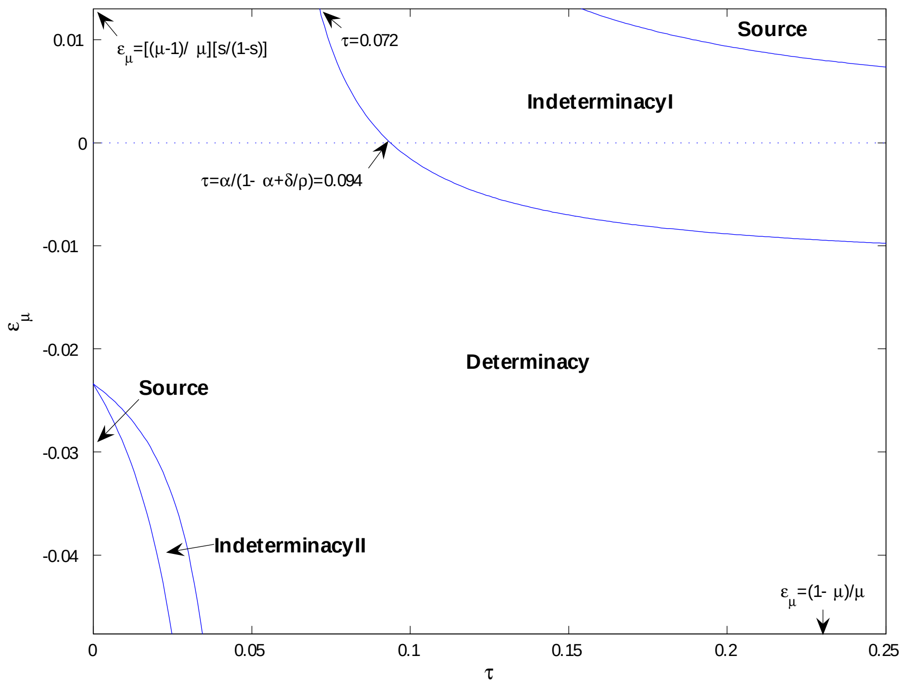

Figure 3: Variable capital utilization, $\mu = 1.05$.

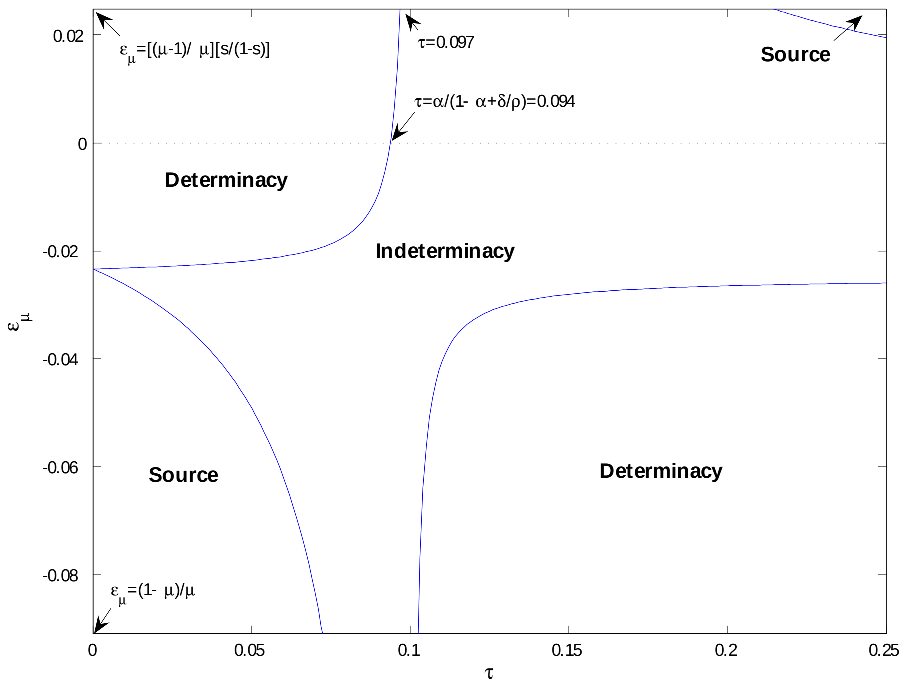

Figure 4: Variable capital utilization, $\mu = 1.10$.

5 Alternative formulation of variety

Here we demonstrate that our results are robust to the formulation of taste for variety. So far, we have assumed that the variety effect does not depend on the degree of elasticity of substitution as suggested by Benassy (1996). Chang et al. (2011) and others assume a different formulation where the variety effect depends on parameters σ and η.[14] For example, consumption is produced via

$$C_t = \left(\int_0^{N_t} y_{i,c,t}^{\sigma} di \right)^{1/\sigma} \qquad \sigma \in (0,1)$$

where the variety effect, ω, is equal to $1/\sigma - 1 > 0$. The intuition for setting the model this way could be that σ and η are an indication of product differentiation and higher product differentiation then implies a greater love for variety. On the other hand, we believe that there is no *a priori* reason for assuming such a strong connection between them.

Figure 5 plots the indeterminacy zone for this model with the markup on the horizontal axis.[15] Formulating variety this way does not change our results: indeterminacy occurs with procyclical and countercyclical markups. The figure also makes it clear that the production externalities in Chang et al. (2011) are not required for indeterminacy: if the markup is constant the sufficient condition for indeterminacy is now $\mu > 1/(1-\alpha)$, which corresponds to Proposition 1's result. If the markup is procyclical, $\varepsilon_\mu > 0$ (i.e. $\sigma > \eta$), then $\mu^{\min} \to 1$, albeit in only a small region. Note that at very low markups there still may be a substantial difference between η and σ. Therefore the variety effect in investment can still be sufficiently large to cause indeterminacy even if μ is very close to unity.[16]

[14] Chang et al. (2011) only consider constant markups, $\eta = \sigma$. See also Devereux et al. (1996).

[15] Capital utilization is set as constant.

[16] This is the reason why we used the more flexible modelling approach in the preceeding Sections.

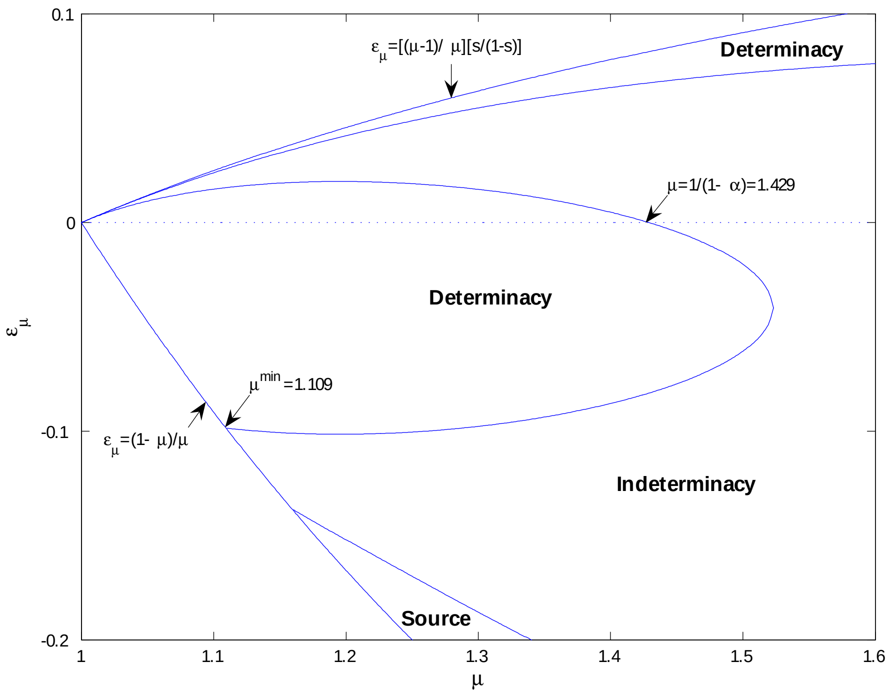

Figure 5: Alternative formulation of variety (constant capital utilization).

6 Business cycle dynamics

We have shown that the taste for variety can generate indeterminacy with procyclical and countercyclical markups. Although that could be viewed as progress alone, this remains void as long as these models cannot replicate at least basic business cycle facts. This is done next by comparing second moments for the US data and the model.

We simulate a discrete time economy under our original formulation of the variety effect by shocking it by i.i.d. sunspot shocks only. For direct comparison, the calibration remains as in Section 4 and the discount factor is $\beta = (1+\rho)^{-1}$. To illustrate that excessive market power is not crucial to our findings, we set the steady state markup to 1.05, which is also the firm level increasing returns to scale, and calibrate its elasticity to $\varepsilon_\mu = 0.01$, which is relatively close to its upper boundary given by the restrictions in Section 2. Since we do not have any clear estimates for the variety effects, τ and ω, we set them to 0.1, which is lower than the production externalities required by Wen (1998).[17] Table 1 presents HP-filtered second moments of the US data and of the artificial economy. The model correctly reproduces the order of relative volatilities and positive correlations. Persistence, measured by the first order autoregressive process, is also well replicated. Moreover, the markup, the number of firms and, unlike in Galí (1994), the investment share are all strongly positively correlated with aggregate economic activity.[18]

Table 1						
	US data			Model		
Variable x	σ_x/σ_Y	$\rho(x,Y)$	AR(1)	σ_x/σ_Y	$\rho(x,Y)$	AR(1)
Y_t	1	1	0.84	1	1	0.88
C_t	0.38	0.71	0.81	0.08	0.67	0.94
P_tX_t	3.62	0.97	0.78	4.50	0.99	0.88
H_t	0.84	0.83	0.90	0.95	0.99	0.88
s_t	2.66	0.94	0.76	3.50	0.99	0.88
Y_t/H_t	0.55	0.54	0.72	0.08	0.67	0.94
N_t	-	-	-	1.58	0.99	0.88
μ_t	-	-	-	0.04	0.99	0.88

See Appendix 8.3 for the source of US data. σ_Y denotes the standard deviation of output and $\rho(x,Y)$ is the correlation of variable x and Y. Blank entries are due to data unavailability.

7 Conclusion

Recent research suggests that markups in the US are largely procyclical or acyclical. While this issue is clearly not settled, it puts doubt on the plausibility of many endogenous business cycle models in which countercyclical markups are the key ingredient. The current paper offers a theory that allows a procyclical markup in endogenous business cycles. In fact, given a

[17] In the discrete time model with constant markups the required τ for indeterminacy is 0.1036. With $\mu = 1.05$ and $\varepsilon_\mu = 0.01$ the required τ is 0.0797.

[18] The low volatility of consumption is the sole outlier with regards to the model's predictions. This problem is the consequence of the additional utilization margin and has been noted by Jaimovich (2007) and Wen (1998). We are able to show that a higher variety effect and/or markup elasticity improves the model's performance in this aspect as it results in a steeper wage-hour locus. A shift of the flat labor supply curve will produce a smaller change in hours the steeper the wage-hour locus.

certain level of product variety effects, a procyclical markup can make it easier for indeterminacy to occur. Taste for variety is critical in generating sunspot equilibria, and hence, variety matters. In comparison to many other studies, especially where increasing returns to scale generate sunspot equilibria, we believe that the mechanism that drives our results is potentially more plausible. First, the variety effect that drives this result does not imply countercyclical marginal costs. Second, a variable markup, and in particular a procyclical markup, means that the required size of the variety effect is significantly lower than the externalities required by many other studies. Finally, the size of the required markups is well within empirical estimates.

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8 Appendix

8.1 Markup movements with respect to output

This Appendix shows that under indeterminacy (i) if $\varepsilon_\mu > 0$ ($\varepsilon_\mu < 0$) the markup is procyclical (countercyclical) and (ii) the investment share, s_t, is always procyclical. This is demonstrated for the model of Section 2 (to keep notation simple; see also Figures 1 and 2); we obtain a relation between aggregate output and the markup[19]

$$\hat{Y}_t = (1+\tau)\alpha\hat{K}_t + \left[\frac{\alpha\delta(1+\tau)(1-\alpha)}{\varepsilon_\mu[\delta(1-\alpha)+\rho]} + \frac{1+\tau-\mu}{\mu-1}\right]\hat{\mu}_t. \tag{A1}$$

[19] We set $\tau = \omega$.

Let us first consider the case of $\tau < \mu - 1$. In (A1), for any $\varepsilon_\mu < 0$ the term in squared bracket is negative and hence the markup is always countercyclical. For a positive ε_μ the markup is procyclical as long as

$$\varepsilon_\mu < \varepsilon_\mu^* \equiv \frac{\alpha\delta(1+\tau)(\alpha-1)(\mu-1)}{[\delta(1-\alpha)+\rho](1+\tau-\mu)}.$$

(Note that $\partial\varepsilon_\mu^*/\partial\tau > 0$.) Indeterminacy requires that

$$\varepsilon_\mu < \varepsilon_\mu^{**} \equiv \frac{\alpha\delta[\alpha+\tau(\alpha-1)](\mu-1)}{[\delta(1-\alpha)+\rho](1+\tau-\mu)}$$

which implies a positively sloped wage-hour locus. It is then easy to see that $\varepsilon_\mu^{**} < \varepsilon_\mu^*$ for any positive $\tau < \mu - 1$. Next, $\tau > \mu - 1$. Here indeterminacy requires that $\varepsilon_\mu > \varepsilon_\mu^{**}$. In this case, the markup is procyclical if $\varepsilon_\mu > 0$ or if $\varepsilon_\mu < \varepsilon_\mu^* < 0$. Clearly $\varepsilon_\mu^{**} > \varepsilon_\mu^*$ for any $\tau > \mu - 1$. Hence the markup is always procyclical (countercyclical) under indeterminacy for any $\varepsilon_\mu > 0$ ($\varepsilon_\mu < 0$). Lastly, from $\widehat{\mu}_t = \varepsilon_\mu \widehat{s}_t$, the investment share is always procyclical.

8.2 Sections 4 and 5 analytics

This Appendix presents the analytical dynamics that underlie the models from Sections 4 and 5 respectively. With variable capital utilization, if $\varepsilon_\mu = 0$, the determinant of $\mathbf{J}$ is

$$\frac{\rho(1-\alpha(1+\tau))(\rho+\delta)(\delta(1-\alpha)+\rho)}{\alpha[\tau(\rho(1-\alpha)+\delta)-\rho\alpha]}$$

and the trace is governed by

$$\frac{\alpha\rho^2(1+\tau)}{\rho\alpha-\tau[\rho(1-\alpha)+\delta]}.$$

If $\varepsilon_\mu \neq 0$, the determinant is

$$\frac{\rho\delta(\mu-1)[\delta(1-\alpha)+\rho](\delta+\rho)[1-\alpha(1+\tau)]}{\gamma\rho^2+\delta^2[\gamma(1-\alpha)+\alpha\tau(\mu-1)]+\rho\delta[2\gamma+\alpha(\tau(\mu-1)-\gamma)-\alpha^2(\mu-1)(1+\tau)]}$$

and the trace is

$$\frac{\rho[\delta\rho(\alpha^2(\mu-1)(1+\tau)+\gamma(\alpha-2))-\gamma\delta^2(1-\alpha)-\gamma\rho^2]}{\rho\delta[\alpha^2(\mu-1)(1+\tau)+\alpha(\gamma+\tau(1-\mu))-2\gamma]-\gamma\rho^2-\delta^2[\gamma(1-\alpha)+\alpha\tau(\mu-1)]}$$

where $\gamma = \varepsilon_\mu(1+\tau-\mu)$.

With the alternative formulation of the variety effect and constant capital utilization (as used in Section 5), if $\varepsilon_\mu = 0$,the determinant of $\mathbf{J}$ is

$$\frac{(1-\alpha\mu)(\rho+\delta)(\delta(1-\alpha)+\rho)}{\alpha[(1-\alpha)\mu-1]}$$

and the trace is given by

$$\frac{\delta(1-\mu)-\rho\mu\alpha}{(1-\alpha)\mu-1}.$$

If $\varepsilon_\mu \neq 0$, the determinant is

$$\frac{\gamma_2[\alpha^2\delta\mu(\mu+\varepsilon_\mu-1)+\varepsilon_\mu\mu(\delta+\rho)-\alpha(\delta(2\varepsilon_\mu\mu+\mu-1)+\varepsilon_\mu\mu\rho)]}{\alpha^3\delta^2\mu(\mu+\varepsilon_\mu-1)+2\alpha\delta\mu\varepsilon_\mu^2(\delta+\rho)-\varepsilon_\mu^2\mu(\delta+\rho)^2-\gamma_3}$$

and the trace is

$$\frac{\alpha^3\delta^2\mu\rho(\mu+\varepsilon_\mu-1)+2\varepsilon_\mu^2\alpha\delta\mu\rho(\delta+\rho)-\varepsilon_\mu^2\mu\rho(\delta+\rho)^2+\gamma_4}{\alpha^3\delta^2\mu(\mu+\varepsilon_\mu-1)+2\varepsilon_\mu^2\alpha\delta\mu(\delta+\rho)-\varepsilon_\mu^2\mu(\delta+\rho)^2-\gamma_3}$$

where $\gamma_2 = \delta[(1-\alpha)\delta+\rho](\delta+\rho)$, $\gamma_3 = \alpha^2\delta[\delta(1+(\varepsilon_\mu^2+\varepsilon_\mu-2)\mu+\mu^2)+\varepsilon_\mu\mu\rho]$, and $\gamma_4 = \alpha^2\delta[\delta^2(\mu-1)^2-\delta\varepsilon_\mu\mu\rho(1+\varepsilon_\mu)-\varepsilon_\mu\mu\rho^2]$. These expressions make obvious why we concentrated on numerical results.

8.3 Data Sources

This Appendix details the source and construction of the data used for calculating US second moments in Section 6. All data is quarterly and for the period 1948:I-2006:IV.

1. Personal Consumption Expenditures, Nondurable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

2. Personal Consumption Expenditures, Services. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Gross Private Domestic Investment. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Gross Domestic Product. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Gross Domestic Product. Seasonally adjusted at annual rates, billions of chained (2005) dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

6. Nonfarm Business Hours. Index 2005=100, seasonally adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006033.

7. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

8. GDP Deflator = (4)/(5).

9. Real Per Capita Consumption, $C_t = [(1) + (2)]/(8)/(7)$.

10. Real Per Capita Investment, $P_t X_t = (3)/(8)/(7)$.

11. Real Per Capita Output, $Y_t = (9) + (10)$.

12. Per Capita Hours Worked, $H_t = (6)/(7)$.

13. Investment Share, $s_t = (10)/(11)$.

14. Labor Productivity, $Y_t/H_t = (11)/(12)$.