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Résumé : ce papier met au point, pour une large classe de modèles d'équilibre général dynamiques stochastiques à anticipations rationnelles, des règles de taux d'intérêt qui non seulement assurent la détermination locale de l'équilibre ciblé au voisinage de l'état stationnaire ciblé, mais aussi empêchent l'économie de quitter progressivement ce voisinage. Nous montrons que dans la plupart des modèles ces règles de taux d'intérêt sont nécessairement prospectives (*i.e.* conditionnent nécessairement le taux d'intérêt aux anticipations des agents privés), alors que dans tous les modèles des règles de taux d'intérêt non prospectives existent qui assurent seulement la détermination locale de l'équilibre ciblé. Nous examinons également la robustesse de l'efficacité de ces règles au relâchement de différentes hypothèses et montrons en particulier qu'elles peuvent encore être efficaces lorsque la banque centrale a une connaissance imparfaite des paramètres structurels du modèle. Nous défendons finalement l'idée que de telles règles pourraient aussi servir de guide utile dans les réflexions sur la meilleure réaction de politique monétaire à des bulles détectées de prix d'actifs ou de taux de change.

Mots-clefs : modèles DSGE, règles de taux d'intérêt, détermination locale, détermination globale, bulles rationnelles.

Codes JEL : E52, E61.

Abstract: this paper designs, for a broad class of rational-expectations dynamic stochastic general-equilibrium models, interest-rate rules which not only ensure the local determinacy of the targeted equilibrium within the neighbourhood of the targeted steady state, but also prevent the economy from gradually leaving this neighbourhood. We show that in most models these interest-rate rules are necessarily forward-looking (*i.e.* make necessarily the interest rate conditional on the private agents' expectations), while in all models non-forward-looking interest-rate rules exist which ensure only the local determinacy of the targeted equilibrium. We also discuss the robustness of the effectiveness of these rules to departures from various assumptions and show in particular that they can still be effective when the central bank has imperfect knowledge of the model's structural parameters. We finally argue that such rules could also serve as a useful guide in the reflections on the best monetary policy reaction to perceived asset-price bubbles or exchange-rate misalignments.

Keywords: DSGE models, interest-rate rules, local determinacy, global determinacy, rational bubbles.

JEL codes: E52, E61.

Résumé non technique : les règles de politique monétaire considérées dans la littérature académique sont le plus souvent des règles de taux d'intérêt satisfaisant le principe dit de Taylor, selon lequel le taux d'intérêt nominal doit réagir plus que proportionnellement au taux d'inflation, ou bien un principe équivalent. De telles règles permettent en théorie à la banque centrale d'éviter le type de fluctuations macroéconomiques qui, selon certains auteurs, se sont produites aux Etats-Unis avant 1979. Le revers de la médaille, comme l'ont montré d'autres auteurs, est que ces règles peuvent laisser les agents privés former des anticipations auto-réalisatrices conduisant l'économie à la trappe à liquidités par exemple, comme cela a pu se produire au Japon dans les années 1990-2000. Ce papier met au point des règles de taux d'intérêt qui permettent à la banque centrale d'éviter tous ces développements indésirables pour une large classe de modèles à anticipations rationnelles. Nous montrons que ces règles conditionnent nécessairement le taux d'intérêt aux anticipations des agents privés dans la plupart des modèles, contrairement aux règles considérées dans la littérature, dans le but de déconnecter la situation économique courante de ces anticipations. Nous montrons aussi qu'en exerçant un effet de levier sur les anticipations des agents privés, ces règles peuvent encore être efficaces dans le cas où la banque centrale a une connaissance imparfaite des valeurs des paramètres structurels du modèle.

Nous défendons finalement l'idée que ces règles pourraient aussi servir de guide utile dans les réflexions sur la meilleure réaction de politique monétaire à des bulles détectées de prix d'actifs ou de taux de change. Dans ce contexte, ces règles viseraient à interrompre une bulle en cours de formation en déconnectant la valeur présente du prix d'actif ou du taux de change des anticipations des agents privés concernant sa valeur future ou bien, lorsque la banque centrale a une connaissance imparfaite des valeurs des paramètres structurels du modèle, en exerçant un effet de levier sur ces anticipations.

Non-technical summary: monetary policy rules considered in the academic literature are typically interest-rate rules satisfying something akin to the so-called Taylor principle, which makes the nominal interest rate react more than one-to-one to the inflation rate. Such rules theoretically enable the central bank to avoid the kind of macroeconomic fluctuations which, according to some authors, occurred in the U.S. before 1979. The other side of the coin, as shown by some other authors, is that these rules can let the private agents form self-fulfilling expectations leading the economy for instance to the liquidity trap, as arguably happened in Japan in the 1990s-2000s.

This paper designs interest-rate rules which enable the central bank to avoid all these undesirable developments for a broad class of rational-expectations models. We show that these rules necessarily make the interest rate conditional on the private agents' expectations in most models, contrary to the rules considered in the literature, in order to disconnect the current economic situation from

these expectations. We also show that by acting as a lever on the private agents' expectations, these rules can still be effective in the case where the central bank has imperfect knowledge of the values of the model's structural parameters.

We finally argue that these rules could also serve as a useful guide in the reflections on the best monetary policy reaction to perceived asset-price bubbles or exchange-rate misalignments. In this context, these rules would aim at interrupting a blooming bubble by disconnecting the current value of the asset price or the exchange rate from the private agents' expectations of its future value or, when the central bank has imperfect knowledge of the values of the model's structural parameters, by acting as a lever on these expectations.

Introduction

By far and large, the most common practice to design monetary policy in rational-expectations dynamic stochastic general equilibrium models is nowadays to linearize the model at hand in the neighbourhood of the targeted steady state and to choose an interest-rate rule both consistent with the targeted local equilibrium and ensuring its local determinacy, *i.e.* making the locally linearized system satisfy Blanchard and Kahn's (1980) condition. Such an interest-rate rule (often, a rule satisfying the so-called Taylor principle) notably enables the central bank to preclude the kind of macroeconomic fluctuations which, according to Clarida, Gali and Gertler (2000) and Lubik and Schorfheide (2004), occurred in the U.S. before 1979. In a series of influential papers, Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b, 2002a, 2002b, 2003) have shown that such interest-rate rules can however leave the door open to non-local equilibria originating locally. For instance, they can let the private agents form self-fulfilling expectations making the economy gradually leave the neighbourhood of the targeted steady state and eventually fall into the neighbourhood of another steady state interpreted as the liquidity trap, as arguably did the Japanese economy in the 1990s-2000s.

This paper designs interest-rate rules which in this context not only are consistent with the targeted local equilibrium and ensure its local determinacy, but also eliminate non-local equilibria originating locally, and hence preclude all the undesirable developments mentioned above. To that aim, we consider a broad class of rational-expectations dynamic stochastic infinite-horizon linear models, meant to represent the locally linearized reduced form of rational-expectations dynamic stochastic general equilibrium models. Given this focus on locally linearized systems, we do not show that non-local equilibria originating locally would exist if the central bank followed an interest-rate rule different from those put forward in this paper. But provided that exogenous disturbances are small enough, a necessary condition for non-local equilibria originating locally to exist is that the locally linearized system admit at least one unstable eigenvalue, *i.e.* one eigenvalue of modulus higher than or equal to one. By removing all unstable eigenvalues from the locally linearized system, the interest-rate rules put forward in this paper thus ensure the absence of non-local equilibria originating locally. And they accordingly manage to ensure the existence and uniqueness of a local equilibrium, *i.e.* to make the locally linearized system satisfy Blanchard and Kahn's (1980) condition, by removing all non-predetermined variables from the locally linearized system. In other words, they turn the so-called saddle path into what could be called a "necklace path". We call them "bubble-free interest-rate rules" because in the corresponding linear system they eliminate all mean-explosive rational bubbles of the type identified by Blanchard (1979) and followers, unlike the interest-rate rules commonly considered in the literature.

Loosely speaking, the way bubble-free interest-rate rules manage to remove all non-predetermined variables from the locally linearized system is by making the interest rate react to the private agents' current expectations of future variables, in a way which mimics their relationship in the locally linearized structural equations (*i.e.* the locally linearized system without the interest-rate rule), so as to disconnect current variables from these expectations. We accordingly show that under certain conditions (likely to be met by most dynamic stochastic general equilibrium models) these interest-rate rules are necessarily forward-looking, *i.e.* necessarily make the current interest rate conditional on the private agents' current expectations of future variables. We also show that for any given local solution of the locally linearized structural equations, there exists a backward-looking interest-rate rule consistent with this solution and ensuring its local determinacy. We therefore conclude that concern for non-local equilibria originating locally provides the missing theoretical justification for the use of forward-looking interest-rate rules. Bernanke and Woodford (1997) have famously warned against following forward-looking interest-rate rules without developing structural models of the economy. We thus go further by arguing for the use of forward-looking interest-rate rules on the basis of a structural model of the economy.

Since loosely speaking bubble-free interest-rate rules mimic the locally linearized structural equations, their coefficients are tied to the structural parameters by equality constraints, rather than by inequality constraints as for the coefficients of interest-rate rules ensuring only local equilibrium determinacy. We however show that the effectiveness of bubble-free interest-rate rules can be robust to departures from the assumption that the central bank has perfect knowledge of the values of the structural parameters. Indeed, these rules then no longer eliminate non-local equilibria originating locally but make them initially more "abrupt" (by using the structural equations as a lever on the private agents' expectations) and hence arguably less likely to be followed by the non-coordinated private agents, while still ensuring local equilibrium determinacy. We also examine or discuss the robustness of the effectiveness of these rules to departures from the assumptions that the central bank has perfect knowledge of the values of the endogenous variables and exogenous shocks, that the central bank can credibly commit to locally following an interest-rate rule and that the private agents form rational expectations.

The monetary policy proposals put forward in the literature to eliminate non-local equilibria originating locally usually consist in switching from an interest-rate rule ensuring local equilibrium determinacy to another rule such as a money growth rate peg (possibly accompanied by a non-Ricardian fiscal policy) when the endogenous variables go outside a specified neighbourhood of the targeted steady state. We argue in the paper that such two-tier policies may however not be completely effective in eliminating all non-local equilibria originating locally for various reasons and therefore that bubble-free interest-rate rules represent a particularly interesting alternative or complement to these two-tier policies. To our knowledge, only three papers make other monetary policy proposals enabling the central bank to eliminate non-local equilibria originating locally. First, Currie and Levine (1993, chap. 4) design "overstable feedback rules" which remove all unstable eigenvalues from linear systems without affecting the number of non-predetermined variables. Applied to locally linearized systems, these rules would eliminate non-local equilibria originating locally but would fail to ensure local equilibrium determinacy. Second, Adão, Correia and Teles (2005) design monetary policy rules ensuring global equilibrium determinacy in a simple non-linear cash-in-advance model. The working mechanism of these rules can be viewed as similar to that of our bubble-free interest-rate rules, although the two papers differ markedly in the presentation of their respective rules as well as in their analytical frameworks (non-linear but specific *vs.* general but locally linearized)[1]. Third, Antinolfi, Azariadis and Bullard (2006) propose in a particular framework interest-rate rules which eliminate non-local equilibria originating locally but fail to ensure local equilibrium determinacy, like those designed by Currie and Levine (1993, chap. 4).

Note finally that bubble-free interest-rate rules make sense only to the extent that the behaviour of private agents is at least partly forward-looking, since equilibrium (in)determinacy would not be an issue otherwise. Most, if not all, rational-expectations dynamic stochastic general equilibrium models based on explicit microeconomic foundations imply such a forward-looking behaviour for the private agents, which has led Woodford (2003, chap. 1) to view the essence of central banking as the management of expectations. But such a forward-looking behaviour is even less disputed for participants in asset markets than for private agents in macroeconomic models. We therefore discuss the possible applications of bubble-free interest-rate rules to asset-price stabilization by central banks. When the central bank precisely knows which asset-price value to target, for instance in the case of an exchange-rate peg, the asset-pricing equation can be linearized in the neighbourhood of this targeted value and bubble-free interest-rate rules then play essentially the same role as previously. When the central bank has no idea about which asset-price value to target, that is to say in most cases, bubble-free interest-rate rules could still serve as a useful guide in the reflections on the best monetary policy reaction to perceived asset-price bubbles or exchange-rate misalignments.

The remaining of the paper is organized as follows. Section 1 presents bubble-free interest-rate rules in a simple framework. Section 2 designs bubble-free policy feedback rules in a general framework. Section 3 discusses the robustness of the effectiveness of bubble-free policy feedback rules to departures from various assumptions. Section 4 discusses the use of bubble-free interest-rate rules for asset-price stabilization. We then conclude and provide a technical appendix.

[1] The two works were conducted independently from each other. The first versions of the present paper go back to Loisel (2003, 2004).

1 Bubble-free interest-rate rules in a simple framework

This section presents bubble-free interest-rate rules in the simple framework of the standard New Keynesian model.

1.1 Type-A and type-B equilibria

In most rational-expectations dynamic stochastic general equilibrium models, the targeted equilibrium (*e.g.* the globally-social-welfare-maximizing equilibrium) is to be found in the neighbourhood of a given steady state within which the model can be approximated by a linearized system of equations. The locally linearized interest-rate rule considered should then ideally eliminate the following two kinds of equilibria:

· **type-A equilibria:** non-targeted local equilibria, which exist if and only if the locally linearized system admits more stable eigenvalues (*i.e.* eigenvalues of modulus strictly lower than one) than required by Blanchard and Kahn's (1980) conditions. For instance, Clarida, Galí and Gertler (2000) and Lubik and Schorfheide (2004) explain the reduction in U.S. macroeconomic volatility from the pre- to the post-1979 period by a change in the Fed interest-rate rule in 1979 from a rule allowing such equilibria to a rule precluding them.

· **type-B equilibria:** non-local equilibria originating locally, which may exist only if the locally linearized system admits at least one unstable eigenvalue, *i.e.* one eigenvalue of modulus higher than or equal to one. For instance, Woodford (1994b, 2003, chap. 2) shows the existence of non-local self-fulfilling inflations and deflations originating locally, either with probability one or with probability strictly between zero and one, while Christiano and Rostagno (2001), Alstadheim and Henderson (2002), Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b, 2002a, 2002b, 2003) and Benhabib and Eusepi (2005) show the existence of non-local equilibria originating locally and converging towards a deterministic cycle, a chaotic cycle or a non-targeted steady state interpreted as the liquidity trap. Benhabib, Schmitt-Grohé and Uribe (2001a, 2002a, 2003) show that such equilibria exist for empirically plausible parameterizations and are robust to wide parameter perturbations, while Benhabib, Schmitt-Grohé and Uribe (2001a, 2002b) argue that such equilibria can account for Japan's fall into the liquidity trap in the 1990s and 2000s.

As mentioned in the introduction, in this paper we do not show the existence of type-B equilibria but focus instead on a necessary condition for their existence when exogenous disturbances are small enough, namely the presence of at least one unstable eigenvalue in the locally linearized system. Note however that Benhabib, Schmitt-Grohé and Uribe (2001a, 2002a, 2002b) provide one reason to suspect the existence of type-B equilibria in many frameworks and in particular in the New Keynesian model. Indeed, they point out that whenever the interest-rate rule respects the zero nominal interest-rate lower bound and makes the interest rate react positively and, at the targeted steady state, more than one-to-one to the inflation rate, there typically exist equilibria originating in the neighbourhood of the targeted steady state and leading to the neighbourhood of a second steady state at which the inflation rate is lower than its targeted value and the interest rate reacts less than one-to-one to the inflation rate.

1.2 The New Keynesian model

This subsection presents the reduced form of the New Keynesian model linearized in the neighbourhood of its commonly-considered steady state, interpreted as its targeted steady state. This reduced form is composed of three equations (the Phillips curve, the IS equation, the interest-rate rule) for three endogenous variables (the inflation rate, the output gap, the short-term nominal interest rate) and two exogenous shocks (the cost-push shock and the natural rate of interest). The Phillips curve, derived from the firms' profit-maximization problem, is written:

$$\pi_t = \beta E_t \left\{\pi_{t+1}\right\} + \kappa x_t + u_t, \tag{1}$$

where π_t denotes the inflation rate and x_t the output gap at date t, $E_t\{.\}$ the private agents' rational-expectation operator conditionally on the information available at date t (which includes the endogenous variables and the exogenous shocks at dates $t-k$ for $k \in \mathbb{N}$), while β and κ are two parameters such that $0 < \beta < 1$ and $\kappa > 0$. This Phillips curve is forward-looking because of the Calvo-type price-setting assumption, as firms know that the price they choose today will remain effective for more than one period on average. The exogenous cost-push shock u of variance $V_u > 0$ is assumed to follow an autoregressive process of order one: $u_t = \rho_u u_{t-1} + \varepsilon_t^u$ with $0 \leq \rho_u < 1$, where ε^u is a white noise.

The IS equation, derived from the representative household's utility-maximization problem, is written:

$$x_t = E_t \left\{x_{t+1}\right\} - \sigma \left(i_t - E_t \left\{\pi_{t+1}\right\} - r_t^n\right), \tag{2}$$

where i_t denotes the short-term nominal interest rate at date t, while σ is a strictly positive parameter. This IS equation is forward-looking due to the usual intertemporal substitution effect. The exogenous natural rate of interest r^n of variance $V_r > 0$ is assumed to follow an autoregressive process of order one: $r_t^n = \rho_r r_{t-1}^n + \varepsilon_t^r$ with $0 \leq \rho_r < 1$, where ε^r is a white noise such that $E\left\{\varepsilon_t^u \varepsilon_{t+k}^r\right\} = 0$ for all $k \in \mathbb{Z}$.

The assumed objective of monetary policy is to minimize the following loss function, which is shown by Woodford (2003, chap. 6) to be linearly and negatively related to the second-order local approximation of the representative household's utility function:

$$L_t = E_t \left\{\sum_{k=0}^{+\infty} \beta^k \left[\left(\pi_{t+k}\right)^2 + \lambda_x \left(x_{t+k} - x^*\right)^2 + \lambda_i \left(i_{t+k} - i^*\right)^2\right]\right\}, \tag{3}$$

where $\lambda_x > 0$, $\lambda_i > 0$, $x^* \geq 0$ and $i^* \geq 0$ are related to the model's structural parameters. In particular, the existence of a nominal interest-rate stabilization objective is due to the existence of transaction frictions and/or, more interestingly in our context, to the consideration of the zero nominal interest-rate lower bound. We finally make the following technical assumption about the parameters:

Assumption 1.1: $\beta \rho_r^2 - (1 + \beta + \kappa\sigma)\rho_r + 1 \neq 0$ *and polynomial*

$$\mathcal{P}(X) \equiv (\beta\lambda_i) X^2 - \left(\lambda_i + \beta\lambda_i + \kappa\lambda_i\sigma + \beta\lambda_x\sigma^2\right) X + \left(\lambda_i + \lambda_x\sigma^2 + \kappa^2\sigma^2\right) \in \mathbb{R}[X]$$

has no root whose modulus is equal to or lower than one (i.e. $\forall X \in \mathbb{C}$, $\mathcal{P}(X) = 0 \Longrightarrow |X| > 1$*).*

1.3 Bubble-free interest-rate rules

Suppose for a moment that the central bank sets the short-term nominal interest rate according to a contemporaneous Taylor rule

$$i_t = \phi + \phi_\pi \pi_t + \phi_x x_t \tag{4}$$

with $(\phi, \phi_\pi, \phi_x) \in \mathbb{R}^3$. The locally linearized system is then made of (1), (2) and (4). As can be easily seen by putting it into Blanchard and Kahn's (1980) form, this system has two non-predetermined variables and two eigenvalues whatever $(\phi_\pi, \phi_x) \in \mathbb{R}^2$. As a consequence, if (ϕ_π, ϕ_x) is chosen so that these two eigenvalues are unstable, then Blanchard and Kahn's (1980) condition is satisfied, *i.e.* type-A equilibria are eliminated, but type-B equilibria may exist. Alternatively, if (ϕ_π, ϕ_x) is chosen so that these two eigenvalues are stable, then the economy jumps out of the frying pan into the fire as type-B equilibria can no longer exist but type-A equilibria do. In other words, contemporaneous Taylor rules make type-A and type-B equilibria the two sides of the same coin. This result naturally holds for any interest-rate rule which is not designed to control the number of non-predetermined variables of the locally linearized system and explains why Benhabib, Schmitt-Grohé and Uribe (2001a, 2002a, 2002b, 2003) find that type-B equilibria exist precisely when type-A equilibria are eliminated by a locally "active" interest-rate rule[2].

By contrast, bubble-free interest-rate rules manage to eliminate both type-A and -B equilibria by removing all non-predetermined variables and all unstable eigenvalues from the locally linearized

[2] In Benhabib, Schmitt-Grohé and Uribe's words: "[i]nterestingly, such [type-B] equilibrium dynamics exist precisely when the target equilibrium is unique from a local point of view" (2002a, p. 72); "[t]he results suggest that central banks that maintain an active monetary policy stance near a given inflation target are more likely to lead the economy into a deflationary spiral (...) than central banks that maintain a globally passive monetary stance such as an interest- or exchange-rate peg" (2001a, p. 43).

system[3]. Such is the case, for instance, of the following kind of interest-rate rules:

$$i_t = \psi + \psi_1 E_t \{\Delta\pi_{t+1}\} + \psi_2 E_t \{\Delta x_{t+1}\} + \psi_3 r_t^n + \psi_4 u_t \tag{5}$$

where Δ denotes the first-difference operator, $\psi \equiv a_\pi$, $\psi_1 \equiv 1$, $\psi_2 \equiv \frac{1}{\sigma}$, $\psi_3 \equiv 1 + b_\pi$ and $\psi_4 \equiv c_\pi$ for $(a_\pi, b_\pi, c_\pi) \in \mathbb{R}^3$, as shown by the following proposition:

Proposition 1.1: *the system made of (1), (2) and (5) taken at dates t to $t+2$ makes (π_t, x_t, i_t) uniquely determined.*

Proof: the replacement of i_t in (2) by the right-hand side of (5) leads to $\pi_t = a_\pi + b_\pi r_t^n + c_\pi u_t$ as the terms in $E_t \{\pi_{t+1}\}$, $E_t \{x_{t+1}\}$ and x_t cancel each other out. Thus, π_t is uniquely determined as a function of the exogenous shocks u_t and r_t^n. The same reasoning conducted one period ahead implies that $E_t \{\pi_{t+1}\}$ is uniquely determined as a function of $E_t \{u_{t+1}\}$ and $E_t \left\{r_{t+1}^n\right\}$ and therefore also uniquely determined as a function of u_t and r_t^n. Then, the replacement of $E_t \{\pi_{t+1}\}$ and π_t in (1) by their expressions as functions of u_t and r_t^n uniquely pins down x_t as a function of u_t and r_t^n. The same reasoning conducted one period ahead implies that $E_t \{x_{t+1}\}$ is uniquely determined as a function of $E_t \{u_{t+1}\}$ and $E_t \left\{r_{t+1}^n\right\}$ and therefore also uniquely determined as a function of u_t and r_t^n. Finally, i_t is then residually and uniquely determined as a function of u_t and r_t^n by (2) or (5). ■

As clear from this proof, rules (5) manage to remove all non-predetermined variables from the locally linearized system by making i_t react to $E_t \{\pi_{t+1}\}$, $E_t \{x_{t+1}\}$ and x_t, in a way which mimics the relationship between these four variables in the locally linearized IS equation (2), so as to disconnect π_t from $E_t \{\pi_{t+1}\}$, $E_t \{x_{t+1}\}$ and x_t in order to pin down π_t, and therefore residually x_t and i_t, uniquely. Given that equation (2) links the expected output-gap variation $E_t \{\Delta x_{t+1}\}$ to the *ex ante* real short-term interest rate $i_t - E_t \{\pi_{t+1}\}$ with an elasticity σ, rules (5) therefore make i_t react to $E_t \{\Delta x_{t+1}\}$ with a coefficient $\frac{1}{\sigma}$ and to $E_t \{\pi_{t+1}\}$ with a coefficient unity. Finally, the coefficient of π_t is for simplicity chosen to be equal to -1, but any other non-zero value would fit the bill as well.

Three points are worth noting at this stage. First, by making i_t react to $E_t \{\pi_{t+1}\}$ with a coefficient unity, rules (5) do not satisfy the so-called Taylor principle, which makes i_t react strictly more than one-to-one to the current or the expected future inflation rate. Two distinct reasons can be put forward to explain this result. The first reason is that, in the standard New Keynesian model considered, the Taylor principle is a necessary condition to eliminate type-A

[3] In the terminology of Loisel (2004), "weak local equilibrium determinacy" refers to the case where type-A equilibria are precluded, while "strong local equilibrium determinacy" refers to the case where both type-A and -B equilibria are precluded.

equilibria for specific parametric families of interest-rate rules, for instance rules of type $i_t = \alpha \pi_t$ or $i_t = \alpha E_t \{\pi_{t+1}\}$ with $\alpha \in \mathbb{R}$, but ceases to be one for slightly more general parametric families of interest-rate rules, as shown *e.g.* by Woodford (2003, chap. 4), and is unlikely to be one for parametric families of interest-rate rules general enough to include rules (5). The second reason is that the coefficient of $E_t \{\pi_{t+1}\}$ in rules (5) is chosen so as to remove all non-predetermined variables from the locally linearized system, which requires that this coefficient be equal to one, while in rules of type $i_t = \alpha E_t \{\pi_{t+1}\}$ for instance, which make the locally linearized system have two non-predetermined variables and two eigenvalues whatever $\alpha \in \mathbb{R}$, the coefficient α is chosen so as to make these two eigenvalues unstable, which requires that the Taylor principle $\alpha > 1$ be satisfied.

Second, there exist an infinity of bubble-free interest-rate rules implementing the same equilibrium as rule (5) for a given $(a_\pi, b_\pi, c_\pi) \in \mathbb{R}^3$. Examples of such rules can be obtained by adding for instance a term of type $\omega (\pi_t - \beta E_t \{\pi_{t+1}\} - \kappa x_t - u_t)$ with $\omega \in \mathbb{R}$, which is actually equal to zero due to (1), to the right-hand side of (5). In particular, unless $(b_\pi, c_\pi) = (-1, 0)$ rules of type (5) do not qualify as "direct rules" in the sense of Giannoni and Woodford (2002) and Woodford (2003, chap. 8) since they make i_t conditional on the exogenous shocks r_t^n and u_t which are no target variables, but a bubble-free direct rule implementing the same equilibrium as rule (5) for a given $(a_\pi, b_\pi, c_\pi) \in \mathbb{R}^3$, provided that $b_\pi \neq \sigma - 1$, can be obtained by replacing in (5) r_t^n by $\frac{1}{\sigma}(x_t - E_t \{x_{t+1}\}) + i_t - E_t \{\pi_{t+1}\}$, as implied by (2), and u_t by $\pi_t - \beta E_t \{\pi_{t+1}\} - \kappa x_t$, as implied by (1), and then solving for i_t. Because such a direct rule typically has some negative and hence "counter-intuitive" coefficients under the calibration considered in the next subsection, we however prefer to focus on rules of type (5).

Third, as bubble-free interest-rate rules are not meant to be effective outside the neighbourhood of the targeted steady state within which the linear approximation is acceptable, proposition 1.1 implies that they eliminate only those type-B equilibria whose paths would remain in this neighbourhood during two periods. In other words, they prevent the economy from gradually leaving the neighbourhood of the targeted steady state, not from abruptly leaving this neighbourhood, where loosely speaking by "gradually" we mean "using one or several eigenvalues whose modulus is larger than but close enough to one, exactly how close to one depending on the size of the neighbourhood of the targeted steady state within which the linear approximation is acceptable". But this limitation does not prevent these rules from eliminating for instance the type-B equilibria obtained under empirically plausible parameterizations by Benhabib, Schmitt-Grohé and Uribe (2001a), along which the inflation rate fluctuates around the targeted steady state for a long period of time before leaving its neighbourhood, in a way which is not inconsistent with observed inflation dynamics[4].

1.4 Numerical application

This subsection uses a standard calibration of the New Keynesian model to put some numerical flesh on the contemporaneous Taylor rules and bubble-free interest-rate rules considered in the previous subsection. To that aim, let us define the equilibrium under discretion E^d as the sequence $\{\pi_t, x_t, i_t\}_{t\in\mathbb{Z}}$ obtained when at each date t the central bank chooses i_t so as to minimize (3) subject to (1) and (2) and the equilibrium under commitment E^c as the sequence $\{\pi_t, x_t, i_t\}_{t\in\mathbb{Z}}$ of type

$$\pi_t = a_\pi + b_\pi r_t^n + c_\pi u_t,\ x_t = a_x + b_x r_t^n + c_x u_t \text{ and } i_t = a_i + b_i r_t^n + c_i u_t, \tag{6}$$

where $(a_\pi, b_\pi, c_\pi, a_x, b_x, c_x, a_i, b_i, c_i) \in \mathbb{R}^9$, minimizing (3) subject to (1) and (2)[5].

Concerning contemporaneous Taylor rules, we obtain the following proposition:

Proposition 1.2: *there exist a unique rule of type (4) consistent with E^d and a unique rule of type (4) consistent with E^c.*

Proof: *cf* appendix A. ■

Let ϕ_π^d, ϕ_x^d (respectively ϕ_π^c, ϕ_x^c) then denote the second and third coefficients of the unique rule of type (4) consistent with E^d (respectively with E^c) and μ^d (respectively μ^c) the lowest modulus of the two eigenvalues of the system made of (1), (2) and this rule. Under Giannoni and Woodford's (2003) calibration:

β	κ	σ	ρ_r	ρ_u	λ_x	λ_i
0,99	0,024	6,25	0,35	0,35	0,048	0,236

assumption 1.1 is satisfied and we get:

ϕ_π^d	ϕ_x^d	μ^d	ϕ_π^c	ϕ_x^c	μ^c
0,64	1,27	1,003	1,71	2,23	1,018

[4] In Benhabib, Schmitt-Grohé and Uribe's (2001a) words: "observed inflation dynamics are in general quite smooth, giving little credence to a model in which movements in inflation at business-cycle frequency are due to jumps from one steady state to another. (...) Interestingly, along both the saddle connection and the limit cycle, the inflation rate fluctuates for long periods of time around the steady-state at which monetary policy is active. Thus, an econometrician using data generated from a saddle connection equilibrium to estimate the slope of the interest rate feedback rule may very well conclude that the economy is displaying stationary fluctuations around the active steady-state, even though the economy is in fact spiraling down a liquidity trap" (p. 43); "the saddle connection is not inconsistent with the observation that the inflation rate fluctuates for long periods of time in a region in which monetary policy is active, as has been argued in the case of the U.S. economy since the Volcker era" (p. 56).

[5] We limit ourselves to sequences of type (6) because we consider contemporaneous Taylor rules (4). Indeed, when the system made of (1), (2) and (4) admits a unique local solution $\{\pi_t, x_t, i_t\}_{t\in\mathbb{Z}}$, this unique solution can be written in the form (6); conversely, for any $(a_\pi, b_\pi, c_\pi, a_x, b_x, c_x, a_i, b_i, c_i) \in \mathbb{R}^9$ such that (6) holds, there exists $(\phi, \phi_\pi, \phi_x) \in \mathbb{R}^3$ such that (4) holds.

Since the values obtained for μ^d and μ^c are higher than one, these two contemporaneous Taylor rules ensure local equilibrium determinacy, *i.e.* eliminate type-A equilibria. But interestingly these values are strikingly close to one, so that if type-B equilibria exist then along these equilibria the endogenous variables remain in the neighbourhood of the targeted steady state for a long period before leaving this neighbourhood, just like in Benhabib, Schmitt-Grohé and Uribe (2001a) as mentioned above.

Concerning bubble-free interest-rate rules, the proof of proposition 1.1 shows that the unique solution of the system made of (1), (2) and (5) for $t \in \mathbb{Z}$ is the unique sequence $\{\pi_t, x_t, i_t\}_{t\in\mathbb{Z}}$ which is of type (6) for some given (a_π, b_π, c_π) and satisfies (1) and (2) for $t \in \mathbb{Z}$. Let us then note ψ_1^d, ψ_2^d, ψ_3^d, ψ_4^d (respectively ψ_1^c, ψ_2^c, ψ_3^c, ψ_4^c) the second to fifth coefficients of the unique rule of type (5) implementing E^d (respectively E^c). Under Giannoni and Woodford's (2003) calibration we get:

ψ_1^d	ψ_2^d	ψ_3^d	ψ_4^d	ψ_1^c	ψ_2^c	ψ_3^c	ψ_4^c
1	0,16	1,03	1,52	1	0,16	1,02	1,50

Note that the values taken by ψ_3^d and ψ_4^d are very close to those taken by ψ_3^c and ψ_4^c respectively, thus making the two rules numerically very close to each other. This result however is simply due to the fact that E^d and E^c are themselves numerically very close to each other.

1.5 Other monetary policy proposals

As mentioned in the introduction, the existing literature has mainly focused on interest-rate rules eliminating only type-A equilibria. Except in Woodford's (1994b) particular model, where an interest-rate peg rules out both type-A and -B equilibria, the existing literature has proposed two-tier policies to eliminate type-B equilibria, in the spirit of Obstfeld and Rogoff's (1983, 1986) fractional-backing proposal to rule out speculative hyperinflations. These two-tier policies, advocated by Clarida, Galí and Gertler (1999, p. 1701), Christiano and Rostagno (2001), Benhabib, Schmitt-Grohé and Uribe (2002a, 2002b, 2003), Woodford (2003, chap. 2) and Evans and Honkapohja (2005), consist in switching from an interest-rate rule eliminating type-A equilibria to another rule such as a money growth rate peg (possibly accompanied by a non-Ricardian fiscal policy) when the endogenous variables go outside a specified neighbourhood of the targeted steady state[6].

These two-tier policies may however not be completely effective in eliminating all type-B equilibria for the following three reasons. First, they may be curative, *i.e.* for instance able to drive the

[6] The non-monotonic interest-rate rule proposed by Alstadheim and Henderson (2002) – at Benhabib, Schmitt-Grohé and Uribe's (2001a, p. 42) suggestion – can also be viewed as such a two-tier policy. The asymmetric interest-rate rule proposed by the same authors departs from the framework adopted by all the other papers (including this one) as it makes the model not locally linearisable.

economy out of the liquidity trap[7], but not necessarily preventive, *i.e.* able to dissuade the private agents from leaving the neighbourhood of the targeted steady state in the first place[8]. Second, the second-tier rule such as a money growth peg might prove itself an additional source of equilibrium indeterminacy, as acknowledged by Christiano and Rostagno (2001). Third, as pointed by Green (2005, pp. 126-127), given their aggressiveness in some circumstances, the credibility and consequently the effectiveness of these policy devices in eliminating type-B equilibria should not be taken for granted. Given that they are immune from these three drawbacks, bubble-free interest-rate rules represent a particularly interesting alternative or complement to these two-tier policies.

Note finally that these two-tier policies may not be completely effective in eliminating all type-B equilibria for a fourth reason: as acknowledged by Benhabib, Schmitt-Grohé and Uribe (2002a, 2002b) indeed, they cannot eliminate equilibrium paths converging towards a deterministic or chaotic cycle within the specified neighbourhood of the targeted steady state. But in such cases, however small this neighbourhood necessarily includes the neighbourhood of the targeted steady state within which the linear approximation of the model is acceptable (since such cycling dynamics require non-linearity), so that the neighbourhood of the targeted steady state within which bubble-free interest-rate rules are meant to be effective may then well be too small for them to be practically useful anyway.

2 Bubble-free policy feedback rules in a general framework

This section designs bubble-free policy feedback rules for a broad class of rational-expectations dynamic stochastic infinite-horizon linear models, meant to represent the locally linearized reduced form of rational-expectations dynamic stochastic general equilibrium models, and shows that bubble-free interest-rate rules are necessarily forward-looking in most of these models, contrary to interest-rate rules ensuring only local equilibrium determinacy.

2.1 A general framework

The economy is made of one policy-maker and many private agents (whether infinitely-lived or in overlapping generations). Time is discrete, indexed by $t \in \mathbb{Z}$. Let $N \in \mathbb{N}^*$[9]. The model is composed of $N + 1$ time-invariant linear equations (N structural equations, which describe the

[7] Their effectiveness in driving the economy out of the liquidity trap should however not be taken for granted, as stressed by Bernanke, Reinhart and Sack (2004, p. 5) who conclude that the best policy still remains to avoid reaching the zero lower bound on the short-term nominal interest rate.

[8] One counter-example is provided by Christiano and Rostagno (2001) who present a model where such a two-tier policy solution acts preventively, *i.e.* nips type-B equilibria in the bud.

[9] In the following, we sometimes use for convenience notations which implicitly assume that $N \geq 2$. In such cases the reader should easily infer the notation rigorously adapted to the case $N = 1$ (*e.g.* replacing $\sum_{i=2}^{N} \{.\}$ by 0).

private agents' behaviour, and one policy feedback rule) for $N+1$ endogenous variables (N non-control variables making up N-dimension vector $\mathbf{Y}_t$ and one control variable or policy instrument z_t) and N exogenous shocks (making up N-dimension vector $\boldsymbol{\xi}_t$). Let L denote the lag operator and $E_t\{.\}$ the private agents' rational-expectation operator conditionally on $\left\{\mathbf{Y}_{t-k}, z_{t-k}, \boldsymbol{\xi}_{t-k}\right\}_{k\geq 0}$. The N structural equations are written as follows:

$$E_t\left\{\mathbf{A}(L)\,\mathbf{Y}_t+\mathbf{B}(L)\,z_t\right\}+\mathbf{C}(L)\,\boldsymbol{\xi}_t=\mathbf{0} \tag{7}$$

$$\text{with } \underset{(N\times N)}{\mathbf{A}(L)} \equiv \sum\nolimits_{k=-m^a}^{n^a}\mathbf{A}_k L^k,\ \underset{(N\times 1)}{\mathbf{B}(L)} \equiv \sum\nolimits_{k=-m^b}^{n^b}\mathbf{B}_k L^k \text{ and } \underset{(N\times N)}{\mathbf{C}(L)} \equiv \sum\nolimits_{k=0}^{n^c}\mathbf{C}_k L^k,$$

where $\left(m^a, m^b, n^a, n^b, n^c\right)\in\mathbb{N}^5$, all $\mathbf{A}_k$, $\mathbf{B}_k$ and $\mathbf{C}_k$ have real numbers as elements and all the eigenvalues of $\mathbf{C}(L)$ are of modulus strictly lower than one. Each exogenous shock is assumed to follow a centered stationary autoregressive process of finite order[10]:

$$\mathbf{D}(L)\,\boldsymbol{\xi}_t=\boldsymbol{\varepsilon}_t \text{ with } \underset{(N\times N)}{\mathbf{D}(L)} \equiv \begin{bmatrix} D_1(L) & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & D_N(L) \end{bmatrix}$$

and $D_i(L)\equiv\sum_{k=0}^{n^d} d_{i,k}L^k$ for $1\leq i\leq N$, where $n^d\in\mathbb{N}$, all $d_{i,k}$ are real numbers, $\mathbf{D}(0)$ is invertible, all the eigenvalues of $\mathbf{D}(L)$ are of modulus strictly lower than one and $\boldsymbol{\varepsilon}_t$ is a N-dimension white noise vector.

For all $i\in\{1,...,N\}$, let $\mathbf{e}_i$ denote the N-element vector whose i^{th} element is equal to one and whose other elements are equal to zero. Let I_A denote the set of $i\in\{1,...,N\}$ such that $\mathbf{e}_i'\mathbf{A}(L)\neq\mathbf{0}$ and I_B the set of $i\in\{1,...,N\}$ such that $\mathbf{e}_i'\mathbf{B}(L)\neq 0$. Let $m_i^a\equiv-\min\left[k\in\{-m^a,...,n^a\},\ \mathbf{e}_i'\mathbf{A}_k\neq\mathbf{0}\right]$ for $i\in I_A$ and $m_i^b\equiv-\min\left[k\in\left\{-m^b,...,n^b\right\},\ \mathbf{e}_i'\mathbf{B}_k\neq 0\right]$ for $i\in I_B$. Finally, let us note

$$\widehat{\mathbf{A}}(L)\equiv\begin{bmatrix}\mathbf{e}_1'L^{m_1^a}\\ \vdots \\ \mathbf{e}_N'L^{m_N^a}\end{bmatrix}\mathbf{A}(L).$$

We make the following two assumptions:

Assumption 2.1: *(i)* $I_A=\{1,...,N\}$*; (ii)* $\forall i\in\{1,...,N\}$, $m_i^a\geq 0$*; (iii)* $\widehat{\mathbf{A}}(0)$ *is invertible.*

Assumption 2.2: *(i)* $1\in I_B$*; (ii)* $m_1^b\geq 0$*; (iii)* $\forall i\in I_B\smallsetminus\{1\}$, $m_i^a-m_i^b>\max\left[0, m_1^a-m_1^b\right]$.

Assumptions 2.1 and 2.2 may seem restrictive at first sight, but appendix B shows that any system of type (7) such that the policy instrument appears in at least one of the structural equations

[10] This assumption is not restrictive in the sense that if each element of $\boldsymbol{\xi}_t$ followed instead a centered stationary finite-order ARMA process, then $\mathbf{C}(L)\,\boldsymbol{\xi}_t$ could easily be rewritten in the form $\underline{\mathbf{C}}(L)\,\underline{\boldsymbol{\xi}}_t$ with $\underline{\mathbf{C}}(L)\equiv\sum_{k=0}^{n^{\underline{c}}}\underline{\mathbf{C}}_k L^k$, where $n^{\underline{c}}\in\mathbb{N}$, all $\underline{\mathbf{C}}_k$ are $N\times N$ matrices with real numbers as elements, all the eigenvalues of $\underline{\mathbf{C}}(L)$ are of modulus strictly lower than one and each element of $\underline{\boldsymbol{\xi}}_t$ follows a centered stationary autoregressive process of finite order.

and that the structural equations are non-redundant implies that holds another system of type (7) satisfying assumptions 2.1 and 2.2.

Three points are worth being made at this stage. First, in monetary policy applications the policy-maker will naturally be the central bank and the policy instrument typically the short-term nominal interest rate. Second, specification (7) is general enough to encompass the locally linearized structural equations of most rational-expectations dynamic stochastic general equilibrium models, including *e.g.* the New Keynesian model considered in the previous section (for which $N = 2$, $m_1^a = m_2^a = 1$ and $m_1^b = 0$) as well as such well-known medium-sized models as Smets and Wouters' (2003). Naturally, this linear approximation is valid only locally, which requires notably that $\boldsymbol{\varepsilon}_t$ have a bounded distribution. Third, our focus on models with only one control variable (which can feature as a lagged, a current and/or an expected future variable in only one or in several structural equations) and no limit condition is without any loss in generality. Indeed, when several control variables, the policy-maker can always exogenize all but one, and the existence of limit conditions would only reduce the set of solutions to the model's structural equations.

Finally, we consider the set of policy rules expressing the current instrument z_t as a finite time-invariant linear combination of past and current endogenous variables and exogenous shocks as well as current expectations of future endogenous variables, *i.e.* the set of policy rules which can be written as follows for $t \in \mathbb{Z}$:

$$E_t \left\{ \mathbf{F}(L) \mathbf{Y}_t \right\} + G(L) z_t + \mathbf{H}(L) \boldsymbol{\xi}_t = \mathbf{0} \tag{8}$$

$$\text{with } \underset{(1\times N)}{\mathbf{F}(L)} \equiv \sum\nolimits_{k=-m^f}^{n^f} \mathbf{F}_k L^k,\ G(L) \equiv \sum\nolimits_{k=0}^{n^g} g_k L^k \text{ and } \underset{(1\times N)}{\mathbf{H}(L)} \equiv \sum\nolimits_{k=0}^{n^h} \mathbf{H}_k L^k,$$

where $\left(m^f, n^f, n^g, n^h\right) \in \mathbb{N}^4$, all g_k are real numbers, $g_0 \neq 0$ and all $\mathbf{F}_k$, $\mathbf{H}_k$ have real numbers as elements.

2.2 Bubble-free policy feedback rules

If $\exists i \in \{1, ..., N\}$, $m_i^a > 0$ and if the policy rule (8) is arbitrarily chosen (*e.g.* if z_t is set exogenously), then the linear model made of (7) and (8) typically leaves the door open to a large variety of sunspot equilibria and/or rational mean-explosive bubbles of the type identified by Blanchard (1979), Blanchard and Watson (1982), Evans (1991) and Froot and Obstfeld (1991). The initial development of these bubbles may correspond to the initial development of type-B equilibria in the dynamic stochastic general equilibrium model considered whose locally linearized reduced form corresponds to this linear model. Now consider a policy feedback rule of the following form:

$$\mathbf{e}_1'\left[E_t\left\{L^{m_1^b}\left[\mathbf{A}(L)\mathbf{Y}_t+\mathbf{B}(L)z_t\right]\right\}+L^{m_1^b}\mathbf{C}(L)\boldsymbol{\xi}_t+\mathbf{O}(L)\mathbf{D}(L)\boldsymbol{\xi}_t\right]$$
$$+\sum_{i=2}^{N}\mathbf{e}_i'L^{m_1^b+\sum_{j=2}^{i}m_j^a}\left[\mathbf{A}(L)\mathbf{Y}_t+\mathbf{B}(L)z_t+\mathbf{C}(L)\boldsymbol{\xi}_t+L^{-m_i^a}\mathbf{O}(L)\mathbf{D}(L)\boldsymbol{\xi}_t\right]$$
$$+L^{m_1^b+\sum_{i=2}^{N}m_i^a}\left[\mathbf{P}(L)\mathbf{Y}_t+Q(L)z_t+\mathbf{R}(L)\mathbf{D}(L)\boldsymbol{\xi}_t\right]=\mathbf{0}, \tag{9}$$

$$\text{with } \underset{(N\times N)}{\mathbf{O}(L)}\equiv\begin{bmatrix}\sum_{k=0}^{m_1^b-1}\mathbf{O}_{1,k}L^k\\ \sum_{k=0}^{m_2^a-1}\mathbf{O}_{2,k}L^k\\ \vdots\\ \sum_{k=0}^{m_N^a-1}\mathbf{O}_{N,k}L^k\end{bmatrix},\ \underset{(1\times N)}{\mathbf{P}(L)}\equiv\sum_{k=0}^{n^p}\mathbf{P}_kL^k \text{ such that } \underset{(N\times N)}{\mathbf{\Omega}}\equiv\begin{bmatrix}\mathbf{P}_0\\ \mathbf{e}_2'\widehat{\mathbf{A}}(0)\\ \vdots\\ \mathbf{e}_N'\widehat{\mathbf{A}}(0)\end{bmatrix}$$
$$\text{is invertible, } Q(L)\equiv\sum_{k=\max\left[0,m_1^a-m_1^b+1\right]}^{n^q}q_kL^k \text{ and } \underset{(1\times N)}{\mathbf{R}(L)}\equiv\sum_{k=0}^{n^r}\mathbf{R}_kL^k,$$

where $(m^p,n^q,n^r)\in\mathbb{N}^3$, all q_k are real numbers and all $\mathbf{O}_{i,k}$, $\mathbf{P}_k$, $\mathbf{R}_k$ have real numbers as elements. We adopt the convention $\sum_{i=u}^{v}\{.\}=0$ for $u>v$. Rules of type (9) belong to the class of rules (8) and qualify as "instrument rules" since their z_t-coefficient $\mathbf{e}_1'\mathbf{B}_{-m_1^b}$ differs from zero.

We first show that rules of type (9) are bubble-free in the sense that they implement a unique equilibrium:

Proposition 2.1 (determinacy): *the system made of (7) and (9) taken at dates t to $t+\max\left[m_1^a,m_1^b\right]+\sum_{i=2}^{N}m_i^a$ makes $\mathbf{Y}_t$ and z_t uniquely determined.*

Proof: *cf* appendix C. ■ As made clear in appendix C, rules of type (9) achieve the existence and uniqueness of the solution (or equivalently the global determinacy of the equilibrium in this linear framework) by disconnecting the current variables from their expected future values. More precisely, the forward-looking part of rules of type (9) is designed to insulate the current variables from the most forward-looking part of the first structural equation $\mathbf{e}_1'(7)$ if $m_1^a>m_1^b$, while their backward-looking part is designed to insulate the current variables from the forward-looking part of the other structural equations $\mathbf{e}_k'(7)$ for all $k\in\{2,...,N\}$ such that $m_k^a>0$. This explains why the time needed by these rules to be effective, specified in the proposition, is a function of the length of the forward-looking part of the structural equations.

We then show that any given VARMA solution of the model's structural equations (not involving white noises other than those featuring in the structural equations) can be uniquely selected by a suitably chosen bubble-free policy feedback rule of type (9):

Proposition 2.2 (controllability): *for any sequence $\{\mathbf{Y}_t,z_t\}_{t\in\mathbb{Z}}$ which satisfies (7) and can be written in a VARMA form*

$$\begin{bmatrix}\mathbf{Y}_t\\ z_t\end{bmatrix}=\mathbf{S}(L)\begin{bmatrix}\mathbf{Y}_t\\ z_t\end{bmatrix}+\mathbf{T}(L)\boldsymbol{\varepsilon}_t \tag{10}$$

$$\textit{with} \underset{((N+1)\times(N+1))}{\mathbf{S}(L)} \equiv \sum_{k=1}^{n^s} \mathbf{S}_k L^k \text{ and } \underset{((N+1)\times N)}{\mathbf{T}(L)} \equiv \sum_{k=0}^{n^t} \mathbf{T}_k L^k$$

where $n^s \in \mathbb{N}^*$, $n^t \in \mathbb{N}$ *and all* $\mathbf{S}_k$ *and* $\mathbf{T}_k$ *have real numbers as elements, there exist* $\mathbf{O}(L)$, $\mathbf{P}(L)$, $Q(L)$ *and* $\mathbf{R}(L)$ *such that* $\{\mathbf{Y}_t, z_t\}_{t\in\mathbb{Z}}$ *is the unique solution of (7) and (9) for* $t \in \mathbb{Z}$.

Proof: *cf* appendix D. ■ Technically speaking, the role of $\mathbf{P}(L)$ and $Q(L)$ is to make the system made of (7) and (9) have the same eigenvalues as $\mathbf{I} - \mathbf{S}(L)$, where $\mathbf{I}$ denotes the $(N+1)\times(N+1)$ identity matrix, so that its unique solution $\{\mathbf{Y}_t, z_t\}_{t\in\mathbb{Z}}$ is such that $[\mathbf{I} - \mathbf{S}(L)] \begin{bmatrix} \mathbf{Y}_t & z_t \end{bmatrix}'$ is equal to a finite-order matrix polynomial in L applied to $\boldsymbol{\varepsilon}_t$; and the role of $\mathbf{O}(L)$ and $\mathbf{R}(L)$ is to ensure that this finite-order matrix polynomial in L is $\mathbf{T}(L)$.

Given that in most rational-expectations dynamic stochastic general equilibrium models the targeted equilibrium (*e.g.* the globally-social-welfare-maximizing equilibrium) is to be found in the neighbourhood of a given steady state and can typically be linearized in a form (10), propositions 2.1 and 2.2 imply that in most rational-expectations dynamic stochastic general equilibrium models there exists a linear interest-rate rule to be followed locally which enables the central bank both to select the targeted local equilibrium to the exclusion of all other local equilibria and to prevent the economy from gradually leaving the neighbourhood of the corresponding steady state, where by "gradually" we mean "in more than $\max\left[m_1^a, m_1^b\right] + \sum_{i=2}^{N} m_i^a$ periods".

2.3 Forward- vs. backward-looking rules

This subsection establishes two propositions about the link between the forward- or backward-looking nature of the policy feedback rule considered and its ability to ensure equilibrium determinacy or, in a more demanding way, to select uniquely a given stationary equilibrium. To that aim, let $\Delta_i(X) \in \mathbb{R}[X]$ for $i \in \{1, ..., N+1\}$ denote the determinant of the $N \times N$ matrix obtained by removing its i^{th} column from $N \times (N+1)$ matrix $X^{\max[n^a, n^b]} \begin{bmatrix} \mathbf{A}(X^{-1}) & | & \mathbf{B}(X^{-1}) \end{bmatrix}$, let $\mathcal{D}(X) \in \mathbb{R}[X]$ denote the greatest common divisor, defined up to a non-zero multiplicative scalar, of all non-zero $\Delta_i(X)$ for $i \in \{1, ..., N+1\}$, and let us make the following assumption:

Assumption 2.3: *all the roots of* $\mathcal{D}(X)$ *have their modulus strictly lower than one.*

Let us also define the concept of μ-bubble-free rules, which will prove useful in the remaining of the paper:

Definition (μ-bubble-free rules): *a rule of type (8) is said to be μ-bubble-free (for a given* $\mu \geq 1$*) when the system made of (7) and this rule admits one unique stationary solution and has no eigenvalue whose modulus is between* 1 *and* μ.

In particular, rules of type (9) are thus $+\infty$-bubble-free when they implement a stationary solution.

A policy feedback rule is said to be forward-looking when it makes the policy variable conditional in particular on the private agents' current expectations of future endogenous variables, and backward-looking when it makes the policy variable conditional only on current and/or past endogenous and/or exogenous variables. In practice, a central bank can extract the private agents' expectations from various sources, such as surveys and prices of financial instruments. If the central bank's forecasts are conditional on the private agents' expectations, then the former should coincide with the latter and therefore the terms "forward-looking" and "forecast-based" can be used interchangeably to qualify an interest-rate rule.

A great deal of attention has been paid in the literature to the issue of whether the interest-rate rule should be forward-looking in order to eliminate type-A and/or type-B equilibria. On the one hand, Woodford (1994a) and Carlstrom and Fuerst (2000, 2001, 2002, 2005) show that some forward-looking rules lead to type-A equilibria, contrary to some current- or backward-looking rules. Similarly, Bernanke and Woodford (1997) show that some forward-looking rules lead to type-A equilibria. On the other hand, Benhabib, Schmitt-Grohé and Uribe (2003) show that some backward-looking rules lead to type-B equilibria, while De Fiore and Liu (2005) show that the backward-looking rules eliminating type-A equilibria in Carlstrom and Fuerst's (2000, 2001, 2002) closed-economy models do not eliminate them in an open-economy model[11].

As suggested by De Fiore and Liu's (2005) results, the conclusions reached by this literature are however very sensitive to the specific structural equations considered as well as to the usually low-dimensional parametric family of rules considered[12]. Indeed, the ability of simple backward-, current- or forward-looking interest-rate rules to preclude type-A equilibria does crucially depend on the structural equations considered, as shown by Benhabib, Schmitt-Grohé and Uribe (2001b), Weder (2006) and Zanetti (2006), as well as on the parametric family of backward-, current- or forward-looking rules considered for the standard New Keynesian model examined in section 1, as shown by Woodford (2003, chap. 4) and Lubik and Marzo (2007).

By contrast, our general setting both for the structural equations and for the interest-rate rule enables us to reach more general conclusions. Given the form of rules (9), one of these conclusions is that all type-A and -B equilibria can be eliminated by some interest-rate rules which are forward-

[11] Besides the rational expectations literature, the adaptive learning literature has also taken part in the forward-looking *vs.* backward-looking interest-rate-rules debate. On the one hand, Evans and Honkapohja (2003, 2006) find that some forward-looking rules make the globally optimal local equilibrium learnable, contrary to the fundamentals-based and hence backward-looking rule consistent with this equilibrium. Similarly, Bullard and Mitra (2002) find that some forward-looking rules make the globally optimal local equilibrium learnable in the absence of type-A equilibria, contrary to some backward-looking rules. On the other hand, Evans and McGough (2005b) find that some forward-looking rules lead to learnable type-A equilibria, and Eusepi (2005) shows that some forward-looking rules precluding type-A equilibria lead to learnable type-B equilibria, while some backward-looking rules preclude both learnable type-A equilibria and learnable type-B equilibria.

[12] One notable exception is Giannoni and Woodford (2002) and Woodford (2003, chap. 8), who design "robustly optimal" interest-rate rules precluding type-A equilibria in a general linear-quadratic framework.

looking if $m_1^a > m_1^b$. Note that the condition $m_1^a > m_1^b$ is typically met if the model includes an Euler equation, which is arguably the case for most rational-expectations dynamic stochastic general-equilibrium models. Proposition 2.3 goes further by showing that actually if $m_1^a > m_1^b$ then all $+\infty$-bubble-free interest-rate rules of type (8) with finite coefficients (*i.e.* all bubble-free interest-rate rules of type (8) with finite coefficients implementing a stationary equilibrium) are forward-looking:

Proposition 2.3: *if $m_1^a > m_1^b$ then no rule of type (8) which is backward-looking (*i.e. *such that $m_f = 0$) and has all its coefficients finite (when g_0 is normalized to one) can be $+\infty$-bubble-free.*

Proof: *cf* appendix E. ■

Proposition 2.4 moreover shows that there always exists a backward-looking interest-rate rule of type (8) precluding the main kind of equilibrium indeterminacy which the existing literature is concerned with, namely local equilibrium indeterminacy (*i.e.* type-A equilibria):

Proposition 2.4: *for any stationary VARMA process of type (10) consistent with (7), there exists a rule of type (8) which is backward-looking (*i.e. *such that $m_f = 0$), ensures local equilibrium determinacy and makes the locally unique equilibrium selected coincide with this VARMA process.*

Proof: *cf* appendix F. ■ Technically speaking, appendix F uses the generalized identity of Bezout to choose $\mathbf{F}(L)$ and $G(L)$ such that the system admits a unique local solution[13] and has the same stable eigenvalues as the targeted stationary VARMA process; the Euclidian division to choose $\mathbf{F}(L)$ such that $m_f = 0$; and Cramer's rule to residually choose $\mathbf{H}(L)$ such that the unique local solution coincides with the targeted stationary VARMA process.

Thus, propositions 2.3 and 2.4 together imply that the desirability of eliminating type-B in addition to type-A equilibria provides the first theoretical justification solely related to equilibrium-indeterminacy concerns so far put forward in the literature for the use of forward-looking interest-rate rules.

Another, closely related branch of the literature has paid attention to the issue of to which extent the interest-rate rule should be forward-looking in order to eliminate type-A equilibria. Batini and Pearlman (2002), Batini, Levine and Pearlman (2004), Batini, Justiniano, Levine and Pearlman (2006) and Leitemo (2006) consider interest-rate rules of type $i_t = \alpha + \beta i_{t-1} + \gamma E_t \{\pi_{t+\theta}\}$ with $(\alpha, \beta, \gamma) \in \mathbb{R}^3$ and $\theta \in \mathbb{N}$, where i_t and π_t respectively denote the short-term nominal interest

[13] If needed, proposition 2.4 could naturally be easily generalized (by choosing $\mathcal{Z}(X)$ without any root of modulus between 1 and μ in appendix F) to show the existence, for any given stationary equilibrium, of backward-looking and μ-bubble-free (instead of 1-bubble-free) rules of type (8) consistent with this equilibrium.

rate and the inflation rate at date t, and find that the more forward-looking the interest-rate rule or equivalently the more distant the forecast horizon (*i.e.* the higher θ), the higher the risks of type-A equilibria or macroeconomic instability − the latter arising when the locally linearized system admits more unstable eigenvalues than required by Blanchard and Kahn's (1980) conditions[14]. This result can be easily explained as follows: the choice of a more forward-looking interest-rate rule (*i.e.* of a higher θ) is most likely to increase the number of eigenvalues and non-predetermined variables of the locally linearized system and hence the risk that no (β, γ) exists such that Blanchard and Kahn's (1980) conditions are satisfied. By contrast, the Calvo-type interest-rate rules put forward by Levine, McAdam and Pearlman (2006), of the kind $i_t = \alpha + \beta i_{t-1} + \gamma E_t \left\{ \sum_{k=0}^{+\infty} \varphi^k \pi_{t+k} \right\}$ with $(\alpha, \beta, \gamma) \in \mathbb{R}^3$ and $\varphi \in]0; 1[$, manage to be infinitely forward-looking while making the locally linearized system have a finite, and possibly even reduced, number of eigenvalues and non-predetermined variables. This property of theirs might well explain why these rules are apparently quite successful, as documented by Levine, McAdam and Pearlman (2006), in eliminating type-A equilibria.

Note also that in most models without monetary policy transmission lags (*i.e.* models with the typical Euler equation featuring the current nominal interest rate), where only short-horizon expected future endogenous variables appear in the forward-looking part of the structural equations, our requirement that rules should eliminate type-A and -B equilibria and Giannoni and Woodford's (2003) requirement that rules should eliminate type-A equilibria and be "robustly optimal" both imply, though for different reasons, that the rule should typically be based on only a-few-period forecasts.

3 Robustness of bubble-free policy feedback rules

This section discusses the robustness of bubble-free policy feedback rules to departures from three assumptions in turn: i) that the policy-maker has perfect knowledge of the data and the structural parameters; ii) that the policy-maker can credibly commit to following a given policy feedback rule; and iii) that the private agents form rational expectations.

3.1 Policy-maker's imperfect knowledge

This subsection examines the sensitivity of propositions 2.1 and 2.2 to the assumption that the policy-maker has perfect knowledge of the data and the model's structural parameters, understood here as the parameters featuring in the structural equations (7). The case where the policy-maker has perfect knowledge of the structural parameters but imperfect knowledge of the data is easily

[14] Levin, Wieland and Williams (2003) reach a similar conclusion while considering a slightly more general family of interest-rate rules.

dealt with:

Proposition 2.5: *if the policy-maker wants to follow a rule of type (9) implementing a given targeted stationary equilibrium but measures at each date t all the endogenous variables $\mathbf{Y}_{t-k}$ for $k \geq 0$, z_{t-k} for $k \geq 0$, $E_t\{\mathbf{Y}_{t+k}\}$ for $k \geq 1$ and all the exogenous shocks $\boldsymbol{\varepsilon}_{t-k}$ for $k \geq 0$ with some exogenous additive measurement errors, each of them randomly drawn from a continuous probability distribution supported on a bounded interval including zero, and accordingly follows the corresponding rule of type (9) based on these measured variables and shocks, then: (i) the system made of (7) and this rule admits one unique solution; (ii) as the length of all the distribution-supporting intervals tends towards zero (*i.e. *as all the measured variables and shocks converge towards the true variables and shocks), this solution converges towards the targeted equilibrium.*

Proof: *cf* appendix G. ■ In short, in case of exogenous additive data-measurement errors proposition 2.1 still holds and proposition 2.2 holds asymptotically, *i.e.* as the size of the errors tends towards zero.

The case where the policy-maker has perfect knowledge of the data but imperfect knowledge of the values of the structural parameters is a little bit more challenging − as well as more interesting, since the coefficients of bubble-free interest-rate rules are tied to the structural parameters by equality constraints, rather than by inequality constraints as for the coefficients of interest-rate rules ensuring only local equilibrium determinacy, as exemplified by the well-known Taylor principle or by Rotemberg and Woodford's (1999) "superinertia principle" (generalized by Woodford, 2003, chap. 8). Given our focus on equilibrium determinacy and controllability, we model the policy-maker's imperfect information on the values of the structural parameters in a structured way, in the form of misspecified dynamics, and not in the form of a non-parametric set of additive model perturbations *à la* Hansen and Sargent[15]. Let us first make the following (admittedly *ad hoc* but in our view intuitive) assumption:

Assumption 2.4: *if the policy-maker follows a μ-bubble-free rule, then as $\mu \longrightarrow +\infty$ the probability that the private agents coordinate on a divergent path tends towards zero.*

We are then ready to state the following proposition:

Proposition 2.6: *if the policy-maker wants to follow a rule of type (9) implementing a given targeted stationary equilibrium but measures all the parameters of the structural equations with some exogenous additive measurement errors, each of them randomly drawn from a continuous*

[15] Onatski and Williams (2003) show indeed that policy feedback rules robust *à la* Hansen and Sargent can for instance lead to unstable dynamics.

*probability distribution supported on a bounded interval including zero, and accordingly follows the corresponding rule of type (9) based on these measured parameters, then, as the length of all the distribution-supporting intervals tends towards zero (*i.e. *as all the measured parameters converge towards the true parameters): (i) the system made of (7) and this rule admits one unique local solution with probability one; (ii) the probability that the private agents coordinate on this solution tends towards one; (iii) this solution converges towards the targeted equilibrium.*

Proof: *cf* appendix H. ■ In other words, if the policy-maker's knowledge of the structural parameters is sufficiently accurate, then rules of type (9) based on the measured parameters are 1-bubble-free. Moreover, as the policy-maker's knowledge of the structural parameters becomes perfect, rules of type (9) based on the measured parameters are μ-bubble-free with $\mu \longrightarrow +\infty$, that is to say that they make diverging paths very steeply sloping and hence (from assumption 2.4) type-B equilibria very unlikely. As made clear in appendix H, these rules manage to be μ-bubble-free with $\mu \longrightarrow +\infty$ while keeping their coefficients finite by using the structural equations as a lever on the private agents' expectations. Finally, the unique equilibrium implemented with probability one gets arbitrarily close to the intended equilibrium as the size of the measurement errors on the parameters of the structural equations tends towards zero.

Hence, both propositions 2.1 and 2.2 hold asymptotically with probability one, at the limit of a convergence process which could take place if the policy-maker gradually learned the true values of the structural parameters and accordingly adjusted its policy feedback rule. Thus, the equality constraints tying the coefficients of rules (9) to the structural parameters (making the policy-maker manoeuvre on a Wicksellian-type razor's edge) prove not as restrictive as they may seem at first sight.

Note finally that bubble-free interest-rate rules may also be useful when the central bank has imperfect knowledge of the values of the structural parameters and for one reason or another only local equilibria matter, *i.e.* non-local equilibria can be safely disregarded. Indeed, in such cases the robustness-concerned central bank may want to adopt an interest-rate rule ensuring local equilibrium determinacy for all admissible parameter values. Conventional interest-rate rules can then fit the bill typically only if some of their coefficients are large enough (in absolute value), that is to say only if they are sufficiently aggressive out-of-equilibrium, which may bring about some undesirable side effects listed in the next subsection. Bubble-free interest-rate rules seem better placed to ensure local equilibrium determinacy for all admissible parameter values without such out-of-equilibrium aggressiveness[16].

[16] This claim is drawn from proposition 2.6. We vainly attempted to illustrate this point in a simple way with contemporaneous Taylor rules using Giannoni's (2002) robustness concept and calibration of the New Keynesian model under uncertainty.

3.2 Policy-maker's inability to commit

If the central bank cannot credibly commit to following a given interest-rate rule, then, in addition to the type-A and -B equilibria described in subsection 1.1, a third kind of unintended equilibria may arise, which the existing literature has so far ignored:

· **type-C equilibria:** non-targeted local equilibria, or non-local equilibria originating locally, which may exist only if the locally linearized system admits at least one unstable eigenvalue. Indeed, the interest-rate rule might well change (possibly before the linear approximation of the structural equations becomes invalid) along a divergent path because the stability-concerned central bank would find it both possible and desirable. More precisely, if a divergent path starts to develop in the neighbourhood of the targeted steady state, then the central bank will sooner or later change its interest-rate rule in order to keep the variables within this neighbourhood or to bring them back into this neighbourhood. Though initially divergent, the resulting path − given the triggered interest-rate-rule adjustment − remains bounded (and possibly even local), hence does not violate the transversality condition typically imposed by the original non-linear model with infinitely-lived utility-maximizing private agents and therefore qualifies as an equilibrium of this model. This "stabilization of last resort" raises a moral hazard problem, as private agents, rightly expecting this reaction from the central bank, can settle on an initially diverging path even though this path would not be an equilibrium if the central bank were compelled to stick to its interest-rate rule − in other words, type-C equilibria can exist even when type-B equilibria do not[17]. Such "boom-and-bust equilibria" may be of practical importance as they could *prima facie* contribute to explain why most post-war U.S. recessions have been due, according to a widespread point of view[18], to a monetary policy tightening putting an end to a period of increasing inflation rate.

These type-C equilibria could be illustrated in a simple way within our general framework. Indeed, suppose for instance that $m_1^a > m_1^b$ and that the central bank's interest-rate-rule choice is arbitrarily limited to the set of backward-looking interest-rate rules. If the central bank initially chooses a backward-looking interest-rate rule ensuring local equilibrium determinacy (along the lines of proposition 2.4), then the economy can embark on a divergent path (as implied by pro-

[17] A parallel could be drawn between this escape-clause approach to interest-rate rules and the escape-clause approach to fixed exchange rate systems (*i.e.* the second-generation models of currency crises). Another related literature, led by Davig and Leeper (2005), examines the consequences of exogenous (as opposed to endogenous) changes in the interest-rate rule on equilibrium determinacy.

[18] This point of view is related by Bernanke (2003) in the following way: "The Fed understood in principle that stabilizing inflation and inflation expectations was important, but − knowing that a slowdown in spending and output (of a magnitude difficult to guess) would be an unwelcome side effect − it was extremely reluctant to tighten monetary policy enough to do the job. The resulting compromise has been appropriately described as 'go-stop' policy. First, over-expansion led to inflation, the 'go' phase. Then, periodically, when inflation became bad enough, the Fed would tighten policy (the 'stop' phase), only to loosen again when the resulting slowdown in the economy began to manifest itself. The net result of this policy pattern was to exacerbate greatly the instability of both inflation and unemployment, while making little progress towards restoring price stability or re-anchoring inflation expectations."

position 2.3). And if the economy does embark on a diverging path, then the central bank may well decide to switch to a backward-looking interest-rate rule precluding divergent paths, at the expense of the possible occurrence of type-A equilibria, in order to keep the variables within the neighbourhood of the targeted steady state (the existence of such rules[19] following from appendix F amended by a choice of $\mathcal{Z}(X)$ whose roots are all of modulus strictly lower than one).

Admittedly, bubble-free interest-rate rules do not enable a central bank which cannot credibly commit to following them to eliminate type-A, -B and -C equilibria. We however argue that bubble-free interest-rate rules have two properties which might make them more credible and hence more effective than conventional interest-rate rules under the no-commitment assumption. First, bubble-free interest-rate rules are fast-acting in-equilibrium in the sense that they are effective provided that at each date the private agents expect them to be followed at the current and the next $\max\left[m_1^a, m_1^b\right] + \sum_{i=2}^N m_i^a$ dates, as indicated in proposition 2.1. By contrast, if $\exists i \in \{1, ..., N\}$, $m_i^a > 0$ then the effectiveness of the rules considered in the existing literature (*i.e.* their ability to select uniquely the targeted equilibrium or in other words to eliminate type-A, -B and -C equilibria) typically rests on the existence of a limit condition and on the assumption that the private agents believe them to be followed permanently, which is a more demanding condition.

Second, bubble-free interest-rate rules are non-aggressive out-of-equilibrium in the sense that by using the structural equations as a lever on the private agents' expectations (as made clear in the previous subsection), these rules manage to be $+\infty$-bubble-free with finite coefficients, thus limiting the out-of-equilibrium sensitivity of the policy instrument to the endogenous variables and in particular to the private agents' expectations. By contast, if $\exists i \in \{1, ..., N\}$, $m_i^a > 0$ then some coefficients of the rules typically considered in the existing literature tend towards infinity when these rules are required to be μ-bubble-free with $\mu \longrightarrow +\infty$ (*i.e.* to push the modulus of the unstable eigenvalues of the dynamic system towards infinity) in order to reduce to zero the probability that the private agents coordinate on a divergent path[20], thus undermining the credibility of these rules by making them costly to follow out-of-equilibrium. The out-of-equilibrium non-aggressive nature of bubble-free interest-rate rules may also enhance their credibility for three additional reasons: first, because a non-agressive rule avoids magnifying the effect of real-time data measurement errors (whose size can be particularly large for the output gap) on the economy; second, because a non-aggressive rule leaves more scope to act before the zero lower bound is reached by saving some interest-rate ammunition; third, because a non-aggressive rule avoids endangering financial stability[21].

[19] These rules correspond to Currie and Levine's (1993, chap. 4) "overstable feedback rules".

[20] This is formally shown by proposition 2.3 for the class of backward-looking rules when $m_1^a > m_1^b$.

[21] These three reasons are mentioned by Bernanke (2004): "the noisier the economic data, the less aggressive policymakers should be (...). Less variable short-term rates reduce the risk that the policy rate will hit the zero lower bound on interest rates; they may also reduce stress in the financial system".

3.3 Private agents' non-rational expectations

One way to relax the rational-expectations assumption is to suppose instead that the private agents form myopic rational expectations, *i.e.* rational expectations up to a given finite horizon. Interestingly, this alternative assumption may make the economy more bubble-prone[22], at least under conventional interest-rate rules. But bubble-free interest-rate rules could then well remain effective in eliminating both type-A and -B equilibria, given their in-equilibrium fast-acting nature (pointed out in the previous subsection).

Another way to relax the rational-expectations assumption is to suppose instead that the private agents form rational expectations only when the central bank follows a simple interest-rate rule. This alternative assumption would make bubble-free interest-rate rules ineffective, because too complex, in case of a large number of forward-looking structural equations. In this case, we would advocate simple μ-bubble-free rules. More precisely, one rule with desirable operating properties would then be the (or one of the several) μ-bubble-free rule(s) with the highest μ in the set of simple rules consistent with the targeted equilibrium, if this set is not empty. Compared to the rules typically advocated by the existing literature, this rule would have the advantage of reducing (even though not completely eliminating) the possibility of type-B equilibria.

More generally, the central bank would then face a trade-off between choosing a simple rule and choosing a μ-bubble-free rule with a high μ, in addition to the trade-off usually considered in the existing literature between choosing a simple rule and choosing a rule consistent with the targeted equilibrium. The contribution of this paper to such cases is to show that a μ-bubble-free rule with an arbitrarily large μ can be obtained as the simplicity requirement is gradually loosened. Of course, a similar trade-off applies when the central bank has an imperfect knowledge of the values of the structural parameters or considers several competing structurally different models of the economy[23], except that for sufficiently high μs no rule − however complex − can then be found which would be μ-bubble-free for all admissible parameter values or for all admissible models.

Naturally, the case for simple μ-bubble-free rules rests on the implicit (and in our view rather intuitive) assumption that private agents are less likely to coordinate on diverging rational-expectations equilibrium paths along which endogenous variables grow at a rate higher than μ, than on diverging rational-expectations equilibrium paths along which endogenous variables grow at a rate lower than μ. The relevance of this assumption could be examined through the lens of the adaptive learning literature. Type-A equilibria have been shown to be learnable by Honkapohja and Mitra (2004)

[22] This point is made *e.g.* by Tirole (1982). In Froot and Obstfeld's (1991, pp. 1193-1994) words: "the theoretical conditions required to rule out rational bubbles assume substantial, perhaps unrealistic, infinite-horizon foresight on the part of economic agents".

[23] Among the many papers considering several competing structurally different models of the economy, Levin, Wieland and Williams' (2003) can be mentioned for its special attention to type-A equilibria.

and Evans and McGough (2005a, 2005b). Type-B equilibria (in the form of paths converging towards the liquidity trap or a cycle) have been shown to be learnable by Bullard and Cho (2005), Evans and Honkapohja (2005) and Eusepi (2005). To our knowledge however, whether steeply sloping divergent paths are less easily learnable than gently sloping ones remains to be seen.

Note finally that μ-bubble-free rules might also be useful when the private agents use "judgement" in their adaptive learning process. In this case indeed, as shown by Bullard, Evans and Honkapohja (2005), non-targeted "exuberance equilibria" can exist under commonly considered interest-rate rules when the system satisfies Blanchard and Kahn's (1980) conditions but admits one or several unstable eigenvalues of modulus relatively close to one. Bullard, Evans and Honkapohja (2005) show that these equilibria can be eliminated by aggressive enough interest-rate rules. This aggressiveness can however have undesirable side effects, as argued in the previous subsection. Given that they move all the system's unstable eigenvalues away from one, μ-bubble-free interest-rate rules might then manage to eliminate these exuberance equilibria without any aggressiveness.

4 Bubble-free interest-rate rules for asset-price stabilization

This section discusses the possible applications of bubble-free interest-rate rules to asset-price stabilization by central banks. To that aim, let us consider the typical asset-pricing equation, written

$$\theta E_t \{p_{t+1}\} - p_t = i_t \tag{11}$$

in its simplest linearized form, where p_t is the asset price, i_t the short-term nominal interest rate and θ a real-number parameter such that $0 < \theta \leq 1$. If i_t is set exogenously, then equation (11) leaves the door open to a large variety of rational mean-explosive bubbles of the type identified by Blanchard (1979), Blanchard and Watson (1982), Evans (1991) and Froot and Obstfeld (1991). These bubbles are likely to be socially undesirable as their formation entails a non-optimal allocation of ressources and their bursting may endanger financial stability. This provides a rationale for policy action to reduce as much as possible, and ideally completely eliminate, their occurrence.

The conventional view, expressed *e.g.* by Bernanke (2002), is that monetary policy should not react to a perceived asset-price bubble for two reasons: first, because to identify correctly an asset-price bubble is difficult, and second, because monetary policy cannot stop an asset-price bubble without causing great damage on the economy. In essence, the second argument is that there is no proportional link between interest rates and asset prices and in particular that asset-price bubbles hardly respond to a modest interest-rate rise and can only be stopped brutally by a interest-rate rise sharp enough to have a large negative impact on the economy[24]. This argument

[24] In Bernanke's (2002) words: "an underlying premise of the lean-against-the-bubble strategists [...] is that the

can be interpreted very simply in the light of equation (11) in the following way. Suppose that an asset-price bubble is in progress and consider an unexpected temporary exogenous rise in the short-term nominal interest-rate i_t. This rise will decrease p_t as intended if it leaves $E_t\{p_{t+1}\}$ unchanged or if it decreases $E_t\{p_{t+1}\}$. But it may also leave p_t unchanged or even increase p_t if it increases $E_t\{p_{t+1}\}$. The elasticity of p_t to i_t thus depends on that of $E_t\{p_{t+1}\}$. A similar reasoning leads to the same conclusion when the exogenous rise in i_t is expected and/or permanent. Now consider an endogenous rise in the short-term nominal interest-rate i_t of the form $i_t = kp_t$ with $k > 0$. Whether temporary or permanent, this rule makes equation (11) at date t become $\theta E_t\{p_{t+1}\} - (1+k)\,p_t = 0$ so that the bubble may then keep on going faster than ever[25]. If (as seems intuitive) the probability that the private agents coordinate on a bubble path depends negatively on the bubble's expected growth rate, then the larger k the more likely the bursting of the bubble. All these considerations are consistent with the argument presented above.

Now consider the following interest-rate rule:

$$i_t = -p_t^* + \theta E_t\{p_{t+1}\} \tag{12}$$

where the exogenous targeted sequence $\{p_t^*\}_{t\in\mathbb{Z}}$ is known to the private agents. This rule eliminates rational bubbles by explicitly tying i_t to $E_t\{p_{t+1}\}$ in a way which mimics their relationship in (11) so as to insulate p_t from $E_t\{p_{t+1}\}$ and ensure $p_t = p_t^*$ for $t \in \mathbb{Z}$. In other words, by disconnecting the elasticity of p_t to i_t from that of $E_t\{p_{t+1}\}$, this rule establishes a fixed proportional link between i_t and p_t.

In such a context, these rules are primarily meant to be curative in the first place, that is to say that they aim at deflating an existing bubble[26]. Moreover, they theoretically enable the

response of incipient bubbles to monetary policy is more or less proportional to the policy action. In other words, [...] a small increase in the federal funds rate must lead to some correspondingly modest decline in the likelihood or size of a bubble. But such a smooth response is not well supported by either theoretical or empirical research on asset price dynamics. [Footnote: Alan Blinder has likened bubble-popping strategies to sticking a needle in a balloon; one cannot count on letting out the air slowly or in a finely calibrated amount.] If a bubble [...] is actually in progress, then investors are presumably expecting outsized returns: 10, 15, 20 percent or more annually. Is it plausible that an increase of $\frac{1}{2}$ percentage point in short-term interest rates, unaccompanied by any significant slowdown in the broader economy, will induce speculators to think twice about their equity investments? All we can conclude with much confidence is that the rate hike will tend to weaken the macroeconomic fundamentals through the usual channels, while the asset bubble, if there is one, may well proceed unchecked. Although neither I nor anyone else knows for sure, my suspicion is that bubbles can normally be arrested only by an increase in interest rates sharp enough to materially slow the whole economy. In short, we cannot practice "safe popping," at least not with the blunt tool of monetary policy." In Greenspan's (2002) words: "the key policy question is: if low-cost, incremental policy tightening appears incapable of deflating bubbles, do other options exist that can at least effectively limit the size of bubbles without doing substantial damage in the process? To date, we have not been able to identify such policies, though perhaps we or others may do so in the future".

[25] This point is similar to that made by Benhabib, Schmitt-Grohé and Uribe (2002b) about the role of monetary policy in equilibrium paths leading to the liquidity trap: "[a]long such equilibrium paths, the central bank, following the prescription of the Taylor rule, continuously eases in an attempt to reverse the persistent decline in inflation. But these efforts are in vain, and indeed counterproductive, for they introduce further downward pressure on inflation" (p. 546).

[26] The rigorous way to consider blooming mean-explosive bubbles (instead of nascent mean-explosive bubbles as in the previous sections) would be to use the original non-linear asset-pricing equation, instead of its linearized version (11) to which we stick for simplicity, but the message would basically remain the same.

central bank to control how gradually to deflate the bubble by choosing the sequence $\{p_t^*\}_{t\in\mathbb{Z}}$ accordingly. In so doing they avoid a further growth and a sharper decline of the bubble, while having the advantage over hypothetical directly preventive solutions to be used in circumstances where bubbles are more easily identifiable. However, if private agents expect the central bank to resort to such a curative solution, then they may be deterred from coordinating on a bubble in the first place and the solution may therefore in effect be preventive as well. Indeed, rational bubbles cannot occur when $\theta < 1$ if the monetary policy intervention ensures that the asset price never goes beyond a given threshold value.

Of course, we are not saying that bubbles can be surgically removed in a routine operation: bubble-free interest-rate rules should not be taken literally. In particular, the effectiveness of rule (12) depends on the relevance of equation (11) which can be derived only under unrealistic symplifying assumptions. More generally, bubble-free interest-rate rules are meant to be effective only if the central bank has correctly detected the presence of a rational bubble in real time and knows the structural equation describing the behaviour of the private agents accurately enough. Given that these two conditions are likely not to be met in practice, resorting to such rules would almost surely trigger unintended volatile market reactions. We nonetheless think that these rules can serve as a useful guide in the reflections on the best monetary policy reaction to perceived asset-price bubbles.

Such considerations could be illustrated with two particular asset prices, namely the stock price and the exchange rate. Indeed, the simplest structural equation governing stock-price dynamics under the assumption of constant exogenous dividends can be written after Campbell and Shiller (1988) in the linearized form (11) with $\theta < 1$, where p_t denotes the logarithm of the ratio price over dividend (up to an additive constant) at date t. In this context, the central bank could infer $E_t\{p_{t+1}\}$ from the futures' price. Note interestingly that a large part of the existing literature, led by Bernanke and Gertler (1999, 2001), models stock-price bubbles as exogenous stochastic processes on which monetary policy has therefore no impact[27]. By contrast, the rational-bubbles framework which we consider (where stock-price bubbles correspond to non-fundamental solutions of the dynamic stock-price equation under rational expectations) leaves the door open to a monetary policy effect on these bubbles − without resorting to an *ad hoc* direct channel between the monetary policy instrument on the one hand and the growth rate and/or the probability of bursting of stock-price bubbles on the other hand, as does another branch of the literature.

Similarly, the simplest structural equation governing exchange-rate dynamics under the assumptions of perfect asset substitutability, capital mobility and risk neutrality, namely the uncovered

[27] In Bernanke and Gertler's (2001, p. 257) words, "[a] deficiency of the literature to date is that the nonfundamental component of stock prices has generally been treated as exogenous."

interest-rate parity (UIP), can be written in the linearized form (11) where p_t then denotes the logarithm of the nominal exchange rate and i_t the difference between the domestic and the foreign short-term nominal interest rates at date t. As reviewed by Sarno and Taylor (2002, chap. 2) for instance, the UIP is usually rejected by the data (the so-called forward bias puzzle) but there is some empirical evidence that the covered interest-rate parity (which replaces $E_t\{p_{t+1}\}$ by the logarithm of the one-period forward rate, *i.e.* the rate agreed at date t for an exchange of currencies at date $t+1$) holds. Hence the UIP's lack of empirical validity needs not undermine the effectiveness of rule (12) provided that the central bank (quite naturally) uses the one-period forward rate as a proxy for $E_t\{p_{t+1}\}$.

Note that in a fully-fledged dynamic rational-expectations general-equilibrium model, the current nominal exchange rate would typically be uniquely pinned down by a long-term condition such as the purchasing power parity, so that no bubble could emerge. As previously discussed, bubbles can however emerge under the alternative assumption of myopic rational expectations which may arguably be more relevant and in particular better reflect the popular notion of market "short-termism". It may even be the case that by clearly presenting them with an arbitrage, the mere announcement of rule (12) by the central bank may induce the private agents who do not usually form rational expectations at all (however myopic), like the noise traders or positive feedback traders considered in the corresponding literature, to form rational expectations.

Bubble-free interest-rate rule (12) could be used under a flexible exchange rate regime to bring back the nominal exchange rate from a bubble value to a value more in line with the fundamentals. This rule would then theoretically enable the central bank to move the nominal exchange rate smoothly along a gradual sequence $\{p_t^*\}_{t\in\mathbb{Z}}$ so as to avoid *e.g.* the sudden collapse of an initially overvalued currency. Bubble-free interest-rate rule (12) could also be used preventively to maintain a (sustainable, *i.e.* fundamentals-consistent) exchange-rate peg. To our knowledge, the only interest-rate rules for an exchange-rate peg to be found in the literature are proposed by Benigno, Benigno and Ghironi (2000) and Benigno and Benigno (2004) in the linearized form $i_t = k(p_t - p^*)$ with $k > 0$, where p^* is the targeted (constant) nominal exchange rate, possibly together with a fractional backing mechanism *à la* Obstfeld and Rogoff (1983, 1986) should the nominal exchange rate embark on a divergent path. The effectiveness of these rules depends on the assumption that the private agents expect them to be followed until the implementation of the fractional backing mechanism, that is to say during a typically long period if k is small. But alternatively if k is large, then these rules are aggressive out-of-equilibrium. By contrast, rule (12) has the advantage of escaping this trade-off between being fast-acting in-equilibrium and being non-aggressive out-of-equilibrium.

Conclusion

This paper aims at giving a new insight into the design of interest-rate rules. The literature has so far focused on interest-rate rules − typically rules satisfying the Taylor principle − precluding unintended fluctuations around the targeted steady state ("type-A equilibria"). As first acknowledged by Benhabib, Schmitt-Grohé and Uribe (2001a, 2001b, 2002a, 2002b, 2003), such rules however do not prevent the economy from embarking on a path gradually leaving the neighbourhood of the targeted steady state and leading for instance to the liquidity trap ("type-B equilibria"). An interest-rate-rule change along such a downwards or upwards path can preclude these undesirable outcomes but may raise a moral hazard problem leading for instance to booms and busts ("type-C equilibria").

We argue that under such rules local equilibrium determinacy and global equilibrium indeterminacy are the two sides of the same coin. Instead, we propose bubble-free interest-rate rules which both preclude unintended fluctuations in the neighbourhood of the targeted steady state and prevent the economy from gradually leaving this neighbourhood. These rules do so by removing all divergent paths from the locally linearized model (if the central bank has perfect knowledge of the values of the structural parameters) or making these divergent paths more abrupt and hence arguably less likely to be followed by the non-coordinated private agents (if the central bank has imperfect knowledge of the values of the structural parameters).

We design these bubble-free interest-rate rules for a broad class of rational-expectations dynamic stochastic infinite-horizon linear models, which encompasses the locally linearized reduced form of many existing rational-expectations dynamic stochastic general equilibrium models. We show that these rules can implement any given VARMA solution of the model's locally linearized structural equations. We take part in the forward-looking *vs.* backward-looking interest-rate rules for equilibrium determinacy debate by showing that in most models bubble-free interest-rate rules are necessarily forward-looking, while in all models there exists a backward-looking rule ensuring local equilibrium determinacy (whatever this equilibrium).

We also argue that given their in-equilibrium fast-acting and out-of-equilibrium non-aggressive nature (as they use the structural equations as a lever on the private agents' expectations), these rules: i) are likely to have better stabilization properties than conventional rules when for robustness concerns local equilibrium determinacy is required for all admissible values of the structural parameters; ii) are likely to be more credible and hence more effective than conventional rules in the absence of a commitment technology; iii) are still effective when the private agents form myopic rational expectations, while on the contrary conventional rules are then more problematic. We finally put forward these rules as a useful guide in the reflections on the best monetary policy reaction to perceived asset-price bubbles or exchange-rate misalignments.

This work could be extended in many interesting ways. In particular, a proper treatment of the credibility of interest-rate rules, showing how the in-equilibrium fast-acting nature and out-of-equilibrium non-agressive nature of a given rule enhance its credibility, would be welcome. Similarly, a proper treatment of the learnability of divergent paths, showing that gently sloping divergent paths are more easily learnable than steeply sloping ones, would opportunely strengthen the case for μ-bubble-free rules. Finally, it would be worth examining whether the results obtained can be further generalized to models which account for monetary policy transmission delays by introducing not past interest rates − as we do to some extent − but the private agents' past expectations of current interest rates in the structural equations, such as Rotemberg and Woodford's (1999) and Giannoni and Woodford's (2003)[28].

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[28] Preliminary investigation suggests that our results could indeed be generalized to such models: for instance, interest-rate rules of type (9) where m_j^a is replaced by $m_j^a + d$ for $2 \leq j \leq N$ (d denoting the monetary policy transmission delay) can easily be shown to be bubble-free in Giannoni and Woodford's (2003) model.

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Appendix

For any system of equations (S), let $L(S)$ and $E_t\{(S)\}$ denote the systems obtained by applying respectively operators L and E_t on both the left-hand side and the right-hand side of each equation of (S). Similarly, for any N-equation system (S), let $\mathbf{e}_i'(S)$ denote the i^{th} equation of (S).

A Proof of proposition 1.2

Let us first determine E^d. Under discretion, at each date t the central bank chooses i_t so as to minimize (3) subject to (1) and (2). Because of the purely forward-looking nature of the model, today's choice of i_t does not depend on yesterday's choices of i_{t-k} for $k \geq 1$. Similarly, tomorrow's choices of i_{t+k} for $k \geq 1$ will not depend on today's choice of i_t. This implies that neither tomorrow's other variables π_{t+k} and x_{t+k} for $k \geq 1$ will depend on today's choice of i_t, because of the purely forward-looking nature of the model again, hence neither will today's private agents' rational expectations $E_t\{\pi_{t+k}\}$, $E_t\{x_{t+k}\}$ and $E_t\{i_{t+k}\}$ for $k \geq 1$, so that the central bank considers these expectations as given when choosing i_t at date t. As a consequence, the central bank chooses i_t so as to minimize $(\pi_t)^2 + \lambda_x (x_t - x^*)^2 + \lambda_i (i_t - i^*)^2 + k_L$ subject to $\pi_t = \kappa x_t + k_\pi$ and $x_t = -\sigma i_t + k_x$, considering k_L, k_π and k_x as given. The first-order condition of this minimization problem is

$$i_t = i^* - \frac{\lambda_x \sigma}{\lambda_i} x^* + \frac{\kappa \sigma}{\lambda_i} \pi_t + \frac{\lambda_x \sigma}{\lambda_i} x_t. \tag{13}$$

Using (13) to replace i_t in (2), and then (1) and the expectation at date t of (1) taken at date $t+1$ to replace x_t and $E_t\{x_{t+1}\}$ in the resulting equation, we get

$$\begin{aligned}\beta \lambda_i E_t\{\pi_{t+2}\} - \left(\lambda_i + \beta\lambda_i + \kappa\lambda_i\sigma + \beta\lambda_x\sigma^2\right) E_t\{\pi_{t+1}\} + \left(\lambda_i + \lambda_x\sigma^2 + \kappa^2\sigma^2\right)\pi_t \\ = \kappa\sigma\left(\lambda_x \sigma x^* - \lambda_i i^*\right) + \kappa\lambda_i\sigma r_t^n + \left[\lambda_i\left(1-\rho_u\right) + \lambda_x\sigma^2\right] u_t.\end{aligned} \tag{14}$$

The expectation at date t of (14) taken at dates $t+k$ for $k \geq 0$ leads to a recurrence equation. Given assumption 1.1, this recurrence equation has a unique local solution:

$$\begin{aligned} E_t\{\pi_{t+k}\} &= \frac{\kappa\sigma\left(\lambda_x\sigma x^* - \lambda_i i^*\right)}{\mathcal{P}(1)} + \frac{\kappa\lambda_i\sigma}{\mathcal{P}(\rho_r)}\rho_r^k r_t^n + \frac{\lambda_i(1-\rho_u)+\lambda_x\sigma^2}{\mathcal{P}(\rho_u)}\rho_u^k u_t \text{ for } k \geq 0 \\ \text{and in particular } \pi_t &= \frac{\kappa\sigma\left(\lambda_x\sigma x^* - \lambda_i i^*\right)}{\mathcal{P}(1)} + \frac{\kappa\lambda_i\sigma}{\mathcal{P}(\rho_r)} r_t^n + \frac{\lambda_i(1-\rho_u)+\lambda_x\sigma^2}{\mathcal{P}(\rho_u)} u_t \end{aligned} \tag{15}$$

with $\mathcal{P}(1) \neq 0$, $\mathcal{P}(\rho_r) \neq 0$ and $\mathcal{P}(\rho_u) \neq 0$ because of assumption 1.1. Equations (1) and (13) then lead to

$$x_t = \frac{\sigma(1-\beta)\left(\lambda_x\sigma x^* - \lambda_i i^*\right)}{\mathcal{P}(1)} + \frac{\lambda_i\sigma(1-\beta\rho_r)}{\mathcal{P}(\rho_r)} r_t^n + \frac{\sigma(\lambda_i\rho_u - \kappa\sigma)}{\mathcal{P}(\rho_u)} u_t \tag{16}$$

$$\text{and } i_t = \frac{\kappa\sigma\left(\lambda_x\sigma x^* - \lambda_i i^*\right)}{\mathcal{P}(1)} + \frac{\sigma^2\left[\kappa^2 + \lambda_x(1-\beta\rho_r)\right]}{\mathcal{P}(\rho_r)} r_t^n + \frac{\sigma\left[\kappa + (\lambda_x\sigma - \kappa)\rho_u\right]}{\mathcal{P}(\rho_u)} u_t. \tag{17}$$

Considered as a rule, (13) is of type (4) and consistent by construction with the implementation of E^d characterized by (15), (16) and (17) for $t \in \mathbb{Z}$. Now let us define

$$\Delta^d \equiv \begin{bmatrix} 1 & 0 & 0 \\ \frac{\kappa\sigma(\lambda_x\sigma x^* - \lambda_i i^*)}{\mathcal{P}(1)} & \frac{\kappa\lambda_i\sigma}{\mathcal{P}(\rho_r)} & \frac{\lambda_i(1-\rho_u)+\lambda_x\sigma^2}{\mathcal{P}(\rho_u)} \\ \frac{\sigma(1-\beta)(\lambda_x\sigma x^* - \lambda_i i^*)}{\mathcal{P}(1)} & \frac{\lambda_i\sigma(1-\beta\rho_r)}{\mathcal{P}(\rho_r)} & \frac{\sigma(\lambda_i\rho_u - \kappa\sigma)}{\mathcal{P}(\rho_u)} \end{bmatrix}$$

so that $\begin{bmatrix} 1 & \pi_t & x_t \end{bmatrix}' = \Delta^d \begin{bmatrix} 1 & r_t^n & u_t \end{bmatrix}'$ at E^d. Computations lead to

$$\left|\Delta^d\right| = \frac{-\lambda_i\sigma}{\mathcal{P}(\rho_r)\mathcal{P}(\rho_u)}\left[\mathcal{P}(1) + \beta\lambda_x\sigma^2(1-\rho_r) + \kappa\lambda_i\sigma(1-\rho_u) + \lambda_i(1-\beta\rho_r)(1-\rho_u)\right].$$

Assumption 1.1 and $\mathcal{P}(0) > 0$ together imply that $\mathcal{P}(1) > 0$, so that we obtain $\left|\Delta^d\right| \neq 0$. Then, $\left|\Delta^d\right| \neq 0$ and the fact that 1, r_t^n and u_t form a base together imply that 1, π_t and x_t form also a base at E^d. As a consequence, any rule of type (4) other than (13) is not consistent with the implementation of E^d.

Let us then determine E^c. Given that 1, r_t^n and u_t form a base, equations (1), (2) and (6) enable us to express a_x, b_x, c_x, a_i, b_i and c_i as functions of a_π, b_π and c_π only:

$$\begin{cases} a_x = \frac{1-\beta}{\kappa} a_\pi \\ b_x = \frac{1-\beta\rho_r}{\kappa} b_\pi \\ c_x = \frac{1-\beta\rho_u}{\kappa} c_\pi - \frac{1}{\kappa} \end{cases} \quad \text{and} \quad \begin{cases} a_i = a_\pi \\ b_i = -\frac{\mathcal{A}(\rho_r)}{\kappa\sigma} b_\pi + 1 \\ c_i = -\frac{\mathcal{A}(\rho_u)}{\kappa\sigma} c_\pi + \frac{1-\rho_u}{\kappa\sigma} \end{cases} \tag{18}$$

where $\mathcal{A}(X) \equiv \beta X^2 - (1+\beta+\kappa\sigma) X + 1$. We then use (6) and (18) to express (3) as a function of a_π, b_π and c_π only. The minimization of this function determines a_π, b_π and c_π:

$$a_\pi = \kappa \frac{(1-\beta)\lambda_x x^* + \kappa\lambda_i i^*}{\kappa^2 + (1-\beta)^2 \lambda_x + \kappa^2 \lambda_i},\ b_\pi = \frac{\kappa\lambda_i \sigma \mathcal{A}(\rho_r)}{\mathcal{B}(\rho_r)} \text{ and } c_\pi = \frac{\lambda_x \sigma^2 (1-\beta\rho_u) + \lambda_i (1-\rho_u)\mathcal{A}(\rho_u)}{\mathcal{B}(\rho_u)}$$

where $\mathcal{B}(X) \equiv \lambda_i [\mathcal{A}(X)]^2 + \lambda_x \sigma^2 (1-\beta X)^2 + \kappa^2\sigma^2 \neq 0$ for all $X \in \mathbb{R}$, while a_x, b_x, c_x, a_i, b_i and c_i are residually determined by (18). Now let us define

$$\Delta^c \equiv \begin{bmatrix} 1 & 0 & 0 \\ a_\pi & b_\pi & c_\pi \\ a_x & b_x & c_x \end{bmatrix}$$

so that $\begin{bmatrix} 1 & \pi_t & x_t \end{bmatrix}' = \Delta^c \begin{bmatrix} 1 & r_t^n & u_t \end{bmatrix}'$ at E^c. Using (18) we get

$$|\Delta^c| = \frac{a_\pi}{\kappa} \left[\beta(\rho_r - \rho_u) b_\pi - 1\right].$$

Assumption 1.1 ensures $\mathcal{A}(\rho_r) \neq 0$ and therefore $a_\pi \neq 0$, so that $|\Delta^c| \neq 0$ if and only if $\beta(\rho_r - \rho_u) b_\pi - 1 \neq 0$. Computations lead then to

$$\left[\beta(\rho_r - \rho_u) b_\pi - 1\right] \mathcal{B}(\rho_u) = -\kappa^2\sigma^2 - \lambda_x \sigma^2 (1-\beta\rho_u)(1-\beta\rho_r) + \lambda_i \mathcal{A}(\rho_u)\,\mathcal{C}(\rho_u, \rho_r) \tag{19}$$

where $\mathcal{C}(X_1, X_2) \equiv \kappa\sigma X_1 - (1-X_1)(1-\beta X_2)$. If $\mathcal{A}(\rho_u)\,\mathcal{C}(\rho_u,\rho_r) \leq 0$ then $\left[\beta(\rho_r - \rho_u) b_\pi - 1\right]$ $\mathcal{B}(\rho_u) < 0$. Alternatively, if $\mathcal{A}(\rho_u)\,\mathcal{C}(\rho_u,\rho_r) > 0$ then two cases can be distinguished. In the case $\mathcal{A}(\rho_u) < 0$ and $\mathcal{C}(\rho_u,\rho_r) < 0$, we use $\mathcal{P}(\rho_u) > 0$ (as implied by assumption 1.1 and $\mathcal{P}(0) > 0$) to get

$$\left[\beta(\rho_r - \rho_u) b_\pi - 1\right] \mathcal{B}(\rho_u) < -\beta\kappa^2\sigma^2\rho_r + \lambda_i \mathcal{A}(\rho_u)\,\rho_u (1 - \beta\rho_r + \kappa\sigma) < 0$$

from (19). In the case $\mathcal{A}(\rho_u) > 0$ and $\mathcal{C}(\rho_u,\rho_r) > 0$, we use $\mathcal{P}(\rho_r) > 0$ (as implied by assumption 1.1 and $\mathcal{P}(0) > 0$) and $\mathcal{C}(\rho_u,\rho_r) > 0$ to get

$$\left[\beta(\rho_r - \rho_u) b_\pi - 1\right] \mathcal{B}(\rho_u) < -\beta\kappa^2\sigma^2\rho_u + \frac{\lambda_i \mathcal{D}(\rho_u, \rho_r)}{1-\rho_u}$$

from (19), where $\mathcal{D}(X_1, X_2) \equiv \kappa\sigma(1-\beta X_1)(X_1 - X_2) + (1-X_1)\mathcal{A}(X_1)\mathcal{C}(X_1, X_2)$. In this case, given that $\mathcal{D}(X_1, X_2)$ is linear in X_2, that $\rho_u < \rho_r < 1$ (as implied by $\mathcal{A}(\rho_u) > 0$ and $\mathcal{C}(\rho_u, \rho_r) > 0$), that $\mathcal{D}(\rho_u, \rho_u) < 0$ (since $\mathcal{C}(\rho_u, \rho_u) = -\mathcal{A}(\rho_u) < 0$) and that

$$\mathcal{D}(\rho_u, 1) = -(1-\rho_u)\left[\kappa^2\sigma^2\rho_u^2 + \kappa\sigma\left[(1-\rho_u)^2 + \rho_u^2(1-\beta\rho_u)\right] + (1-\beta)(1-\rho_u)^2(1-\beta\rho_u)\right] < 0,$$

we obtain $\mathcal{D}(\rho_u, \rho_r) < 0$ and hence $[\beta(\rho_r - \rho_u)b_\pi - 1]\mathcal{B}(\rho_u) < 0$. Thus, in all cases we have $[\beta(\rho_r - \rho_u)b_\pi - 1]\mathcal{B}(\rho_u) < 0$, which implies $\beta(\rho_r - \rho_u)b_\pi - 1 \neq 0$ and therefore $|\Delta^c| \neq 0$. Then, $|\Delta^c| \neq 0$ and the fact that 1, r_t^n and u_t form a base together imply that 1, π_t and x_t form also a base at E^c. As a consequence, any rule of type (4) other than

$$i_t = \begin{bmatrix} a_i & b_i & c_i \end{bmatrix} (\Delta^c)^{-1} \begin{bmatrix} 1 \\ \pi_t \\ x_t \end{bmatrix}$$

is not consistent with the implementation of E^c.

B On assumptions 2.1 and 2.2

Consider the following two assumptions, which state that the structural equations are non-redundant and that the policy instrument appears in at least one of the structural equations:

Assumption 2.1': *$\forall(\alpha_1, ..., \alpha_N) \in \mathbb{R}^N$, if there exist $(k_1, ..., k_N) \in \mathbb{N}^{*N}$ and $(\gamma_{i,j}, \delta_{i,j}) \in \mathbb{R}^* \times \mathbb{N}$ for $i \in \{1, ..., N\}$ and $j \in \{1, ..., k_i\}$ such that (i) $\forall i \in \{1, ..., N\}$, $\forall(j, j') \in \{1, ..., k_i\}^2$, $\delta_{i,j} = \delta_{i,j'} \Longrightarrow j = j'$, (ii) $\sum_{i=1}^N \alpha_i \mathbf{e}_i' \sum_{j=1}^{k_i} \gamma_{i,j} L^{-\delta_{i,j}} \mathbf{A}(L) = \mathbf{0}$, then $(\alpha_1, ..., \alpha_N) = (0, ..., 0)$.*

Assumption 2.2': *$I_B \neq \varnothing$.*

This appendix shows the following proposition:

Proposition 2.7: *any system of type (7) satisfying assumptions 2.1' and 2.2' implies that holds another system of type (7) satisfying assumptions 2.1 and 2.2.*

Proof: consider a system S of type (7) satisfying assumptions 2.1' and 2.2'. This system can be rewritten in the following six steps, where for simplicity we keep the same notations at each step.
Step 1: note that assumption 2.1' ensures that S satisfies assumption 2.1.i.
Step 2: if $\widehat{\mathbf{A}}(0)$ is invertible then S satisfies assumption 2.1.iii. Otherwise, if $\widehat{\mathbf{A}}(0)$ is not invertible then there exists $(\alpha_1, ..., \alpha_N) \in \mathbb{R}^N$ such that $\sum_{i=1}^N \alpha_i \mathbf{e}_i' \widehat{\mathbf{A}}(0) = 0$ and $(\alpha_1, ..., \alpha_N) \neq (0, ..., 0)$. Note $\widehat{I}$ the set of $i \in \{1, ..., N\}$ such that $\alpha_i \neq 0$ and consider

$$\widehat{\imath} \in \arg\max_{i \in \widehat{I}} (m_i^a).$$

Replace $\mathbf{e}'_{\widehat{i}}(S)$ by $\sum_{i=1}^{N}\alpha_i E_t\left\{L^{m_i^a-m_{\widehat{i}}^a}\mathbf{e}'_i(S)\right\}$. Compared to the previous system, the resulting system has the same n^a and m_i^a for $i\in\{1,...,N\}\setminus\left\{\widehat{i}\right\}$ and a strictly lower $m_{\widehat{i}}^a$ (whose existence is secured by assumption 2.1'). Repeat this (sub)step again and again as long as $\widehat{\mathbf{A}}(0)$ is not invertible. At each (sub)step there exists $i\in\{1,...,N\}$ such that m_i^a is strictly decreased. Given that $\forall i\in\{1,...,N\}$, $m_i^a\geq n^a$, this process must end at some point. Assumption 2.1' ensures that $\widehat{\mathbf{A}}(0)$ is invertible at the end of this process, that is to say that the final system satisfies assumption 2.1.iii.

Step 3: re-order the equations such that $1\in I_B$, so that assumption 2.2.i is satisfied, and $\forall i\in I_B\setminus\{1\}$, $m_i^a-m_i^b\geq m_1^a-m_1^b$.

Step 4: $\forall i\in I_B\setminus\{1\}$ such that $m_i^a-m_i^b=m_1^a-m_1^b$, replace $\mathbf{e}'_i(S)$ by

$$\mathbf{e}'_i\mathbf{B}_{-m_i^b}E_t\left\{L^{\min\left[0,m_1^b-m_i^b\right]}\mathbf{e}'_1(S)\right\}-\mathbf{e}'_1\mathbf{B}_{-m_1^b}E_t\left\{L^{\min\left[0,m_i^b-m_1^b\right]}\mathbf{e}'_i(S)\right\}. \tag{20}$$

Steps 2 and 3 ensure that the resulting system satisfies $\forall i\in I_B\setminus\{1\}$, $m_i^a-m_i^b>m_1^a-m_1^b$.

Step 5: if $m_1^a-m_1^b\geq 0$ then $\forall i\in I_B\setminus\{1\}$, $m_i^a-m_i^b>0$. Otherwise, if $m_1^a-m_1^b<0$ then $\forall i\in I_B\setminus\{1\}$ such that $m_i^a-m_i^b\leq 0$, replace $\mathbf{e}'_i(S)$ by (20). Given step 4, this operation lowers m_i^b without affecting m_i^a. Repeat this (sub)step again and again as long as $\exists i\in I_B\setminus\{1\}$ such that $m_i^a-m_i^b\leq 0$. The resulting system satisfies $\forall i\in I_B\setminus\{1\}$, $m_i^a-m_i^b>0$ and therefore, given step 4, assumption 2.2.iii.

Step 6: replace $\mathbf{e}'_1(S)$ by $E_t\left\{L^{\min\left[0,m_1^a,m_1^b\right]}\mathbf{e}'_1(S)\right\}$ and $\mathbf{e}'_i(S)$ by $E_t\left\{L^{\min[0,m_i^a]}\mathbf{e}'_i(S)\right\}$ for $i\in\{2,...,N\}$ so as to satisfy assumptions 2.1.ii and 2.2.ii.

By construction, the system obtained at the end of this six-step process is of type (7) and satisfies assumptions 2.1 and 2.2. ■

If the initial system and the final system are equivalent to each other, then propositions 2.1 to 2.6 still hold when assumptions 2.1 and 2.2 are replaced by assumptions 2.1' and 2.2', so that the consideration of assumptions 2.1 and 2.2 in the main text, instead of assumptions 2.1' and 2.2', is without any loss in generality.

Otherwise, since the initial system implies the final system, any solution of the former is also a solution of the latter. As they can select uniquely any solution of the final system (propositions 2.1 and 2.2), bubble-free rules can therefore also select uniquely any solution of the initial system, so that propositions 2.1 and 2.2 still hold when assumptions 2.1 and 2.2 are replaced by assumptions 2.1' and 2.2'. So do proposition 2.4, as can be easily shown by adjusting the number of roots of $\mathcal{Z}(X)$ whose modulus is higher than or equal to one in appendix F if the final system together with a backward-looking policy feedback rule has more non-predetermined variables than the initial system together with a backward-looking policy feedback rule, and proposition 2.5. However, propositions 2.3 and 2.6 then no longer hold.

C Proof of proposition 2.1

The substraction of $\mathbf{e}_1'(7)$ from $E_t\left\{L^{-m_1^b}(9)\right\}$ leads to equation $(\overrightarrow{1})$:

$$\sum_{i=2}^{N}\mathbf{e}_i' L^{\sum_{j=2}^{i} m_j^a}\left[\mathbf{A}(L)\,\mathbf{Y}_t+\mathbf{B}(L)\,z_t+\mathbf{C}(L)\,\boldsymbol{\xi}_t+L^{-m_i^a}\mathbf{O}(L)\,\mathbf{D}(L)\,\boldsymbol{\xi}_t\right] \\ +L^{\sum_{i=2}^{N} m_i^a}\left[\mathbf{P}(L)\,\mathbf{Y}_t+Q(L)\,z_t+\mathbf{R}(L)\,\mathbf{D}(L)\,\boldsymbol{\xi}_t\right]=0. \quad (\overrightarrow{1})$$

Similarly, $\forall k\in\{2,...,N\}$, equation $(\overrightarrow{k})$ can be derived from equation $(\overrightarrow{k-1})$ by substracting $\mathbf{e}_k'(7)$ from $E_t\left\{L^{-m_k^a}(\overrightarrow{k-1})\right\}$:

$$\sum_{i=k+1}^{N}\mathbf{e}_i' L^{\sum_{j=k+1}^{i} m_j^a}\left[\mathbf{A}(L)\,\mathbf{Y}_t+\mathbf{B}(L)\,z_t+\mathbf{C}(L)\,\boldsymbol{\xi}_t+L^{-m_i^a}\mathbf{O}(L)\,\mathbf{D}(L)\,\boldsymbol{\xi}_t\right] \\ +L^{\sum_{i=k+1}^{N} m_i^a}\left[\mathbf{P}(L)\,\mathbf{Y}_t+Q(L)\,z_t+\mathbf{R}(L)\,\mathbf{D}(L)\,\boldsymbol{\xi}_t\right]=0 \quad (\overrightarrow{k})$$

and in particular

$$\mathbf{P}(L)\,\mathbf{Y}_t+Q(L)\,z_t+\mathbf{R}(L)\,\mathbf{D}(L)\,\boldsymbol{\xi}_t=0. \quad (\overrightarrow{N})$$

$\forall k\in\{2,...,N\}$, the substraction of $L^{m_k^a}(\overrightarrow{k})$ from $(\overrightarrow{k-1})$ leads to equation $(\overleftarrow{k})$:

$$\mathbf{e}_k'\left[L^{m_k^a}\left[\mathbf{A}(L)\,\mathbf{Y}_t+\mathbf{B}(L)\,z_t+\mathbf{C}(L)\,\boldsymbol{\xi}_t\right]+\mathbf{O}(L)\,\mathbf{D}(L)\,\boldsymbol{\xi}_t\right]=0. \quad (\overleftarrow{k})$$

Equations $\overrightarrow{N}$ and $\overleftarrow{k}$ for $k\in\{2,...,N\}$ together can be re-written as follows:

$$\mathbf{U}(L)\,\mathbf{Y}_t+\mathbf{V}(L)\,z_t+\mathbf{W}(L)\,\boldsymbol{\xi}_t=\mathbf{0} \quad (21)$$

with $\underset{(N\times N)}{\mathbf{U}(L)}=\begin{bmatrix}\mathbf{P}(L)\\ \mathbf{e}_2'\widehat{\mathbf{A}}(L)\\ \vdots\\ \mathbf{e}_N'\widehat{\mathbf{A}}(L)\end{bmatrix}\equiv\sum_{k=0}^{n^u}\mathbf{U}_k L^k$, $\underset{(N\times 1)}{\mathbf{V}(L)}=\begin{bmatrix}Q(L)\\ \mathbf{e}_2' L^{m_2^a}\mathbf{B}(L)\\ \vdots\\ \mathbf{e}_N' L^{m_N^a}\mathbf{B}(L)\end{bmatrix}\equiv\sum_{k=\max\left[0,m_1^a-m_1^b+1\right]}^{n^v}\mathbf{V}_k L^k$

due to assumption 2.2.iii and $\underset{(N\times N)}{\mathbf{W}(L)}=\begin{bmatrix}\mathbf{R}(L)\,\mathbf{D}(L)\\ \mathbf{e}_2'\left[L^{m_2^a}\mathbf{C}(L)+\mathbf{O}(L)\,\mathbf{D}(L)\right]\\ \vdots\\ \mathbf{e}_N'\left[L^{m_N^a}\mathbf{C}(L)+\mathbf{O}(L)\,\mathbf{D}(L)\right]\end{bmatrix}\equiv\sum_{k=0}^{n^w}\mathbf{W}_k L^k$,

where $(n^u,n^v,n^w)\in\mathbb{N}^3$ and all $\mathbf{U}_k$, $\mathbf{V}_k$, $\mathbf{W}_k$ have real numbers as elements. Since $\mathbf{U}(0)=\boldsymbol{\Omega}$ is invertible, (21) can be used to express $\mathbf{Y}_t$ as a function of $\mathbf{Y}_{t-1-k}$, $z_{t-\max\left[0,m_1^a-m_1^b+1\right]-k}$ and $\boldsymbol{\xi}_{t-k}$ for $k\geq 0$ and $t\in\mathbb{Z}$. If $m_1^a\geq m_1^b$, this expression can be used sequentially to replace $E_t\left\{\mathbf{Y}_{t+j}\right\}$ for $j\in\left\{0,...,m_1^a-m_1^b\right\}$ in (9) and thus get z_t as a function of $\mathbf{Y}_{t-1-k}$, z_{t-1-k} and $\boldsymbol{\xi}_{t-k}$ for $k\geq 0$. Alternatively, if $m_1^a<m_1^b$ then (9) directly expresses z_t as a function of $\mathbf{Y}_{t-1-k}$, z_{t-1-k} and $\boldsymbol{\xi}_{t-k}$ for $k\geq 0$. In both cases, the system made of this equation for z_t and equation (21) for $\mathbf{Y}_t$ is therefore backward-looking and non-degenerate and hence makes $\mathbf{Y}_t$ and z_t predetermined.

D Proof of proposition 2.2

Suppose that (7) and (10) hold for $t \in \mathbb{Z}$. Then there exists $\mathbf{e}_1'\mathbf{O}(L)$ such that

$$E_t\left\{L^{m_1^b}\mathbf{e}_1'\left[\mathbf{A}(L)\mathbf{Y}_t+\mathbf{B}(L)z_t\right]\right\}+L^{m_1^b}\mathbf{e}_1'\mathbf{C}(L)\boldsymbol{\xi}_t+\mathbf{e}_1'\mathbf{O}(L)\mathbf{D}(L)\boldsymbol{\xi}_t=0$$

(result 1) and $\mathbf{e}_i'\mathbf{O}(L)$ for all $i \in \{2,...,N\}$ such that $\mathbf{e}_i'$ (21) holds (result 2). Moreover, if $m_1^a \leq m_1^b$ then $\max\left[0, m_1^a - m_1^b + 1\right] \in \{0,1\}$ and hence there exists a linear combination of the first N lines of (10) which can be written in the form $\mathbf{P}(L)\mathbf{Y}_t+Q(L)z_t+\mathbf{R}(L)\boldsymbol{\varepsilon}_t=0$. Alternatively, if $m_1^a > m_1^b$ then $\max\left[0, m_1^a - m_1^b + 1\right] \geq 2$, the $(N+1)^{th}$ line of (10) can be used sequentially to remove z_{t-k} for $k \in \left\{1,...,m_1^a - m_1^b\right\}$ from the first N lines of (10) and there exists a linear combination of the resulting N equations which can be written in the form $\mathbf{P}(L)\mathbf{Y}_t+Q(L)z_t+\mathbf{R}(L)\boldsymbol{\varepsilon}_t=0$. In both cases, the invertibility of $\boldsymbol{\Omega}$ is made possible by assumption 2.1.iii. Hence, whether $m_1^a \leq m_1^b$ or $m_1^a > m_1^b$, there exist $\mathbf{P}(L)$, $Q(L)$ and $\mathbf{R}(L)$ such that $\mathbf{e}_1'$ (21) holds (result 3). Results 1, 2 and 3 together imply that there exist $\mathbf{O}(L)$, $\mathbf{P}(L)$, $Q(L)$ and $\mathbf{R}(L)$ such that (9) holds for $t \in \mathbb{Z}$. From proposition 2.1 we then conclude that for any VARMA of type (10) consistent with (7) there exist $\mathbf{O}(L)$, $\mathbf{P}(L)$, $Q(L)$ and $\mathbf{R}(L)$ such that this VARMA is the unique solution of (7) and (9).

E Proof of proposition 2.3

Suppose $m_1^a > m_1^b$ and consider a rule of type (8) which is backward-looking (*i.e.* such that $m_f = 0$). Given assumptions 2.1 and 2.2, the system made of (7) and this rule has at least one non-predetermined variable and therefore must admit at least one eigenvalue of infinite modulus for the rule considered to be $+\infty$-bubble-free. Now this system's non-zero eigenvalues are those of the corresponding perfect-foresight deterministic system

$$\boldsymbol{\Psi}(L)\begin{bmatrix}\mathbf{Y}_t\\ z_t\end{bmatrix}=0 \text{ where } \underset{((N+1)\times(N+1))}{\boldsymbol{\Psi}(L)} \equiv \sum_{k=0}^{n^\psi}\boldsymbol{\Psi}_k L^k \text{ with } \boldsymbol{\Psi}_0=\left[\begin{array}{c|c} & 0\\ \widehat{\mathbf{A}}(0) & \vdots\\ & 0\\ \hline \mathbf{F}_0 & g_0\end{array}\right],$$

$n^\psi \in \mathbb{N}$, all $\boldsymbol{\Psi}_k$ have real numbers as elements and the zero elements in the last column of $\boldsymbol{\Psi}_0$ come from assumptions $m_1^a > m_1^b$ and 2.2.iii. Assumption 2.1.iii and the normalization $g_0 = 1$ (made without any loss in generality since $g_0 \neq 0$) make $\boldsymbol{\Psi}_0$ invertible, so that according to a standard matricial result of time series analysis (*cf. e.g.* Hamilton, 1994, chap. 10, prop. 10.1) this system's eigenvalues are the roots of polynomial $\Phi(X) \equiv \left|X^{n^\psi}\boldsymbol{\Psi}\left(X^{-1}\right)\right| \in \mathbb{R}[X]$. The coefficient $|\boldsymbol{\Psi}_0| = \left|\widehat{\mathbf{A}}(0)\right| g_0 = \left|\widehat{\mathbf{A}}(0)\right|$ of $X^{(N+1)n^\psi}$ in $\Phi(X)$ is non-zero and independent of the rule's coefficients. In order to make $\Phi(X)$ admit at least one root whose modulus tends towards infinity, the rule must therefore make the absolute value of at least one of the coefficients of X^k for $k \in \left\{0,...,(N+1)n^\psi - 1\right\}$ tend towards infinity, which implies that the absolute value of at least one of the rule's coefficients must tend towards infinity.

F Proof of proposition 2.4

If $m_1^a \leq m_1^b$ then bubble-free rules (9) are backward-looking and ensure the local determinacy of any given stationary VARMA process of type (10). The remaining of the proof therefore deals with the case where $m_1^a > m_1^b$. We proceed in five steps: first, we show that any system of type (7) together with a backward-looking rule of type (8) can be written in Blanchard and Kahn's (1980) form; second, we construct some particular $\mathbf{F}(L)$ and $G(L)$; third, we show that $m_f = 0$, so that whatever $\mathbf{H}(L)$ the corresponding rule (8) is backward-looking; fourth, we show that Blanchard and Kahn's (1980) condition is satisfied, so that whatever $\mathbf{H}(L)$ this rule ensures local equilibrium determinacy; fifth, we show that a suitable choice of $\mathbf{H}(L)$ makes the locally unique equilibrium selected coincide with the targeted stationary VARMA process.

Step 1: consider a system (S) of type (7) and a backward-looking rule (R) of type (8). Let us rewrite (S) step by step and keep for simplicity the same notation (S) at each step. Re-order the lines of (S) so that $m_1^a \geq ... \geq m_N^a$. Let $K \in \{1, ..., N\}$ and $\{i_1, ..., i_K\} \in \{1, ..., N\}^K$ be such that $m_1^a = ... = m_{i_1}^a > m_{i_1+1}^a = ... = m_{i_2}^a > ... > m_{i_{K-1}+1}^a = ... = m_{i_K}^a = m_N^a$. Re-order the elements of $\mathbf{Y}_t$ and accordingly the columns of $\mathbf{A}(L)$ so that $\forall i \in \{1, ..., N-1\}$, the $(N-i) \times (N-i)$ matrix noted $\mathbf{M}_i$ obtained by removing the first i lines and the first i columns from $\widehat{\mathbf{A}}(0)$ is invertible, this re-ordering being made possible by the invertibility of $\widehat{\mathbf{A}}(0)$. Replace $\mathbf{e}_i'(S)$ by

$$\mathbf{e}_i' \widehat{\mathbf{A}}(0)^{-1} E_t \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & L^{m_2^a - m_1^a} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & L^{m_N^a - m_1^a} \end{bmatrix} (S)$$

for $i \in \{1, ..., i_1\}$. If $K = 1$ then replace sequentially $E_t \left\{ z_{t+m_{i_1}^a - k} \right\}$ for $k \in \left\{1, ..., m_{i_1}^a\right\}$ (if they appear) in (S) by their expressions in $E_t \left\{ L^{k - m_{i_1}^a} (R) \right\}$. The resulting system (S) is equivalent to the original one and together with (R) can easily be written in Blanchard and Kahn's (1980) form with $i_1 m_{i_1}^a = \sum_{i=1}^N m_i^a$ non-predetermined variables. Otherwise (*i.e.* if $K \geq 2$), let us set $k = 1$. Replace $E_t \left\{ z_{t+m_{i_1}^a - k} \right\}$ (if it appears) in $\mathbf{e}_i'(S)$ for $i \in \{1, ..., i_1\}$ by its expression in $E_t \left\{ L^{k - m_{i_1}^a} (R) \right\}$. Then, replace $E_t \left\{ \mathbf{e}_i' \mathbf{Y}_{t+m_{i_1}^a - k} \right\}$ for $i \in \{i_1 + 1, ..., N\}$ (if they appear) in $\mathbf{e}_i'(S)$ for $i \in \{1, ..., i_1\}$ by their expression in

$$\mathbf{M}_i^{-1} \left[\begin{array}{cccc|cccc} 0 & 0 & \cdots & 0 & L^{m_{i_1+1}^a - m_{i_1}^a + k} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & L^{m_N^a - m_{i_1}^a + k} \end{array} \right] (S).$$

If $m_{i_1}^a > m_{i_2}^a + 1$ then repeat these last two steps sequentially for $k \in \left\{2, ..., m_{i_1}^a - m_{i_2}^a\right\}$. Proceed in a similar way as previously to transform $\mathbf{e}_i'(S)$ for $i \in \{i_1 + 1, ..., i_2\}$, then (if $K \geq 3$) $\mathbf{e}_i'(S)$

for $i \in \{i_2+1,...,i_3\}$ and so on up to $\mathbf{e}_i'(S)$ for $i \in \{i_{K-1}+1,...,i_K\}$. The final system (S) is equivalent to the initial one and together with (R) can easily be written in Blanchard and Kahn's (1980) form with $\sum_{j=1}^{K} i_j m_{i_j}^a = \sum_{i=1}^{N} m_i^a$ non-predetermined variables. We have thus shown that any system of type (7) together with a backward-looking rule of type (8) could be written in a Blanchard and Kahn's (1980) form with a number $m \equiv \sum_{i=1}^{N} m_i^a$ of non-predetermined variables. Note finally that this number m of non-predetermined variables does not depend on the particular backward-looking rule of type (8) considered.

Step 2: the generalized identity of Bezout implies that there exists $(\mathcal{U}_1(X),...,\mathcal{U}_{N+1}(X)) \in \mathbb{R}[X]^{N+1}$ such that

$$\sum_{i=1}^{N+1} \mathcal{U}_i(X)\,\Delta_i(X) = \mathcal{D}(X). \tag{22}$$

Let $\Theta(X) \in \mathbb{R}[X]$ denote the polynomial, defined up to a non-zero multiplicative scalar, which has the same roots (whose modulus is strictly lower than one) with the same multiplicity as the eigenvalues of the system $\mathbf{I} - \mathbf{S}(L)$ corresponding to the autoregressive part of the targeted stationary VARMA process (10). Let $\mathcal{Z}(X) \in \mathbb{R}[X]$ be a given polynomial which: i) has exactly m roots (taking into account their multiplicity) whose modulus is higher than or equal to one; and ii) is such that $\Theta(X)$ is a divisor of $\mathcal{Z}(X)\mathcal{D}(X)$. Let $n \in \mathbb{N}$ be such that $n \geq 2d_{\Delta_{N+1}} - d_{\mathcal{D}} + \max_{i\in\{1,...,N+1\}}(d_{\mathcal{U}_i}) - d_{\mathcal{Z}}$, where for any $\mathcal{H}(X) \in \mathbb{R}[X]$, $d_{\mathcal{H}}$ denotes the degree of $\mathcal{H}(X)$. Let $\mathcal{Q}(X) \in \mathbb{R}[X]$ and $\mathcal{R}(X) \in \mathbb{R}[X]$ be respectively the quotient and the remainder of the Euclidian division of $X^n\mathcal{Z}(X)$ by $\Delta_{N+1}(X)$, *i.e.* the unique polynomials such that $X^n\mathcal{Z}(X) = \Delta_{N+1}(X)\mathcal{Q}(X) + \mathcal{R}(X)$ with $d_{\mathcal{R}} < d_{\Delta_{N+1}}$. Multiplying the left-hand side and the right-hand side of (22) by $\mathcal{R}(X)$, we obtain

$$\mathcal{R}(X)\sum_{i=1}^{N+1} \mathcal{U}_i(X)\,\Delta_i(X) = \mathcal{R}(X)\,\mathcal{D}(X)$$

and therefore

$$\sum_{i=1}^{N} [\mathcal{R}(X)\,\mathcal{U}_i(X)]\,\Delta_i(X) + [\mathcal{R}(X)\,\mathcal{U}_{N+1}(X) + \mathcal{Q}(X)\,\mathcal{D}(X)]\,\Delta_{N+1}(X) = X^n\mathcal{Z}(X)\,\mathcal{D}(X).$$

Let us note $\mathcal{F}_i(X) \equiv \mathcal{R}(X)\,\mathcal{U}_i(X)$ for $i \in \{1,...,N\}$ and $\mathcal{G}(X) \equiv \mathcal{R}(X)\,\mathcal{U}_{N+1}(X) + \mathcal{Q}(X)\,\mathcal{D}(X)$. The choice of $\mathbf{F}(L)\,\mathbf{e}_i = (-1)^{N+1-i} L^{d_{\mathcal{G}}}\mathcal{F}_i\left(L^{-1}\right)$ for $i \in \{1,...,N\}$ and $G(L) = L^{d_{\mathcal{G}}}\mathcal{G}\left(L^{-1}\right)$ is admissible as it satisfies the requirements $G(X) \in \mathbb{R}[X]$ and $g_0 \neq 0$.

Step 3: we have

$$\begin{aligned}
&\left\{\begin{array}{l}
n \geq 2d_{\Delta_{N+1}} - d_{\mathcal{D}} + \max\limits_{i\in\{1,...,N+1\}}(d_{\mathcal{U}_i}) - d_{\mathcal{Z}} \\
n = d_{\Delta_{N+1}} + d_{\mathcal{Q}} - d_{\mathcal{Z}} \\
d_{\Delta_{N+1}} > d_{\mathcal{R}}
\end{array}\right. \\
&\Longrightarrow d_{\mathcal{Q}} + d_{\mathcal{D}} > d_{\mathcal{R}} + \max_{i\in\{1,...,N+1\}}(d_{\mathcal{U}_i}) \\
&\Longrightarrow d_{\mathcal{G}} = d_{\mathcal{Q}} + d_{\mathcal{D}} > \max_{i\in\{1,...,N\}}(d_{\mathcal{F}_i})
\end{aligned}$$

so that the $\mathbf{F}(L)$ constructed at step 2 is such that $\forall i \in \{1, ..., N\}$, $\mathbf{F}(X)\,\mathbf{e}_i \in \mathbb{R}[X]$, in other words $m_f = 0$, *i.e.* any rule (8) with the $\mathbf{F}(L)$ and $G(L)$ constructed at step 2 is backward-looking.

Step 4: the non-zero eigenvalues of the system made of (7) and any rule (8) with the $\mathbf{F}(L)$ and $G(L)$ constructed at step 2 are those of the corresponding perfect-foresight deterministic system

$$\boldsymbol{\Psi}(L)\begin{bmatrix}\mathbf{Y}_t \\ z_t\end{bmatrix} = 0 \text{ where } \underset{((N+1)\times(N+1))}{\boldsymbol{\Psi}(L)} \equiv \sum\nolimits_{k=0}^{n^{\psi}} \boldsymbol{\Psi}_k L^k = \left[\begin{array}{c|c} & L^{m_1^a}\mathbf{e}_1'\mathbf{B}(L) \\ \widehat{\mathbf{A}}(L) & \vdots \\ & L^{m_N^a}\mathbf{e}_N'\mathbf{B}(L) \\ \hline \mathbf{F}(L) & G(L)\end{array}\right],$$

$n^{\psi} \in \mathbb{N}$ and all $\boldsymbol{\Psi}_k$ have real numbers as elements. Given that $m_1^a > m_1^b$, assumptions 2.1.iii, 2.2.iii and $g_0 \neq 0$ make

$$\boldsymbol{\Psi}_0 = \left[\begin{array}{c|c} & 0 \\ \widehat{\mathbf{A}}(0) & \vdots \\ & 0 \\ \hline \mathbf{F}(0) & g_0\end{array}\right]$$

invertible, so that according to a standard matricial result of time series analysis (*cf. e.g.* Hamilton, 1994, chap. 10, prop. 10.1) this system's eigenvalues are the roots of polynomial $\left|X^{n^{\psi}}\boldsymbol{\Psi}\left(X^{-1}\right)\right| \in \mathbb{R}[X]$, where $|.|$ denotes the determinant operator. As a consequence, the system's non-zero eigenvalues are the non-zero roots of

$$\sum\nolimits_{i=1}^{N}\left[(-1)^{N+1-i}\,\mathbf{F}\left(X^{-1}\right)\mathbf{e}_i\right]\Delta_i(X) + \left[G\left(X^{-1}\right)\right]\Delta_{N+1}(X)$$

and hence, by construction of $\mathbf{F}(L)$ and $G(L)$, the non-zero roots of $\mathcal{Z}(X)\,\mathcal{D}(X)$. Given assumption 2.3 and by definition of $\mathcal{Z}(X)$, there are exactly m non-zero roots of $\mathcal{Z}(X)\,\mathcal{D}(X)$ whose moduli are higher than or equal to one. Given steps 1 and 3, this implies that Blanchard and Kahn's condition (1980) is satisfied, that is to say that any rule (8) with the $\mathbf{F}(L)$ and $G(L)$ constructed at step 2 ensures local equilibrium determinacy.

Step 5: if the targeted stationary VARMA process (10) holds for $t \in \mathbb{Z}$, then: i) there exists a unique $\underset{(1\times N)}{\boldsymbol{\Xi}(L)} \equiv \sum_{k=0}^{+\infty}\boldsymbol{\Xi}_k L^k$, where all $\boldsymbol{\Xi}_k$ have real numbers as elements, such that

$$\mathbf{F}(L)\,\mathbf{Y}_t + G(L)\,z_t + \boldsymbol{\Xi}(L)\,\boldsymbol{\varepsilon}_t = \mathbf{0} \tag{23}$$

where $\mathbf{F}(L)$ and $G(L)$ are the ones constructed at step 2; and ii) there exists a unique

$$\underset{(N\times N)}{\boldsymbol{\Pi}(L)} \equiv \begin{bmatrix}\sum_{k=0}^{m_1^a-1}\boldsymbol{\Pi}_{1,k}L^k \\ \sum_{k=0}^{m_2^a-1}\boldsymbol{\Pi}_{2,k}L^k \\ \vdots \\ \sum_{k=0}^{m_N^a-1}\boldsymbol{\Pi}_{N,k}L^k\end{bmatrix},$$

where all $\boldsymbol{\Pi}_{i,k}$ have real numbers as elements, such that

$$\left[\begin{array}{c|c} & L^{m_1^a}\mathbf{e}_1'\mathbf{B}(L) \\ \widehat{\mathbf{A}}(L) & \vdots \\ & L^{m_N^a}\mathbf{e}_N'\mathbf{B}(L)\end{array}\right]\begin{bmatrix}\mathbf{Y}_t \\ z_t\end{bmatrix} + \begin{bmatrix}L^{m_1^a}\mathbf{e}_1'\mathbf{C}(L) \\ \vdots \\ L^{m_N^a}\mathbf{e}_N'\mathbf{C}(L)\end{bmatrix}\boldsymbol{\xi}_t + \boldsymbol{\Pi}(L)\,\boldsymbol{\varepsilon}_t = \mathbf{0}. \tag{24}$$

For any $p \in \mathbb{N}^*$ and $\digamma(X) \in \mathbb{R}[X]$, let us note $\mathbf{d}_p[\digamma(L)]$ the $p \times p$ matrix whose diagonal elements are all equal to $\digamma(L)$ and whose non-diagonal elements are all equal to 0. Multiplying (23) by $D(L) \equiv \prod_{i=1}^{N} D_i(L)$ and (24) by $\mathbf{d}_N[D(L)]$ leads to

$$D(L)\mathbf{F}(L)\mathbf{Y}_t + D(L)G(L)z_t + D(L)\boldsymbol{\Xi}(L)\boldsymbol{\varepsilon}_t = \mathbf{0} \text{ and} \tag{25}$$

$$\mathbf{d}_N[D(L)]\left[\begin{array}{c|c} & L^{m_1^a}\mathbf{e}_1'\mathbf{B}(L) \\ \widehat{\mathbf{A}}(L) & \vdots \\ & L^{m_N^a}\mathbf{e}_N'\mathbf{B}(L)\end{array}\right]\left[\begin{array}{c}\mathbf{Y}_t \\ z_t\end{array}\right] + \left[\begin{array}{c}L^{m_1^a}\mathbf{e}_1'\mathbf{C}(L) \\ \vdots \\ L^{m_N^a}\mathbf{e}_N'\mathbf{C}(L)\end{array}\right]$$
$$\left[\begin{array}{cccc}\prod_{i=2}^{N} D_i(L) & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \prod_{i=1}^{N-1} D_i(L)\end{array}\right]\boldsymbol{\varepsilon}_t + \mathbf{d}_N[D(L)]\boldsymbol{\Pi}(L)\boldsymbol{\varepsilon}_t = \mathbf{0}, \tag{26}$$

since as a multiple of the $N \times N$ identity matrix, $\mathbf{d}_N[D(L)]$ is such that $\mathbf{d}_N[D(L)]\mathbf{K}(L) = \mathbf{K}(L)\mathbf{d}_N[D(L)]$ for any $N \times N$ matrix $\mathbf{K}(L)$ whose elements are polynomials in L with real-number-valued coefficients. The system made of (25) and (26) is backward-looking (since $m_1^a > m_1^b$) and non-degenerate (since $D(0) = |\mathbf{D}(0)| \neq 0$ and $|\boldsymbol{\Psi}_0| \neq 0$). Cramer's rule then implies that there exist $(n_1, ..., n_{N+1}) \in \mathbb{N}^{N+1}$ with $n_i \geq d_{\Delta_i}$ for $i \in \{1, ..., N+1\}$ and $\underset{((N+1)\times N)}{\boldsymbol{\Upsilon}_1(L)} \equiv \sum_{k=0}^{n^{\upsilon_1}} \boldsymbol{\Upsilon}_{1,k}L^k$, where $n^{\upsilon_1} \in \mathbb{N}$ and all $\boldsymbol{\Upsilon}_{1,k}$ have real numbers as elements, such that this system can be rewritten

$$\mathbf{d}_{N+1}\left[D(L)L^{d_{\mathcal{Z}}+d_{\mathcal{D}}}\mathcal{Z}\left(L^{-1}\right)\mathcal{D}\left(L^{-1}\right)\right]\left[\begin{array}{c}\mathbf{Y}_t \\ z_t\end{array}\right] =$$
$$\boldsymbol{\Upsilon}_1(L)\boldsymbol{\varepsilon}_t + \mathbf{d}_{N+1}[D(L)]\left[\begin{array}{c}L^{n_1}\Delta_1\left(L^{-1}\right)\boldsymbol{\Xi}(L)\boldsymbol{\varepsilon}_t \\ \vdots \\ L^{n_{N+1}}\Delta_{N+1}\left(L^{-1}\right)\boldsymbol{\Xi}(L)\boldsymbol{\varepsilon}_t\end{array}\right] \tag{27}$$

given step 4. But Cramer's rule also implies that there exists $\underset{((N+1)\times N)}{\boldsymbol{\Upsilon}_2(L)} \equiv \sum_{k=0}^{n^{\upsilon_2}} \boldsymbol{\Upsilon}_{2,k}L^k$, where $n^{\upsilon_2} \in \mathbb{N}$ and all $\boldsymbol{\Upsilon}_{2,k}$ have real numbers as elements, such that the targeted stationary VARMA process (10) can be rewritten

$$\mathbf{d}_{N+1}\left[L^{d_\Theta}\Theta\left(L^{-1}\right)\right]\left[\begin{array}{c}\mathbf{Y}_t \\ z_t\end{array}\right] = \boldsymbol{\Upsilon}_2(L)\boldsymbol{\varepsilon}_t,$$

which implies

$$\mathbf{d}_{N+1}\left[D(L)L^{d_{\mathcal{Z}}+d_{\mathcal{D}}}\mathcal{Z}\left(L^{-1}\right)\mathcal{D}\left(L^{-1}\right)\right]\left[\begin{array}{c}\mathbf{Y}_t \\ z_t\end{array}\right] = \mathbf{d}_{N+1}\left[D(L)\frac{L^{d_{\mathcal{Z}}+d_{\mathcal{D}}}\mathcal{Z}\left(L^{-1}\right)\mathcal{D}\left(L^{-1}\right)}{L^{d_\Theta}\Theta\left(L^{-1}\right)}\right]\boldsymbol{\Upsilon}_2(L)\boldsymbol{\varepsilon}_t \tag{28}$$

where $\frac{X^{d_{\mathcal{Z}}+d_{\mathcal{D}}}\mathcal{Z}\left(X^{-1}\right)\mathcal{D}\left(X^{-1}\right)}{X^{d_\Theta}\Theta\left(X^{-1}\right)} \in \mathbb{R}[X]$ by definition of $\mathcal{Z}(X)$. Given that $\Delta_{N+1}(X) \neq 0$ due to assumption 2.1.iii, the identification of (27) with (28) shows that $\exists n^\xi \in \mathbb{N}$, $\forall k > n^\xi$, $\boldsymbol{\Xi}_k = \mathbf{0}$. The choice of $\mathbf{H}(L) = \boldsymbol{\Xi}(L)\mathbf{D}(L)$ is therefore admissible. We have thus shown that for any given stationary VARMA process (10) consistent with (7) there exist $\mathbf{F}(L)$, $G(L)$ and $\mathbf{H}(L)$, with $m_f = 0$, such that this stationary VARMA process is the locally unique solution of (7) and (8).

G Proof of proposition 2.5

As straightforward from appendix C, rules of type (9) based on the measured variables and shocks still ensure the existence and uniqueness of the equilibrium. Besides, assumption 2.3 implies the following extension of proposition 2.2: *whatever the targeted stationary VARMA process of type (10) consistent with (7),* i.e. *whatever the targeted VARMA process of type (10) satisfying (7) and such that all the eigenvalues of the systems $\mathbf{I} - \mathbf{S}(L)$ and $\mathbf{T}(L)$ are of modulus strictly lower than one, there exist $\mathbf{O}(L)$, $\mathbf{P}(L)$, $Q(L)$ and $\mathbf{R}(L)$ such that this VARMA process is the unique solution of (7) and (9) and such that the system*

$$\begin{bmatrix} \mathbf{P}(L) & Q(L) \\ \mathbf{A}(L) & \mathbf{B}(L) \end{bmatrix}$$

has all its eigenvalues of modulus strictly lower than one, as can be easily shown along the lines of appendix F. In turn, this extension of proposition 2.2 and appendix D straightforwardly imply together that as the size of data measurement errors tends towards zero, the unique equilibrium implemented by the rule of type (9) based on the measured variables and shocks converges towards the stationary equilibrium implemented by the corresponding rule based on the true variables and shocks[29].

H Proof of proposition 2.6

Let us first define the metric d which we use to characterize convergence processes by

$$\left(\underset{(N_1 \times N_2)}{\mathbf{X}_1(L)}, \underset{(N_1 \times N_2)}{\mathbf{X}_2(L)} \right) \equiv \left(\sum_{k=-m^x}^{n^x} \mathbf{X}_{1,k} L^k, \sum_{k=-m^x}^{n^x} \mathbf{X}_{2,k} L^k \right)$$
$$\mapsto d\left(\mathbf{X}_1(L), \mathbf{X}_2(L)\right) = \max_{-m^x \leq k \leq n^x} \left[\max_{1 \leq i \leq N_1} \left(\max_{1 \leq j \leq N_2} \left| \mathbf{e}'_{1,i} \left(\mathbf{X}_{1,k} - \mathbf{X}_{2,k} \right) \mathbf{e}_{2,j} \right| \right) \right],$$

where $(m^x, n^x) \in \overline{\mathbb{N}}^2$, $(N_1, N_2) \in \mathbb{N}^{*2}$, all $\mathbf{X}_{1,k}$, $\mathbf{X}_{2,k}$ have real numbers as elements and for $h \in \{0,1\}$ and $l \in \{1, ..., N_h\}$ $\mathbf{e}_{h,l}$ is the N_h-element vector whose l^{th} element is equal to one

[29] Note that assumption 2.3 is not only sufficient, but also necessary for this extension of proposition 2.2 to hold. Similarly, this extension of proposition 2.2 is not only sufficient, but also necessary for the unique equilibrium implemented by the rule of type (9) based on the measured variables and shocks to converge towards the given stationary equilibrium implemented by the corresponding rule based on the true variables and shocks as the size of data measurement errors tends towards zero. Indeed, given that the probability distributions of the exogenous additive measurement errors are assumed to be continuous, the probability that all roots of $\mathcal{D}(X)$ are active eigenvalues of the system made of (7) and the rule of type (9) based on the measured variables and shocks is equal to one, where by "active eigenvalue" we mean an eigenvalue associated with a non-zero coefficient in the analytical expression of the system's solution. If at least one of the roots of $\mathcal{D}(X)$ were of modulus higher than one, then with probability one the system's unique solution would be divergent and hence infinitely distant from the targeted stationary equilibrium. We therefore conlude that assumption 2.3 is not only sufficient, but also necessary for proposition 2.5 to hold.

and whose other elements are equal to zero. Suppose that the policy-maker wrongly believes the structural equations to be

$$E_t\left\{\widetilde{\mathbf{A}}(L)\mathbf{Y}_t+\widetilde{\mathbf{B}}(L)z_t\right\}+\widetilde{\mathbf{C}}(L)\boldsymbol{\xi}_t=\mathbf{0}$$

with $\widetilde{\mathbf{D}}(L)\boldsymbol{\xi}_t=\boldsymbol{\varepsilon}_t$, though without being mistaken on the values of m_1^b and m_i^a for $1\le i\le N$[30], and accordingly follows the policy feedback rule $(\widetilde{R})$ corresponding to (9) where $\mathbf{A}(L)$, $\mathbf{B}(L)$, $\mathbf{C}(L)$ and $\mathbf{D}(L)$ are respectively replaced by $\widetilde{\mathbf{A}}(L)$, $\widetilde{\mathbf{B}}(L)$, $\widetilde{\mathbf{C}}(L)$ and $\widetilde{\mathbf{D}}(L)$. Noting

$$\varepsilon\equiv\max\left[d\left(\widetilde{\mathbf{A}}(L),\mathbf{A}(L)\right),d\left(\widetilde{\mathbf{B}}(L),\mathbf{B}(L)\right),d\left(\widetilde{\mathbf{C}}(L),\mathbf{C}(L)\right),d\left(\widetilde{\mathbf{D}}(L),\mathbf{D}(L)\right)\right],$$

we proceed in three steps: first, we show that the system made of (7) and $(\widetilde{R})$ can be written in Blanchard and Kahn's (1980) form with probability one; second, we show that for ε sufficiently close to zero Blanchard and Kahn's (1980) condition is satisfied, so that there is one unique local solution, and that as $\varepsilon\longrightarrow 0$ the moduli of the system's unstable eigenvalues tend towards infinity; third, we show that as $\varepsilon\longrightarrow 0$ this unique local solution converges towards the unique solution of (7) and (9).

Step 1: consider a given system (S) of type (7). Replace $E_t\left\{z_{t+m_1^b}\right\}$ in $\mathbf{e}_1'(S)$ by its expression in $E_t\left\{L^{-m_1^b}(\widetilde{R})\right\}$; if $m_1^a<m_1^b$, in which case $(\widetilde{R})$ is backward-looking, then replace sequentially $E_t\left\{z_{t+m_1^b-k}\right\}$ for $k\in\left\{1,...,m_1^b-m_1^a\right\}$ (if they appear) in the resulting equation by their expressions in $E_t\left\{L^{k-m_1^b}(\widetilde{R})\right\}$; note $(\widetilde{E})$ the resulting equation. Consider

$$(\widetilde{S})\equiv\left\{\begin{array}{c}(\widetilde{E})\\ \mathbf{e}_2'(S)\\ \vdots\\ \mathbf{e}_N'(S)\end{array}\right.\quad\text{and }\widehat{\widetilde{\boldsymbol{A}}}(L)\equiv\left[\begin{array}{c}\mathbf{e}_1'L^{m_1^a}\\ \vdots\\ \mathbf{e}_N'L^{m_N^a}\end{array}\right]\widetilde{\boldsymbol{A}}(L)$$

where $\widetilde{\boldsymbol{A}}(L)$ is defined by writing $(\widetilde{S})$ in the form $E_t\left\{\widetilde{\boldsymbol{A}}(L)\mathbf{Y}_t+\widetilde{\boldsymbol{B}}(L)z_t\right\}+\widetilde{\boldsymbol{C}}(L)\boldsymbol{\xi}_t=\mathbf{0}$[31]. Given that the probability distributions of the exogenous additive measurement errors are assumed to be continuous, the probability that $\widehat{\widetilde{\boldsymbol{A}}}(0)$ is invertible is equal to one. In the remaining of the proof we therefore assume that $\widehat{\widetilde{\boldsymbol{A}}}(0)$ is invertible. Rewrite $(\widetilde{S})$ in a similar way as in step 1 of appendix F, with $(\widetilde{S})$, $\widetilde{\boldsymbol{A}}(L)$, $\widehat{\widetilde{\boldsymbol{A}}}(0)$ and $(\widetilde{R})$ playing the roles of (S), $\mathbf{A}(L)$, $\widehat{\mathbf{A}}(0)$ and (R) respectively. If $m_1^a\le m_1^b$ then this rewriting enables us to put the system made of $(\widetilde{S})$ and $(\widetilde{R})$ in Blanchard and Kahn's (1980) form since $(\widetilde{R})$ is backward-looking and since $m_i^a>m_i^b$ for $i\in I_B\smallsetminus\{1\}$ due to assumption 2.2.iii. Alternatively, if $m_1^a>m_1^b$ then this rewriting also enables us to put the system made of $(\widetilde{S})$ and $(\widetilde{R})$ in Blanchard and Kahn's (1980) form, even though $(\widetilde{R})$ is forward-looking, because $m_i^a-m_i^b>m_1^a-m_1^b$ for $i\in I_B\smallsetminus\{1\}$ due to assumption 2.2.iii and because the only variable

[30] As should become clear by the end of the proof, the case where the policy-maker is also mistaken on the values of m_1^b and m_i^a for $1\le i\le N$ can be easily dealt with in the same way but at the expense of expositional clarity.

[31] $\widetilde{\boldsymbol{A}}(L)$, $\widetilde{\boldsymbol{B}}(L)$ and $\widetilde{\boldsymbol{C}}(L)$ should not be confused with $\widetilde{\mathbf{A}}(L)$, $\widetilde{\mathbf{B}}(L)$ and $\widetilde{\mathbf{C}}(L)$.

of type $E_t\{z_{t+k}\}$ with $k\in\mathbb{N}$ appearing in the system made of the rewritten system $(\widetilde{S})$ and $(\widetilde{R})$ is z_t in $(\widetilde{R})$. In both cases the number of non-predetermined variables is equal to $m\equiv\sum_{i=1}^{N} m_i^a=0$. Since the system made of (S) and $(\widetilde{R})$ is equivalent to the system made of $(\widetilde{S})$ and $(\widetilde{R})$, we have thus shown that with probability one, the system made of (S) and $(\widetilde{R})$ can be written in Blanchard and Kahn's (1980) form with m non-predetermined variables.

Step 2: for any system or equation (x), let $(\overline{x})$ denote the perfect-foresight deterministic form of (x). The same reasoning as the one conducted at the beginning of appendix C, this time starting from $(\widetilde{E})$ instead of $(\overrightarrow{1})$ and using $(\widetilde{R})$ instead of (R), leads to an equation $(\widetilde{\overrightarrow{N}})$, corresponding to equation $(\overrightarrow{N})$ in appendix C, such that $(\overline{\widetilde{\overrightarrow{N}}})$ is of the form

$$\widetilde{\mathbf{P}}(L)\,\mathbf{Y}_t+\widetilde{Q}(L)\,z_t=0 \text{ with } \underset{(1\times N)}{\widetilde{\mathbf{P}}(L)}\equiv\sum_{k=-m}^{n^{\widetilde{p}}}\widetilde{\mathbf{P}}_kL^k \text{ and } \widetilde{Q}(L)\equiv\sum_{k=-m+1+\max\left[m_1^a-m_1^b,\,0\right]}^{n^{\widetilde{q}}}\widetilde{q}_kL^k,$$

where $\left(n^{\widetilde{p}},n^{\widetilde{q}}\right)\in\mathbb{N}^2$, all $\widetilde{\mathbf{P}}_k$ have real numbers as elements, all $\widetilde{q}_k$ are real numbers, $\widetilde{\mathbf{P}}_{-m}=\mathbf{e}_1'\widehat{\widehat{\mathbf{A}}}(0)$ and $(\widetilde{\mathbf{P}}(L),\widetilde{Q}(L))\longrightarrow(\mathbf{P}(L),Q(L))$ as $\varepsilon\longrightarrow 0$. The non-zero eigenvalues of the system made of (S) and $(\widetilde{R})$ are those of the system made of $(\overline{S})$ and $(\overline{\widetilde{R}})$ which in turn are those of the system made of $(\overline{S})$ and $(\overline{\widetilde{\overrightarrow{N}}})$. The latter system can be rewritten

$$\widetilde{\mathbf{\Gamma}}_1(L)\begin{bmatrix}\mathbf{Y}_t\\ z_t\end{bmatrix}=\mathbf{0} \text{ with } \widetilde{\mathbf{\Gamma}}_1(L)\equiv\begin{bmatrix} L^m\widetilde{\mathbf{P}}(L) & L^m\widetilde{Q}(L)\\ \mathbf{e}_1'L^{\max\left[m_1^a,m_1^b\right]}\mathbf{A}(L) & \mathbf{e}_1'L^{\max\left[m_1^a,m_1^b\right]}\mathbf{B}(L)\\ \mathbf{e}_2'L^{m_2^a}\mathbf{A}(L) & \mathbf{e}_2'L^{m_2^a}\mathbf{B}(L)\\ \vdots & \vdots\\ \mathbf{e}_N'L^{m_N^a}\mathbf{A}(L) & \mathbf{e}_N'L^{m_N^a}\mathbf{B}(L)\end{bmatrix}\equiv\sum_{k=0}^{n^{\widetilde{\gamma}_1}}\widetilde{\mathbf{\Gamma}}_{1,k}L^k$$

where $n^{\widetilde{\gamma}_1}\in\mathbb{N}$ and all $\widetilde{\mathbf{\Gamma}}_{1,k}$ have real numbers as elements. Let us define

$$\underset{(N\times(N+1))}{\mathbf{J}_1}\equiv\begin{bmatrix}1&0&0&\cdots&0\\0&0&1&\ddots&\vdots\\\vdots&\ddots&\ddots&\ddots&0\\0&\cdots&0&0&1\end{bmatrix},\quad \underset{((N+1)\times N)}{\mathbf{J}_2}\equiv\begin{bmatrix}1&0&\cdots&0\\0&\ddots&\ddots&\vdots\\\vdots&\ddots&\ddots&0\\0&\cdots&0&1\\0&\cdots&\cdots&0\end{bmatrix},$$

$$\underset{((N+1)\times 1)}{\mathbf{J}_3}\equiv\begin{bmatrix}0\\\vdots\\0\\1\end{bmatrix} \text{ and } \underset{(N\times(N+1))}{\mathbf{J}_4}\equiv\begin{bmatrix}0&1&0&\cdots&0\\0&\ddots&\ddots&\ddots&\vdots\\\vdots&\ddots&\ddots&\ddots&0\\0&\cdots&0&0&1\end{bmatrix}.$$

If $m_1^a\geq m_1^b$ then replace sequentially $\mathbf{Y}_{t-k}$ for $k\in\left\{0,...,m_1^a-m_1^b\right\}$ in the second line of $\widetilde{\mathbf{\Gamma}}_1(L)\begin{bmatrix}\mathbf{Y}_t & z_t\end{bmatrix}'=\mathbf{0}$ by its expression in $(\mathbf{J}_1\widetilde{\mathbf{\Gamma}}_{1,0}\mathbf{J}_2)^{-1}\mathbf{J}_1\widetilde{\mathbf{\Gamma}}_1(L)\,L^k\begin{bmatrix}\mathbf{Y}_t & z_t\end{bmatrix}'=\mathbf{0}$, given that $|\mathbf{J}_1\widetilde{\mathbf{\Gamma}}_{1,0}\mathbf{J}_2|=|\widehat{\widehat{\mathbf{A}}}(0)|\neq 0$, and note $\widetilde{\mathbf{\Gamma}}_2(L)\begin{bmatrix}\mathbf{Y}_t & z_t\end{bmatrix}'=\mathbf{0}$ the resulting system, with $\widetilde{\mathbf{\Gamma}}_2(L)\equiv\sum_{k=0}^{n^{\widetilde{\gamma}_2}}\widetilde{\mathbf{\Gamma}}_{2,k}L^k$ where $n^{\widetilde{\gamma}_2}\in\mathbb{N}$ and all $\widetilde{\mathbf{\Gamma}}_{2,k}$ have real numbers as elements ($\widetilde{\mathbf{\Gamma}}_2(L)=\widetilde{\mathbf{\Gamma}}_1(L)$ if $m_1^a<m_1^b$). Given that $\mathbf{J}_1\widetilde{\mathbf{\Gamma}}_{1,k}\mathbf{J}_3=\mathbf{0}$ for $k\in\left\{0,...,\max\left[m_1^a-m_1^b,0\right]\right\}$ due to assumption 2.2.iii,

we have

$$\widetilde{\mathbf{\Gamma}}_{2,0} = \begin{bmatrix} \mathbf{e}_1' \widehat{\widehat{\mathbf{A}}}(0) & 0 \\ \mathbf{0} & \mathbf{e}_1' \mathbf{B}_{-m_1^b} \\ \mathbf{e}_2' \widehat{\widehat{\mathbf{A}}}(0) & 0 \\ \vdots & \vdots \\ \mathbf{e}_N' \widehat{\widehat{\mathbf{A}}}(0) & 0 \end{bmatrix}.$$

Since $\widehat{\widehat{\mathbf{A}}}(0)$ is invertible, $\widetilde{\mathbf{\Gamma}}_{2,0}$ is invertible as well so that according to a standard matricial result of time series analysis (*cf. e.g.* Hamilton, 1994, chap. 10, prop. 10.1) the non-zero eigenvalues of $\widetilde{\mathbf{\Gamma}}_2(L)$, which are those of $\widetilde{\mathbf{\Gamma}}_1(L)$, are the roots of polynomial $\widetilde{\mathcal{E}}(X) \equiv \left| X^{n^{\widetilde{\gamma}_2}} \widetilde{\mathbf{\Gamma}}_2\left(X^{-1}\right)\right| \in \mathbb{R}[X]$. Now $\widetilde{\mathcal{E}}(X) = \widetilde{\mathcal{E}}_1(X) + \widetilde{\mathcal{E}}_2(X)$ where

$$\widetilde{\mathcal{E}}_1(X) \equiv \left| \begin{array}{cc} X^{n^{\widetilde{\gamma}_2}-m} \sum_{k=-m}^{-1} \widetilde{\mathbf{P}}_k X^{-k} & X^{n^{\widetilde{\gamma}_2}-m} \sum_{k=-m}^{-1} \widetilde{q}_k X^{-k} \\ \multicolumn{2}{c}{\mathbf{J}_4 X^{n^{\widetilde{\gamma}_2}} \widetilde{\mathbf{\Gamma}}_2\left(X^{-1}\right)} \end{array} \right|$$

$$\text{and } \widetilde{\mathcal{E}}_2(X) \equiv \left| \begin{array}{cc} X^{n^{\widetilde{\gamma}_2}-m} \sum_{k=0}^{n^{\widetilde{p}}} \widetilde{\mathbf{P}}_k X^{-k} & X^{n^{\widetilde{\gamma}_2}-m} \sum_{k=0}^{n^{\widetilde{q}}} \widetilde{q}_k X^{-k} \\ \multicolumn{2}{c}{\mathbf{J}_4 X^{n^{\widetilde{\gamma}_2}} \widetilde{\mathbf{\Gamma}}_2\left(X^{-1}\right)} \end{array} \right|.$$

If $m = 0$ then $\widetilde{\mathcal{E}}_1(X) = 0$. Otherwise the degree of $\widetilde{\mathcal{E}}_1(X)$ is equal to $n^{\widetilde{\gamma}_2}(N+1)$ since the coefficient of $X^{n^{\widetilde{\gamma}_2}(N+1)}$ in $\widetilde{\mathcal{E}}_1(X)$ is $\left|\widetilde{\mathbf{\Gamma}}_{2,0}\right| \neq 0$. For ε sufficiently close to 0, the degree of $\widetilde{\mathcal{E}}_2(X)$ is equal to $n^{\widetilde{\gamma}_2}(N+1) - m$ since the coefficient of $X^{n^{\widetilde{\gamma}_2}(N+1)-m}$ in $\widetilde{\mathcal{E}}_2(X)$ is

$$\begin{vmatrix} \widetilde{\mathbf{P}}_0 & \widetilde{q}_0 \\ \mathbf{0} & \mathbf{e}_1' \mathbf{B}_{-m_1^b} \\ \mathbf{e}_2' \widehat{\mathbf{A}}(0) & 0 \\ \vdots & \vdots \\ \mathbf{e}_N' \widehat{\mathbf{A}}(0) & 0 \end{vmatrix} \longrightarrow (-1)^{N+1} \mathbf{e}_1' \mathbf{B}_{-m_1^b} |\mathbf{\Omega}| \neq 0 \text{ as } \varepsilon \longrightarrow 0.$$

Let us note $\widetilde{x}_1, ..., \widetilde{x}_{n^{\widetilde{\gamma}_2}(N+1)}$ the roots of $\widetilde{\mathcal{E}}(X)$, ranked first by increasing modulus (*i.e.* $|\widetilde{x}_1| \leq ... \leq \left|\widetilde{x}_{n^{\widetilde{\gamma}_2}(N+1)}\right|$) and second by increasing complex argument (*i.e.* if $\exists i \in \left\{1, ..., n^{\widetilde{\gamma}_2}(N+1) - 1\right\}$, $|\widetilde{x}_i| = |\widetilde{x}_{i+1}|$, then $\varphi(\widetilde{x}_i) \leq \varphi(\widetilde{x}_{i+1})$, where $\varphi : \mathbb{C} \longrightarrow [0; 2\pi[$ denotes the complex argument function). Similarly, let us note $x_1, ..., x_n$ the non-zero eigenvalues of system

$$\begin{bmatrix} \mathbf{P}(L) & Q(L) \\ \mathbf{A}(L) & \mathbf{B}(L) \end{bmatrix}$$

ranked first by increasing modulus and second by increasing complex argument, which are all of modulus strictly lower than one due to assumption 2.3 as pointed out in appendix G. Since $\widetilde{\mathcal{E}}_1(X) \longrightarrow 0$ as $\varepsilon \longrightarrow 0$, we have

$$\left(\widetilde{x}_1, ..., \widetilde{x}_{n^{\widetilde{\gamma}_2}(N+1)-m}\right) \longrightarrow (0, ..., 0, x_1, ..., x_n) \text{ as } \varepsilon \longrightarrow 0$$

$$\text{and } \forall k \in \{0, ..., m-1\}, \left|\widetilde{x}_{n^{\widetilde{\gamma}_2}(N+1)-k}\right| \longrightarrow +\infty \text{ as } \varepsilon \longrightarrow 0,$$

which implies: i) given step 1, that the system made of (S) and $(\widetilde{R})$ satisfies Blanchard and Kahn's (1980) condition and therefore admits a unique convergent solution for ε sufficiently close to zero; ii) given assumption 2.4, that the probability that the private agents coordinate on this solution tends towards one as $\varepsilon \longrightarrow 0$.

Step 3: let us note

$$\begin{bmatrix} \mathbf{Y}_t \\ z_t \end{bmatrix} = \mathbf{J}(L)\,\boldsymbol{\varepsilon}_t, \text{ with } \underset{((N+1)\times N)}{\mathbf{J}(L)} \equiv \sum_{k=0}^{+\infty} \mathbf{J}_k L^k, \tag{29}$$

where all $\mathbf{J}_k$ have real numbers as elements, the unique solution of the system made of (S) and (R). Let us similarly note

$$\begin{bmatrix} \mathbf{Y}_t \\ z_t \end{bmatrix} = \widetilde{\mathbf{J}}(L)\,\boldsymbol{\varepsilon}_t, \text{ with } \underset{((N+1)\times N)}{\widetilde{\mathbf{J}}(L)} \equiv \sum_{k=0}^{+\infty} \widetilde{\mathbf{J}}_k L^k, \tag{30}$$

where all $\widetilde{\mathbf{J}}_k$ have real numbers as elements, this unique convergent solution of the system made of (S) and $(\widetilde{R})$ for ε sufficiently close to zero. This last step of the proof shows that $\widetilde{\mathbf{J}}(L) \longrightarrow \mathbf{J}(L)$ as $\varepsilon \longrightarrow 0$.

Substep 3.1: let us write equations $(\overrightarrow{k})$ for $k \in \{1, ..., N\}$, obtained in appendix C, in the form

$$\overrightarrow{\mathbf{U}}(L)\,\mathbf{Y}_t + \overrightarrow{\mathbf{V}}(L)\,z_t + \overrightarrow{\mathbf{W}}(L)\,\boldsymbol{\xi}_t = \mathbf{0}$$

$$\text{with } \underset{(N\times N)}{\overrightarrow{\mathbf{U}}(L)} \equiv \sum_{k=0}^{n^{\overrightarrow{u}}} \overrightarrow{\mathbf{U}}_k L^k,\ \underset{(N\times 1)}{\overrightarrow{\mathbf{V}}(L)} \equiv \sum_{k=\max\left[0, m_1^a - m_1^b + 1\right]}^{n^{\overrightarrow{v}}} \overrightarrow{\mathbf{V}}_k L^k \text{ and } \underset{(N\times N)}{\overrightarrow{\mathbf{W}}(L)} \equiv \sum_{k=0}^{n^{\overrightarrow{w}}} \overrightarrow{\mathbf{W}}_k L^k,$$

where $\left(n^{\overrightarrow{u}}, n^{\overrightarrow{v}}, n^{\overrightarrow{w}}\right) \in \mathbb{N}^3$ and all $\overrightarrow{\mathbf{U}}_k$, $\overrightarrow{\mathbf{V}}_k$, $\overrightarrow{\mathbf{W}}_k$ have real numbers as elements. For all $(i,j) \in \{1, ..., N\}^2$ such that $i \leq j$, let $\zeta_{i,j}$ be defined by $\zeta_{i,j} = 1$ if $\forall k \in \{i, ..., j\}$, $m_k^a = 0$ and $\zeta_{i,j} = 0$ otherwise. We then have

$$\overrightarrow{\mathbf{U}}(0) = \begin{bmatrix} \zeta_{2,N} & 1 & \zeta_{2,2} & \cdots & \zeta_{2,N-1} \\ \vdots & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \zeta_{N-1,N-1} \\ \zeta_{N,N} & 0 & \cdots & 0 & 1 \\ 1 & 0 & \cdots & \cdots & 0 \end{bmatrix} \mathbf{U}(0),$$

so that since $\mathbf{U}(0) = \boldsymbol{\Omega}$ is invertible, $\overrightarrow{\mathbf{U}}(0)$ is invertible as well. In this case, the same reasoning as the one conducted at the end of appendix C, this time using $\overrightarrow{\mathbf{U}}(L)$, $\overrightarrow{\mathbf{V}}(L)$ and $\overrightarrow{\mathbf{W}}(L)$ instead of $\mathbf{U}(L)$, $\mathbf{V}(L)$ and $\mathbf{W}(L)$, leads to a system of the form

$$E_t\left\{\boldsymbol{\Lambda}_1(L)\begin{bmatrix} \mathbf{Y}_t \\ z_t \end{bmatrix} + \boldsymbol{\Lambda}_2(L)\,\boldsymbol{\xi}_t\right\} = \mathbf{0} \tag{31}$$

$$\text{with } \underset{(N+1)\times(N+1)}{\boldsymbol{\Lambda}_1(L)} \equiv \sum_{k=0}^{n^{\lambda_1}} \boldsymbol{\Lambda}_{1,k} L^k \text{ and } \underset{(N+1)\times N}{\boldsymbol{\Lambda}_2(L)} \equiv \sum_{k=0}^{n^{\lambda_2}} \boldsymbol{\Lambda}_{2,k} L^k,$$

where $\left(n^{\lambda_1}, n^{\lambda_2}\right) \in \mathbb{N}^2$, all $\mathbf{\Lambda}_{1,k}$, $\mathbf{\Lambda}_{2,k}$ have real numbers as elements, $\mathbf{\Lambda}_{1,0}$ is invertible and all eigenvalues of $\mathbf{\Lambda}_1(L)$ are of modulus strictly lower than one. Since (31) is equivalent to the system made of (S) and (R), (29) is the unique solution of (31).

Similarly, let us follow the same reasoning as the one conducted at the beginning of appendix C, this time starting from $(\widetilde{E})$ instead of $(\overrightarrow{1})$ and using $(\widetilde{R})$ instead of (R), to get equations $(\widetilde{\overrightarrow{2}})$ to $(\widetilde{\overrightarrow{N}})$ corresponding to equations $(\overrightarrow{2})$ to $(\overrightarrow{N})$ in appendix C. Equations $(\widetilde{E})$ and $(\widetilde{\overrightarrow{k}})$ for $k \in \{2, ..., N\}$ can then be re-written in the form

$$E_t \left\{ \widetilde{\overrightarrow{\mathbf{U}}}(L)\, \mathbf{Y}_t + \widetilde{\overrightarrow{\mathbf{V}}}(L)\, z_t + \widetilde{\overrightarrow{\mathbf{W}}}(L)\, \boldsymbol{\xi}_t \right\} = \mathbf{0}$$

$$\text{with } \underset{(N\times N)}{\widetilde{\overrightarrow{\mathbf{U}}}(L)} \equiv \sum_{k=-m^{\widetilde{\overrightarrow{u}}}}^{n^{\widetilde{\overrightarrow{u}}}} \widetilde{\overrightarrow{\mathbf{U}}}_k L^k,\ \underset{(N\times 1)}{\widetilde{\overrightarrow{\mathbf{V}}}(L)} \equiv \sum_{k=-m^{\widetilde{\overrightarrow{v}}}}^{n^{\widetilde{\overrightarrow{v}}}} \widetilde{\overrightarrow{\mathbf{V}}}_k L^k \text{ and } \underset{(N\times N)}{\widetilde{\overrightarrow{\mathbf{W}}}(L)} \equiv \sum_{k=-m^{\widetilde{\overrightarrow{w}}}}^{n^{\widetilde{\overrightarrow{w}}}} \widetilde{\overrightarrow{\mathbf{W}}}_k L^k,$$

where $(m^{\widetilde{\overrightarrow{u}}}, m^{\widetilde{\overrightarrow{v}}}, m^{\widetilde{\overrightarrow{w}}}, n^{\widetilde{\overrightarrow{u}}}, n^{\widetilde{\overrightarrow{v}}}, n^{\widetilde{\overrightarrow{w}}}) \in \mathbb{N}^6$, all $\widetilde{\overrightarrow{\mathbf{U}}}_k$, $\widetilde{\overrightarrow{\mathbf{V}}}_k$, $\widetilde{\overrightarrow{\mathbf{W}}}_k$ have real numbers as elements and $(\widetilde{\overrightarrow{\mathbf{U}}}(L), \widetilde{\overrightarrow{\mathbf{V}}}(L), \widetilde{\overrightarrow{\mathbf{W}}}(L)) \longrightarrow (\overrightarrow{\mathbf{U}}(L), \overrightarrow{\mathbf{V}}(L), \overrightarrow{\mathbf{W}}(L))$ as $\varepsilon \longrightarrow 0$. Given that $\widetilde{\overrightarrow{\mathbf{U}}}(0) \longrightarrow \overrightarrow{\mathbf{U}}(0)$ as $\varepsilon \longrightarrow 0$, $\widetilde{\overrightarrow{\mathbf{U}}}(0)$ is invertible for ε sufficiently small, so that the same reasoning as the one conducted at the end of appendix C leads to a system of the form

$$E_t \left\{ \widetilde{\mathbf{\Lambda}}_1(L) \begin{bmatrix} \mathbf{Y}_t \\ z_t \end{bmatrix} + \widetilde{\mathbf{\Lambda}}_2(L)\, \boldsymbol{\xi}_t \right\} = \mathbf{0} \tag{32}$$

$$\text{with } \underset{(N+1)\times(N+1)}{\widetilde{\mathbf{\Lambda}}_1(L)} \equiv \sum_{k=-m^{\widetilde{\lambda}_1}}^{n^{\widetilde{\lambda}_1}} \widetilde{\mathbf{\Lambda}}_{1,k} L^k \text{ and } \underset{(N+1)\times N}{\widetilde{\mathbf{\Lambda}}_2(L)} \equiv \sum_{k=-m^{\widetilde{\lambda}_2}}^{n^{\widetilde{\lambda}_2}} \widetilde{\mathbf{\Lambda}}_{2,k} L^k,$$

where $\left(m^{\widetilde{\lambda}_1}, m^{\widetilde{\lambda}_2}, n^{\widetilde{\lambda}_1}, n^{\widetilde{\lambda}_2}\right) \in \mathbb{N}^4$, all $\widetilde{\mathbf{\Lambda}}_{1,k}$, $\widetilde{\mathbf{\Lambda}}_{2,k}$ have real numbers as elements and $(\widetilde{\mathbf{\Lambda}}_1(L), \widetilde{\mathbf{\Lambda}}_2(L)) \longrightarrow (\mathbf{\Lambda}_1(L), \mathbf{\Lambda}_2(L))$ as $\varepsilon \longrightarrow 0$. Since (32) is implied by the system made of (S) and $(\widetilde{R})$, (30) is one solution of (32).

Substep 3.2: let us consider a given sequence of $\left(\widetilde{\mathbf{A}}(L), \widetilde{\mathbf{B}}(L), \widetilde{\mathbf{C}}(L), \widetilde{\mathbf{D}}(L)\right)$ converging towards $(\mathbf{A}(L), \mathbf{B}(L), \mathbf{C}(L), \mathbf{D}(L))$. This sequence corresponds to a unique sequence of ε converging towards zero and a unique sequence of $\widetilde{\mathbf{J}}(L)$. If $\widetilde{\mathbf{J}}_0$ did not converge towards $\mathbf{J}_0$ along this sequence of $\widetilde{\mathbf{J}}(L)$, then there would exist a strictly positive real number θ_0 and an extracted sequence of $\left(\widetilde{\mathbf{A}}(L), \widetilde{\mathbf{B}}(L), \widetilde{\mathbf{C}}(L), \widetilde{\mathbf{D}}(L)\right)$ such that $\left\|\widetilde{\mathbf{J}}_0 - \mathbf{J}_0\right\| \geq \theta_0$ for every element of the corresponding extracted sequence of $\widetilde{\mathbf{J}}(L)$, where $\|.\|$ denotes a given norm on matrices. From (31) and (32) it is easy to see, but tedious to show formally, that for any element of this extracted sequence of $\left(\widetilde{\mathbf{A}}(L), \widetilde{\mathbf{B}}(L), \widetilde{\mathbf{C}}(L), \widetilde{\mathbf{D}}(L)\right)$ sufficiently close to $(\mathbf{A}(L), \mathbf{B}(L), \mathbf{C}(L), \mathbf{D}(L))$ there would then exist a strictly increasing sequence extracted from the sequence $\left(\left\|\widetilde{\mathbf{J}}_k\right\|\right)_{k\in\mathbb{N}}$ corresponding to this element, which is impossible given that $\widetilde{\mathbf{J}}_k \longrightarrow \mathbf{0}$ as $k \longrightarrow +\infty$, so that we conclude that $\widetilde{\mathbf{J}}_0 \longrightarrow \mathbf{J}_0$ along the sequence of $\widetilde{\mathbf{J}}(L)$ considered. By the same reasoning we obtain that $\forall k \in \mathbb{N}$, if $\left(\widetilde{\mathbf{J}}_0, ..., \widetilde{\mathbf{J}}_k\right) \longrightarrow (\mathbf{J}_0, ..., \mathbf{J}_k)$ along the sequence of $\widetilde{\mathbf{J}}(L)$ considered then $\widetilde{\mathbf{J}}_{k+1} \longrightarrow \mathbf{J}_{k+1}$ along this sequence. By recurrence on $k \in \mathbb{N}$ we therefore conclude that $\forall k \in \mathbb{N}$, $\widetilde{\mathbf{J}}_k \longrightarrow \mathbf{J}_k$ along this sequence. Given that there exists $(\overline{p}, \overline{q}) \in \mathbb{N}^2$ such that every element of the sequence of $\widetilde{\mathbf{J}}(L)$ considered is the Wold form of a VARMA(p, q) process with $p \leq \overline{p}$ and $q \leq \overline{q}$, as implied by Blanchard and Kahn's (1980) results in our context, this "simply continuous" convergence ($\forall k \in \mathbb{N}$, $\widetilde{\mathbf{J}}_k \longrightarrow \mathbf{J}_k$) implies in turn the "absolutely continuous" convergence $\widetilde{\mathbf{J}}(L) \longrightarrow \mathbf{J}(L)$ along the sequence of $\widetilde{\mathbf{J}}(L)$ considered.

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