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Abstract

This paper models the relationship between growth and volatility for G7 economies in the time period 1960-2009. It delivers for the first time estimates of this relationship based on a logarithm variant of stochastic volatility in mean (SV-M) models. The relationship appears significantly positive in Germany and Italy, but insignificant in other countries. We also show that output volatility has increased in all countries since the beginning of the financial crisis, which illustrates the end of the great moderation. For comparison, the paper also delivers estimates based on a logarithm variant of GARCH in mean (log-GARCH-M) models, the class of time series models previously used in the literature to estimate the growth-volatility relationship. We show that SV-M models deliver results preferable to those of log-GARCH-M models, despite the high computational cost of their estimation. SV-M models fit generally better data than log-GARCH-M ones. As their residuals do not violate distribution assumptions, they do not deliver dubious conclusions concerning the significance of the relationship, which is the case of the log-GARCH-model for France, the UK and the US. Finally, SV-M models suggest a positive impact of unexpected volatility on output growth, which is not taken into account by log-GARCH-M models.
Keywords: growth, volatility, sequential Monte-Carlo methods.
JEL code: O40, E32, C15.

Résumé

Cet article modélise la relation entre la croissance et la volatilité des économies du G7 au cours de la période 1960-2009. Il fournit pour la première fois des estimations de cette relation fondée sur une variante en logarithme des modèles à volatilité stochastique en moyenne (SV-M). La relation s'avère significativement positive pour l'Allemagne et l'Italie, mais non significative pour les autres pays. La volatilité du PIB a augmenté dans tous les pays depuis le début de la crise, ce qui illustre la fin de la grande modération. L'article fournit aussi, à titre de comparaison, des estimations fondées sur une variante en logarithme des modèles GARCH en moyenne (log-GARCH-M). Ceux-ci étaient utilisés jusqu'à présent dans la littérature pour estimer la relation volatilité-croissance. Les modèles SV-M fournissent des résultats préférables à ceux des log-GARCH-M, en dépit d'un temps de calcul plus important pour mener à bien les estimations. Ils témoignent d'abord d'une meilleure adéquation aux données et respectent mieux les hypothèses concernant les distributions des résidus, ce qui évite de tirer des conclusions infondées sur la significativité de la relation. Enfin, les modèles SV-M suggèrent que la volatilité non anticipée aurait un impact positif sur la croissance du PIB, impact qui n'est pas pris en compte par les modèles log-GARCH-M.
Mots-clés: croissance, volatilité, méthodes de Monte-Carlo séquentielles.
Classification JEL: O40, E32, C15.

Introduction

The deep recession caused by the 2007-2008 financial crisis seems to end the period of the Great Moderation. This large decline of output volatility occurred in the mid-1980s, as shown for the US by McConnell and Perez-Quiros (2000) and Blanchard and Simon (2001). This decline also occurred for most of other G7 economies (Stock and Watson 2005). Three explanations have been suggested for the moderation: good monetary policy (good policies), improved inventory management (good practices) or a decline in the volatility of exogenous shocks (good luck). A frequency-domain analysis, carried out by Ahmed et al. (2004), concluded that good luck was the most likely explanation. Recent shocks associated to the crisis confirm this conclusion. Canarella et al. (2008) find preliminary evidence that US and UK economies have entered in a new regime of high volatility. Such a high volatility has put foreground the debate about the relationship between growth and volatility. Indeed, this issue plays a crucial role in assessing the usefulness of short-run stabilization policies (see for example Blackburn 1999 or Aghion et al. 2009).

The aim of our paper is to determine the empirical relationship between output volatility and output growth. We propose a new empirical approach based on the stochastic volatility in mean (SV-M) model, the SV counterpart of GARCH-M models. SV-M models incorporate output volatility as an explanatory variable of output growth. As we do not have an analytical expression of their likelihood, we estimate these models with Sequential Monte-Carlo (SMC) methods detailed in Doucet et al. (2001).

As far as we know, the SV-based approach adopted here has not been carried out for studying the relationship between output volatility and output growth. Following Caporale and McKiernan (1998), time series studies of this relationship are generally based on GARCH in mean (GARCH-M) models. The SV-M model, created by Koopman and Uspensky (2002), is applied in their paper to international stock index returns. Berument et al. (2009) applied this model to measure the effect of inflation uncertainty on inflation.

Here, we estimate a logarithm variant of SV-M models of G7 output series in the time period 1960-2009. We relate the growth to the log-volatility instead of the volatility itself, as the skewness of output growth is low. This avoids the problem of autocorrelation of residuals encountered in Koopman and Uspensky (2002). We find a significantly positive relationship for Germany and Italy and insignificant relations in other countries. For the sake of comparison, we also estimate a logarithm variant of GARCH-M models (log-GARCH-M), which were previously applied in the literature. Our results appear preferable relative to those of log-GARCH-M models for three reasons. SV-M models fit generally better data than log-GARCH-M models. As their residuals do not violate distribution assumptions, they do not deliver dubious conclusions concerning the significance of the relationship, which is the case of the log-GARCH-M model for France, the UK and the US. Finally, SV-M models suggest a positive impact of unexpected volatility on output growth, which is not taken into account by log-GARCH-M models.

The paper proceeds as follows. In the first section, we survey the theoretical debate and the previous empirical evidence about the relationship between the output volatility and the long-run growth. In the second section, we present our SV-M model, in comparison with the log-GARCH-M model, and the estimation methodology. In a third section, we comment our empirical results. In a last section, we conclude.

1. Volatility and growth

1.1. **The theoretical debate.** Firstly, output volatility might have two opposite effects on investment behavior. On one hand, a high uncertainty should decrease investment, because such expenditures are irreversible and can be delayed (Pindyck 1991). This effect is reinforced, when the risk increases with time (Hassler 1996; Bloom et al. 2009). On the other hand, Cooper et al. (1999) show that, when shocks are positively serially correlated and adjustment costs are fixed, replacement investment is procyclical.

Then, the impact on growth can be analyzed, by incorporating the investment behavior in endogenous growth models. For example, with convex adjustment costs of the capital stock, the long-term growth of a AK model should be negatively correlated to output volatility (Barlevy 2004). Without adjustment costs, Jones et al. (2005) show that this relationship is negative, only if the risk aversion of households is lower than one. In the opposite case, an increase in volatility raises the average growth rate with the saving rate because of a precautionary comportment.

Finally, following the Schumpeterian view, some endogenous growth models underline the role of productivity improving activities (PIA). Aghion and Saint-Paul (1998) show that recessions should have a cleansing effect, i.e. an incentive to reorganize and innovate. The opportunity cost of long-term productivity enhancing investments, relative to short-term ones, is lower during recessions than during expansions and their share in total investment should thus be countercyclical. However, credit market imperfections hamper innovation and reorganization during recessions (Aghion et al. 2005). Credit constraints have an asymmetrical effect: they imply a reduction of R&D expenditures during recessions, but no augmentation during expansions. Moreover, this effect is stronger for R&D expenditures than physical investments because such expenditures have a positive return at a longer term.

1.2. **The previous empirical evidence.** If the theoretical debate has not determined what should be the sign of this growth-volatility relationship, empirical evidence is also mixed and divided between studies based on panel datasets and those based on time series.

Ramey and Ramey (1995), the reference paper in the literature on this subject, find a negative relationship for 92 countries in the period 1960-1985, which is robust to various controls[1]. They relate output growth $y_{i,t}$ to control variables $X_{i,t}$ and to the standard deviation σ_i of the residuals, by estimating the following equation:

$$y_{i,t} = \delta\sigma_i + \theta X_{i,t} + \sigma_i \varepsilon_{i,t}$$

where $\varepsilon_{i,t} \sim \mathcal{NID}(0,1)$. On the contrary, they find a positive effect of the volatility of expected growth, when they distinguish it from the volatility of innovations. This point would explain the opposite result found

[1]Although some earlier papers like Kormendi and Meguire (1985) found the opposite result, this result has then been confirmed by Martin and Rogers (2000), Kneller and Young (2001) and Rafferty (2005).

previously in Kormendi and Meguire (1985), where the standard deviation of monetary shocks, correlated with output innovations, might capture the negative effect.

However, this empirical approach based on panel datasets relies on a rough description concerning the evolution of output volatility. This one is generally measured as the standard deviation of the regression residuals in the whole time period or in fixed-length windows and the cross-section dimension is in this case more important than the time dimension. Thus, the heterogeneity of the countries included in the sample might affect these results. For example, Imbs (2007) has shown with a sectoral dataset covering 47 countries that a positive relationship at the sectoral level is hidden by a negative relationship at the aggregated level.

Some papers tried to deal with this heterogeneity issue, by modeling the growth-volatility relationship with GARCH-M models estimated on time series datasets. In these models, a country's expected output growth linearly depends on the conditional volatility and the conditional volatility follows an ARMA process, whose innovations are the square of the innovations of output growth. As far as we know, this model was applied to the estimation of the growth-volatility relationship for the first time by Caporale and McKiernan (1996). They modelled the industrial production (IP) of the United Kingdom in the time period 1948.01-1991.09. This paper finds a positive effect, but results are mixed in other following applications of GARCH-M models to GDP (or IP) time series[2].

2. Modeling the growth-volatility relationship

After a brief description of the SV-M model applied here, we compare it to a log variant of GARCH-M models which were previously applied to the same issue. Then, we explain the methodology chosen here for the estimation of the SV-M model.

2.1. **SV-M models.** We consider the following SV-M model, which is very close to the model proposed in Koopman and Uspensky (2002):

$$\left\{ \begin{array}{lcl} y_t & = & c + \sum_{i=1}^{p} \alpha_i y_{t-i} + \delta h_t + \sigma^* \exp\left(\frac{h_t}{2}\right) \varepsilon_t \\ h_t & = & \phi h_{t-1} + \eta_t \end{array} \right. \tag{2.1}$$

with two independent noises $\varepsilon_t \sim \mathcal{NID}(0,1)$ and $\eta_t \sim \mathcal{NID}(0,\sigma_\eta^2)$. The mean equation of output growth y_t contains lagged terms, the log-volatility h_t and the innovation, equal to a standardised noise ε_t times the squared root of the volatility $\exp\left(\frac{h_t}{2}\right)$ and the volatility level σ^*. The parameter ϕ is constrained to lie in the interval]-1;1[, in order to model the log-volatility as a stationary AR(1) process.

The only difference with the model of Koopman and Uspensky (2002) consists in including the log-volatility in the mean, instead of the volatility itself. In both models, the log-volatility is assumed Gaussian and centered. Thus, the volatility of output growth has a high skewness, i.e. a high asymetry caused by the the zero bound. As the skewness of output growth is quite low (e.g. -0.20 for the US), a relation to the log-volatility seems likelier

[2]For the US, Caporale and McKiernan (1998) and Fountas and Karanasos (2006) estimated a positive effect with annual data in a time period which starts in the nineteenth century. However, if the model takes into account the great moderation with a structural break, the effect is not significant (Fang and Miller 2008).

than a relation to the volatility itself. If the true model includes the log-volatility, estimating a misspecified model with the volatility shoud cause autocorrelation of residuals. This is the case in Koopman and Uspensky (2002): Box-Ljung statistics exceed their critical values in most estimated models. As shown in section 4, this problem seems to be solved with our specification, as residuals do not show significant auto-correlation.

In a reduced form, if we define the innovations by $u_t = \sigma^* \exp\left(\frac{h_t}{2}\right)\varepsilon_t$, the logarithm of their square follows an ARMA(1,1) process:

$$\log(u_t^2) = (1-\phi)\log(\sigma^{*2}) + \phi\log(u_{t-1}^2) + \left(\eta_t + \log(\varepsilon_t^2) - \phi\log(\varepsilon_{t-1}^2)\right)$$

because the last term is a non-Gaussian MA(1) process.

Finally, the expected growth $y_{t|t-1}$ depends on the expected log-volatility, called here $h_{t|t-1}$ and equal to ϕh_{t-1}:

$$y_{t|t-1} = c + \sum_{i=1}^{p}\alpha_i y_{t-i} + \delta h_{t|t-1},$$

but output growth also depends on the unexpected volatility $(h_t - h_{t|t-1})$ which is equal to the innovation η_t:

$$y_t = c + \sum_{i=1}^{p}\alpha_i y_{t-i} + \delta h_{t|t-1} + \delta\left(h_t - h_{t|t-1}\right) + \sigma^*\exp\left(\frac{h_t}{2}\right)\varepsilon_t.$$

2.2. **Comparison with log-GARCH-M models.** The previous SV-M model has a specification close to the following log-GARCH(1,1)-M model:

$$\begin{cases} y_t &= c + \sum_{i=1}^{p}\alpha_i y_{t-i} + \delta\log(\sigma_t^2) + u_t \\ \log(\sigma_t^2) &= \omega + \xi\log(\sigma_{t-1}^2) + \psi\log(u_{t-1}^2) \end{cases}$$

with σ_t^2 the conditional volatility of the innovation u_t. Such a log variant of the GARCH model was proposed without any in-mean effect in Geweke (1986).

For the sake of comparison with the SV-M model, this model is rewritten with standardised residuals $\varepsilon_t \sim \mathcal{NID}(0,1)$ and a centered log-volatility h_t:

$$\begin{cases} y_t &= c + \sum_{i=1}^{p}\alpha_i y_{t-i} + \delta h_t + \sigma^*\exp\left(\frac{h_t}{2}\right)\varepsilon_t \\ h_t &= \phi h_{t-1} + \psi\eta_{t-1} \end{cases}$$

with $\phi = \xi + \psi$ and $\eta_t = \log(\varepsilon_t^2) - E\left[\log(\varepsilon_t^2)\right]$. The mean of $\log(\varepsilon_t^2)$ is known to be approximately equal to -1.27. The stationarity of the log-volatility requires the persistence parameter to be lower than one in absolute value ($|\phi| < 1$). This log-GARCH-M model has the same mean equation as the previous SV-M model with the same independent variables : the lags of the dependent variable and the log-volatility of the innovation.

Importantly, while the log-volatility is modeled in the SV-M as an AR(1) process with a second independent innovation, the log-volatility is in the log-GARCH-M model an expected log-volatility based on the residuals of the mean equation.

As in the SV-M model, the logarithm of the squared innovations follows an ARMA(1,1) process:

$$\log(u_t^2) = (1-\phi)\log(\sigma^{*2}) + \phi\log(u_{t-1}^2) + \left(\log(\varepsilon_t^2) + (\psi-\phi)\log(\varepsilon_{t-1}^2)\right)$$

because the last term is also a non-Gaussian MA(1) process.

Finally, the nature of output volatility is different in both models. In the log-GARCH(1,1)-M model, the expected output growth $y_{t|t-1}$ depends on the expected log-volatility:

$$y_{t|t-1} = c + \sum_{i=1}^{p} \alpha_i y_{t-i} + \delta h_t.$$

and there is no unexpected log-volatility. Thus, an important advantage of SV-M relative to GARCH-M models consists in relating output growth to both components of the volatility, through the coefficient δ.

3. Estimation methodology

Both GARCH and SV are non-linear models. In log-GARCH-M models, log-volatility is directly observed given a set of parameters θ and past observations $y_1, \ldots y_{t-1}$ (we will denote this $y_{0:t-1}$ in a more condensed way). Thus, likelihood has an analytical expression available and can be directly maximized with numerical methods, at a low computational cost. This is not the case for SV-M models, where parameters and volatility have to be estimated and where likelihood has no analytical form.

To handle this, simulation techniques allow to simulate the posterior distribution of the volatility and its minimum mean squared error (MMSE) estimate. The maximisation of the likelihood based on the estimated volatility delivers parameters estimates, but this maximisation has a high time cost. Monte-Carlo Markov chain (MCMC) or sequential Monte-Carlo (SMC) methods, described respectively in Robert and Casella (2004) and Doucet et al. (2001), allow to sample from posterior distributions. Here, we choose the second approach, which is better suited to our recursive problem. This approach has already been applied by Fernandez-Villaverde and Rubio-Ramirez (2005) to the estimation of state variables and parameters of dynamic stochastic general equilibrium (DSGE) models. Our estimation methodology differs from theirs for the estimation of parameters.

3.1. **Filtered estimation of the states.** State space models are characterized by a measurement equation, where the observed variables are expressed as a function of the contemporaneous state variables, and a transition equation, that expresses the dynamics of the state variables. The SV-M model as specified in equation 2.1 is already written in a state-space form.

SMC methods are based on the Bayes law and use iterative algorithms, that take advantage of this description. Their aim is to simulate sequentially according to the *filtered* distribution of the unobserved variable, i.e. the conditional distribution of h_t given past observations $y_{0:t-1}$. The sampling with importance re-sampling (SIR) algorithm that we use, proceeds sequentially for all dates t in the two following steps:

(i) Suppose a sample $(h^i_{t|t-1})_{i=1..N}$ distributed according the conditional law of h_t given past observations is available (we will denote it $h^{1:N}_{t|t-1}$). Using observation y_t and measurement equation, the sample is re-sampled into $h^{1:N}_{t|t}$, according to measurement likelihood weights given by [3]

[3]In this part, we denote $p(a|b)$ the conditional probability density function of a given b for all random variables a and b.

$$\begin{aligned}\ell_t(h_t, y_{0:t};\theta) &\triangleq p(y_t|h_t, y_{t-p:t-1};\theta)\\ &= \frac{1}{\sqrt{2\pi}\sigma^* \exp\left(\frac{h_t}{2}\right)}\exp\left(-\frac{(y_t - c - \sum_{i=1}^{p}\alpha_i y_{t-i} - \delta h_t)^2}{2(\sigma^*)^2\exp(h_t)}\right)\end{aligned}$$

with $\theta = (c, \alpha_1, ..., \alpha_p, \delta, \sigma^*, \phi, \sigma_\eta)'$ the vector of parameters. This resampling step generates a sample distributed according to the conditional law of h_t given past and present observations. The SIR algorithm cannot be executed without the existence of these densities.

(ii) Then, some transition noises are drawn according to the law of η_t specified in the model. They are used together with the sample $h_{t|t}^{1:N}$ and the transition equation to generate a sample $h_{t+1|t}^{1:N}$ distributed according to the conditional law of h_{t+1} given past observations.

To initialize at date $t = 0$, N particles $h_{0|-1}^{1:N}$ are drawn from an *a priori* initial law of the unobserved log-volatility h_0. Here, we choose the uniform law $\mathcal{U}([-1;1])$[4]. Then operations (i) and (ii) are sequentially repeated for all dates $t > 0$, till the last observation y_T.

The generated samples are used to get filtered estimates of the unobserved variables, since the MMSE filtered estimate of h_t is given by

$$\mathbb{E}[h_t|y_{0:t};\theta] \approx \frac{1}{N}\sum_{i=1}^{N} h_{t|t}^i$$

3.2. **Smoothed estimation of the states.** In the previous subsection, we only explain how we estimate unobserved variables given past and current observations. The estimate given all observations, including future ones, is called *smoothed* estimate and is computed from an extra smoothing algorithm. Here, we implement the non-linear forward-backward smoother of Godsill et al. (2004), that consists in re-weighting the previous particles into $h_{t|T}^{1:N}$, using the transition likelihood, given by

$$p(h_t|h_{t-1};\theta) = \frac{1}{\sqrt{2\pi}\sigma_\eta}\exp\left(-\frac{(h_t - \phi h_{t-1})^2}{2\sigma_\eta^2}\right)$$

The existence of these densities is another necessary assumption, for performing smoothed estimation. MMSE smoothed estimates of h_t, given by $\mathbb{E}[h_t|y_{0:T};\theta]$, are again computed as the empirical mean of smoothed samples.

The 95% confidence intervals of both filtered and smoothed estimates are directly computed from the samples distributed according to the corresponding distributions.

3.3. **Maximum likelihood estimation of parameters.** We use maximum likelihood inference to estimate the parameters. SIR algorithm described previously provides a way to compute the likelihood $\mathcal{L}(\theta)$ of the model,

[4]As in any Bayesian inference, this choice has an impact on the simulated sample $h_{t|t}^{1:N}$, but this impact decrease quickly with the date t.

applying Monte Carlo approximation, since

$$
\begin{aligned}
(3.1) \qquad \mathcal{L}(\theta) &= \mathbb{E}_h[\ell_0(h_0, y_0, \theta)|\theta] \prod_{t=1}^{T} \mathbb{E}_h[\ell_t(h_t, y_{0:t}, \theta)|y_{0:t-1}; \theta] \\
&\approx \prod_{t=0}^{T} \frac{1}{N} \sum_{i=1}^{N} \ell_t(h^i_{t|t-1}, y_{0:t}; \theta)
\end{aligned}
$$

However, this computation of likelihood is very noisy, due to Monte Carlo approximation errors. This makes usual gradient maximization algorithms inefficient.

To circumvent this issue, we maximize an approximated likelihood $\mathcal{L}(\theta, \theta_0)$ using a unique filtered or smoothed sample of particles computed with a unique parameter θ_0. This technique, developped in Hürzeler and Künsch (2001), provides a smooth approximate of the likelihood which we can maximize with usual gradient maximization algorithms. This method has two advantages relative to simulated annealing, used for example in Fernandez-Villaverde and Rubio-Ramirez (2005). It does not require to calibrate technical parameters, e.g. the speed of cooling, and it delivers standard errors of parameters estimates.

The filter-based method relies on the importance sampling principle. Importance weights are defined as follows,

$$
\pi_t(h_t, \theta, \theta_0) = \frac{p(h_t|y_{0:t-1}; \theta)}{p(h_t|y_{0:t-1}; \theta_0)}
$$

Then, the likelihood can be computed from equation (3.1), using

$$
\begin{aligned}
\mathbb{E}_h[\ell_t(h_t, y_{0:t}, \theta)|y_{0:t-1}; \theta] &= \mathbb{E}_h[\ell_t(h_t, y_{0:t}, \theta_0)\pi_t(h_t, \theta, \theta_0)|y_{0:t-1}; \theta_0] \\
&\approx \frac{1}{N} \sum_{i=1}^{N} \ell_t(h^i_{t|t-1}, y_{0:t}, \theta_0)\, \pi_t(h^i_{t|t-1}, \theta, \theta_0)
\end{aligned}
$$

It is also possible to approximate likelihood using a smoothed sample, defining importance weights as

$$
\pi^{0:T}(h_{0:T}, \theta, \theta_0) = \frac{p(h_{0:T}, y_{0:T}; \theta)}{p(h_{0:T}, y_{0:T}; \theta_0)}
$$

Then the following identity can be used

$$
\begin{aligned}
\mathcal{L}(\theta) &= \mathcal{L}(\theta_0)\, \mathbb{E}_h\left[\pi^{0:T}(h_{0:T}, \theta, \theta_0)\middle|\, y_{0:T}; \theta_0\right] \\
&\approx \mathcal{L}(\theta_0)\, \frac{1}{N} \sum_{i=1}^{N} \pi^{0:T}(h^i_{0:T|T}, \theta, \theta_0)
\end{aligned}
$$

The whole computation methodoly for π_t and $\pi^{0:T}$ is explained in Hürzeler and Künsch (2001).

This method enables in both versions to remove noises due to multiple samples computations for different θ. It allows then to use classical optimization tools. Here we use sequential quadratic programming, a variant of Newton's method for contrained maximization problems.

However, approximation using importance sampling is not accurate for θ too far from θ_0. So we need to make θ_0 converging toward the optimum, in order to get a good approximation of the likelihood at the optimum. In practice, this leads to use an iterative scheme with two steps:

(i) Maximization of the importance sampling approximated likelihood

(ii) Update of θ_0 with the maximum computed in (i).

This converges to the maximum of the exact likelihood. Then, standard errors of parameters estimates are computed from the inverse of the Hessian matrix at the optimum.

Computation with filtered samples is a global approximation of the likelihood. However it has a high $O(N^2)$ complexity. On the contrary, computation with smoothed samples is $O(N)$, but it is only a local approximation. So in the first global iteration, we maximize filter based computed likelihood. This enables to find a first maximum, much better than the arbitrary initial condition. Then we switch to smoother based computation, which converges much faster.

However, the global convergence issue remains, since we still deal with an approximated likelihood. The only solution to increase precision, is to rise the number N of particles. In practice, those computations are very time consuming, so we use $N = 500$ particles in the first iteration and $N = 2000$ in the following ones. Running those algorithms with Matlab on an AMD Athlon Dual Core Processor (2.59 GHz), a filtering step is achieved in 1 second, a smoothing step is achieved in 2 minutes and about 2 hours are needed to perform a full estimation of the model, with in average 15 global iterations and 150 likelihood computations in each iteration.

4. Empirical results

In this section, SV-M models are applied to the output of G7 countries and their results are compared to those of log-GARCH-M models. The estimation of parameters should give information about the sign of the growth-volatility relationship. The estimation of the log-volatility should help to describe the end of the great moderation.

4.1. **Data.** We consider quarterly GDP growth rate series of G7 countries (Germany, France, Italy, UK, US, Japan and Canada). Time series start in 1960:Q2, except for Germany for which quarterly data are available only from 1968:Q1 and for Canada for which quarterly data are available only from 1961:Q2. They all finish in 2009:Q2. Thus, last observations are preliminary estimates, which should be revised in the future, and results should be cautiously interpreted at the end of the sample. The series come from the Eurostat national accounts, except for American, Japanese and Canadian series which come from OECD main economic indicators database. They are seasonally adjusted and expressed in the value of their national currency, using chained-price indexes. They have been retropolated using historical series of the OECD business statistics database (BSDB).

4.2. **Specification tests.** The number of lags in the mean equation is determined for SV-M and log-GARCH-M models by minimising the Schwarz information criterion (SIC). We also impose that our residuals should be independent, as they are specified to be i.i.d in the model. When the Q statistic of order 12 is too high, the Ljung-Box test rejects the independence of residuals and the number of lags is increased until the independence of residuals is accepted by the test. As expected in section 2, our specification with the log-volatility in the mean equation, instead of the volatility itself (specification of Koopman and Uspensky 2002), provides satisfactory results concerning the autocorrelation of residuals. Table 1 and 2 show that we cannot reject the null hypothesis of no autocorrelation for all countries either for SV-M and log-GARCH-M models.

We also check for the normality of the residuals. For all SV-M models, the Jarque-Bera statistic indicates that the null hypothesis cannot be rejected at the 5% level. On the contrary, the null hypothesis is rejected for France and UK with log-GARCH-M models. This might be due to fat tail effects. Using a Student's t-distribution for residuals could take this effect into account, but such a variant of the log-GARCH-M model would be less parsimonious than the SV-M model.

Finally, we find a better fit (higher log-likelihood and lower SIC) of SV-M models relative to log-GARCH-M ones. Such a result was also found by Koopman and Uspensky (2002) with financial time series. Kim et al. (1998) had previously shown with Monte-Carlo tests that SV models have a fit significantly better than that of GARCH models, but insignificantly different from that of less parsimonious t-GARCH models.

4.3. **Parameters estimates.** Table 1 reports the SV-M estimations results for the seven countries in the full sample period.

Table 1. Estimation results for the SV-M model

	de	fr	it	uk	us	jp	ca
Mean equation							
c	0.54	0.36	0.41	0.24	0.48	0.32	0.31
	0.12	*0.08*	*0.12*	*0.11*	*0.09*	*0.13*	*0.10*
α_1	0.11	0.25	0.46	0.10	0.28	0.24	0.44
	0.08	*0.07*	*0.07*	*0.07*	*0.07*	*0.07*	*0.08*
α_2		0.24	0.11	0.11	0.23	0.15	0.00
		0.07	*0.08*	*0.07*	*0.08*	*0.07*	*0.08*
α_3			-0.03	0.19	-0.10	0.28	0.14
			0.08	*0.07*	*0.07*	*0.07*	*0.07*
α_4			-0.12	0.11			
			0.07	*0.08*			
δ	0.21	0.08	0.16	-0.07	-0.02	0.08	-0.02
	0.08	*0.05*	*0.07*	*0.04*	*0.07*	*0.17*	*0.06*
Volatility equation							
$\ln \sigma^*$	-0.07	-0.58	-0.29	-0.40	-0.40	-0.03	-0.35
	0.15	*0.08*	*0.12*	*0.09*	*0.12*	*0.10*	*0.06*
φ	0.93	0.95	0.95	0.93	0.94	0.82	0.94
	0.04	*0.02*	*0.03*	*0.03*	*0.03*	*0.07*	*0.03*
$\ln \sigma_\eta$	-1.10	-1.25	-1.27	-0.62	-1.21	-0.92	-1.18
	0.10	*0.07*	*0.19*	*0.10*	*0.16*	*0.16*	*0.07*
Criteria and tests							
SIC	2.89	1.90	2.43	2.42	2.39	3.07	2.39
Jacque-Bera	2.86	0.69	0.66	4.16	2.89	2.69	3.55
	0.24	0.71	0.72	0.13	0.24	0.26	0.17
Q(12)	13.53	11.32	5.00	9.27	12.96	14.31	12.68
	0.33	0.50	0.96	0.68	0.37	0.28	0.39

Legend: standard errors are written in italic and p-values at the 5% level are underlined.

Volatility is highly persistent in all countries with estimated ϕ close to 0.94, except for Japan where ϕ is a bit lower (0.82). Germany and Japan show the highest volatility level ($\sigma^* \approx 0.95$) during the period, almost twice as high as France's level, which is the lowest ($\sigma^* = 0.56$). The variations in the volatility process are the largest for the UK ($\sigma_\eta = 0.54$) and the smallest for France, Italy and the US with a value close to 0.29. Different patterns about the relationship between growth and volatility arise from the estimations. Germany and Italy present a significant and positive relationship, with respectively $\delta = 0.21$ and $\delta = 0.16$. For other countries, δ is not significant, but we may distinguish France and Japan where the value is positive ($\delta = 0.08$ for both), from the UK and the US where the value is negative ($\delta = -0.07$ and $\delta = -0.02$ respectively). Hence, the SV-M model gives us insights that output volatility can be positively related to output growth, although this relationship is not significant for all countries.

TABLE 2. Estimation results for the log-GARCH-M model

	de	fr	it	uk	us	jp	ca
Mean equation							
c	**0.34**	**0.46**	**0.40**	**0.18**	**0.48**	**0.16**	**0.38**
	0.04	*0.05*	*0.03*	*0.00*	*0.04*	*0.05*	*0.00*
α_1	**0.10**	**0.32**	**0.33**	**0.16**	**0.13**	**0.28**	**0.51**
	0.03	*0.03*	*0.01*	*0.00*	*0.02*	*0.05*	*0.02*
α_2	**0.12**	**0.17**	**0.00**	**0.08**	**0.24**	**0.25**	
	0.06	*0.02*	*0.02*	*0.00*	*0.04*	*0.05*	
α_3	**-0.04**			**0.12**	**-0.07**	**0.22**	
	0.05			*0.00*	*0.03*	*0.04*	
α_4	**0.07**					**-0.04**	
	0.03					*0.03*	
δ	**0.09**	**0.07**	**0.13**	**-0.15**	**-0.07**	**-0.09**	**0.00**
	0.03	*0.03*	*0.03*	*0.00*	*0.03*	*0.11*	*0.02*
Volatility equation							
$\ln \sigma^*$	**-0.11**	**0.30**	**-0.04**	**-0.01**	**-0.19**	**0.06**	**-0.40**
	0.11	*0.15*	*0.11*	*0.00*	*0.10*	*0.12*	*0.19*
φ	**0.82**	**1.00**	**0.96**	**0.85**	**0.91**	**0.92**	**0.94**
	-	-	-	-	-	-	-
ψ	**0.17**	**0.04**	**0.10**	**0.19**	**0.11**	**0.09**	**0.14**
	0.05	*0.01*	*0.02*	*0.00*	*0.03*	*0.05*	*0.04*
Criteria and tests							
SIC	**2.91**	**1.95**	**2.52**	**2.72**	**2.43**	**3.18**	**2.33**
Jarque-Bera	**5.4**	**24.5**	**3.7**	**84.7**	**4.0**	**2.8**	**2.1**
	0.07	0.00	0.15	0.00	0.14	0.25	0.36
Q(12)	**10.9**	**11.6**	**10.9**	**14.9**	**15.6**	**14.4**	**21.7**
	0.53	0.48	0.54	0.25	0.21	0.28	0.04

Legend: standard errors are written in italic and p-values at the 5% level are underlined.

To challenge these results we confront them to log-GARCH-M estimation results, which are presented in table 2. First, as in the SV-M model, the estimate of ϕ indicates high persistence levels for all countries, but the French log-volatility is nearly integrated (its persistence is close to one) in the log-GARCH-M model. The high variations in volatility for the UK also arise from the large ψ.

Second, except for Canada, estimates of δ, the coefficient of the growth-volatility relationship, are always lower in the log-GARCH-M model than in the SV-M model. This difference suggests that the SV-M model would take into account a positive impact of unexpected volatility on output growth. If we look at standard errors of estimates in the SV-M model, we see that this conclusion is only suggestive as this difference is not significant for all countries. Although we would theoretically expect to get different results from both models due the presence of unexpected volatility in the SV-M model only, the δ coefficient expresses the aggregated impact of expected and unexpected volatilities, which may be very close to the impact of the expected one only. Indeed, when persistence is very high, $1 - \phi^2 \ll \phi^2$ and then

$$var(\phi h_{t-1}) = \frac{\phi^2}{1-\phi^2}(\sigma^*)^2 \gg (\sigma^*)^2 = var(\eta_t)$$

This means that the expected volatility is very large compared to the unexpected volatility. In practice, the expected volatility accounts for 85% to 90% of total volatility, except for Japan where it is lower. This probably explains why parameter estimates obtained with SV-M and log-GARCH-M models are not significantly different. However, there were *a priori* no reasons to discard the unexpected volatility *ex ante*. Our results simply suggest that doing so turns out to be a legitimate approximation.

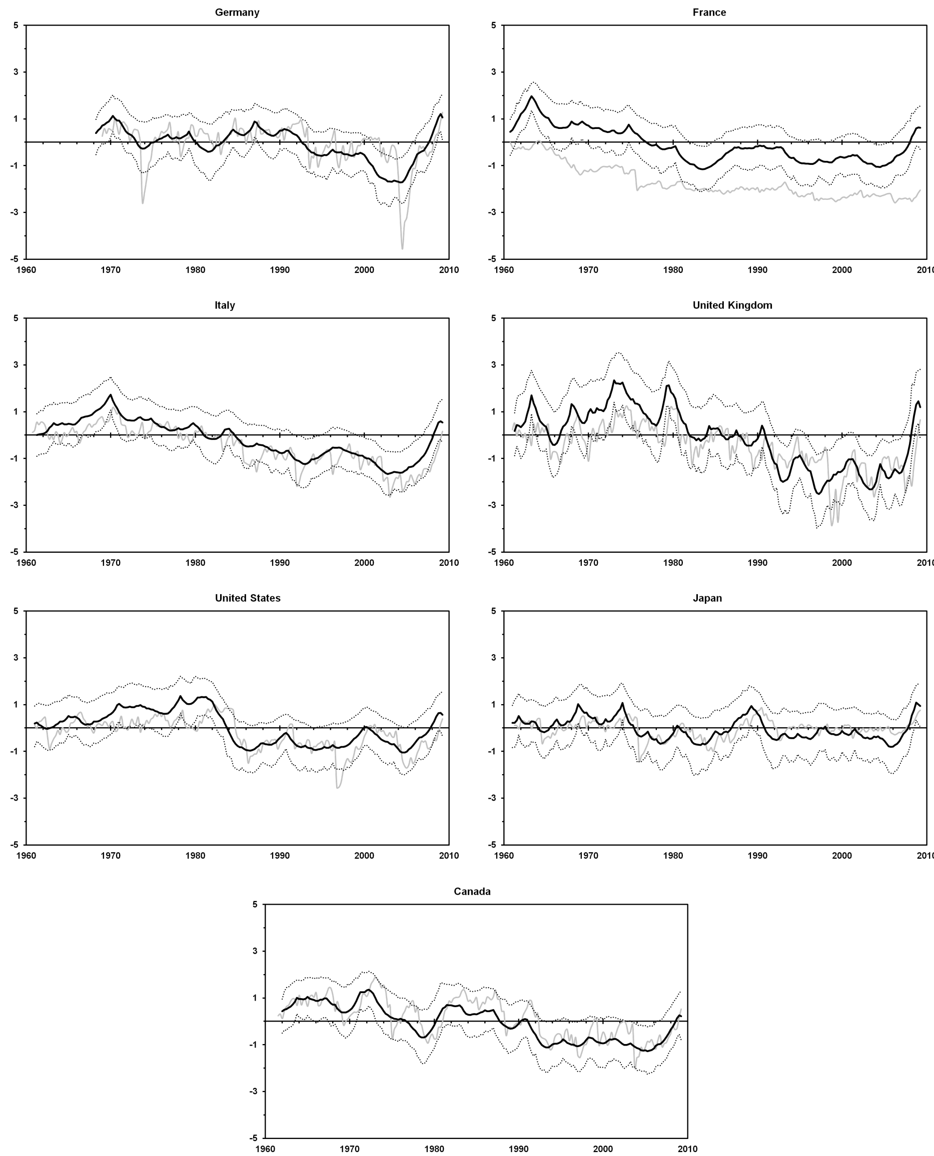
Third, while estimates are only significant for Germany and Italy in the SV-M model (with a positive sign), estimates are also significant for France, the UK and the US (with respectively a positive, a negative and a negative sign) in the log-GARCH-M model. This could be a misleading result. In the log-GARCH-M case, contrary to the SV-M case, the normality of residuals is not warranted and Student tests can deliver false conclusions about the significance of the growth-volatility relationship. Indeed, p-values of Jarque-Bera tests are lower than 10% for three countries (Germany, France and the UK), lower than 15% for two countries (Italy and the US) and higer than 25% for the two last countries (Japan and Canada). In conclusion, SV-M models deliver more robust conclusions than log-GARCH-M models, because of the better quality of their residuals.

4.4. **Estimates of log-volatilities.** Figure 1 plots the estimates of stochastic volatility. For all countries except Japan, we observe high volatility levels in the 1960's and 1970's, which tend to decrease from the 1980's until the beginning of the current crisis. The decrease is prompt for the US around 1983, which is consistent with the standard "great moderation" story. For other countries, the decrease is more gradual.

If we consider 95% confidence intervals of state estimates, we notice that the lowest upper bound of the interval in the whole sample is lower than the highest lower bound of the interval. Thus, for all countries, the decrease in volatility is significant in the last 50 years. Recent data gives us precious indications on a clear increase of the volatility in the last two years, implied by the the financial crisis of 2007-2008. This illustrates the end of the great moderation.

We can compare our results with those obtained by Stock and Watson (2005). They estimate the volatility in an AR model of the G7 output growth rates with non-stationary SV innovations. Their sample covers the period going from 1960 to 2002. Our results are similar for the US, the UK and Italy. Some divergences occur

Figure 1. Estimates of the log-volatility in G7 economies



Legend: for the SV-M model, smoothed estimates of the log-volatility (black line) with their 95% confidence intervals (dotted lines); for the log-GARCH-M model, log-volatilities (grey line).

for France, Germany and Japan, but global conclusions are the same, although their time period does note include the end of the great moderation.

5. Conclusion

This paper applies for the first time the SV-M model to describe the relationship between growth and volatility. This relationship appears significantly positive in Germany and Italy, but insignificant in other countries. Output volatility has increased in all countries since the beginning of the financial crisis, which illustrates the end of the great moderation. SV-M models are also compared with log variants of GARCH-M models, the class of time series models previously used in the literature to address this issue. We show three advantages of SV-M models. They fit generally better data than log-GARCH-M models. As their residuals do not violate distribution assumptions, they do not deliver dubious conclusions concerning the significance of the relationship, which is the case of the log-GARCH-model for France, the UK and the US. Finally, they deliver a positive impact of the unexpected volatility, which is not taken into account in log-GARCH-M models.

In further researches, the model of the growth-volatility relationship could first be enriched in two directions: by disentangling the unexpected from the expected volatility; by distinguishing the long-run from the short-run fluctuations of output growth. These distinctions could be incorporated by merging the SV-M model with an unobserved component framework. Then, the model could also be extended with a multivariate dimension, in order to encompass the model of Ramey and Ramey (1995). Such an extended model would be more difficult to estimate, but it would help to characterise more precisely the growth-volatility relationship.

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