NOTES D'ÉTUDES

ET DE RECHERCHE

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THE USE OF MARKOV-SWITCHING

MULTIFRACTAL MODELS

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Long term vs. short term comovements in stock markets :

the use of Markov-switching multifractal models.

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Abstract: Empirical techniques to assess market comovements are numerous from cointegration to dynamic conditional correlations. This paper uses the fractal properties of asset returns and presents estimations of Markov switching multifractal models [as MSM] to give new insights about short and long run dependencies in stock returns. The main advantage of the model is to allow for the derivation of several indicators of comovements on heterogenous lasting horizons. Empirical applications are performed for four stock indices (CAC DAX FTSE NYSE) at daily frequency between 1996 and 2008.

keywords : Multivariate volatility models, Markov switching multifractal model, transmission, comovements.

JEL Classification : C32 F36 G15

Résumé: Les techniques empiriques pour rendre compte des comouvements sont nombreuses, de la cointégration aux modèles de corrélations dynamiques. Cet article utilise les propriétés fractales des rendements d'actifs financiers et présente des estimations de modèles markoviens multifractals à changements de régimes [MSM] pour dériver de nouveaux indicateurs concernant la dépendance de court et long terme entre indices boursiers. Le principal avantage du modèle est de permettre la dérivation d'indicateurs de comouvements sur des horizons de durées hétérogènes. Des applications empiriques sont effectuées pour quatre indices (CAC DAX FTSE NYSE) à fréquence journalière entre 1996 et 2008.

Mots-Clé : Modèles multivariés de volatilité; Modèles de Markov switching multifractals; transmission; comouvements.

Classification JEL : C32 F36 G15

Non Technical Summary

Recent developments in financial markets have shown the importance for market participants of the measure of risk and of comovements.

The following paper develops several new indicators for assessing volatility and comovements between stock markets from long-run to the very short-term dependence. Several techniques in econometrics have already been developed such as cointegration, multivariate volatility models or dynamic conditional correlations. A new set of indicators is derived from a Markov-Switching Multifractal model [MSM] in a bivariate form as initiated by Mandelbrot et al. (1997) and Calvet et al. (2006).

The use of fractal mathematics as a tool relies on the simple idea that dependence may differ and occur at different horizons. This straightforward idea stems from the diversity of investors and the diversity of information they react to. For instance, some investors may react to the regular disclosure of accounting information or the publication of national statistics, while others may react to a fall in a foreign market or to the "buy dynamics" observed on a specific asset. This leads to dependencies at different frequencies, which may affect price dynamics differently.

The MSM model clearly improves the view of this strata structure of investment decisions and enables us to disentangle the links between markets at several horizons that are endogenously determined.

This paper presents an empirical application for four indices between 1996 and April 2008: CAC, FTSE, DAX and NYSE. A structure of three superimposed cycles is estimated by the model. The NYSE appears to be the most resilient with a very short-term cycle of 15 days, while European markets' shortest cycles are between 20 and 47 days. The medium-term cycle varies from 47 days for the NYSE to 101 days for the FTSE. Finally the longest cycle is around 146 days for the NYSE and up to 392 days for the CAC.

The first indicator is the probability of crisis. This is defined as the probability of being in the highest state of volatility on the considered market for all horizons. The second indicator is the probability of extreme comovements, defined as the probability of being in crisis for one market, knowing that another market is in crisis. Finally long-term high volatility cycles are derived and correspond to the highest state of volatility at the longest horizon.

The main financial crises are detected by the model: the Asian crisis, the Russian crisis,

March 2000, 11 September, the US accounting scandals, the Second Iraq War and January 2008. Note that in 2007 the subprime crisis is not considered as a crisis since the long-term volatility component did not jump to a high value. However, the short- and medium-term components moved to high volatility, which weakened the market and increased the threat of a crisis occurring. This crisis is finally witnessed during the Black Monday, January 21, 2008.

Extreme comovement phenomena have evolved over time in the sample and differ between markets. They appear very strong in the euro area between France and Germany and less pronounced but strong in Europe (i.e. between the UK and France or the UK and Germany). Finally, extreme comovement probability conditional on the United States appears stronger for the UK than for France and Germany. However, overall, US comovements conditional on other markets appear quite erratic and sudden when it occurs.

Lastly, we consider long-term cycles of volatility. It appears that the Asian crisis is a key event in the sample since it generated, until the end of 2003, a long-term high volatility cycle. This is observed for all markets. However the period from 2004 until the end of 2006 is remarkably characterised by a very low volatility long-run cycle. However, as mentioned previously, 2007 appears to be a transition period with a rise in the short- and medium-term component of volatility. This has clearly weakened the long-term low volatility cycle and made market participants more vigilant about market price dynamics. Finally, 2008 has opened a new high volatility long term cycle.

Résumé non technique

La dynamique récente des marchés financiers a de nouveau rappelé l'intérêt des participants de marchés pour les mesures du risque et de comouvements. Le présent papier développe plusieurs nouveaux indicateurs pour rendre compte de la volatilité et de ses comouvements entre marchés depuis la dépendance de long terme jusqu'à la dépendance de court terme. De nombreuses techniques en économétrie ont déjà été développées : cointégration, modèles multivariés hétéroscédastiques, modèles de corrélations dynamiques. Une nouvelle série d'indicateurs est dérivée d'un modèle multifractal à régime markovien [MSM], dans sa forme bivariée, initiée par Mandelbrot et al. (1997) et Calvet et al. (2006).

L'utilisation des mathématiques fractales repose sur l'idée simple que la dépendance peut différer et se produire à différents horizons. Ceci est une conséquence directe de la diversité des investisseurs sur les marchés ou des différentes incitations informationnelles. Par exemple, certains investisseurs réagissent aux publications comptables, alors que d'autres arbitrent à haute fréquence entre les marchés. Ainsi sont obtenues des dépendances à différentes fréquences qui affectent les dynamiques de prix de façon hétérogène.

Le modèle MSM améliore clairement cette vision en strates du marché et nous permet de dénouer le court du long-terme sur les marchés pour les différents horizons qui sont déterminés de façon endogène. Le papier présente une application empirique pour quatre indices entre 1996 et avril 2008 : CAC, FTSE, DAX and NYSE. Une structure de trois cycles superposés est estimée par le modèle. Le NYSE apparaît être le plus résilient avec un court-terme de l'ordre de 15 jours, alors que les cycles courts des marchés européens sont entre 20 et 47 jours. Le cycle de moyen-terme est autours de 47 jours pour le NYSE jusqu'à 101 jours pour le FTSE. Finalement, le plus long terme est de 146 jours pour le NYSE jusqu'à 392 jours sur le CAC.

Le premier indicateur est une probabilité de crise. Ceci est défini comme la probabilité d'être dans un état haut de volatilité sur tous les horizons. Le second indicateur est la probabilité de comouvements extrêmes. Ceci est défini comme la probabilité d'être en crise sur un marché conditionnellement à ce que l'autre marché soit en situation de crise. Enfin, les cycles de long-terme de forte volatilité sont tirés du modèle et correspondent à l'état haut de la volatilité sur la fréquence la plus basse.

Les principales crises financières sont détectées par le modèle : la crise asiatique, la crise russe, mars 2000, le 11 septembre, les scandales de la comptabilité aux Etats-Unis, la guerre en Irak et janvier 2008. Notons qu'en 2007, la crise dite des "subprimes" n'est pas considérée comme une crise car la composante de long terme de la volatilité est restée dans un état de basse volatilité. Cependant, les composantes de court et moyen-terme ont migré vers un état de forte volatilité ce qui a fragilisé le marché et augmenté la menace d'une crise. Cette crise est finalement observée durant le lundi noir du 21 janvier 2008.

Les phénomènes de comouvements extrêmes ont évolué au cours du temps et diffèrent entre les marchés. Ils apparaissent très forts en zone euro entre la France et l'Allemagne, et moins prononcés mais forts en Europe (i.e. entre le Royaume-Uni et l'Allemagne ou la France). Enfin, la probabilité de comouvements extrêmes conditionnelle au NYSE est plus forte pour le Royaume-Uni que pour la France et l'Allemagne. Les comouvements pour le marché américain, conditionnellement aux autres marchés apparaissent erratiques et soudains lorsqu'ils se produisent.

Finalement, sont considérés les cycles de long terme de la volatilité. Il apparaît que la crise asiatique est un événement clé sur l'échantillon car a généré jusqu'en 2003 un cycle de forte volatilité sur le long terme. Ceci est observé sur tous les marchés. Néanmoins, la période 2004-2006 est remarquablement caractérisée par une très faible volatilité de long terme. Cependant, comme mentionné précédemment, 2007 apparaît comme une année charnière avec une hausse des composantes de court et moyen-terme. Ceci a fragilisé le cycle de long terme de basse volatilité et augmenté la vigilance des participants de marché. Enfin, 2008 a ouvert un nouveau cycle de forte volatilité de long terme et ce à partir du lundi noir.

1 Introduction

This article proposes a new set of indicators gauging comovement risk and volatility spillovers between financial prices. Derived from the bivariate Markov-Switching multifractal model of asset returns initiated by Calvet et al. (2006) these indicators latter described are :

- the volatility cycles or periods;

- a crisis probability;

- a probability of extreme comovements;

- and probabilities of long term high (or low) volatility cycles.

Quantitative methods for assessing risk transmission and comovements are numerous in the literature. Kasa (1994) through cointegration analysis using Johansen (1992) tests tries better understanding stock market integration. In this strand of cointegration analysis, Kanas (1998) performs a similar analysis. More recently, with the idea there exists several types of comovements between markets (as Forbes and Rigobon (2002) dichotomy between integration and contagion), Billio Lo Duca and Pellizon (2005) in a Vector Error Correction Mechanism (VECM) framework introduce regime switching to address break in the integration process or contagion process. Idier (2006), using as well a VECM framework, separates transmission between the first and second moments by expanding the model with a multivariate General Auto Regressive Conditional Heteroskedastic model, making the bridge between cointegration analysis and the wide class of multivariate GARCH models. The use of multivariate GARCH model has also been widely explored by researchers in assessing risk and volatility transmission (survey by Bauwens et al. (2006) or Engle and Sheppard (2007)). From the Baba-Engle-Kraft and Kroner (as BEKK) model to the Dynamic Conditional Correlation of Engle et Sheppard (2001) several improvements have been introduced in these models as asymmetries or structural breaks. Recently, Billio and Caporin (2005) have introduced a Markov switching DCC model with the main improvement that correlations may jump assuming different states in correlations. Other approaches concerning the use of high frequency data and realized volatilities or realized variance-covariance matrices specifications have also been applied : the Heterogenous Autoregressive model of Corsi (2006)

7

or the paper by Bauer and Vorkink (2007) modelizing the realized Bipower variance matrices issued from the work of Barndnorff-Nielson and Shephard (2004).

In the strand of multivariate volatility models, a last class of specifications, recently developed by Calvet, Fisher and Mandelbrot (1997), Calvet and Fisher (2001,2002,2004) or Calvet, Fisher and Thompson (2006) uses the fractal properties of asset returns.

Fractal properties of asset returns may be related to information cascade occurrence on a market. An information cascade is the disability of market agents to move first rationally. In other words, once one first has moved randomly to a particular decision (for example buy an asset), the others take into account this action to move subsequently. Since subsequent players do not have any other information than witnessing this first move on the market, they draw rationale incentives from this limited set of information to move on the market in the same direction as the first agent. As an ex post consequence, buy the asset was rationale since the market is now upward. The changes in the asset price are subsequently accelerating, since the network of agents witnessing agents moves is growing.

Information on the market arrive very often and the population of market participants may be very heterogeneous for certain classes of assets. As underlined by Zumbach and Lynch (2001), from hedge funds with substantive positions to small individual traders, market moves are motivated by different types of information arrivals and launch relatively long or short periods of volatility clustering. There is no uninformed traders moving on the market, but at least one rationale in a limited set of information to move on the market: information disclosure, statistics, arbitrage between markets, market moves etc.

It is thus expected from an empirical model to consider this heterogeneity in news, and this strata structure of information revelation in price processes. In this direction, fractal properties of asset returns may be useful. More than thinking about time dependency in the evolution of the market, it is more accurate to think in terms of frequencies. Statistics are published at a regular frequency and the same, for example, concerning dividend distribution or firm information disclosure. Generally, it may be assumed that different types of traders, use different types of information, at different frequencies, and so the market moves in terms of frequencies.

Calvet, Fisher and Mandelbrot (1997) have shown on exchange rate data that returns satisfied scale properties of fractal objects. From these observations, they have developed a Markov

switching multifractal model taking into account these properties. To analyze risk transmission, a Bivariate model has been developed, in a similar fashion to the univariate one. These models always applied to exchange rate data give very satisfactory insight concerning comovements since it estimates a strata structure of transmission cycles of different lengths. The model also exerts a probabilistic structure on a wide range of volatility states that largely improves the view of the nature and of the degree of transmission between returns. Concerning stock market prices, Fillol (2003) analyses the fractal properties of asset returns for the French CAC40 index that also satisfy fractal scale properties. Lux and Kaizoji (2007) studies the behavior of prices in the Japan stock market using this model. However, empirical applications of this model, in his bivariate form, stay relatively scarce.

Following the distinction of Forbes and Rigobon (2002) concerning integration and contagion, the model estimates a probabilistic structure concerning the several cycles in prices. These cycles organized as strata are an illustration of scale properties of fractal objects, and help distinguishing long term versus short term links in index returns. The model thus allows for non discrimination between short term comovements and long term comovement, but for a discrete scale of potential shifts affecting volatilities at different frequencies. Advantages are twofold. First, this graduation in the different horizons is endogenous and not imposed by the model. Second, the structure of the model results for a relatively reasonable number of parameters to a wide set of potential volatility and comovement states.

An empirical application is done for four stock indices, CAC FTSE DAX and NYSE at daily frequency from 01/01/1996 to 24/04/2008. In this paper the four indices are coupled to each other in Bivariate Markov switching models. The estimations, by maximum likelihood, permit to identify the different cycles, with different durations, state varying correlations and a probabilistic structure concerning comovements. This allows for a new way to detect the crises, that it is opposed to the long term cycles identified in index returns. Finally it gives a complete new set of indicators concerning links on several horizons between markets.

The following section presents the MSM model first in a univariate framework. The third section presents the bivariate form and the derivations of the comovement indicators. Section 4 presents the empirical application of the MSM models for stock indexes. Finally section 5 concludes.

2 The Univariate multifractal model of asset returns

2.1 The model

This modelisation combines persistent changes in the value of the asset and very short lasting shifts. Major news are considered to have long lasting effects while minor news are considered short lasting in effects. From Calvet and Fisher (2002), the returns are formalized as:

$$R_t = \left(\prod_{k=1}^{\bar{k}} M_{k,t} \right)^{1/2} \sigma \varepsilon_t \tag{1}$$

with σ the unconditional standard error and ε a residual following a standard Gaussian distribution (0,1). Returns are specified as the product of $\bar{k}$ components M_k. These components are drawn at each date from a binomial distribution taking values $m_0 \in [1; 2]$ and 2-m_0 with equal probability so that $E(M_k) = 1$, to guaranty a conservative mass measure. The binomial distribution is considered to be state and time invariant : if an information arrival occurs, the new multiplier M_k is drawn from the time invariant M binomial distribution but the M_k differ in the occurrence of information arrivals, in other words in their frequency γ_k. The index k, corresponds to several horizons so that for $k = 1$, a short lasting shift is obtained while for $k = \bar{k}$ it is observed a long lasting shift. Horizons of each component is defined similarly as in Calvet and Fisher (2004). The frequencies to which components actually jump, indexed by k, are defined as:

$$\gamma_k = 1 - (1 - \gamma_1)^{b^{(k-1)}} \tag{2}$$

where $\gamma_1 \in [0; 1]$ is the highest frequency of information arrivals (and so the shortest horizon) and $b \in]0; 1]$ so that $\gamma_k \in [0; 1]$ for all k[1].

Some components M_k take a high value m_0 quite often and come back to a low value $m_1 = 2$-m_0 while others may change and stay at a high level for longer time. The heterogeneity in traders and news give a more complex dependency on the market than simple time dependency. Publications of GDP bring a certain type of traders on the market during these days, with a certain behaviour, which are not the same behaviors for example than people who trade in

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[1] This differs from the original model since here $b \in]0; 1]$ for computationnal interest in bounding the b parameter.

the European markets when the US market moves to a certain extent. A superposition of different trading cycles is obtained with different shocks and different persistences. Low frequency components can be attributed to biggest events in the market while highest frequency components would be algorithm trading for example. It is not considered that people are informed or not in the market, but that people are interested in different types of information and thus behave differently.

2.2 Univariate estimation procedure

Calvet and Fisher (2004) use a maximum likelihood optimization procedure to estimate the set of parameters $\Omega = (m_0, \sigma, b, \gamma_1) \in \mathbb{R}^4$. Since M_k follows a binomial distribution, it is obtained $2^{\bar{k}}$ volatility states. A volatility state is defined as a vector $m^i = (M_1, M_2, ...M_{\bar{k}})$ of dimension $\bar{k}$. Updating the probability state vector Π_t of elements $\Pi_t^j = \Pr(M_t = m^j \mid R_1, R_2, ...R_t)$ consists in recursively calculating the probabilities of the $2^{\bar{k}}$ possible states in volatilities. The transition matrix A of the Markov chain has elements a_{ij} defined as:

$$
\begin{aligned}
a_{i,j} &= \Pr(M_{t+1} = m^j \mid M_t = m^i) \\
&= \prod_{k=1}^{\bar{k}} \left[(1 - \gamma_k) \Im_{\{m_k^i = m_k^j\}} + \gamma_k \times \Pr(M = m_k^j) \right],
\end{aligned}
\tag{3}
$$

with $\Im$ a variable taking value one if $m_k^i = m_k^j$ and zero otherwise. The conditional density of returns in period t is

$$
f_{R_t}(R \mid M_t = m^i) = \left[\left(\prod_{k=1}^{\bar{k}} M_{k,t} \right)^{1/2} \sigma \right]^{-1} \times \varphi \left(R \times \left[\left(\prod_{k=1}^{\bar{k}} M_{k,t} \right)^{1/2} \sigma \right]^{-1} \right)
\tag{4}
$$

with φ the density of a standard Gaussian distribution (0,1). Considering the vector ω_t of dimension $2^{\bar{k}}$ of element $f_{R_t}(R \mid M_t = m^i)$ with $i = 1$ to $2^{\bar{k}}$. Calvet and Fisher (2004) show that the updated probability Π_{t+1} is obtained as:

$$
\Pi_{t+1} = \frac{\omega(R_{t+1}) * \Pi_t A}{[\omega(R_{t+1}) * \Pi_t A] \; \iota'},
\tag{5}
$$

and the log likelihood is:

$$\ln(L(x_1, ...x_t \mid m_0, \sigma, b, \gamma_1)) = \sum \ln(\omega(R_t).\Pi_{t-1} A) \tag{6}$$

We notice that the vector Π_t in the estimation procedure is initialized in Π_0 such that $\Pi_0^i = \prod_{k=1}^{\bar{k}} \Pr(M_t = m^i)$ for all i.

3   Market comovements and the bivariate MSM model

Market integration and stability analysis needs models that take into account internationally transmitted information which may differ in effects. It follows the strand in literature focused in links between markets as Longin and Solnik (1995), Harris et al. (1995), Masih and Masih (2001), Avouyi-Dovi and Netto (2003), Kearney and Poti (2005) or Kallberg and Pasquarello (2007).

Each type of news can be characterized by the correlation in the components of the same frequency k between several places. It may help understanding if very high level of comovements for example is observed in transient components, or in the most persistent ones. This can be linked with the usual distinction between integration and contagion, done in Forbes and Rigobon (2002), with the major improvement that a graduate scale from the short common changes up to persistent shifts is defined. To do so the MSM may be expressed as a bivariate binomial model to analyze the links between two markets (Calvet et al.(2006)).

Let define the vector of returns as $x_t = \binom{R_t^\alpha}{R_t^\beta}$ for markets α and β. The vector of the components at the k-th frequency is $M_{k,t} = \binom{M_{k,t}^\alpha}{M_{k,t}^\beta}$. The period t volatility is characterized by the $(2,\bar{k})$ matrix $M_t = \left(M_{1,t}; M_{2,t}; ...M_{\bar{k},t} \right)$ where $\bar{k}$ is the index for the lowest frequency. Each row stands for a market indexed by c=$\{\alpha, \beta\}$, while each column for a frequency k=$\{1,2,...\bar{k}\}$. Consistently with the previous section, the vector returns may be written as:

$$x_t = \binom{M_t^{\alpha^{1/2}}}{M_t^{\beta^{1/2}}} * \varepsilon_t \tag{7}$$

with * the Hadamard product, $M_t^c = \prod\limits_{k=1}^{\bar{k}} M_{k,t}^c$ and $\varepsilon \in \mathbb{R}^2$ the vector of residuals which are IID Gaussian $(0, \sum)$ with

$$\sum = \begin{pmatrix} \sigma_\alpha^2 & \rho_\varepsilon \sigma_\alpha \sigma_\beta \\ \rho_\varepsilon \sigma_\alpha \sigma_\beta & \sigma_\beta^2 \end{pmatrix}. \tag{8}$$

The $\rho_\varepsilon \in [0;1]$ represents unconditional correlation between the residuals. It is the first source of correlation between the two markets. A second source of correlation is the correlation between jumps: in period t, each returns α or β may be hit by an information arrival at frequency γ_k on each corresponding k component. The correlation between information arrivals is represented by a new $\lambda \in [0;1]$ coefficient as follows.

Let consider the dummy variables D_k^α and D_k^β which take values 1 if an information arrival (jump) occurs on component k of series α or β and 0 otherwise.

The vector $D_k = \binom{D_k^\alpha}{D_k^\beta}$ is specified as IID and, as in Calvet et al. (2006), it satisfies few conditions. The arrival vector needs to be symmetric which means that $\binom{D_k^\alpha}{D_k^\beta} \overset{d}{=} \binom{D_k^\beta}{D_k^\alpha}$.

Then, to be consistent with the univariate case we set

$$\Pr(D_k^\alpha = 1) = \gamma_k = 1 - (1 - \gamma_1)^{b^{(k-1)}} \tag{9}$$

with $\gamma_1 \in [0;1]$ is the highest frequency of jump and $b \in]0;1]$ so that $\gamma_k \in [0;1]$ for all k; and

$$\Pr(D_k^\beta = 1 \mid D_k^\alpha = 1) = (1 - \lambda)\gamma_k + \lambda. \tag{10}$$

Then, in line with the previous univariate case, the component $M_{k,t}^c$ is drawn from a binomial distribution taking value m^c and $2\text{-}m^c$ with the same probability if an information arrival occurs and stays constant otherwise, therefore:

$$M_{k,t}^c \overset{d}{=} M_{k,t-1}^c + D_{k,t}^c * (M - M_{k,t-1}^c) \tag{11}$$

where * is the Hadamard product and M the vector-component distribution.

Finally, a last parameter of the dependency structure is the correlation between M^α and M^β under the bivariate binomial distribution M.

13

The matrix $(p_{i,j})_k = \Pr(M_k = (m_i^\alpha, m_j^\beta))$ with $i,j = \{H, L\}$ for High and Low value is defined as

$$\begin{bmatrix} p_{LL} & p_{LH} \\ p_{HL} & p_{HH} \end{bmatrix}_k = \begin{bmatrix} \frac{1+\rho_m^*}{4} & \frac{1-\rho_m^*}{4} \\ \frac{1-\rho_m^*}{4} & \frac{1+\rho_m^*}{4} \end{bmatrix}_k \tag{12}$$

where $\rho_m^* \in [0; 1]$ is the correlation between components of frequency k of series α and β. Since it is set that the binomial distribution is the same for all component $M_{k,t}^c$ whatever is k, or stage invariant as in the univariate case, the k index may be omitted.

3.1 Comovements structure and typology

In this section comovement indicators are derived and discussed. These indicators are drawn from the dependency structure of the model given by the parameters λ, ρ_ε and ρ_m^*. The parameter λ gives the unconditional correlation between jumps on the markets. ρ_ε gives the unconditional correlation between the residuals of the models. Finally, ρ_m^* gives the unconditional correlation of the multipliers M_k^α and M_k^β under the bivariate binomial distribution M.

3.1.1 Variances and conditional correlations

Contrary to the wide class of multivariate GARCH models where the Σ matrix is characterized by time varying elements, the MSM accounts for a fixed elements matrix. Time varying correlations in this framework are obtained from the dynamics of the states of the $\bar{k}$ components. The conditional covariance between returns is as follows:

$$Cov_t\left(x_t^\alpha, x_t^\beta\right) = \rho_\varepsilon \sigma_\alpha \sigma_\beta \prod_{k=1}^{\bar{k}} E\left[(M_{k,t}^\alpha M_{k,t}^\beta)^{\frac{1}{2}}\right] \tag{13}$$

and the conditional variance for series c, c=$\{\alpha, \beta\}$ as:

$$Var_t(x_t^c) = \sigma_c^2 E(M_t^c) \tag{14}$$

14

so that the correlations are written as

$$Corr_t\left(x_t^\alpha, x_t^\beta\right) = \rho_\varepsilon \prod_{k=1}^{\bar{k}} \frac{E\left[(M_{k,t}^\alpha M_{k,t}^\beta)^{\frac{1}{2}}\right]}{\left(E(M_t^\alpha)E(M_t^\beta)\right)^{\frac{1}{2}}} \tag{15}$$

These correlations are clearly supposed to be much less flexible than correlation issued from pure time varying volatility models. Since the number of states is limited without pure time dependency between correlations, it is expected these correlations on the one hand to present some jumps (as components are jumping) but to be more rigid on a global perspective.

3.1.2 States Probabilities

Given the transition probability matrix A (see appendix A), each state may be assigned a probability Π_t^j, for j=1 to d= $4^{\bar{k}}$.

$$\Pi_t^j = \Pr\left(M_t = m^j \mid X_t\right) \tag{16}$$

with $X_t \equiv \{x_s\}_{s=1}^t$ the history of past returns. Π_t is calculated recursively by Bayesian updating as follows.

Let consider $\Pi_t = \left(\Pi_t^1, \Pi_t^2 ... \Pi_t^d\right)$, the probability state determined for time t. The returns in t+1 are observed and are assumed to follow a bivariate Gaussian density conditional on the volatility state $f_{x_{t+1}}(x_{t+1} \mid M_{t+1} = m^j)$ with variance covariance matrix H_j of this distribution:

$$H_j = \begin{bmatrix} \sigma_\alpha^2 M_{t+1}^\alpha & \rho_\varepsilon \sigma_\alpha \sigma_\beta \left(M_{t+1}^\alpha M_{t+1}^\beta\right)^{1/2} \\ \rho_\varepsilon \sigma_\alpha \sigma_\beta \left(M_{t+1}^\alpha M_{t+1}^\beta\right)^{1/2} & \sigma_\beta^2 M_{t+1}^\beta \end{bmatrix} \tag{17}$$

The updated probability is a function of actual returns and the history of past probabilities is given by

$$\Pi_{t+1}^j = \frac{f(x_{t+1}) * \Pi_t A}{[(f(x_{t+1}) * \Pi_t A)\, \iota']} \tag{18}$$

with * the Hadamard product, ι a $(1 \times 4^{\bar{k}})$ vector of ones, A the transition matrix and $f(x_{t+1})$ a $(1, 4^{\bar{k}})$ vector of elements $f_{x_{t+1}}(x_{t+1} \mid M_{t+1} = m^j)$. The derivation of the comovement indicators exploits this probabilistic structure.

15

3.1.3   Periods

An indicator of interest is the more general notion of periods or cycles. The multifrequency setting of the model allows for the identification of the different superposed cycles in the asset returns. This is defined as the inverse of the frequency of change γ_k in the different lasting components $M_{k,t}^c$. While in the univariate case this is only the cycles of single series of returns, in the bivariate cases it is the shared cycles between two series. It is defined as follows:

$$\tau_k = \frac{1}{\left(1 - (1 - \gamma_1)^{b^{(k-1)}}\right)} \tag{19}$$

The number of cycles depends on the number of $\bar{k}$ frequencies considered in the model. To determine the optimal number of frequencies, the Vuong Test from Calvet and Fisher (2004) is further applied as a selection model test.

3.1.4   Probability of extreme comovements

To latter identify crises and crises comovements between markets, joint probability to be in the highest volatility state in two markets is of interest. It is defined as follows:

$$
\begin{aligned}
\Pr(crisis)_t &= \Pr(M_{1,t}^\alpha =_{...} M_{\bar{k},t}^\alpha = m_0^\alpha \text{ and } M_{1,t}^\beta =_{...} M_{\bar{k},t}^\beta = m_0^\beta) \\
&= \Pi_t . \delta_1
\end{aligned}
\tag{20}
$$

with δ_1 a vector of dimension $4^{\bar{k}}$ with dirac elements $\delta_{1,i} = 1_{\left\{M_{1,t}^\alpha =_{...} M_{\bar{k},t}^\alpha = m_0^\alpha\right\}} \times 1_{\left\{M_{1,t}^\beta =_{...} M_{\bar{k},t}^\beta = m_0^\beta\right\}}$ for i=1 to $4^{\bar{k}}$, given that each component for a given series follows the same binomial distribution taking high value m_0^c for c=$\{\alpha, \beta\}$ or low value $2 - m_0^c$. In this setting a crisis is identified when all components are at their highest values for all horizons.

Moreover, it is defined the conditional probability to be in a high state of volatility in market α given that market β is in a high volatility state. This represents the conditional probability of

extreme comovements between two markets and is defined as:

$$
\begin{aligned}
\Pr(extreme\ comov)_t &= \Pr(M_{1,t}^\alpha =_{...} M_{\bar{k},t}^\alpha = m_0^\alpha \mid M_{1,t}^\beta =_{...} M_{\bar{k},t}^\beta = m_0^\beta) \\
&= \frac{\Pr(M_{1,t}^\alpha =_{...} M_{\bar{k},t}^\alpha = m_0^\alpha \text{ and } M_{1,t}^\beta =_{...} M_{\bar{k},t}^\beta = m_0^\beta)}{\Pr(M_{1,t}^\beta =_{...} M_{\bar{k},t}^\beta = m_0^\beta)} \\
&= \frac{\Pi_t.\delta_1}{\Pi_t.\delta_2} \tag{21}
\end{aligned}
$$

with δ_2 a vector of dimension $4^{\bar{k}}$ with dirac elements $\delta_{2,i} = 1_{\left\{M_{1,t}^\beta =_{...} M_{\bar{k},t}^\beta = m_0^\beta\right\}}$ for i=1 to $4^{\bar{k}}$. This gives insights about how a market is influenced by the others and if high volatility states are actually common between markets.

3.1.5 Long term cycles

Other indicators of interest are the long run cycles in volatility (high or low) that are shared between returns. To identify the low common long run cycles in volatility, the states for which the components with the lowest frequency of jump ($k = \bar{k}$) for the two series have both a low value $2 - m_0^c$ are considered. It means that the series may be hit on shorter run cycles by shocks but the longest cycle stays however low. This probability to be in a low long run cycle is thus written as:

$$
\begin{aligned}
\Pr(LLRC)_t &= \Pr(M_{\bar{k},t}^\alpha = M_{\bar{k},t}^\beta = 2 - m_0^\alpha) \\
&= \Pi_t.\delta_3 \tag{22}
\end{aligned}
$$

with δ_3 a vector of dimension $4^{\bar{k}}$ with dirac elements $\delta_{3,i} = 1_{\left\{M_{\bar{k},t}^\alpha = 2 - m_0^\beta\right\}} \times 1_{\left\{M_{\bar{k},t}^\alpha = 2 - m_0^\beta\right\}}$ and inversely, the probability to be in high long run volatility cycle is:

$$
\begin{aligned}
\Pr(HLRC)_t &= \Pr(M_{\bar{k},t}^\alpha = M_{\bar{k},t}^\beta = m_0^\alpha) \\
&= \Pi_t.\delta_4 \tag{23}
\end{aligned}
$$

with δ_4 a vector of dimension $4^{\bar{k}}$ with dirac elements $\delta_{4,i} = 1_{\left\{M_{\bar{k},t}^\alpha = m_0^\beta\right\}} \times 1_{\left\{M_{\bar{k},t}^\alpha = m_0^\beta\right\}}$. This completely new set of indicators help understanding the nature of comovement and the effects

of several events on different markets.

3.2 The Maximum likelihood estimation

Calvet et al. (2006) develop a maximum likelihood optimization procedure to estimate the set of parameters $\Omega' = (\sigma_\alpha, \sigma_\beta, m_0^\alpha, m_0^\beta, b, \gamma_1, \rho_\varepsilon, \lambda, \rho^*) \in \mathbb{R}^9$. Since it is considered $\bar{k}$ components, it is obtained $4^{\bar{k}}$ volatility states. This geometrical growth in volatility states makes the computation quite heavy but take a very wide view of the different possible states in volatility. A GMM alternative method as developed by Lux (2006) may also be applied.

The econometrician only observes the history of past returns $X_t \equiv \{x_s\}_{s=1}^t$ and does not observe the states of volatilities. The Π_t vector in empirical application, as in Calvet et al. (2006) is initialized at its ergodic distribution and updated as presented previously. The logarithm of the likelihood function is

$$l(x_{1...}x_T; \Omega') = \sum_{t=1}^{T} \ln(f(x_t \mid x_{t-1}, x_{t-2}, ...x_1)) \tag{24}$$

with

$$f(x_t \mid x_{t-1}, x_{t-2}, ...x_1) = \sum_{j=1}^{4^{\bar{k}}} f(x_t \mid M_{t-1} = m^j) \Pr(M_{t-1} = m^j \mid x_{t-1}, x_{t-2}, ...x_1)$$

so that the log likelihood is finally

$$l(x_{1...}x_T; \Omega') = \sum_{t=1}^{T} \ln(f(x_t).(\Pi_{t-1}A)) \tag{25}$$

4 Empirical applications

The dataset comprises four market indices : CAC, DAX, FTSE and NYSE. Daily prices are prices at 3pm GMT when all considered markets are opened simultaneously. The sample spans 12 years of market data from 01/01/1996 to 24/04/2008 at daily frequency. Univariate estimations of the MSM model are first provided to give an insight concerning volatility cycles τ_k, frequencies γ_k, and sample correlations between the components. Then the bivariate model estimations are

provided and discussed for each pair of indices. All the programs and routines are written using the MatLab software and data are obtained from the Reuters datascope tick history database.

4.1   Univariate MSM

Index returns are computed as $R_t = \ln(P_t/P_{t-1})$. The MSM($\bar{k}$) model is estimated for $\bar{k} = 1$ to 8 by maximizing the likelihood derived in equation 6. This corresponds for each estimation to a set of $2^{\bar{k}}$ states in the volatility process. Tables 1-4 in appendix B present the eight model estimations for each of the four series.

It is obtained (as in Calvet et al. (2006)) that component m_0 are decreasing in the number of frequencies. This is consistent with the idea that heterogeneity in volatility states is less required with an increase in the number of states (i.e. frequencies). It also appears that the frequencies γ_k are lower than frequencies obtained in the exchange rate market by Calvet et al. (2006) so that longer cycles are predominant.

A stabilization of the likelihood is observed from $\bar{k} = 4$ for all series. To formalize the selection model procedure, a likelihood ratio test is performed to test systematically a model with $\bar{k}$ components against a model with $\bar{k} + 1$ components. This tests developed in Calvet and Fisher (2004) is adjusted for correlations in the addends (Vuong-HAC test) and is presented in appendix C.

The Vuong HAC tests to select the appropriate model gives the MSM(3) model as an optimal choice. However, it is also tested MSM(3) for each index against the model with $\bar{k} = 5, 6, 7$ and 8 (appendix C). The trade-off between increasing the number of states in volatility by increasing $\bar{k}$ against selecting MSM(3) advocates for staying with $\bar{k} = 3$, with insignificant gains in likelihoods.

Peaks in volatility (Figure 1) are obtained for well-known dates identified as major events on financial markets. Concerning the subprime crisis, it mainly concerns the NYSE index while the others in Europe are less impacted. Details will be given in the bivariate forms estimations. Then the three periods (or volatility cycles) are computed as the inverse of the frequencies γ_k (Table 1).
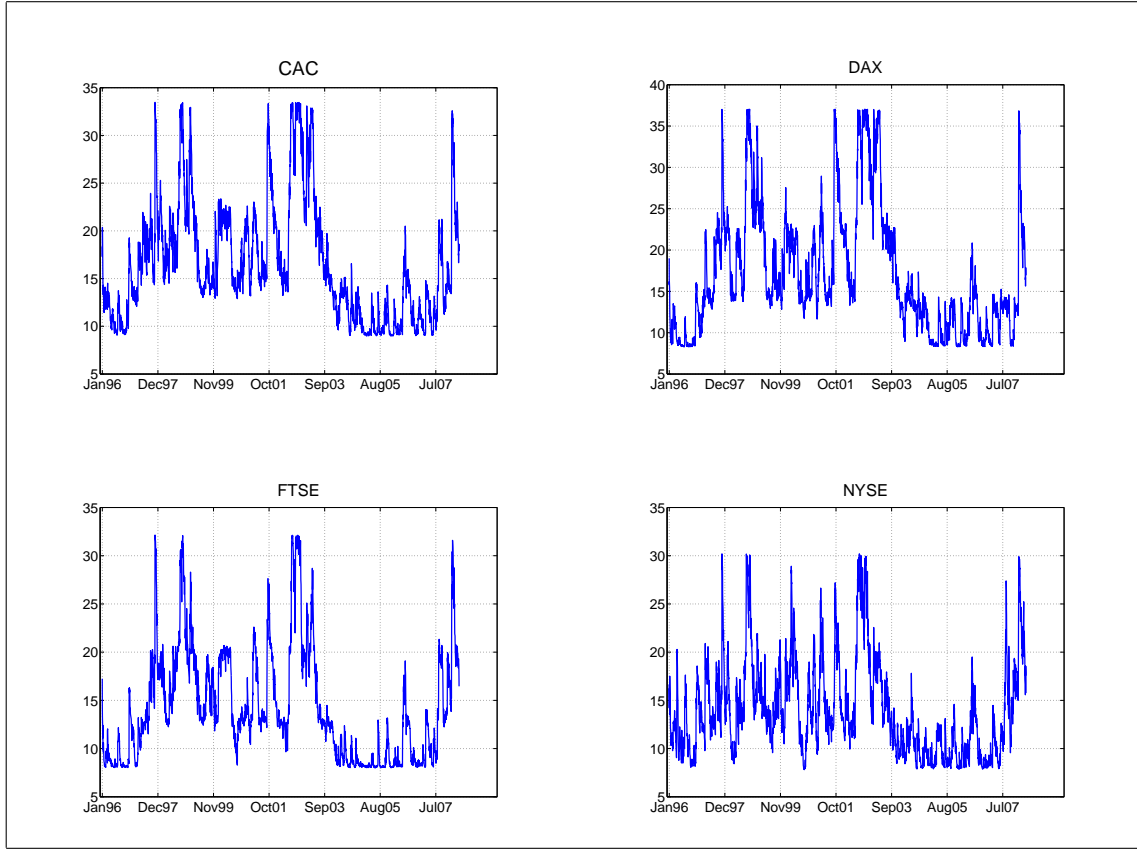
Figure 1: volatility MSM(3)

	CAC	DAX	FTSE	NYSE
τ_1	22.7	25.3	47.6	15.7
τ_2	93.8	95.0	101.3	47.7
τ_3	392.7	360.1	215.8	146.5

Table 1: Volatility cycles from MSM(3) in days

The shortest one is a cycle from 15 days (NYSE) up to 47 days (FTSE). The short run resiliency of the NYSE index is thus the most efficient. Short term is somehow between one and 2 months and a half. Medium term is between 3 and 5 months. Finally, long term is between 10 and 20 months.

This rises two questions. First of all, from a transmission perspective, are comovements

effectively higher in period of high volatility or not. Shocks are obtained on volatility at the same date, but cycles in volatility are different. This means that if a shock in volatility increases correlations at one point in time it should perturb comovements in the following days but in an heterogenous way since the shock may be on a short lasting component, or on a long lasting one. Second, if a shock hits different lasting components of volatility in two indexes, the resilience to the shock becomes very different between places and a decrease in comovement should even be observed after a sudden rise. Typically, it is expected that correlations between the NYSE and the other indexes are weakened by this difference in the length of the cycles. To first gauge where transmission occurs, it is presented the correlations between the $\bar{k}$ components obtained from the decomposition derived form the MSM(3), for the four returns series.

	M_1^{cac}	M_2^{cac}	M_3^{cac}	M_1^{dax}	M_2^{dax}	M_3^{dax}	M_1^{ftse}	M_2^{ftse}	M_3^{ftse}	M_1^{nyse}	M_2^{nyse}	M_3^{nyse}
M_1^{cac}	1	0.69	0.36	0.86	0.60	0.34	0.79	0.63	0.46	0.74	0.75	0.53
M_2^{cac}	0.69	1	0.71	0.62	0.92	0.67	0.79	0.89	0.79	0.54	0.79	0.85
M_3^{cac}	0.36	0.71	1	0.33	0.76	0.98	0.57	0.78	0.92	0.33	0.56	0.82
M_1^{dax}	0.86	0.62	0.33	1	0.64	0.34	0.72	0.59	0.45	0.69	0.65	0.49
M_2^{dax}	0.60	0.92	0.76	0.64	1	0.77	0.69	0.82	0.79	0.48	0.68	0.83
M_3^{dax}	0.34	0.67	0.98	0.34	0.77	1	0.54	0.75	0.90	0.32	0.52	0.80
M_1^{ftse}	0.79	0.79	0.57	0.72	0.69	0.37	1	0.88	0.70	0.72	0.86	0.72
M_2^{ftse}	0.63	0.89	0.78	0.59	0.82	0.75	0.88	1	0.90	0.58	0.81	0.89
M_3^{ftse}	0.46	0.79	0.92	0.45	0.79	0.90	0.70	0.90	1	0.43	0.65	0.90
M_1^{nyse}	0.74	0.54	0.33	0.69	0.48	0.32	0.72	0.58	0.43	1	0.80	0.54
M_2^{nyse}	0.75	0.79	0.56	0.65	0.68	0.52	0.86	0.81	0.65	0.80	1	0.79
M_3^{nyse}	0.53	0.85	0.82	0.49	0.53	0.80	0.72	0.89	0.90	0.54	0.79	1

Table 2: Correlations between components MSM(3)

Correlations (Table 2) are not surprisingly stronger between components of the same returns series and also stronger at the same frequency between two different series. An interesting feature observed is that correlations are higher for long term components (k=3): even if there are some arbitrages on the shorter run (smaller correlations), there is convergence in the long run for

market risk. This exactly show why it is important to consider several frequencies in the data since results on market links may really depend on the frequency of the data used.

However, at this stage comovement are not explicitly implemented in the univariate models. The bivariate model presented in section 3 is thus estimated for each pair of indices.

4.2 Bivariate MSM estimations and comovements structure

The previous section shows that for the four indexes, MSM(3) is optimal. Estimations and results are thus provided in this section for this model. However, complete estimations of bivariate models for $\bar{k}=2$ to 5 are presented in appendix D. Since it is estimated by pair, six models are estimated. The following table gives estimations of bivariate MSM(3) models.

	CAC-DAX	CAC-FTSE	CAC-NYSE	DAX-FTSE	DAX-NYSE	FTSE-NYSE
m_0^α	1.419 (0.0175)	1.414 (0.0171)	1.426 (0.0186)	1.467 (0.0183)	1.470 (0.0166)	1.433 (0.0163)
σ^α	1.549 (0.0592)	1.556 (0.0589)	1.534 (0.0531)	1.715 (0.0646)	1.709 (0.0620)	1.285 (0.0693)
m_0^β	1.469 (0.0167)	1.436 (0.0181)	1.437 (0.0179)	1.435 (0.0176)	1.434 (0.0178)	1.432 (0.0192)
σ^β	1.716 (0.0620)	1.339 (0.0711)	1.257 (0.0543)	1.337 (0.0725)	1.264 (0.0577)	1.263 (0.0644)
b	0.253 (0.0718)	0.314 (0.0962)	0.278 (0.0961)	0.313 (0.102)	0.306 (0.082)	0.330 (0.086)
γ_1	0.041 (0.0093)	0.032 (0.0095)	0.051 (0.0139)	0.033 (0.0083)	0.045 (0.0086)	0.044 (0.0125)
λ	0.956 (0.482)	0.984 (0.487)	0.899 (0.461)	0.903 (0.437)	0.788 (0.614)	0.903 (0.453)
ρ_ε	0.909 (0.0031)	0.837 (0.0055)	0.784 (0.0077)	0.814 (0.0065)	0.787 (0.0077)	0.779 (0.0079)
lnL	-7132.4	-6982.8	-7201.9	-7357.5	-7405.0	-6544.5

Table 3: Models estimations for bivariate $MSM(3)$

First, estimations of the component m_0 and σ for each of the series are close to the estimations in the univariate cases and stable across models. The estimations are constrained with $\rho_m{}^*=1$ because it is not rejected by the data. Calvet et al. (2006) always consider this constraint in the estimation procedure as well for exchange rate data. Ex ante, for stock indexes, both types of estimations (constraint and unconstraint) are performed. It is confirmed even for stock prices that is $\rho_m{}^*$ not different from unity. Coming back to the model, the $\rho_m{}^*$ parameter gives the

unconditional correlation of the multipliers $M_k^{\alpha} = \{m_0^{\alpha}; 2 - m_0^{\alpha}\}$ and $M_k^{\beta} = \{m_0^{\beta}; 2 - m_0^{\beta}\}$ under the bivariate binomial distribution M (equation 12). Therefore, it means that the probability to be in instant t in two opposite states in these two places (for example very high volatility state in α and very low volatility state in β) is null.

Turning to the comovement structure, estimated parameter λ gives the unconditional correlation between information arrivals (jumps) on markets (equation 10). Estimates are very high: from 0.78 for DAX-NYSE to 0.98 between the CAC and the FTSE.

The unconditional correlations between the residuals, ρ_{ε}, also appears quite high (equation 8). This correlation is the lowest for the NYSE whatever is the other index (around 0.76). A ranking in market correlations is obtained. The highest ones are between two places sharing the same currency (CAC-DAX). The second one in level is between European countries (FTSE-CAC and FTSE-DAX). The last and lowest ones are between European places and the NYSE index.

4.2.1 Shared cycles and correlations between indices

From equation 19 it is calculated the shared volatility cycles for each pair. This gives one more piece of information than previously since it is a shared cycle between the two considered indices.

	CAC-DAX	CAC-FTSE	CAC-NYSE	DAX-FTSE	DAX-NYSE	FTSE-NYSE
τ_1	24.3	30.6	19.2	30.2	21.9	22.7
τ_2	94.4	96.4	67.9	95.6	70.5	67.8
τ_3	371.1	305.5	242.7	304.5	229.2	204.5

Table 4: Volatility shared cycles length between indices (days)

The shared volatility cycles are the shortest one for the NYSE, whatever is the other index, CAC, DAX or FTSE. Their length is closed from what is observed in the univariate case.

From the set of estimated parameters, the varying correlations are obtained. However, these correlations are not strictly speaking "time varying" correlations as in the class of multivariate GARCH models, but daily state dependent correlations. Figure 2 gives the historical evolution of the varying correlations for each of the indexes obtained from equation 15.

Persistent changes in correlation states are more observed for the NYSE index than for correlations between the CAC and the DAX ; correlations with the FTSE being an intermediate
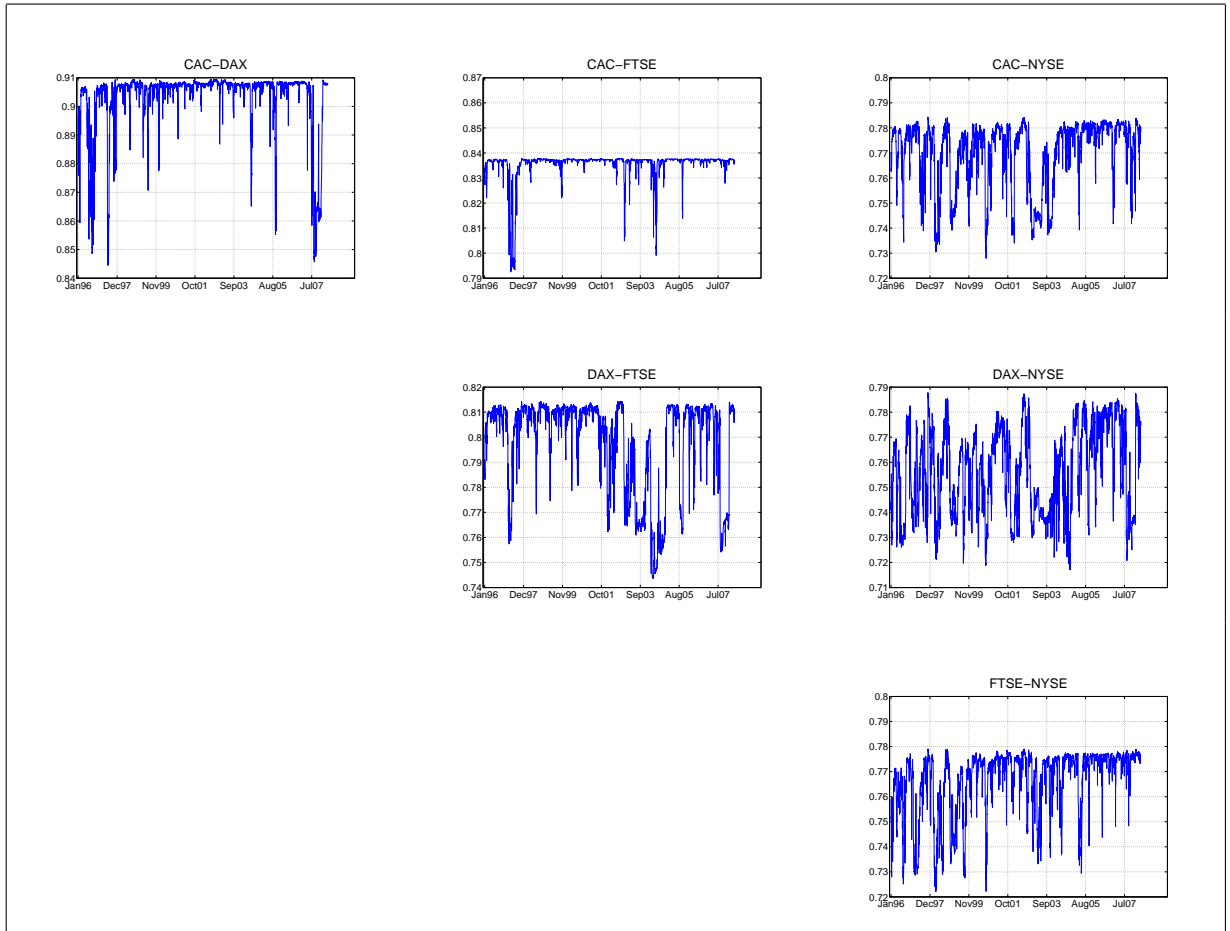
Figure 2: Bivariate MSM(3) Correlations

case. These correlations are quite rigid but exert some negative or positive sudden shocks driven by jumps in the heterogeneous lasting volatility components.

4.2.2 Crises, extreme comovements and long term volatility cycles

Crises are detecting by an increase in the probability to be for both markets in the highest volatility state defined by equation 20 (Figure 3).

Several periods of trouble are detected for all indices. The Asian crisis lasts two days and all indices are concerned: 27/10/97 and 28/10/97. The Russian crisis in august 1998, starts a weak confidence period for market participants. Moreover this period is the period of crisis with LTCM (Long term capital management), coupled with the three successive cuts in the fed funds
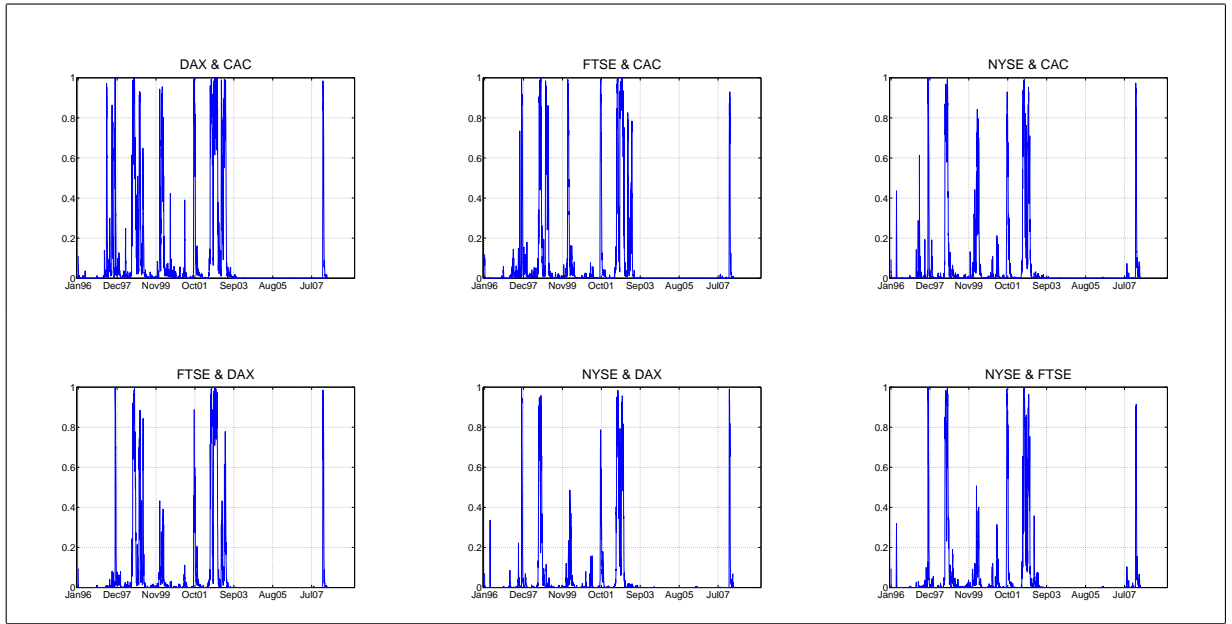
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Figure 3: Bivariate MSM(3) joint crisis probability

rate to avoid market fears (29/09; 15/10; 29/11). March 2000 is also observed with a pick in the crisis probability at 0.82. The effects of 09.11 are also detected by the probability crisis. Then, the period between July the 15th 2002 and August the 10th 2002, is also characterized by high common peaks in volatility and so a rise in the probability of joint crisis. This period is a trouble period concerning the involvement of different countries (and particularly England) in the Iraqi War II. It is also marked by the high volatility on the US market due to bankruptcy of large companies as Wordcom and Anro and the beginning of the US accounting scandals. Finally, the agreement from the US senate in October 2002 for the Iraqi war and the entry of troops in March 2003 in Iraq rise volatilities. We notice after this a very long period of very low probability crisis until 2007. The end of 2007 and 2008 is marked by a peak in the probability of joint crisis, due to the subprime episode. This peak appeared in the early 2008 when contagion really appeared in the markets.

A complementary key feature of the model is to give the probability of volatility transmission between markets. It is presented the conditional probabilities to be in the highest volatility state on market α knowing that market β is in the highest volatility state (equation 21), defined as the probability of crisis transmission. Figure 4 concerns the transmission phenomena involving

25

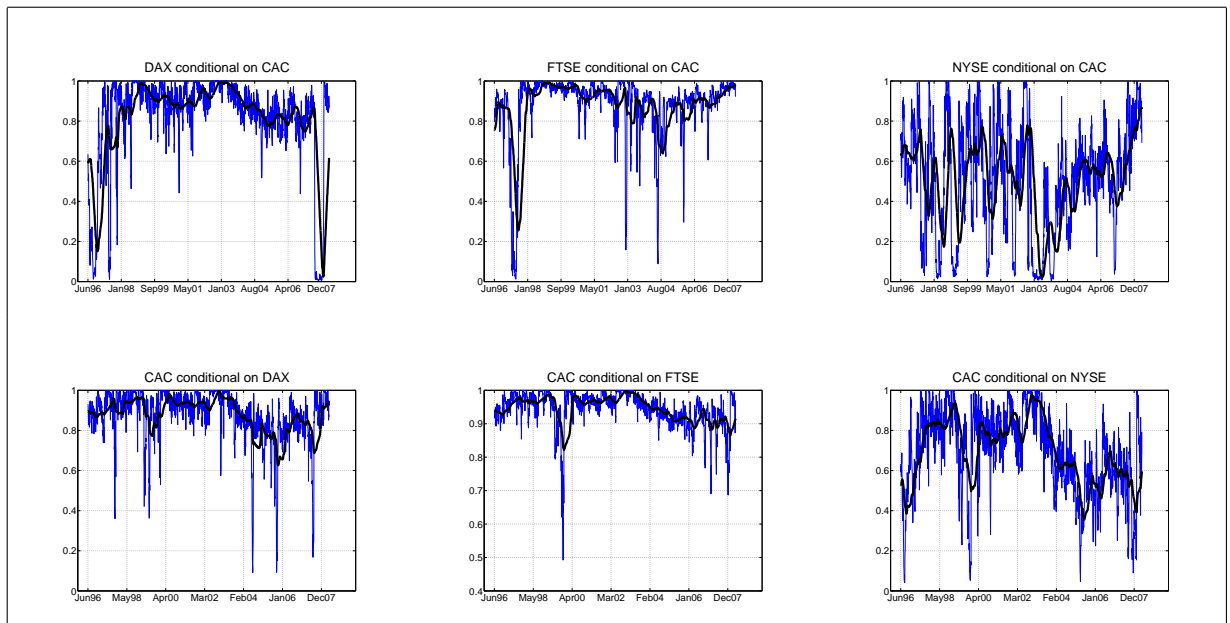the CAC index. All the remaining graphs are reported in the appendix E.

Figure 4: Bivariate MSM(3) : Conditional probability of extreme comovements (50 days MA in bold)

Structurally, it appears that conditional extreme comovement probability is higher between European places. With the US, it is higher from the NYSE to the CAC than the reverse, which is not surprising. However, we notice one particular negative shock in 1999, the year of the euro area creation but this was transient. Concerning extreme comovement for the CAC conditional on the NYSE, peaks are observed during trouble periods. This is confirmed at the end of the sample which is linked with the subprime crisis. This is also observed for the other indexes.

For the DAX conditional on the CAC, this is also strong except after August 2007 where a huge break is obtained for the DAX with all other indices : this shows relative strength of the German market to the recent events. This is notably due to the fact that financial industry is not as weighted in the DAX than in other countries like France and United Kingdom ; and 2007 performances of the German economy was also better.

For the FTSE, extreme comovements are stronger conditional on the CAC than on the DAX, and an intermediate case with the NYSE. However, on this market, the risk probability with the three others is rising since 2005.

From a long term perspective, and not only a crisis perspective, it is drawn the timing of the long term cycles in volatility from equations 22 and 23. Long term cycles are captured by the volatility components with the lowest frequency of jump. It thus corresponds to the longest periods.

To analyze this, it is considered the probability for having a low value for both component $\bar{k}$ in market α and β and the probability to have a high value for both components $\bar{k}$. For each pair of indices, Figure 5 gives the probability to be in a common long term high volatility cycle, and the probability to be in a long term low volatility cycle. All graphs are reported in appendix F.
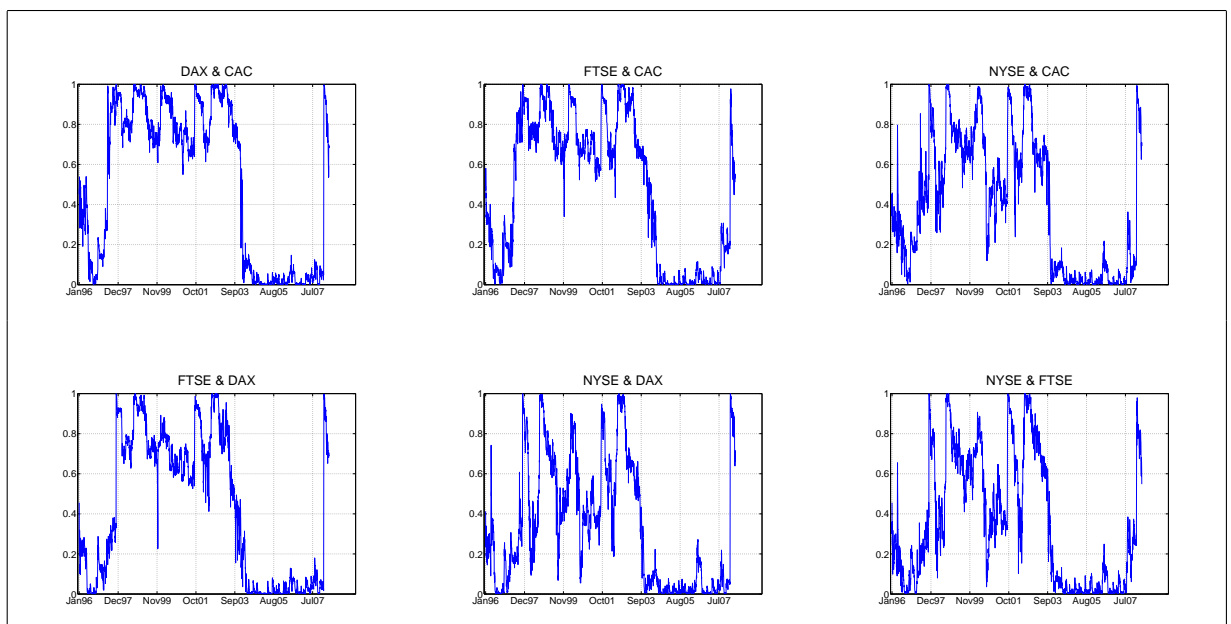
Figure 5: Bivariate MSM(3) : Probability of long term high volatility cycle

The probability of long term high common cycle for the pair of indices is very high in all cases between end-1997 to end-2003. The probability to be in a common low volatility state is high for the period 1994-1997 and 2003-2006. This is a new indicator about comovement and help understanding historical evolution of common long term cycles in asset prices. It appears that the Asian crisis had globally launched from 1997 a high volatility cycle.

Typically, the subprime crisis does not appear as a crisis before 2008 because it occurs during

a long term low volatility cycle, and did not reverse this cycle to a high long term volatility cycle. This is key since the only switch of short and medium term cycle do not generate on their own a crisis since the long term cycle is still at the low level for volatility. However, in the early 2008, the long term cycle has clearly jumped to the high value on all markets. The probability of crisis transmission has jumped to unity for all cases. The contagion phenomena is clearly at the heart of the 2008 crisis.

5 Conclusions

The paper presents the Multifractal Markov Switching model for index returns on four major places: Paris, Frankfurt, London and New-York. From this empirical model, it is defined a set of indicators that help understanding the nature of comovements, cycles and correlations. First, it is defined a state varying correlation between indices that depends on a graduate scale of several volatility states. From this, periods are defined and exert a three volatility cycles strata structure of comovements. Then, from the probability structure assigned to these volatility scales, it is calculated probabilistic indicators about crisis and long term cycles. A crisis is newly defined as a rise in the joint probability in being in the highest state of volatility. In other words, it corresponds when the three identified cycles are respectively in their highest states. Extreme high volatility comovements are then defined as a probability of highest volatility state conditional on the volatility of another market.

This is a main contribution of the MSM model for identifying crises, comovements and long run dependency since the number of cycles, and the volatility states are not imposed. Moreover a probabilistic structure is estimated which is more accurate in assessing the degree and the nature of commonality during periods of trouble.

The model has detected several joint crises (end 1997, August and September 1998, September 2001, July 2002, October 2002, March 2003 and January 2008) occurring during a high long term volatility cycle. From the model it is shown that between 1996 and 2007, three long term cycles are detected: 1996-1997 and 2003-2007 are both predominantly low volatility long term cycles and 1997-2003 a high one. However, 2008 appears as a start in a new high volatility long term cycle, this launched by the identified January 2008 crisis. Conditional probabilities of extreme

comovements have also been defined, which give the potential risk for one market to move in crisis time with another market. These features have shown how for example the DAX index has resisted to troubles during 2007 perturbations, even if one main reason is the low financial industry weighting in the German index compared to others.

To a methodology perspective, it would be interesting in further research to recover a stronger time dependency in the correlations and the MSM model. This would lead to an intermediate model coupling time varying correlations and the specification of the returns based on the product of several long lasting components. To summarize, the use of this empirical model gives a set of new indicators about comovements, other than correlations and complements views about market integration, market comovements and crisis.

Appendix A. Transition matrix

The probability that one piece of information arrives at the same time on both market is given by

$$d_{11,k} = \Pr(D^{\alpha}_{k,t} = 1 = D^{\beta}_{k,t}) = \Pr(D^{\beta}_{k,t} = 1 \mid D^{\alpha}_{k,t} = 1).\Pr(D^{\alpha}_{k,t} = 1) \tag{26}$$

and similarly for the probability that only one piece of information arrives on one of the two markets, and no information arrival on both market. These different probabilities give the following d_k matrices, with element $d_{ij,k}$ where $i=D^{\alpha}_{k,t}$ and $j=D^{\beta}_{k,t}$:

$$d_k = \begin{bmatrix} d_{11,k} & d_{10,k} \\ d_{01,k} & d_{00,k} \end{bmatrix} = \begin{bmatrix} [(1-\lambda)\gamma_k + \lambda]\,\gamma_k & (1-\gamma_k)\,(1-\lambda)\gamma_k \\ (1-\gamma_k)\,(1-\lambda)\gamma_k & [1 - \gamma_k(1-\lambda)]\,(1-\gamma_k) \end{bmatrix} \tag{27}$$

Since it is considered a bivariate binomial model, it is obtained for each k that the random vector $M_{k,t}$ can take four possible states: $s_1^k = \left(m_0^{\alpha}, m_0^{\beta}\right)$, $s_2^k = \left(m_0^{\alpha}, m_1^{\beta}\right)$, $s_3^k = \left(m_1^{\alpha}, m_0^{\beta}\right)$, $s_4^k = \left(m_1^{\alpha}, m_1^{\beta}\right)$ with $m_1^c = 2 - m_0^c$. The d_k matrix allows for the calculation of the transition matrix T_k of the multipliers vector $M_{k,t} = \binom{M^{\alpha}_{k,t}}{M^{\beta}_{k,t}}$ where each element is defined as

$$t_{ij} = \Pr(s^k_{t+1} = s^k_j \mid s^k_t = s^k_i) \tag{28}$$

with $i,j = \{1,2,3,4\}$. All calculations give:

$$T_k = \begin{pmatrix} \Psi_k & \Phi_k\text{-}\left(d_{00,k} + \frac{d_{01,k}}{2}\right) & \Phi_k\text{-}\left(d_{00,k} + \frac{d_{01,k}}{2}\right) & \Psi_k\text{-}(d_{00,k} + d_{01,k}) \\ \Psi_k\text{-}\left(d_{00,k} + \frac{d_{01,k}}{2}\right) & \Phi_k & \Phi_k\text{-}(d_{00,k} + d_{01,k}) & \Psi_k\text{-}\left(d_{00,k} + \frac{d_{01,k}}{2}\right) \\ \Psi_k\text{-}\left(d_{00,k} + \frac{d_{01,k}}{2}\right) & \Phi_k\text{-}(d_{00,k} + d_{01,k}) & \Phi_k & \Psi_k\text{-}\left(d_{00,k} + \frac{d_{01,k}}{2}\right) \\ \Psi_k\text{-}(d_{00,k} + d_{01,k}) & \Phi_k\text{-}\left(d_{00,k} + \frac{d_{01,k}}{2}\right) & \Phi_k\text{-}\left(d_{00,k} + \frac{d_{01,k}}{2}\right) & \Psi_k \end{pmatrix} \tag{29}$$

with

$$\Psi_k = d_{00} + d_{01} + d_{11}\left(\frac{1 + \rho_m^*}{4}\right)$$

$$\Phi_k = d_{00} + d_{01} + d_{11}\left(\frac{1 - \rho_m^*}{4}\right).$$

Finally, depending on the choice of $\bar{k}$, the number of frequencies in the model, the volatility

state transition matrix of asset returns A with elements (a_{ij}) with $1{\leqslant}$i,j${\leqslant}4^{\bar{k}}$ is given by:

$$a_{ij} = \Pr(S_{t+1} = S^j \mid S_t = S^i)$$

with S=$\left(s^1, s^2, ..., s^{\bar{k}}\right)$, the vector of frequency states so that the number of states grows geometrically with the number of frequencies.

Appendix B. Univariate model estimations

	$\bar{k}=1$	2	3	4	5	6	7	8
m_0^{CAC}	1.649 (0.017)	1.542 (0.016)	1.421 (0.018)	1.391 (0.018)	1.341 (0.017)	1.336 (0.023)	1.337 (0.024)	1.388 (0.019)
σ^{CAC}	1.572 (0.043)	1.636 (0.046)	1.546 (0.056)	1.389 (0.065)	1.508 (0.073)	1.314 (0.061)	1.138 (0.053)	2.428 (0.021)
b^{CAC}	—	0.212 (0.11)	0.238 (0.095)	0.387 (0.118)	0.564 (0.126)	0.586 (0.142)	0.528 (0.100)	0.365 (0.130)
γ_{CAC}	0.022 (0.0057)	0.021 (0.0057)	0.044 (0.0118)	0.037 (0.011)	0.029 (0.0125)	0.029 (0.0139)	0.033 (0.0133)	0.037 (0.012)
lnL	-4782.3	-4697.3	-4690.1	-4687.4	-4686.5	-4686.9	-4688.0	-4689.0

Table 1: MSM(k) estimations by MLE for the CAC index

	$\bar{k}=1$	2	3	4	5	6	7	8
m_0^{DAX}	1.686 (0.015)	1.598 (0.015)	1.469 (0.014)	1.389 (0.0189)	1.395 (0.014)	1.352 (0.019)	1.346 (0.022)	1.355 (0.017)
σ^{DAX}	1.760 (0.043)	1.677 (0.037)	1.716 (0.0529)	1.592 (0.097)	1.988 (0.135)	1.801 (0.103)	1.139 (0.069)	2.817 (0.187)
b^{DAX}	—	0.081 (0.0569)	0.262 (0.108)	0.432 (0.127)	0.403 (0.093)	0.581 (0.104)	0.499 (0.097)	0.489 (0.092)
γ_{DAX}	0.018 (0.0046)	0.030 (0.0041)	0.039 (0.0099)	0.039 (0.0117)	0.040 (0.014)	0.035 (0.0135)	0.044 (0.0157)	0.043 (0.0170)
lnL	-4991.7	-4883.7	-4872.0	-4872.4	-4873.6	-4872.1	-4873.0	-4874.4

Table 2: MSM(k) estimations by MLE for DAX index

	$\bar{k}=1$	2	3	4	5	6	7	8
m_0^{FTSE}	1.676 (0.015)	1.553 (0.017)	1.438 (0.015)	1.412 (0.022)	1.349 (0.021)	1.331 (0.029)	1.295 (0.029)	1.296 (0.034)
σ^{FTSE}	1.171 (0.029)	1.353 (0.041)	1.378 (0.060)	1.170 (0.044)	1.213 (0.076)	1.079 (0.070)	1.041 (0.088)	1.188 (0.109)
b^{FTSE}	—	0.326 (0.141)	0.468 (0.225)	0.491 (0.168)	0.574 (0.132)	0.610 (0.127)	0.642 (0.127)	0.667 (0.142)
γ_{FTSE}	0.019 (0.0038)	0.021 (0.0068)	0.021 (0.009)	0.025 (0.0105)	0.031 (0.014)	0.032 (0.0151)	0.038 (0.017)	0.035 (0.0167)
lnL	-4192.3	-4107.0	-4096.5	-4091.1	-4090.4	-4091.2	-4091.4	-4091.6

Table 3: MSM(k) estimations by MLE for FTSE index

	$\bar{k}=1$	2	3	4	5	6	7	8
m_0^{NYSE}	1.663 (0.014)	1.526 (0.015)	1.436 (0.018)	1.379 (0.020)	1.348 (0.022)	1.422 (0.024)	1.293 (0.018)	1.438 (0.021)
σ^{NYSE}	1.165 (0.028)	1.234 (0.037)	1.227 (0.070)	1.161 (0.0615)	1.072 (0.0558)	1.159 (0.059)	1.039 (0.085)	2.04 (0.168)
b^{NYSE}	–	0.558 (0.52)	0.323 (0.119)	0.349 (0.112)	0.497 (0.121)	0.377 (0.129)	0.597 (0.082)	0.252 (0.120)
γ_{NYSE}	0.043 (0.008)	0.030 (0.014)	0.063 (0.022)	0.108 (0.046)	0.092 (0.046)	0.058 (0.017)	0.112 (0.051)	0.069 (0.026)
lnL	-4018.5	-3959.8	-3952.1	-3950.5	-3951.2	-3952.2	-3949.7	-3953.7

$Table$ 4: MSM(k) estimations by MLE for NYSE index

Appendix C. HAC Vuong Test

	CAC			DAX			FTSE			NYSE		
#k	LRstat	c.v	prob	LRstat	c.v	prob	LRstat	c.v	prob	LRstat	c.v	prob
2vs1	1.575*	0.464	0.99	2.00*	0.51	0.99	1.471*	0.504	0.99	0.94*	0.382	0.99
3vs2	0.193*	0.235	0.92	0.21*	0.24	0.93	0.204*	0.216	0.96	0.134*	0.164	0.92
4vs3	0.049	0.111	0.77	0.007	0.136	0.53	0.065	0.151	0.76	0.035	0.111	0.69
5vs4	0.009	0.083	0.58	0.023	0.046	0.79	0.016	0.089	0.62	0.019	0.048	0.74
6vs5	0.008	0.04	0.62	0.028	0.072	0.74	0.001	0.046	0.49	0.058	0.081	0.88
7vs6	0.015	0.024	0.84	0.012	0.023	0.82	0.006	0.062	0.56	0.027	0.049	0.53
8vs7	0.026	0.097	0.67	0.003	0.03	0.51	0.003	0.0034	0.92	0.072	0.103	0.87
3vs5	0.011	0.101	0.664	0.003	0.138	0.498	0.108	0.153	0.787	0.0175	0.112	0.61
3vs6	0.003	0.127	0.52	0.055	0.154	0.721	0.094	0.169	0.778	0.002	0.124	0.49
3vs7	0.012	0.138	0.554	0.042	0.165	0.763	0.091	0.155	0.844	0.044	0.105	0.76
3vs8	0.038	0.110	0.714	0.041	0.136	0.698	0.087	0.142	0.845	0.028	0.062	0.77

$HAC-Vuong$ test. Null Hypothesis: models are equivalent.

Appendix D. Bivariate model estimations

	CAC-DAX	CAC-FTSE	CAC-NYSE	DAX-FTSE	DAX-NYSE	FTSE-NYSE
m_0^1	1.544 (0.018)	1.542 (0.018)	1.543 (0.018)	1.596 (0.0176)	1.598 (0.0179)	1.555 (0.017)
σ^1	1.642 (0.051)	1.637 (0.0485)	1.644 (0.049)	1.678 (0.038)	1.681 (0.041)	1.355 (0.047)
m_0^2	1.596 (0.0162)	1.553 (0.0174)	1.526 (0.0181)	1.558 (0.0171)	1.527 (0.0182)	1.526 (0.0179)
σ^2	1.677 (0.038)	1.351 (0.0468)	1.228 (0.0418)	1.347 (0.044)	1.219 (0.044)	1.232 (0.042)
b	0.129 (0.061)	0.262 (0.139)	0.311 (0.151)	0.157 (0.0803)	0.196 (0.097)	0.395 (0.174)
γ_1	0.026 (0.004)	0.021 (0.0048)	0.026 (0.0056)	0.026 (0.0056)	0.032 (0.0053)	0.025 (0.0059)
λ	0.949 (0.480)	0.988 (0.491)	0.891 (0.427)	0.789 (0.365)	0.713 (0.400)	0.922 (0.453)
ρ_ε	0.905 (0.003)	0.840 (0.0052)	0.787 (0.0072)	0.811 (0.0069)	0.782 (0.0073)	0.781 (0.0068)
lnL	-7251.3	-7011.9	-7242.6	-7442.0	-7470.7	-6571.1

Bivariate $MSM(2)$

	CAC-DAX	CAC-FTSE	CAC-NYSE	DAX-FTSE	DAX-NYSE	FTSE-NYSE
m_0^1	1.389 (0.018)	1.392 (0.0168)	1.385 (0.0187)	1.388 (0.0177)	1.389 (0.0163)	1.381 (0.017)
σ^1	1.391 (0.067)	1.387 (0.063)	1.399 (0.068)	1.591 (0.083)	1.594 (0.082)	1.209 (0.059)
m_0^2	1.389 (0.018)	1.404 (0.024)	1.424 (0.0188)	1.388 (0.026)	1.423 (0.021)	1.424 (0.0194)
σ^2	1.592 (0.089)	1.175 (0.0588)	1.064 (0.042)	1.202 (0.074)	1.065 (0.043)	1.065 (0.0422)
b	0.413 (0.078)	0.428 (0.079)	0.415 (0.083)	0.432 (0.093)	0.440 (0.090)	0.424 (0.098)
γ_1	0.038 (0.0091)	0.032 (0.0076)	0.045 (0.0113)	0.037 (0.0103)	0.045 (0.0117)	0.044 (0.0112)
λ	0.966 (0.479)	0.982 (0.487)	0.893 (0.432)	0.912 (0.436)	0.826 (0.402)	0.927 (0.454)
ρ_ε	0.908 (0.0034)	0.838 (0.0054)	0.783 (0.0087)	0.810 (0.006)	0.783 (0.0074)	0.776 (0.0067)
lnL	-7094.7	-6971.1	-7198.2	-7340.2	-7385.5	-6538.1

Bivariate $MSM(4)$

	CAC-DAX	CAC-FTSE	CAC-NYSE	DAX-FTSE	DAX-NYSE	FTSE-NYSE
m_0^1	1.343 (0.021)	1.340 (0.0193)	1.329 (0.019)	1.421 (0.029)	1.349 (0.0163)	1.340 (0.021)
σ^1	1.514 (0.088)	1.505 (0.074)	1.473 (0.090)	1.902 (0.111)	1.505 (0.068)	1.178 (0.072)
m_0^2	1.423 (0.024)	1.350 (0.021)	1.360 (0.022)	1.352 (0.021)	1.358 (0.021)	1.359 (0.022)
σ^2	1.91 (0.102)	1.215 (0.067)	1.080 (0.072)	1.221 (0.067)	1.081 (0.076)	1.081 (0.078)
b	0.512 (0.081)	0.570 (0.098)	0.507 (0.091)	0.527 (0.096)	0.521 (0.096)	0.514 (0.077)
γ_1	0.030 (0.0083)	0.031 (0.0091)	0.056 (0.019)	0.031 (0.009)	0.056 (0.016)	0.055 (0.015)
λ	0.925 (0.458)	0.977 (0.518)	0.909 (0.446)	0.879 (0.445)	0.867 (0.410)	0.935 (0.449)
ρ_ε	0.908 (0.0031)	0.836 (0.0063)	0.791 (0.0067)	0.816 (0.0058)	0.789 (0.008)	0.784 (0.0073)
lnL	-7128.9	-6951.8	-7184.0	-7356.7	-7374.2	-6523.8

Bivariate MSM(5)

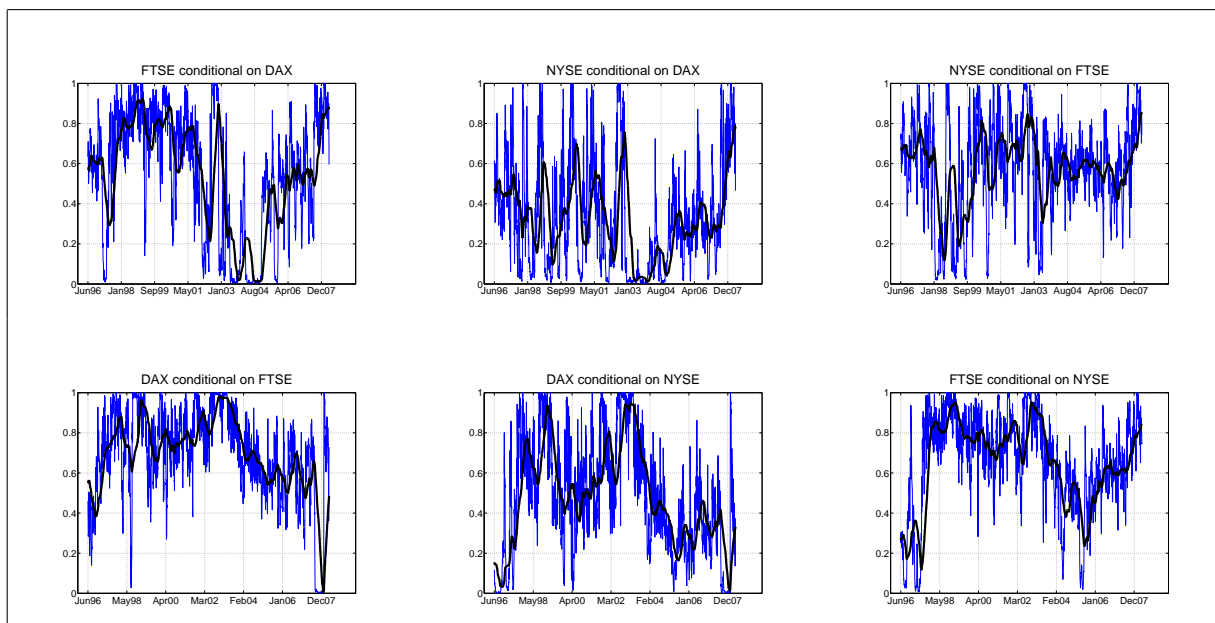## Appendix E. Conditional extreme comovements

Figure 6: Bivariate MSM(3) : conditional probability of extreme comovements (50 days MA in bold)

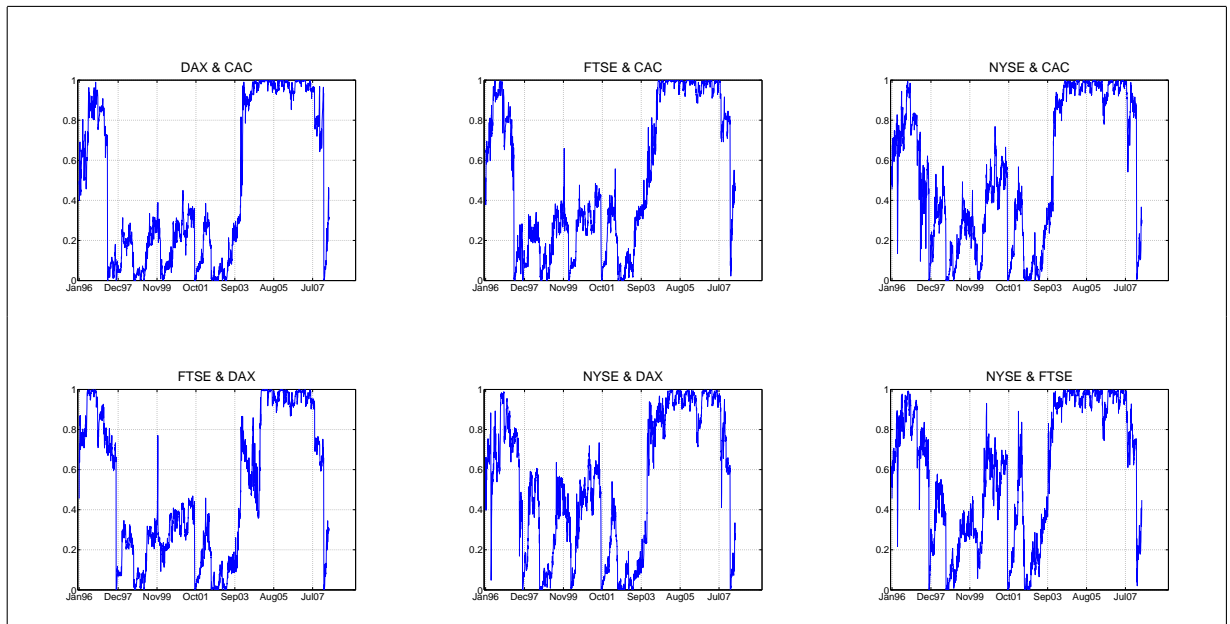Appendix F. Long run volatility cycles

Figure 7: Bivariate MSM(3) : Probability of long term low volatility cycle

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