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The Credibility of Certifiers

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Abstract

It is often argued that certifiers have an incentive to offer inflated certificates, although they deny it. In this paper, we study a model in which a certifier is paid by sellers, and may offer them inflated certificates, but incurs costs if doing so. We find that the certifier may face a commitment problem: The certifier offers inflated certificates if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for it. However, in equilibrium, the buyers cannot be fooled. The certifier would hence make a higher profit if the certifier did not offer inflated certificates and the buyers believed it. The number of inflated certificates, which the certifier offers in equilibrium, depends on the costs of offering inflated certificates. Yet, the certifier may oppose an increase in the costs of offering inflated certificates. We show that whether a certifier welcomes tighter regulation or lobbies against it, may depend on whether the new regulation only imposes higher costs, or also reduces the certifier's commitment problem significantly.

1 Introduction

It is often argued that certifiers have an incentive to offer inflated certificates. Recently, credit rating agencies have attracted a lot of attention. They have been accused of assigning inflated ratings to structured products, such as mortgage-backed securities and collateralized-debt obligations. In the last decade, auditors have also been repeatedly in the spotlight. Especially in the wake of the Enron scandal, they have faced allegations of being lapdogs instead of watchdogs.

The problem, it is argued, lies in the business model. Certifiers are typically paid by sellers who are interested in favorable certificates. An old proverb says: 'He who pays the piper calls the tune.'

Yet, certifiers may not profit from offering inflated certificates. The reason is twofold. First, certifiers typically incur costs if they offer inflated certificates. Often they have to spend time and money to obscure that they offer inflated certificates. In addition, an inflated certificate is usually detected with some probability. And if caught, certifiers are usually punished. Often they have to pay a fine. Moreover, they occasionally lose some of their business. Sometimes, they even lose all of their business, as in the case of Arthur Andersen for its role in the Enron scandal, though this was a rare event. Secondly, the market may take into account that certifiers have an incentive to offer inflated certificates.

In this paper, we examine a certifier's incentive to offer inflated certificates. We consider a model in which a certifier is paid by sellers. While the certifier can observe the type of a good, the buyers cannot. The buyers can only observe whether the seller owns a certificate, and if so, the type of the certificate. The sellers are hence interested in favorable certificates. The certifier may offer them inflated certificates, but incurs costs if doing so. We suppose that the costs of offering inflated certificates increase with the number of inflated certificates.

We find that if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for it, the certifier always offers at least one inflated certificate. However, the certifier does not profit from offering inflated certificates, because in equilibrium, the buyers cannot be fooled: They believe correctly that the certifier offers a certain amount of inflated certificates, and take this into account in their willingness-to-pay for a good. And the sellers take this into account in their willingness-to-pay for a certificate. The certifier would hence make a higher profit if the certifier did not offer inflated certificates and the buyers believed it.

The certifier hence faces a commitment problem. It would be best off if it did not offer inflated certificates, and the buyers believed it. But if the buyers actually believed that the certifier does not offer inflated certificates, the certifier would have an incentive to do so.

The number of inflated certificates obviously depends on the costs the certifier incurs when offering inflated certificates. If the costs increase, the certifier offers less inflated certificates in equilibrium.

Yet, the certifier may oppose an increase in the costs of offering inflated certificates, because an increase in the costs has two opposing effects on the certifier's profit. On the one hand, an increase in the costs reduces the number of inflated certificates, and thereby indirectly increases the certifier's profit. On the other hand, however, an increase in the costs directly reduces the certifier's profit.

There is a growing literature on certification intermediaries. Several papers study under which conditions a certifier does not have an incentive to offer inflated certificates. However, there is also a growing number of papers which show that a certifier may offer inflated certificates in equilibrium.

Strausz (2005) studies a model in which buyers detect any inflated certificate ex post, and the certifier goes out of business if caught. He shows that honest certification requires a patient certifier, high prices, and constitutes a natural monopoly. In our model, we take a different approach. We consider a situation in which a certifier offers inflated certificates in equilibrium, and show that the certifier faces a commitment problem.

Peyrache and Quesada (2011) also study a model in which an inflated certificate is detected with probability one, and the certifier goes out of business forever if caught. They focus, however, on an equilibrium in which a certifier may offer an inflated certificate: They find that an impatient certifier may offer an inflated certificate with some probability. However, in their model, even a very impatient certifier does not offer an inflated certificate with probability one. In our model, we find that a certifier always offers at least one inflated certificate if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for it.

Mathis et al. (2009) study a reputation model to analyze the incentive of a credit rating agency to assign inflated ratings. They assume that a credit rating agency is either committed to tell the truth, or maximizes its payoff. They show that a credit rating agency which maximizes its payoff may first build a reputation of being committed, and then 'cash in' on its reputation by assigning inflated ratings. Our model, however, does not rely on

the assumption that the buyers believe that the certifier is committed to tell the truth with positive probability. In our model, the buyers know that the certifier maximizes its profit.

Bolton et al. (2011) also study the incentive of a credit rating agency to offer inflated ratings. They assume that a fraction of investors are naive and take ratings at face value. They find that a credit rating agency may offer inflated ratings if there are many naive investors and the expected punishment is light. In contrast to Bolton et al. (2010), our model does not rely on the assumption that a fraction of buyers are naive. We show that even if buyers cannot be fooled in equilibrium, a certifier may have an incentive to offer inflated certificates.

In Stolper (2009), we study a model in which a regulator can observe the default rate within a rating category for each credit rating agency. The default rate may, however, be influenced by a common shock, and the credit rating agencies may collude to offer inflated ratings. As a result, the regulator cannot detect whether high default rates are due to collusion or the common shock. In Stolper (2009), we hence suggest that a regulator should not only deter a credit rating agency from unilaterally offering inflated ratings, but also provide an incentive to deviate from a collusive agreement to offer inflated ratings. In our model of a certifier's credibility, we do not consider the possibility of a collusive agreement. Instead, we show that a certifier may face a commitment problem.

There are several papers which study related commitment problems in other settings. Barro and Gordon (1983), for instance, study a model in which a central bank tries to decrease unemployment by increasing inflation. When making its decision, the central bank takes inflation expectations as given, and chooses an excessive rate of inflation. However, in equilibrium, people correctly anticipate the central bank's decision. The central bank hence cannot reduce unemployment, and would be better off if both inflation and the expectation of inflation were zero. Yet, if the expectation of inflation was actually zero, the central bank would have an incentive to choose a positive rate of inflation.

Signal-jamming models (first analyzed by Holmström, 1982) often make a similar point. Stein (1989), for instance, studies a model in which a manager tries to increase the stock price by pumping up earnings. However, in equilibrium, the manager cannot fool investors because they take into account that earnings are inflated. The manager would hence be better off if he (or she) did not manipulate earnings, and the investors did not suspect that earnings are manipulated. But if the investors actually believed that the manager does not manipulate earnings, the manager would have an incentive to do so.
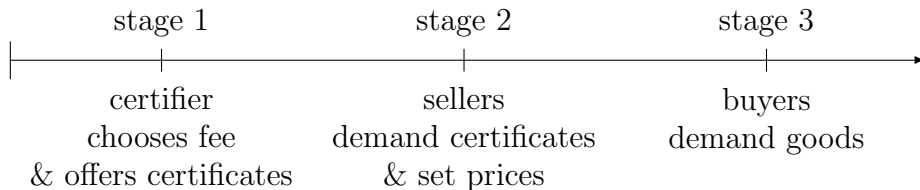
Figure 1: The time structure.

The rest of the paper is organized as follows. In section 2, we present the model. In section 3, we consider the certifier's decision whether to offer inflated certificates. We show that the certifier faces a commitment problem and analyze the effect of an increase in the costs of offering inflated certificates. In section 4, we conclude. Proofs are provided in the appendix.

2 The Model

Consider a model with many sellers, many buyers, and a certifier. Each seller owns one good, of which there are two types, A and B. Let a denote the number of sellers who own a type-A good, and let b denote the number of sellers who own a type-B good. Each seller faces one buyer. A buyer's utility of a type-A good is u_A, and a buyer's utility of a type-B good is u_B, where $u_A > u_B$. The buyers cannot, however, observe the type of a good. The certifier, by contrast, can observe the type of a good. The buyers can only observe whether the seller owns a certificate, and if so, the type of the certificate. There are two types of certificates, again $\mathcal{A}$ and $\mathcal{B}$. A type-$\mathcal{A}$ certificate indicates that a good is of type A, and a type-$\mathcal{B}$ certificate indicates that a good is of type B.

The time structure is as follows. At stage 1, the certifier chooses a (uniform) fee and offers each seller a certificate. (Results do not change if the certifier chooses different fees for different types of certificates.) At stage 2, each seller decides whether to buy the certificate and sets a price for his or her good. At stage 3, each buyer decides whether to buy the good. Figure 1 illustrates the time structure.

At stage 1, the certifier offers every seller, who owns a type-A good, a type-$\mathcal{A}$ certificate, but may offer sellers, who own a type-B good, a type-$\mathcal{B}$ as well as a type-$\mathcal{A}$ certificate. Let x denote the number of sellers who are offered an inflated certificate.

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If the certifier offers sellers an inflated certificate, the certifier incurs costs. Typically, a certificate which indicates the wrong type is detected with some probability, and if detected, the certifier is punished. The certifier may, for instance, have to pay a fine or be ignored by buyers in future periods. In addition, the certifier may have to spend time and money to obscure that it offers inflated certificates. We suppose that the probability of detection, the punishment, or the cost of obfuscation increases with the number of inflated certificates. We hence assume that the certifier incurs costs $K(x)$, where $K(0) = 0$ and $K'(x) > 0$. Moreover, we assume $K''(x) \geq 0$. We suppose that if the probability of detection or the punishment varies for different sellers, a certifier first offers an inflated certificate to sellers, for whom the detection probability or the punishment is low.

3   The Credibility of Certifiers

In this section, we consider the certifier's decision whether to offer inflated certificates. First, we show that the certifier faces a commitment problem. Then, we analyze the effect of an increase in the costs of offering inflated certificates.

3.1   A Commitment Problem

To consider the certifier's commitment problem, we solve the model backwards. We begin with the buyers' decision whether to buy the good.

At stage 3, the buyers buy the good if and only if the price is less than or equal to their willingness-to-pay, which depends on the certificate the seller owns. If a seller owns a type-$\mathcal{B}$ certificate, the buyer knows that the seller owns a type-B good,[1] and is hence only willing to pay u_B. If a seller either owns a type-$\mathcal{A}$ certificate or does not own a certificate at all, the buyer's willingness-to-pay depends on the buyer's beliefs.

Suppose the buyers believe that every seller, who is offered a type-$\mathcal{A}$ certificate, buys the certificate, and no seller, who is offered a type-$\mathcal{B}$ certificate, buys the certificate. Let x^e denote the buyers' belief about the number of sellers who are offered an inflated certificate. Then, if a seller owns a type-$\mathcal{A}$ certificate, the buyer believes that the seller owns a type-A good with probability $\frac{a}{a+x^e}$ and a type-B good with probability $\frac{x^e}{a+x^e}$, and is hence willing to

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[1]Because the certifier may only offer sellers, who own a type-B good, a type-$\mathcal{B}$ certificate.

pay $\frac{au_A + x^e u_B}{a + x^e}$ for the good. If a seller does not own a certificate, the buyer believes that the seller owns a type-B good, and is hence only willing to pay u_B.

At stage 2, each seller, with or without a certificate, sets a price equal to the buyer's willingness-to-pay. The difference in prices determines the sellers' willingness-to-pay for a certificate. The sellers are willing to pay zero for a type-$\mathcal{B}$ certificate. The willingness-to-pay for a type-$\mathcal{A}$ certificate again depends on the buyers' beliefs.

Suppose again the buyers believe that every seller, who is offered a type-$\mathcal{A}$ certificate, buys the certificate, and no seller, who is offered a type-$\mathcal{B}$ certificate, buys the certificate. Then, the sellers are willing to pay $\frac{au_A + x^e u_B}{a + x^e} - u_B$ for a type-$\mathcal{A}$ certificate, which is equivalent to $\frac{a(u_A - u_B)}{a + x^e}$.

The sellers buy the offered certificate if and only if the certifier chose a fee less than or equal to their willingness-to-pay. Sellers, who are offered a type-$\mathcal{A}$ certificate, buy the certificate if and only if the fee is less than or equal to their willingness-to-pay for the certificate. Sellers, who are offered a type-$\mathcal{B}$ certificate, buy the certificate if and only if the fee is zero.

At stage 1, the certifier hence chooses a fee equal to the sellers' willingness-to-pay for a type-$\mathcal{A}$ certificate. As a result, every seller, who is offered a type-$\mathcal{A}$ certificate, buys the certificate, and no seller, who is offered a type-$\mathcal{B}$ certificate, buys the certificate.

Suppose in the following that the buyers believe correctly that every seller, who is offered a type-$\mathcal{A}$ certificate, buys the certificate, and no seller, who is offered a type-$\mathcal{B}$ certificate, buys the certificate. Then, as shown above, the sellers are willing to pay $\frac{a(u_A - u_B)}{a + x^e}$ for a type-$\mathcal{A}$ certificate, and the certifier chooses a fee equal to this amount. Let $WTP(x^e)$ denote the sellers' willingness-to-pay for a type-$\mathcal{A}$ certificate, where

$$WTP(x^e) = \frac{a(u_A - u_B)}{a + x^e}. \tag{1}$$

The certifier's profit $\Pi(x|x^e)$ is given by

$$\Pi(x|x^e) = WTP(x^e)(a + x) - K(x). \tag{2}$$

The certifier chooses x to maximize its profit Π given the buyers' belief x^e. (In equilibrium, the buyers' beliefs are correct, but, when making its decision, the certifier takes the buyers' beliefs as given.)
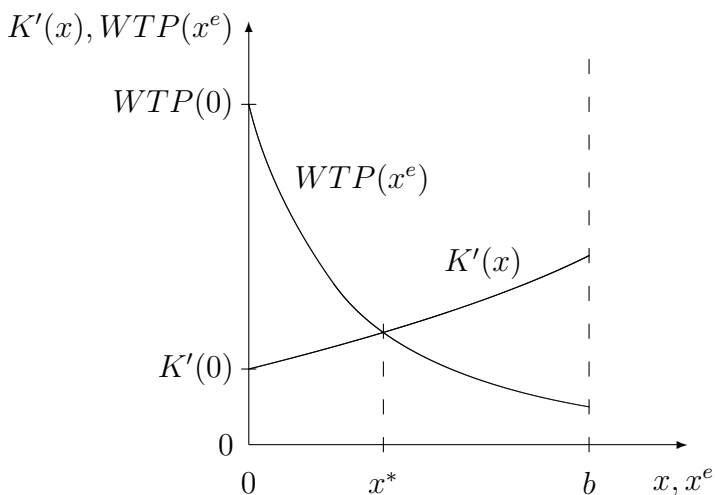
Figure 2: An equilibrium with an interior solution.

Suppose there exists an equilibrium with an interior solution. Let x^* denote the number of sellers who are offered an inflated certificate in equilibrium. Then, x^* is given by

$$\Pi'(x = x^*|x^e = x^*) = 0. \tag{3}$$

x^* is hence given by

$$K'(x^*) = WTP(x^*). \tag{4}$$

That is, in an equilibrium with an interior solution, two conditions must hold. First, the marginal costs of offering an inflated certificate are equal to the sellers' willingness-to-pay for a certificate ($K'(x) = WTP(x^e)$). Secondly, the buyers believe correctly that the certifier offers a certain number of inflated certificates ($x^e = x$). Figure 2 illustrates this. x^* is given by the intersection of $K'(x)$ and $WTP(x^e)$.

In figure 2, we can also see:

Proposition 1 *If $K'(x = 0) < WTP(x^e = 0)$, then, in equilibrium, the certifier offers at least one seller an inflated certificate ($x^* > 0$).*

That is, if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for this certificate (given the buyers believe the certifier does not offer inflated certificates), then, in equilibrium, the certifier offers at least one seller an inflated certificate.

The certifier does not, however, profit from offering inflated certificates:

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Proposition 2 *Suppose $x^* > 0$. Then, the certifier would make a higher profit than in equilibrium if the certifier did not offer inflated certificates ($x = 0$), and the buyers believed it ($x^e = 0$).*

The intuition for this result is straightforward. In equilibrium, the buyers cannot be fooled. They believe correctly that the certifier offers a certain number of inflated certificates, and take this into account in their willingness-to-pay for a good. The sellers, in turn, take this into account in their willingness-to-pay for a certificate. By offering inflated certificates, the certifier hence only incurs costs, but cannot increase its revenue as compared to the situation in which the certifier does not offer inflated certificates, and the buyers believe it.

If $x^* > 0$, the certifier hence faces a commitment problem. The certifier would make a higher profit if the certifier did not offer inflated certificates, and the buyers believed it. But if the buyers actually believed the certifier did not offer inflated certificates, the certifier would have an incentive to do so.

3.2 The Effect of an Increase in Costs

The number x^* of inflated certificates in equilibrium and the profit $\Pi(x = x^* | x^e = x^*)$ the certifier makes in equilibrium depend on the costs $K(x)$ of offering inflated certificates. Let

$$K(x) \equiv ck(x), \tag{5}$$

where $k(0) = 0$, $k'(x) > 0$ and $k''(x) \geq 0$. Suppose an equilibrium with an interior solution exists, and consider an increase in c.

First, consider the effect of an increase in c on x^*. If c increases, the certifier obviously offers less inflated certificates in equilibrium.

Proposition 3 *Suppose $0 < x^* < b$. Then, x^* decreases if c increases.*

Figure 3 illustrates this. An increase in c shifts $ck'(x)$ up. As a result, the certifier offers less inflated certificates in equilibrium.

Now consider the effect of an increase in c on $\Pi(x = x^* | x^e = x^*)$. In equilibrium, the buyers cannot be fooled ($x^e = x^*$). The profit the certifier makes in equilibrium is hence
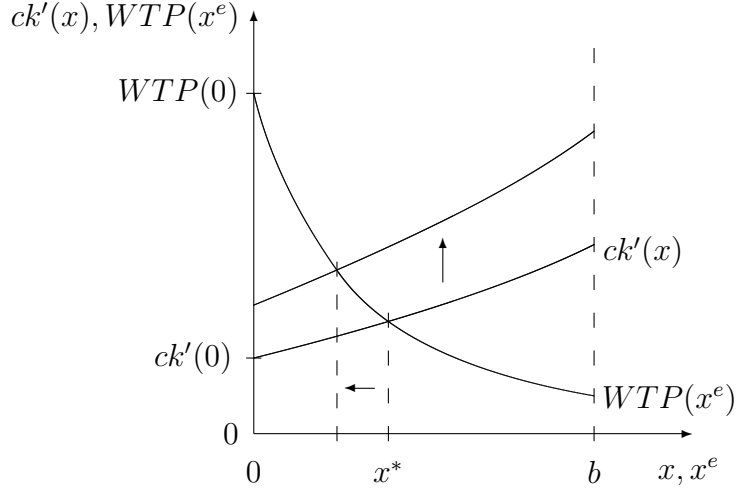
Figure 3: The effect of an increase in costs.

given by

$$\Pi(x = x^* | x^e = x^*) = \frac{a(u_A - u_B)}{a + x^*}(a + x^*) - ck(x^*) \tag{6}$$

which is equivalent to

$$\Pi(x = x^* | x^e = x^*) = a(u_A - u_B) - ck(x^*). \tag{7}$$

The profit the certifier makes in equilibrium thus depends on the costs $ck(x^*)$ the certifier incurs in equilibrium.

An increase in c has two opposing effects on the costs $ck(x^*)$ the certifier incurs in equilibrium. On the one hand, an increase in c directly increases the costs the certifier incurs. On the other hand, an increase in c reduces the number of inflated certificates, and thus indirectly reduces the costs the certifier incurs. The effect of c on the costs $ck(x^*)$ is given by

$$\frac{\mathrm{d}[ck(x^*(c))]}{\mathrm{d}c} = k(x^*) + c\frac{\mathrm{d}k(x^*)}{\mathrm{d}x}\frac{\mathrm{d}x^*}{\mathrm{d}c}. \tag{8}$$

The effect of an increase in c on the profit the certifier makes in equilibrium hence depends on the properties of the cost function. We get:

Proposition 4 *Suppose $0 < x^* < b$. Then, the profit the certifier makes in equilibrium increases with c if*

$$\frac{k''(x^*)}{k'(x^*)^2} < \frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)}, \tag{9}$$

and decreases with c if the inequality is reversed.

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The intuition for this result is as follows. If $k(x^*)$ is small, the direct effect of an increase in c on the costs $ck(x^*)$ is small. If $k'(x^*)$ is large, the effect of a decrease in x^* on the costs $ck(x^*)$ is large. And if $k''(x^*)$ is small, x^* decreases a lot if c increases. Hence, if $k(x^*)$ is small, $k'(x^*)$ is large, and $k''(x^*)$ is small, the profit the certifier makes in equilibrium increases with c. And vice versa.

Cost functions are often assumed to be linear, quadratic, or exponential. If the certifier faces a linear cost function, the marginal costs of offering inflated certificates are constant. If the certifier faces a quadratic or exponential cost function, the marginal costs of offering inflated certificates increase. We get:

Proposition 5 *Suppose $0 < x^* < b$. Then, if the cost function is linear or quadratic, the profit the certifier makes in equilibrium increases with c. If the cost function is exponential, the profit the certifier makes in equilibrium decreases with c.*

The costs $ck(x)$ may, for example, be interpreted as the expected fine the certifier has to pay. c may be interpreted as the fine the certifier has to pay if caught, and $k(x)$ may be interpreted as the probability of being caught (which increases with the number of inflated certificates). In this case, to use a more adequate notation, c could be substituted by f and $k(x)$ by $p(x)$. Then, the certifier would face an expected fine of $p(x)f$ for offering x inflated certificates. (Alternatively, c may be interpreted as the fine the certifier has to pay for each inflated certificate which is detected, and $k(x)$ may be interpreted as the expected number of inflated certificates which are detected. In this case, to use a more adequate notation, c could be substituted by f, and $k(x)$ by $n(x)$. Then, the certifier would expect to pay a total fine of $n(x)f$ for offering x inflated certificates.)

Suppose now that the certifier has to pay a fine f if caught, and that the certifier is caught with probability $p(x)$. (That is, the certifier faces an expected fine of $p(x)f$ for offering x inflated certificates.) Then, an increase in the fine has two effects. On the one hand, an increase in the fine increases the punishment if caught. On the other hand, an increase in the fine decreases the number of inflated certificates, and thus decreases the probability of being caught. Proposition 5 suggests that the certifier would welcome an increase in the fine if the probability of being caught increases linearly or quadratically with the number of inflated certificates. The certifier would, however, oppose an increase in the fine if the probability of being caught increases exponentially with the number of inflated certificates.

4   Conclusion

Certifiers often claim that they have no incentive to offer inflated certificates. The message of this paper, however, is that certifiers face a commitment problem if the costs of offering the first inflated certificate are lower than the sellers' willingness-to-pay for it: A certifier does not profit from offering inflated certificates, because in equilibrium, the buyers cannot be fooled. The certifier would make a higher profit if the certifier did not offer inflated certificates, and the buyers believed it. But if the buyers actually believed the certifier did not offer inflated certificates, the certifier would have an incentive to do so.

In addition, there may be a welfare loss. The certifier may, for instance, have to spend time and money to obscure that it offers inflated certificates. Moreover, if there is a fine for offering inflated certificates, the fine may not be a pure transfer. If there are costs of imposing the fine, a part of the fine is 'lost'. Besides, there may also be a welfare loss which is not captured by our simple model. For example, it is often argued that credit rating agencies contributed to the subprime crisis by assigning inflated ratings to mortgage-backed securities and collateralized debt obligations.

It seems that the costs of offering the first inflated certificate are indeed often lower than the sellers' willingness-to-pay for it. Typically, an inflated certificate is detected with some probability, and if detected, the certifier is usually punished. Yet, it seems that if a certifier is only a bit too lax, the probability of detection is low, and if the certifier is caught nonetheless, the punishment is not too harsh.

In the model, we find that the number of inflated certificates depends on the costs a certifier incurs if it offers inflated certificates. If the costs are low, a certifier offers a lot of inflated certificates in equilibrium.

It could be argued that credit rating agencies had a strong incentive to assign inflated ratings to mortgage-backed securities and collateralized-debt obligations, because they got off lightly for assigning inflated ratings. Granted, they now face (somewhat) tighter regulation and lost some business, but none of the major credit rating agencies went out of business, and it is unclear whether they lost more business than they attracted by offering inflated ratings in the first place.

In the model, we show that a certifier may yet oppose an increase in the costs of offering inflated certificates. On the one hand, an increase in the costs reduces the number of inflated certificates, and thus indirectly increases the certifier's profit. On the other hand, however,

an increase in the costs directly reduces the certifier's profit. Hence, only if the increase in costs reduces the number of inflated certificates by a large amount, the certifier's profit increases with the costs of offering inflated certificates. If, however, the increase in costs does not reduce the number of inflated certificates significantly, the certifier's profit decreases with the costs of offering inflated certificates.

This suggests that whether a certifier welcomes tighter regulation or lobbies against it may depend on whether the new regulation only imposes higher costs on the certifier, or also helps to reduce the certifier's commitment problem significantly.

Appendix

Proof of Proposition 1

Suppose $K'(x=0) < WTP(x^e = 0)$. Then, there does not exist an equilibrium in which the certifier does not offer inflated certificates: If the buyers believe the certifier does not offer inflated certificates ($x^e = 0$), the certifier offers at least one seller an inflated certificate ($x > 0$), because

$$\Pi'(x = 0 | x^e = 0) > 0 \tag{10}$$

if

$$K'(x = 0) < WTP(x^e = 0). \tag{11}$$

There exists either an equilibrium with an interior solution (as characterized in section 3.1), or there exists an equilibrium with a corner solution, where $x^* = b$.

Hence, if $K'(x=0) < WTP(x^e = 0)$, the certifier offers at least one seller an inflated certificate in equilibrium ($x^* > 0$). ■

Proof of Proposition 2

$$\Pi(x = 0 | x^e = 0) > \Pi(x = x^* | x^e = x^*), \tag{12}$$

if

$$\frac{a(u_A - u_B)}{a + 0}(a + 0) - K(0) > \frac{a(u_A - u_B)}{a + x^*}(a + x^*) - K(x^*) \tag{13}$$

which is equivalent to

$$K(x^*) > 0 \tag{14}$$

or

$$x^* > 0. \tag{15}$$

Hence, if $x^* > 0$, the certifier would make a higher profit than in equilibrium if the certifier did not offer inflated certificates ($x = 0$), and the buyers believed it ($x^e = 0$). ■

Proof of Proposition 3

Suppose $0 < x^* < b$. Then, x^* is given by

$$ck'(x^*) = \frac{a(u_A - u_B)}{a + x^*}. \tag{16}$$

Applying the implicit function theorem yields

$$\frac{\mathrm{d}x^*}{\mathrm{d}c} = -\frac{k'(x^*)}{ck''(x^*) + \frac{a(u_A - u_B)}{(a+x^*)^2}}. \tag{17}$$

Because $k'(x) > 0$ and $k''(x) \geq 0$,

$$\frac{\mathrm{d}x^*}{\mathrm{d}c} < 0. \tag{18}$$

Hence, if $0 < x^* < b$, x^* decreases if c increases. ∎

Proof of Proposition 4

The certifier's profit is given by

$$\Pi = \frac{a(u_A - u_B)}{a + x^e}(a + x) - ck(x). \tag{19}$$

In equilibrium, the buyers cannot be fooled ($x^e = x^*$). Hence, in equilibrium, the certifier's revenue is given by

$$\frac{a(u_A - u_B)}{a + x^*}(a + x^*) = a(u_A - u_B). \tag{20}$$

The effect of c on $\Pi(x = x^* | x^e = x^*)$ thus depends on the effect of c on $ck(x^*(c))$, which is given by

$$\frac{\mathrm{d}[ck(x^*(c))]}{\mathrm{d}c} = k(x^*) + c\frac{\mathrm{d}k(x^*)}{\mathrm{d}x}\frac{\mathrm{d}x^*}{\mathrm{d}c}. \tag{21}$$

Suppose $0 < x^* < b$. Then, using (16) and (17) yields

$$\frac{\mathrm{d}[ck(x^*(c))]}{\mathrm{d}c} = k(x^*) - \frac{1}{\frac{k''(x^*)}{k'(x^*)^2} + \frac{c}{a(u_A - u_B)}}. \tag{22}$$

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The effect of c on $\Pi(x = x^* | x^e = x^*)$ is thus given by

$$\frac{\mathrm{d}\Pi(x = x^* | x^e = x^*)}{\mathrm{d}c} = -k(x^*) + \frac{1}{\frac{k''(x^*)}{k'(x^*)^2} + \frac{c}{a(u_A - u_B)}}. \qquad (23)$$

Hence,

$$\frac{\mathrm{d}\Pi(x = x^* | x^e = x^*)}{\mathrm{d}c} > 0 \qquad (24)$$

if

$$\frac{k''(x^*)}{k'(x^*)^2} < \frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)}. \qquad (25)$$

Note that

$$\frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)} > 0 \qquad (26)$$

if

$$a(u_A - u_B) - ck(x^*) > 0 \qquad (27)$$

or

$$\Pi(x = x^* | x^e = x^*) > 0. \qquad (28)$$

Finally,

$$\frac{\mathrm{d}\Pi(x = x^* | x^e = x^*)}{\mathrm{d}c} < 0 \qquad (29)$$

if

$$\frac{k''(x^*)}{k'(x^*)^2} > \frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)}. \qquad (30)$$

Hence, if $0 < x^* < b$, the profit the certifier makes in equilibrium increases with c if

$$\frac{k''(x^*)}{k'(x^*)^2} < \frac{1}{k(x^*)} - \frac{c}{a(u_A - u_B)}, \qquad (31)$$

and decreases with c if the inequality is reversed. ∎

Proof of Proposition 5

Suppose $0 < x^* < b$. First consider a linear cost function, then consider a quadratic and an exponential cost function.

First, suppose

$$K(x) \equiv cx. \qquad (32)$$

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Then, x^* is given by

$$c = \frac{a(u_A - u_B)}{a + x^*}, \tag{33}$$

or

$$x^* = \frac{a(u_A - u_B - c)}{c}. \tag{34}$$

$\Pi(x = x^* | x^e = x^*)$ is thus given by

$$\Pi = a(u_A - u_B) - c\frac{a(u_A - u_B - c)}{c} \tag{35}$$

or

$$\Pi = ac. \tag{36}$$

If $K(x) = cx$, $\Pi(x = x^* | x^e = x^*)$ hence increases with c.

Now, suppose

$$K(x) \equiv cx^2. \tag{37}$$

Then, x^* is given by

$$2cx = \frac{a(u_A - u_B)}{a + x^*}, \tag{38}$$

or

$$x^* = \sqrt{\left(\frac{a}{2}\right)^2 + \frac{a(u_A - u_B)}{2c}} - \frac{a}{2}. \tag{39}$$

$\Pi(x = x^* | x^e = x^*)$ is thus given by

$$\Pi = a(u_A - u_B) - c\left(\sqrt{\left(\frac{a}{2}\right)^2 + \frac{a(u_A - u_B)}{2c}} - \frac{a}{2}\right)^2 \tag{40}$$

or

$$\Pi = \frac{a(u_A - u_B)}{2} - \frac{a^2 c}{2} + \sqrt{\left(\frac{a^2 c}{2}\right)^2 + \frac{a^3 c(u_A - u_B)}{2}}. \tag{41}$$

$\frac{d[\Pi(x = x^* | x^e = x^*)]}{dc}$ is hence given by

$$\frac{d\left[\Pi(x = x^* | x^e = x^*)\right]}{dc} = -\frac{a^2}{2} + \frac{1}{2}\frac{\left(\frac{a^2}{2}\right)^2 2c + \frac{a^3(u_A - u_B)}{2}}{\sqrt{\left(\frac{a^2 c}{2}\right)^2 + \frac{a^3 c(u_A - u_B)}{2}}}. \tag{42}$$

Hence,

$$\frac{d\left[\Pi(x = x^* | x^e = x^*)\right]}{dc} > 0 \tag{43}$$

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if

$$u_A > u_B, \tag{44}$$

which holds by assumption. If $K(x) = cx^2$, $\Pi(x = x^*|x^e = x^*)$ hence increases with c.

Finally, suppose

$$K(x) \equiv ce^x. \tag{45}$$

Then, using (23) yields

$$\frac{\mathrm{d}\Pi(x = x^*|x^e = x^*)}{\mathrm{d}c} = -e^{x^*} + \frac{1}{\frac{e^{x^*}}{(e^{x^*})^2} + \frac{c}{a(u_A - u_B)}}. \tag{46}$$

Hence,

$$\frac{\mathrm{d}\Pi(x = x^*|x^e = x^*)}{\mathrm{d}c} < 0 \tag{47}$$

if

$$\frac{1}{e^{x^*}} < \frac{e^{x^*}}{(e^{x^*})^2} + \frac{c}{a(u_A - u_B)} \tag{48}$$

or

$$\frac{c}{a(u_A - u_B)} > 0, \tag{49}$$

which holds by assumption. If $K(x) = ce^x$, $\Pi(x = x^*|x^e = x^*)$ hence decreases with c.

Hence, if $0 < x^* < b$ and the cost function is linear or quadratic, the profit the certifier makes in equilibrium increases with c. If the cost function is exponential, the profit the certifier makes in equilibrium decreases with c. ∎

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