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How Important is Technology? A Counterfactual Analysis^{*}

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Abstract

The multiplier effect of total factor productivity on aggregate output in the one-sector neoclassical growth model is well known, but what about the effects of regional productivity levels on the aggregate output as well as other national and regional variables? This paper studies the impact of productivity changes in the goods sector and the transportation sector in a general equilibrium trade model where agents in each location produce different varieties of a common set of goods. Wages are assumed to be equalized in nominal terms across locations, with differences in purchasing power (due to trade costs) offset by agents' preferences for particular locations in the initial steady-state. Instead of assuming iceberg costs, a transportation sector is modeled to allow an efficient distribution of workers across the production and transportation sectors. The state level data from the U.S. support the model, and the comparative statics exercises have several implications on the national and state-level variables of the U.S. economy. It is shown that if the national production technology level (i.e., the production technology level in each region) is doubled, the national output increases by 5 times, the price dispersion across regions increases by 20%, the population dispersion across regions decreases by 1%, and the ratio of production labor force to transportation labor force increases by 10 times. As the transportation costs approach zero, the national output increases by more than 10 times, the price dispersion across regions decreases by 20%, the population dispersion across regions increases by 1%, and the ratio of production labor force to transportation labor force increases by 5 times.

JEL Classification: R12, R13, R32

Key Words: Technology, Trade, Intermediate Inputs, Transportation

1. Introduction

The positive effect of technology on an economy is a well known phenomenon. But, how important is technology? How much do different types of technology changes affect macroeconomic variables, such as consumption, production, trade, price levels, and labor market? What are the relevant implications for the same variables at regional levels? How are different sectors affected by these technology changes (at both national and regional levels)? How are regional population levels (i.e., migration) affected by these technology changes? This paper attempts to answer these questions by considering the magnitudes of the effects of production and transportation technologies on the U.S. national and regional variables. In particular, an M + 1-factor, M-industry (i.e., M-good), N-variety, N-region general equilibrium trade model, which considers the distributions of both production and consumption within a country (or a union) at a disaggregate level, is introduced by considering the role of intermediate inputs. As in Armington (1969), it is assumed that each variety is differentiated by its location of production. In this sense, we present the economy of a country (or a union) consisting of a finite number of regions, where there are finite numbers of individuals and firms. The microfoundations of the model result in explaining the exports of an industry (in a region) depending on the geographical location of the region, the industry-specific relative marginal

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costs of production, income, price level and production level of all other regions. The heterogeneities across regions (i.e., geographical location and population differences) and across sectors (i.e., transportation technology levels and region specific production technology levels) are the main motivations of the model. This heterogeneity gives the model more flexibility and makes it more realistic compared to the models in the literature.

By using our model, we attempt to find the effects of different technology types on the bilateral differences of the following variables across regions, at both aggregate and disaggregate levels: i) price levels, ii) real wage rates, iii) consumption levels, iv) production levels, v) bilateral trade volumes, vi) population levels. Moreover, we investigate the motivation behind the distribution of labor across production and transportation sectors. In particular, by having an analytical solution, the model has the following implications:

- 1. The ratio of factory gate price of a good across regions is inversely related to the ratio of region specific technology levels.
- 2. The ratio of transaction price for a good across regions is determined by the ratio of the weighted average of technology levels, where weights are determined according to the geographical location, and taste parameters.
- 3. The ratio of the cost-of-living index across regions is given by the ratio of the weighted average of technology levels, where weights are determined according to the geographical location, factor shares, and taste parameters.
- 4. The real wage in each region differs by its cost-of-living index. Moreover, as the factor share of labor increases, the real wage in each region increases.
- 5. The relative consumption of a region for a good increases with its population level and decreases with its distance from other regions (i.e., remoteness).
- 6. The relative production of a region is directly related to its production technology and inversely related to the distance of the region from other regions.
- 7. The ratio of transportation workers to production workers increases as the distance (i.e., the transportation cost) across regions gets higher (i.e., as the regions get dispersed), or as the labor share in transportation increases, or as the labor share in the production sector decreases, *ceteris paribus*.
- 8. A region imports more goods (measured in values) from the higher technology regions and fewer goods from the more distant regions.

We show that the model is capable of explaining the patterns of consumption, production, trade, and price levels within the U.S. under the parameter values borrowed from the literature. The model is also successful in predicting the bilateral ratios of consumption, production, trade, and price levels within the U.S., at the state level. Finally, we simulate the model on the U.S. economy to find the effects of changes in different technology types on variables such as regional output, national output, price dispersion across regions, and regional population levels. Our simulations have several implications on U.S. macroeconomic variables. In particular, the counterfactual analysis suggests the following results for the U.S. economy:

- 1. An increase (a decrease) in the production technology of all sectors in all regions leads to higher (lower) national output. In particular, if the national production technology is doubled (halved), the national output increases (decreases) by about 5 times.
- 2. The cost-of-living index dispersion across regions increases as the level of technology increases. In other words, even when there is no regional or sectoral technology change, the economy can create such price dispersions through technology changes at the national level. Nevertheless, the magnitude of the effect of technology on price dispersion is not great: if we double (halve) the national production technology, the price dispersion increases (decreases) by only 20%.
- 3. An increase (a decrease) in the national production technology leads to a higher (lower) ratio of production to transportation workers in the labor market. If we double (halve) the national production technology, this ratio increases (decreases) by more than 10 times. Thus, national production technology plays a big role in the determination of this ratio.
- 4. Population dispersion across regions decreases (increases) as the national production technology increases (decreases). If we double (halve) the national production technology, population dispersion decreases (increases) by around 1%.

- 5. The national output decreases (increases) as the transportation costs increase (decrease). As transportation costs approach zero, the national output increases by more than 10 times, which suggests that decreasing regional barriers (which correspond to national borders in an international trade context) leads to welfare gains in a multiplicative manner.
- 6. The cost-of-living index dispersion across regions increases (decreases) as the transportation costs get higher (lower). If we double the transportation costs, the cost-of-living index dispersion increases by 80%. As transportation costs approach zero, the cost-of-living index dispersion decreases by 20%. Thus, transportation costs are not significant sources of cost-of-living index dispersion according to our analysis.
- 7. The price dispersion across regions at the commodity level increases (decreases) as the transportation costs get higher (lower). However, the effect of a change in the national transportation technology on the price dispersion at the commodity level is different from the one on the cost-of-living index dispersion, in terms of both magnitudes and second derivatives. This result justifies the need for a disaggregate level analysis to understand the underlying reasons of price dispersion.
- 8. As the transportation costs get higher (i.e., as the transportation technology gets lower) the ratio between production and transportation workers gets lower, which means that relatively more labor is needed in the transportation sector. In particular, if we double the transportation costs, the ratio decreases by 50%. However, as transportation costs approach zero, the ratio increases by 5 times. Thus, the labor force allocated to production is significantly affected by transportation costs. This result also helps us understand the magnitude of the effect of transportation costs on the national output.
- 9. Population dispersion across regions increases (decreases) as the transportation costs decrease (increase). If we double the transportation costs, population dispersion decreases by around 1%. As the transportation costs approach zero, populatin dispersion increases by 1%. Thus, transportation costs are not significant sources of population dispersion according to our analysis.
- 10. Region-specific technology changes have smaller effects on national output compared to the effects of national technological changes.
- 11. While a technology change in some regions increases the price dispersion, a technology change in others decreases it. This result is true also for the price dispersion at the commodity level. Thus, geography matters for price dispersion.
- 12. Different sectoral technology changes have different effects on the national output level. In particular, while some sectors such as food-beverage and gasoline have higher effects on the national output, some others have less effect on it.
- 13. A technological increase in non-durable goods mostly increases the price dispersion across regions while a technological increase in durable goods mostly decreases it.
- 14. While a technological change in some sectors such as gasoline, coal petroleum, chemical products a have higher effect on the ratio of production to transportation workers, other have a lesser effect on it.

Related Literature

The relation between technology and trade has been extensively analyzed ever since David Ricardo published his *Principles of Political Economy*. Grossman and Helpman (1995) make an excellent survey of this literature on technology and trade. Although the relation between geography and trade is another well known phenomenon, modeling the relation between trade and geography is still in progress. Krugman (1980, 1991) provides an introduction to the relation between geography and trade using the economies of scale with transportation costs as the main motivations behind trade. The influential paper by Eaton and Kortum (2002) build a Ricardian model in which the bilateral trade around the world is related to the parameters of geography and technology.¹ Recently, Alvarez and Lucas (2007) studied a variation of the Eaton–Kortum model to investigate the determinants of the cross-country

¹Rossi-Hansberg (2005) builds a spatial Ricardian model, in which, as in Eaton and Kortum (2002), trade is related to the parameters of geography and technology; but this time the technological differences are endogenous and determined by spatial specialization patterns through production externalities. However, for the question asked in this paper, the best strategy is to keep technology levels as exogenous, so that the pure effects of technology changes can be analyzed effectively. As Kehoe (2003) perfectly puts, the point is not that we should want to take the level of technology as exogenous. In fact, the point is exactly the opposite: If a model with technology treated as exogenous accounts for most regional and macroeconomic fluctuations, then we know that it is changes in the technology that we need to be able to explain.

distribution of trade volumes, such as size, tariffs and distance, by using a general equilibrium analysis. Both Eaton and Kortum (2002), and Alvarez and Lucas (2007) have competitive models. However, as Alvarez and Lucas (2007) suggest, although it is easy to work with competitive models, they ignore monopoly rents, which are present in reality. Thus, to consider monopoly rents is a challenge in terms of modeling in a general equilibrium analysis framework. Besides having an analytical solution, this paper considers these monopoly rents and thus has a more realistic model that can also be calibrated and used for a general equilibrium analysis.

The theoretical studies based on gravity equations, such as Anderson (1979), Bergstrand (1985, 1989), among many others, also analyze the effects of geography on trade by considering the relation between distance and economic activity across regions. These studies are popular mostly due to their empirical successes.² In particular, the first attempt to provide a microeconomic foundation for the gravity models belongs to Anderson (1979). The main motivation behind Anderson's (1979) gravity model of is the assumption that each region is specialized in the production of only one good.³ Despite its empirical success, as Anderson and van Wincoop (2003) point out, the specialization assumption suppresses finer classifications of goods, and thus makes the model useless in explaining the trade data at disaggregate levels. By having a disaggregate level analysis, the model of this paper *can* be used in explaining variables at disaggregate level. In particular, we show that Anderson's (1979) gravity model, which he presents in the Appendix of his paper and which is used by Anderson and van Wincoop (2003), is just a special case of our model. Moreover, by having a closed form solution, our model goes beyond the gravity models and finds the main motivations behind the regional trade as the heterogeneity across regions that we have mentioned above.

Another deficiency of in Anderson's (1979) gravity model is the lack of the production side. Bergstrand (1985) bridges this gap by introducing a one-factor, one-industry, N-country general equilibrium model in which the production side is considered. In his following study, Bergstrand (1989) extends his earlier gravity model to a two-factor, two-industry, N-country gravity model.⁴ In terms of modeling, this paper introduces an M + 1-factor, M-industry (i.e., M-good), N-variety, N-region general equilibrium trade model, which is more realistic compared to those models. The model of this paper is also useful for simulations in order to analyze possible interactions across locations and sectors, since it is in more disaggregate terms.

Moreover, none of the studies mentioned above investigate the magnitude of the impact of production and transportation technologies on the national, regional, and sectoral variables of a country. This paper also bridges this gap by analyzing the U.S. economy at disaggregate level. By having a closed form solution (rather than a numerical solution), the model of this paper goes beyond the other models and makes everything more transparent in analytical terms.

The rest of the paper is organized as follows. Section 2 introduces our regional trade model. Section 3 presents the closed form solution of the model together with its implications. Section 4 makes an empirical analysis of our model and obtains the parameters to be used in our simulation. Section 5 depicts the results of our simulation. Section 6 concludes. The proofs are given in Appendix A, and data are depicted in Appendix B.

2. The Model

We model the economy of a country (or a union) consisting of finite number of regions, where there are finite number of individuals and firms.⁵ We make our analysis for a typical region, r. The total number of regions is R. Each good is denoted by j = 1, ..., J, where J is the number of available goods, and it may be produced in each region. Each variety is denoted by i, which is also the notation for the region producing that variety. An individual is denoted by h, and total number of individuals in region r is H_r . In the model, generally speaking, $X_{a,b}(h, j)$ stands for the variable X, where a is related to the region of consumption, b is related to the variety (and thus, the region of production), h is related to the individual, and j is related to the good.

²Deardorff (1984) reviews the earlier gravity literature. For recent applications, see Wei (1996), Jensen (2000), Rauch (1999), Helpman (1987), Hummels and Levinsohn (1995), and Evenett and Keller (2002).

³In appendix of his paper, Anderson (1979) extends his basic model to a model in which multiple goods are produced in each region. ⁴Also see Suga (2007) for a monopolistic-competition model of international trade with external economies of scale, Lopez et al. (2006) for an analysis on home-bias on U.S. imports of processed food products, and Gallaway et al. (2003) as an empirical study to estimate short-run and long-run industry-level U.S. Armington elasticities.

⁵The model is similar to those continuum-of-goods models that are typical in international trade and open economy macroeconomics studies such as Dornbusch et al. (1977, 1980), Eaton and Kortum (2002), Erceg et al. (2000), Corsetti and Pesenti (2005), Gali and Monacelli (2005), Matsuyama (2000), and Yilmazkuday (2007, 2008a, 2008b).

2.1. Individuals and Labor Market

A typical individual h in region r maximizes:

$$U(C_{r}(h), N_{r}(h)) \equiv \log(C_{r}(h)) + \log(Z - N_{r}(h)) + \log(\zeta_{r}(h))$$
(2.1)

where $C_r(h)$ is a composite consumption index, $N_r(h)$ is the hours of labor supplied by each individual, Z is the total amount of hours, and $\zeta_r(h)$ is a (per capita) region specific utility of the individual out of living in region r^{6} . The composite consumption index is defined as:

$$C_{r}(h) = \left(\sum_{j} \left(\beta_{r}(j)\right)^{\frac{1}{\varepsilon}} \left(C_{r}(h,j)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $C_r(h, j)$ is the consumption of good j given by the CES function:

$$C_r(h,j) \equiv \left(\sum_i \left(\theta_{r,i}\right)^{\frac{1}{\eta}} \left(C_{r,i}(h,j)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

where $C_{r,i}(h,j)$ is the variety i of good j produced in region i; $\varepsilon > 0$ is the elasticity of substitution across goods; $\eta > 1$ is the elasticity of substitution across varieties; and finally, $\beta_r(j)$ and $\theta_{r,i}$ are taste parameters.

To make it clearer, consider the following matrix showing the feasible consumption set of a typical individual:

[Insert Figure 2.1]

Each column of the matrix in Figure 2.1 shows a specific good, whereas each row of it shows a specific variety. According to this matrix, roughly speaking, orange is considered as a good, whereas Florida orange and California orange are considered as varieties. Moreover, since each variety is produced by a specific region, ith row of this matrix shows the goods produced in region i. In this context, for an individual in region r, the taste parameter $\beta_r(j)$ is related to j'th column of the matrix and the taste parameter $\theta_{r,i}$ is related to i'th row of the matrix. Note that some cells of this matrix may be empty, implying that there is no production made for a specific variety of a specific good (i.e., that specific good is not produced in a specific region).

Besides the labor income, each individual also receives $\Gamma(h)$ as profit income, independent of her location of residence. In this context, the individual in region r maximizes Equation 2.1 subject to the following budget constraint:

$$\sum_{j} P_r(j) C_r(h,j) \le W N_r(h) + \Gamma(h)$$
(2.2)

where $P_r(j)$ is the price index of good j; and W is the unique hourly nominal wage determined in the national labor market.

The optimal allocation of any given expenditure within each variety of goods yields the following demand functions:

$$C_{r,i}(h,j) = \theta_{r,i} \left(\frac{P_{r,i}(j)}{P_r(j)}\right)^{-\eta} C_r(h,j)$$

and

$$C_{r}(h,j) = \beta_{r}(j) \left(\frac{P_{r}(j)}{P_{r}}\right)^{-\varepsilon} C_{r}(h)$$

where $P_r(j) \equiv \left(\sum_i \theta_{r,i} P_{r,i}(j)^{1-\eta}\right)^{\frac{1}{1-\eta}}$ is the price index of the good j (which is composed of different varieties), and $P_r \equiv \left(\sum_{j} \beta_r(j) P_r(j)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$ is the cost-of-living index in region r. It follows from the equations above that $\sum_{j} P_r(j) C_r(h, j) = P_r C_r(h)$.

Thus, the optimality condition for the individual is given by:

$$\frac{C_r(h)}{Z - N_r(h)} = \frac{W}{P_r}$$
(2.3)

⁶Considering such a (per capita) region specific utility is important in terms of migration issues, as we will see below.

If we combine Equations 2.2 and 2.3, we can obtain:

$$N_r(h) = Z - \frac{\Gamma(h)}{W}$$
(2.4)

which implies that $N_r(h)$ is constant in all regions. Then, $N_r(h) = N(h)$, together with Equation 2.3, implies that:

$$\frac{C_r(h)}{C_i(h)} = \frac{P_i}{P_r} \tag{2.5}$$

which means that the individual specific consumption ratio is inversely related to ratio of price levels across regions r and i.

2.2. Firms

In each region, there are two types of firms: production firms and transportation firms.

2.2.1. Production Firms

A monopolistically competitive production firm in region r produces variety r of good j by using labor and intermediate inputs purchased from other firms in the economy. In particular, we have the following constant returns to scale (CRS) production function:

$$Y_r(j) = A_r(j) \left[L_r(j) \right]^l \left[G_r^j \right]^g$$
(2.6)

where $A_r(j)$ represents good and region specific technology, $L_r(j)$ represents labor, G_r^j represents the composite intermediate input, and finally, l and g represent the factor shares which are the same across production firms.

The firm chooses $L_r(j)$ and each G_r^j , taking the wage rates and the price of intermediate goods as given. The cost minimization problem of the firm is as follows:

$$\min_{L_r(j), G_r^j} L_r(j) W + G_r^j P^j$$

s.t. $Y_r(j) = A_r(j) [L_r(j)]^l [G_r^j]^g$

which implies that the marginal cost of producing good j in region r is given by:

$$MC_r(j) = \left[\frac{W}{l}\right]^l A_r(j)^{-1} \left[\frac{P^j}{g}\right]^g$$
(2.7)

Intermediate goods that are used in the production of good j in region r are given by the following indices:

$$G_{r}^{j} = \left(\sum_{m} \left(\omega^{j}\left(m\right)\right)^{\frac{1}{\varepsilon}} \left(G_{r}^{j}\left(m\right)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$G_{r}^{j}\left(m\right) = \left(\sum_{i} \left(\lambda_{r,i}^{m}\right)^{\frac{1}{\eta}} \left(G_{r,i}^{j}\left(m\right)\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

where $G_r^j(m)$ is the composite index for the intermediate input of good m; $G_{r,i}^j(m)$ is the intermediate input of variety i of good m (which is imported from region i); $\omega^j(m)$ and $\lambda_{r,i}^m$ are production specific and production/region specific taste parameters of the firms, respectively. The optimality of the firm that produces good j gives the following demand functions:

$$G_{r,i}^{j}(m) = \lambda_{r,i}^{m} \left(\frac{P_{r,i}(m)}{P^{j}(m)}\right)^{-\eta} G_{r}^{j}(m)$$
(2.8)

$$G_r^j(m) = \omega^j(m) \left(\frac{P^j(m)}{P^j}\right)^{-\varepsilon} G_r^j$$
(2.9)

where $P^{j}(m) \equiv \left(\sum_{i} \lambda_{r,i}^{m} P_{r,i}(m)^{1-\eta}\right)^{\frac{1}{1-\eta}}$ is the intermediate input price index of the good variety j, and $P^{j} \equiv \left(\sum_{m} \omega^{j}(m) P^{j}(m)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$ is the intermediate input price index in region r. Note that both $P^{j}(m)$ and P^{j} are

production specific and *not* region specific. We achieve this by setting $\lambda_{r,i}^m = \frac{1}{(1+\tau_{r,i}(m))^{1-\eta}}$ for each r and i.⁷ Thus, we have:

$$P^{j}(m) \equiv \left(\sum_{i} P_{i,i}(m)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

$$(2.10)$$

which implies that:

$$P^{j} \equiv \left(\sum_{m} \omega^{j}(m) \left(\sum_{i} P_{i,i}(m)^{1-\eta}\right)^{\frac{1-\varepsilon}{1-\eta}}\right)^{\frac{1-\varepsilon}{1-\varepsilon}}$$
(2.11)

It also follows from the equations above that $\sum_{m} P^{j}(m) G_{r}^{j}(m) = P^{j}G_{r}^{j}$. Moreover, since P^{j} is good specific, the marginal cost of production (for a specific good) differs in each region according to its technology level.

2.2.2. Transportation Firms

We have to define our trade cost first. Anderson and van Wincoop (2004) categorize the trade costs under two names, costs imposed by policy (tariffs, quotas, etc.) and costs imposed by the environment (transportation, wholesale and retail distribution, insurance against various hazards, etc.). Since we analyze trade within a country, we ignore the first category and focus on the second one. In particular, we assume that trade between regions is subject to a transportation cost:

$$P_{i,r}(j) = (1 + \tau_{i,r}(j)) \left(P_{r,r}(j)\right)$$
(2.12)

where $\tau_{i,r}(j) > 0$ is the net transportation cost for variety r of good j produced in region r and consumed to region i. Equation 2.12 says that the price of the variety j that is produced in region r is more expensive in region $i (\neq r)$ than it is in region r. This assumption is commonly used in the literature (see Anderson and van Wincoop 2003, 2004).

The transportation service is produced by a competitive transportation firm that transports variety r of good j from region r to region i by the following CRS production function:⁸

$$P_{r,r}(j) C_{i,r}(j) D_{i,r} = A (D_{i,r}, j) [L_r(t)]^{l(t)} [G_r^t]^{g(t)}$$
(2.13)

where $A(D_{i,r}, j)$ represents the good specific transportation technology that depends on the distance of transportation, $L_r(t)$ represents labor used in transportation, G_r^t represents the composite intermediate input (analogous to G_r^j in the production process of the production firms), and finally, l(t) and g(t) represent the factor shares in transportation, which are the same across the transportation of different goods. The cost minimization problem of the transportation firm is as follows:

$$\min_{L_{r}(j),G_{r}^{t}} L_{r}(t) W + G_{r}^{*} P^{*}$$
s.t. $P_{r,r}(j) C_{i,r}(j) D_{i,r} = A(D_{i,r},j) [L_{r}(t)]^{l(t)} [G_{r}^{t}]^{g(t)}$

which implies that the marginal cost of transporting good j from region r to region i is given by:

$$MC_{i,r}^{j}(t) = \frac{1}{A(D_{i,r},j)} \left[\frac{W}{l(t)}\right]^{l(t)} \left[\frac{P^{t}}{g(t)}\right]^{g(t)}$$
(2.14)

where P^t is the intermediate input price index (analogous to P^j in the production process of the production firms) for the transportation firms in all regions.

2.3. Equilibrium

This section describes the aggregate properties of the model.

 $^{^{7}}$ As we will show in our closed form solution, the implication of this assumption is that the firms are willing to buy more intermediate inputs from closer regions.

 $^{^{8}}$ We consider a competitive transportation firm (rather than a monopolistically competitive one) because of its implications on the actual transportation prices, as we will se below.

2.3.1. Consumption

We assume that individuals of a typical region share the same tastes. Thus, in region r, the demand function for variety i of good j is given by:

$$C_{r,i}(j) = \theta_{r,i} \left(\frac{P_{r,i}(j)}{P_r(j)}\right)^{-\eta} C_r(j)$$

$$(2.15)$$

and

$$C_r(j) = \beta_r(j) \left(\frac{P_r(j)}{P_r}\right)^{-\varepsilon} C_r$$
(2.16)

where $C_{r,i}(j) = \sum_{h=1}^{H_r} C_{r,i}(h,j) = H_r C_{r,i}(h,j)$ is the total demand for variety *i* of good *j* (in region *r*); $C_r(j) = \sum_{h=1}^{H_r} C_r(h,j) = H_r C_r(h,j)$ is the total demand for good *j* (in region *r*); $C_r = \sum_{h=1}^{H_r} C_r(h) = H_r C_r(h)$ is the total demand (in region *r*); and H_r is the population (in region *r*).

2.3.2. Labor Market

The total labor supply of the individuals in all regions, N, is equal to the sum of the labor demands of the production and transportation firms in all regions, L, i.e.:

$$L = \sum_{r} L_{r} = \sum_{r} \sum_{j} L_{r}(j) + \sum_{r} \sum_{t} L_{r}(t) = \sum_{r} \sum_{h=1}^{H_{r}} N(h) = N(h) \sum_{r} H_{r} = N$$
(2.17)

2.3.3. Profits

The total amount of profit in all regions is equally distributed among the households in all regions, who owns an equal share of all firms, i.e.:

$$\sum_{r} \sum_{j} \Gamma_{r} (j) = \sum_{r} \sum_{h=1}^{H_{r}} \Gamma (h) = \Gamma (h) \sum_{r} H_{r}$$
(2.18)

2.3.4. Regional Resource Constraint

Now, the budget constraint of region r can be written as:

$$P_r C_r \le N_r W + \Gamma_r \tag{2.19}$$

where WN_r is the total wage in region r. Note that, by using Equations 2.17 and 2.18, we can say that $P_rC_r = \gamma_r PC$, where $\gamma_r = \frac{H_r}{\sum_r H_r}$ and PC is the total income in the country (or in the union). Thus, the ratio of the income levels of two regions (say, i and r) is given by:

$$\frac{P_i C_i}{P_r C_r} = \frac{H_i}{H_r} \tag{2.20}$$

This result has a very realistic implication. In particular, it says that the location at which the income earned does not matter. The important thing that determines the income of a region is its population level. In other words, each region receives an income proportional to its population. Thus, individuals are labeled according to their residency, not their office.

Remark 1. The income ratio across regions is equal to their population ratio.

2.3.5. Region Specific Utility, Population Levels, and Migration

In equilibrium, in order to have no migration across regions, the (per capita) utility should be the same in all regions. Considering Equations 2.1 and 2.4, we achieve this by the following assumption:

$$\frac{\zeta_r(h)}{\zeta_i(h)} = \frac{C_i(h)}{C_r(h)}$$
(2.21)

for all r and i, which implies that the utilities are in fact the same across regions r and i, i.e., $U(C_r(h), N_r(h)) = U(C_i(h), N_i(h))$ for all r and i. Equation 2.21 suggests that a possible difference between any two regions in terms of per capita consumption is compensated by the difference in terms of per capita region specific utility, and thus no migration takes place. The migration implications will be clearer when we perform counterfactual exercises below.

We assume that the total amount of region specific utility is equal to a fixed (exogenous or God given) endowment in each region. In other words, for region r, we have:

$$\sum_{h=1}^{H_r} \zeta_r \left(h \right) = H_r \zeta_r \left(h \right) = \zeta_r \tag{2.22}$$

By combining Equations 2.5, 2.21, and 2.22, we can write:

$$\frac{H_r}{H_i} = \frac{P_i \zeta_r}{P_r \zeta_i} \tag{2.23}$$

which means that the ratio of population across two regions is inversely related by the price ratio and positively related by the total fixed region specific utility.

We will use this expression to consider the effects of technology changes on migration in our counterfactual analysis, below. In particular, having a fixed (exogenous or God given) total endowment of region specific utility is going to be the main tool deriving migration when we will have changes in different types of technologies. For instance, according to Equations 2.3 and 2.4, after a possible increase in relative prices across regions, the utility of the individual in the higher price region will reduce through consumption. This utility reduction will force some of the individuals to migrate toward lower price regions to have more utility. However, the reduced population in the higher price region (after migration) will lead having more per capita region specific utility due to Equation 2.22. Thus, the new equilibrium will be achieved when Equation 2.23 holds again, this time with different population and price levels in each region.

2.3.6. Market Clearing Condition

For each variety r of good j (produced in region r), market clearing condition implies:

$$Y_{r}(j) = \sum_{i} C_{i,r}(j) + \sum_{i} \sum_{m} G_{i,r}^{m}(j)$$
(2.24)

where $C_{i,r}(j)$ is the demand of region *i* for the variety *r* of good *j* produced in region *r*; and $\sum_{m} G_{i,r}^{m}(j)$ is the total demand of the producers in region *i* for the variety *r* of good *j* (produced in region *r*). Equation 2.24 basically says that the variety *r* of good *j* produced in region *r* is either consumed locally or by other regions, either for consumption or further production. By using Equations 2.12, 2.8, 2.9, 2.16 and 2.15, we can rewrite the market clearing condition for variety *r* of good *j* as:

$$Y_{r}(j) = \sum_{i} \frac{\theta_{i,r}\beta_{i}(j) (P_{i}(j))^{\eta-\varepsilon} (P_{i})^{\varepsilon} C_{i}}{P_{r,r}(j)^{\eta} (1+\tau_{i,r}(j))^{\eta}} + \sum_{i} \frac{\sum_{m} \omega^{m}(j) (P^{m})^{\varepsilon} G_{i}^{m}}{(1+\tau_{i,r}(j)) (P_{r,r}(j))^{\varepsilon}}$$

This implies that the gross regional product (the total value of production plus transportation) in region r is given by:

$$GRP_{r} = \sum_{i} \frac{\theta_{i,r}\beta_{i}(j)(P_{i}(j))^{\eta-\varepsilon}(P_{i})^{\varepsilon}C_{i}}{P_{r,r}(j)^{\eta-1}(1+\tau_{i,r}(j))^{\eta-1}} + \sum_{i} \frac{\sum_{m} \omega^{m}(j)(P^{m})^{\varepsilon}G_{i}^{m}}{(P_{r,r}(j))^{\varepsilon-1}}$$
(2.25)

By including the transportation sector, we can also write the total income (which is equal to total expenditure) of region r as follows:

$$P_{r}C_{r} = \underbrace{\gamma_{r}\sum_{i}\sum_{j}\tau_{i,r}\left(j\right)\left(\frac{\theta_{i,r}\beta_{i}\left(j\right)\left(P_{i}\left(j\right)\right)^{\eta-\varepsilon}\left(P_{i}\right)^{\varepsilon}C_{i}}{P_{r,r}\left(j\right)^{\eta-1}\left(1+\tau_{i,r}\left(j\right)\right)^{\eta}}\right)}_{\text{Final Transportation Income}} + \underbrace{\gamma_{r}\sum_{r}\sum_{i}\sum_{j}\left(\frac{\theta_{i,r}\beta_{i}\left(j\right)\left(P_{i}\left(j\right)\right)^{\eta-\varepsilon}\left(P_{i}\right)^{\varepsilon}C_{i}}{P_{r,r}\left(j\right)^{\eta-1}\left(1+\tau_{i,r}\left(j\right)\right)^{\eta}}\right)}_{\text{Final Production Income}} = \gamma_{r}\sum_{r}\sum_{i}\sum_{j}\left(\frac{\theta_{i,r}\beta_{i}\left(j\right)\left(P_{i}\left(j\right)\right)^{\eta-\varepsilon}\left(P_{i}\right)^{\varepsilon}C_{i}}{P_{r,r}\left(j\right)^{\eta-1}\left(1+\tau_{i,r}\left(j\right)\right)^{\eta-1}}\right)$$

$$(2.26)$$

Note that income comes only from the final good production/transportation to avoid double counting.

2.4. Price Setting

The production firms maximize their profits by using their market power, while the competitive transportation firms set their prices equal to their marginal costs.

2.4.1. Production Firms

In region r, we assume that a typical firm that produces a variety r of good j faces the following profit maximization problem:

$$\max_{P_{r,r}(j)} Y_r(j) \left[P_{r,r}(j) - MC_r(j) \right]$$

subject to Equation 2.24. The first order condition for this problem is as follows⁹:

$$Y_{r}(j)\left[1 - \frac{\eta}{P_{r,r}(j)} \left(P_{r,r}(j) - MC_{r}(j)\right)\right] = 0$$

$$P_{r,r}(j) = \frac{\eta}{\eta - 1} MC_{r}(j)$$
(2.27)

which implies that:

where $\frac{\eta}{\eta-1}$ represents the gross mark-up. Together with Equation 2.7, Equation 2.27 implies that, for a specific good, the factory price of the product differs in each region only because of the region specific technology levels.

2.4.2. Transportation Firms

Since each transportation firm is competitive, the price of transportation is set equal to its marginal cost. Thus, we have: $1 = \int W r^{l(t)} f r r r r^{q(t)}$

$$t_{i,r}(j) = \frac{1}{A(D_{i,r},j)} \left[\frac{W}{l(t)}\right]^{l(t)} \left[\frac{P^t}{g(t)}\right]^{g(t)}$$

where $t_{i,r}(j) = \frac{\tau_{i,r}(j)}{D_{i,r}}$ is the price per unit of distance transported by the transportation firm for a unit currency worth of good j. If we set $A(D_{i,r}, j) = \frac{D_{i,r}}{(D_{i,r})^{\delta(j)}-1}$ and normalize the price of intermediate goods for the transportation sector, P^t , such that $\left(\frac{W}{l(t)}\right)^{l(t)} \left(\frac{P^t}{g(t)}\right)^{g(t)} = 1$, we can write¹⁰:

$$1 + \tau_{i,r}(j) = 1 + D_{i,r}t_{i,r}(j) = (D_{i,r})^{\delta(j)}$$

⁹Notice that the firm takes the aggregate consumption (C_i) , the consumer price index (P_i) , and $(P_i(j))^{\eta-\varepsilon}$ in each region as given in the optimization problem.

¹⁰Setting $A(t,j) = \frac{D_{i,r}}{(D_{i,r})^{\delta(j)}-1}$ means that the technology level used for transportation increases (decreases) as the distance increases, given that $\delta(j) < 1$ and $D_{i,r} > (<) \left(\frac{1}{1-\delta(j)}\right)^{1/\delta(j)}$. Setting $\left(\frac{W}{l(t)}\right)^{l(t)} \left(\frac{P^t}{g(t)}\right)^{g(t)} = 1$ implies that $P^t = \left(\frac{l(t)}{W}\right)^{\frac{l(t)}{g(t)}} g(t)$; which basically puts additional restrictions on the transportation sector specific parameters $\lambda_{r,i}^t$'s since we have $P^t(m) \equiv \left(\sum_i \lambda_{r,i}^t P_{r,i}(m)^{1-\eta}\right)^{\frac{1}{1-\eta}}$ and $P^t \equiv \left(\sum_m \omega^j(m) P^t(m)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$.

which is the mostly used *ad hoc* gross transportation cost in the literature (see Anderson and van Wincoop 2003, 2004, among others). The reason for us to not follow the literature (i.e., the reason for not directly assuming $1 + \tau_{i,r}(j) = (D_{i,r})^{\delta(j)}$) is that we want to show the effects of the transportation sector on the national labor market.

2.5. Interregional (Bilateral) Trade

According to our model, the export of the good j from region r to region i is given by $C_{i,r}(j) + \sum_{m} G_{i,r}^{m}(j)$. By using region specific demand functions, we can write it as follows:

$$C_{i,r}(j) + \sum_{m} G_{i,r}^{m}(j) = \frac{\theta_{i,r}\beta_{i}(j) (P_{i}(j))^{\eta-\varepsilon} (P_{i})^{\varepsilon} C_{i}}{P_{r,r}(j)^{\eta} (1+\tau_{i,r}(j))^{\eta}} + \frac{\sum_{m} \omega^{m}(j) (P^{m})^{\varepsilon} G_{i}^{m}}{(1+\tau_{i,r}(j)) (P_{r,r}(j))^{\varepsilon}}$$
(2.28)

Note that the first term stands for the final good trade, and the second term stands for the intermediate input trade. In a special case in which there is only one good produced in each region ($\beta_i(j) = 1$ and $\theta_{i,r} = \theta_r$), and in which there is no intermediate input trade at all, after standard calculations, we obtain¹¹:

$$(1+\tau_{i,r}) X_{i,r}(j) = \theta_i \frac{(P_i)^{\eta-1} P_i C_i}{(P_{r,r}(j))^{\eta-1} ((1+\tau_{i,r}(j)))^{\eta-1}}$$
(2.29)

where $(1 + \tau_{i,r}) X_{i,r}(j) = (1 + \tau_{i,r}) \left(C_{i,r}(j) + \sum_{m} G_{i,r}^{m}(j)\right) P_{r,r}(j)$ is the nominal value of the exports from region r to region i measured in region i, and P_iC_i is the total income in region i. Equation 2.29 is the main equation (FOC) that is used to find the gravity equation of Anderson-van Wincoop (2003) model. Regardless of the number of goods produced in each region, compared to our model, Anderson-van Wincoop (2003) model ignores information coming from the intermediate input trade, composite price of a good, $(P_i(j))^{\varepsilon-\eta}$, the distribution parameter $\beta_i(j)$, and the difference between ε and η . Hence, we can say that the gravity model of Anderson and van Wincoop (2003) is a special case of our model. This result supports Deardorff's (1998) remark, "I suspect that just about any plausible model of trade would yield something very like the gravity equation...".

It is also important to note that our bilateral trade equation (Equation 2.28) includes region and good specific fixed effects that are commonly used in empirical gravity studies, such as Harrigan (1996), Hummels (1999), Redding and Venables (2004), Rose and van Wincoop (2001) and Anderson and van Wincoop (2003). The next proposition gives further information about the bilateral trade implications of our model.

Proposition 1. The bilateral trade of good j across two regions can be explained by:

- Geographical location, i.e., $(1 + \tau_{i,r}(j)) = (D_{i,r})^{\delta(j)}$, j = 1, ..., J and i = 1, ..., R.
- Population level, i.e., H_i , i = 1, .., R.
- Taste parameters, i.e., $\beta_i(j)$ and $\theta_{r,i}$, i = 1, ..., R and j = 1, ..., J.
- Good specific transportation technologies, i.e., $\delta(j)$, j = 1, ..., J.
- Good/region specific production technologies, i.e., $A_i(j), j = 1, ..., J$ and i = 1, ..., R.
- **P roof.** See Equation 3.26 in the next section.

In order to go one step further and find the motivation behind our model, we have to obtain an analytical solution. The derivation of the closed form solution is given in the next section.

3. Analytical Solution and Implications

In this section, we present the closed form solution of our model and its implications on price levels, consumption levels, production levels, bilateral trade levels, and the distribution of labor across production and transportation sectors. We have to restrict ourselves to the special case in which $\varepsilon = 1$ to achieve a closed form solution.¹² To obtain the solution, first, we solve for the price levels; then, we solve for other variables in terms of G_r^j 's and C_r 's; and finally, we find expressions for G_r^j 's and C_r 's in terms of exogenous variables.

¹¹In another special case in which $\eta = \varepsilon$, after ignoring the supply side of our model (and thus, intermediate input trade) at all, we obtain the gravity model in Appendix of Anderson (1979).

¹²Another closed form solution can be obtained by ignoring the intermediate input trade in the model. In such a case, we would not need the assumption of $\varepsilon = 1$.

3.1. Price Levels and Real Wage

By using Equations 2.7 and 2.27, the price at the factory gate can be written as:

$$P_{i,i}(j) = \frac{\eta}{\eta - 1} \left[\frac{W}{l}\right]^l A_i(j)^{-1} \left[\frac{P^j}{g}\right]^g$$
(3.1)

Equation 3.1 implies that the ratio of factory gate price of good j across varieties (regions) a and b is given by:

$$\frac{P_{a,a}\left(j\right)}{P_{b,b}\left(j\right)} = \frac{A_{b}\left(j\right)}{A_{a}\left(j\right)}$$

which is inversely related to the variety (region) specific technology levels.

Remark 2. The ratio of factory gate price of a good across varieties (regions) is inversely related to the ratio of variety (region) specific technology levels.

We can write the price index for good j in region r as:

$$P_r(j) = \frac{\eta}{\eta - 1} \left[\frac{W}{l}\right]^l \left[\frac{P^j}{g}\right]^g \left(\sum_i \theta_{r,i} \left(\frac{(D_{r,i})^{\delta(j)}}{A_i(j)}\right)^{1 - \eta}\right)^{\frac{1}{1 - \eta}}$$
(3.2)

Equation 3.2 implies that the ratio of the price of good j across regions a and b is given by:

$$\frac{P_{a}\left(j\right)}{P_{b}\left(j\right)} = \frac{\left(\sum_{i} \theta_{a,i} \left(\frac{\left(D_{a,i}\right)^{\delta\left(j\right)}}{A_{i}\left(j\right)}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\sum_{i} \theta_{b,i} \left(\frac{\left(D_{b,i}\right)^{\delta\left(j\right)}}{A_{i}\left(j\right)}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}}$$

which is basically the ratio of the weighted average technology levels, where weights are determined according to the geographical location and taste parameters.

Remark 3. The ratio of price index for a good across regions is determined by the ratio of the weighted average technology levels, where weights are determined according to the geographical location and taste parameters.

Claim 1. The price of intermediate input for good j, P^{j} , is given by:

$$P^{j} = \frac{W}{l} \left(\frac{\eta}{\eta - 1}\right)^{\frac{1}{l}} (1 - l)^{\frac{l-1}{l}} \prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i}(m)^{-1}\right)^{1-\eta}\right)^{\frac{\omega^{s}(m)}{1-\eta}}\right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}}$$
(3.3)

where $\boldsymbol{\omega}$ is the matrix consisting of ω^{j} 's multiplied by g; \mathbf{I} is the identity matrix; and $(\mathbf{I} - \boldsymbol{\omega})_{js}^{-1}$ is the j'th row of s'th column in $(\mathbf{I} - \boldsymbol{\omega})^{-1}$.

P roof. See Appendix A. \blacksquare

By using Equations 3.1, 3.2 and 3.3, we can write $P_{i,i}(j)$ and $P_r(j)$ as follows:

$$P_{i,i}(j) = \frac{W}{l} \left(\frac{\eta}{\eta - 1}\right)^{\frac{1}{l}} \frac{(1 - l)^{\frac{l-1}{l}}}{A_i(j)} \prod_s \left(\prod_m \left(\sum_i \left(A_i(m)^{-1}\right)^{1 - \eta}\right)^{\frac{\omega^s(m)}{1 - \eta}}\right)^{(\mathbf{I} - \boldsymbol{\omega})_{j_s}^{-1}g}$$
(3.4)

and

$$P_{r}(j) = \frac{W}{l} \left(\frac{\eta}{\eta-1}\right)^{\frac{1}{l}} (1-l)^{\frac{l-1}{l}} \prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i}(m)^{-1}\right)^{1-\eta}\right)^{\frac{\omega^{s}(m)}{1-\eta}}\right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}g} \times \left(\sum_{i} \theta_{r,i} \left(\frac{(D_{r,i})^{\delta(j)}}{A_{i}(j)}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$
(3.5)

Finally, we can write the cost-of-living index for region r as follows:

$$P_{r} \equiv \frac{W}{l} \left(\frac{\eta}{\eta - 1}\right)^{\frac{1}{l}} (1 - l)^{\frac{l-1}{l}} \prod_{j} \left(\prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i} \left(m\right)^{-1}\right)^{1-\eta} \right)^{\frac{\omega^{s}(m)}{1-\eta}} \right)^{(\mathbf{I} - \boldsymbol{\omega})_{js}^{-1}g} \right)^{\beta_{r}(j)} \times \left(\sum_{i} \theta_{r,i} \left(\frac{(D_{r,i})^{\delta(j)}}{A_{i}(j)} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \right)^{(\mathbf{I} - \boldsymbol{\omega})_{js}^{-1}g} \right)^{\beta_{r}(j)}$$
(3.6)

Equation 3.6 implies that the ratio of the price of living index across regions a and b is given by:

$$\frac{P_{a}}{P_{b}} = \frac{\prod_{j} \left(\prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i} \left(m \right)^{-1} \right)^{1-\eta} \right)^{\frac{\omega^{s}(m)}{1-\eta}} \right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}g} \left(\sum_{i} \theta_{a,i} \left(\frac{(D_{a,i})^{\delta(j)}}{A_{i}(j)} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \right)^{\beta_{a}(j)}}{\prod_{j} \left(\prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i} \left(m \right)^{-1} \right)^{1-\eta} \right)^{\frac{\omega^{s}(m)}{1-\eta}} \right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}g} \left(\sum_{i} \theta_{b,i} \left(\frac{(D_{b,i})^{\delta(j)}}{A_{i}(j)} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \right)^{\beta_{b}(j)}}$$

which is basically the ratio of the weighted average of technology levels, where weights are again determined according to the geographical location and taste parameters.

Remark 4. The ratio of the price of living index across regions is given by the ratio of the weighted average of technology levels, where weights are determined according to the geographical location, factor shares and taste parameters.

By Equation 3.6, the real wage in region r can be found as:

$$\frac{W}{P_{r}} = \frac{l\left(\frac{\eta-1}{\eta}\right)^{\frac{1}{l}} (1-l)^{\frac{1-l}{l}}}{\prod_{j} \left(\prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i}(m)^{-1}\right)^{1-\eta}\right)^{\frac{\omega^{s}(m)}{1-\eta}}\right)^{(\mathbf{I}-\boldsymbol{\omega})^{-1}_{js}g} \left(\sum_{i} \theta_{r,i} \left(\frac{(D_{r,i})^{\delta(j)}}{A_{i}(j)}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{\beta_{r}(j)}} \tag{3.7}$$

which says that if the factor share of labor l increases, the real wage in each region increases.

Remark 5. While nominal wage is the same across regions, the real wage in each region differs by its cost-of-living index. Moreover, as the factor share of labor *l* increases, the real wage in each region increases.

3.2. Consumption

By using Equations 2.20 and 3.6, we can write the ratio of consumption across regions a and b as follows:

$$\frac{C_{a}}{C_{b}} = \frac{H_{a} \prod_{j} \left(\prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i} \left(m \right)^{-1} \right)^{1-\eta} \right)^{\frac{\omega^{s}(m)}{1-\eta}} \right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}g} \left(\sum_{i} \theta_{b,i} \left(\frac{(D_{b,i})^{\delta(j)}}{A_{i}(j)} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \right)^{\beta_{b}(j)}}{H_{b} \prod_{j} \left(\prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i} \left(m \right)^{-1} \right)^{1-\eta} \right)^{\frac{\omega^{s}(m)}{1-\eta}} \right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}g} \left(\sum_{i} \theta_{a,i} \left(\frac{(D_{a,i})^{\delta(j)}}{A_{i}(j)} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \right)^{\beta_{a}(j)}$$
(3.8)

which is basically the ratio of the ratio of the population levels multiplied by the weighted average of technology levels, where weights are determined according to the geographical location and taste parameters. Individual C_i 's will be found later on.

Similarly, the ratio of good j consumption across regions a and b is given by:

$$\frac{C_{a}\left(j\right)}{C_{b}\left(j\right)} = \frac{H_{a}\beta_{a}\left(j\right)\left(\sum_{i}\theta_{b,i}\left(\frac{\left(D_{b,i}\right)^{\delta\left(j\right)}}{A_{i}\left(j\right)}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}}{H_{b}\beta_{b}\left(j\right)\left(\sum_{i}\theta_{a,i}\left(\frac{\left(D_{a,i}\right)^{\delta\left(j\right)}}{A_{i}\left(j\right)}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}}$$

which says that the relative consumption of a region increases with its population level and decreases with its distance to other regions.

Remark 6. The relative consumption of a region for a good increases with its population level and decreases with its distance to other regions.

3.3. Production

By using Equations 3.3, 3.4, 3.5 and 3.6, we can write Equation 2.24 as:

$$Y_{r}(j) = \sum_{i} M_{i,r}(j) C_{i} + \sum_{i} \sum_{m} V_{i,r}^{m}(j) G_{i}^{m}$$
(3.9)

where

$$M_{i,r}(j) = \frac{\theta_{i,r}\beta_{i}(j) A_{r}(j)^{\eta} \prod_{j} \left(\prod_{s} \left(\prod_{m} \left(\sum_{k} \left(A_{k}(m)^{-1} \right)^{1-\eta} \right)^{\frac{\omega^{s}(m)}{1-\eta}} \right)^{(\mathbf{I}-\omega)_{js}^{-1}g} \right)^{\beta_{i}(j)}{\times \left(\sum_{m} \theta_{i,m} \left(\frac{(D_{i,m})^{\delta(j)}}{A_{m}(j)} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}}} \left((D_{i,r})^{\delta(j)} \right)^{\eta} \prod_{s} \left(\prod_{m} \left(\sum_{k} \left(A_{k}(m)^{-1} \right)^{1-\eta} \right)^{\frac{\omega^{s}(m)}{1-\eta}} \right)^{(\mathbf{I}-\omega)_{js}^{-1}g} \times \left(\sum_{m} \theta_{i,m} \left(\frac{(D_{i,m})^{\delta(j)}}{A_{m}(j)} \right)^{1-\eta} \right)^{(\mathbf{I}-\omega)_{js}^{-1}g} \right)^{(\mathbf{I}-\omega)_{js}^{-1}g}$$
(3.10)

and

$$V_{i,r}^{m}(j) = \frac{\omega^{m}(j) A_{r}(j)}{\left((D_{i,r})^{\delta(j)}\right)} \prod_{s} \left(\prod_{j} \left(\sum_{i} \left(A_{i}(j)^{-1} \right)^{1-\eta} \right)^{\frac{\omega^{s}(j)}{1-\eta}} \right)^{(\mathbf{I}-\boldsymbol{\omega})_{ms}^{-1}l}$$
(3.11)

In a special case in which $\theta_{i,a} = \theta_{i,b}$, this implies that the production ratio of good j across regions a and b is given by:

$$\frac{Y_{a}(j)}{Y_{b}(j)} = \frac{\sum_{i} M_{i,a}(j) C_{i} + \sum_{i} \sum_{m} V_{i,a}^{m}(j) G_{i}^{m}}{\sum_{i} M_{i,b}(j) C_{i} + \sum_{i} \sum_{m} V_{i,b}^{m}(j) G_{i}^{m}}$$

which says, according to Equations 3.10 and 3.11, that the relative production of a region is directly related to its production technology and inversely related to the distance of the region to other regions.

Remark 7. When $\theta_{i,a} = \theta_{i,b}$, the relative production of a region is directly related to its production technology and inversely related to the distance of the region to other regions.

3.4. Resource Constraint - Intermediate Inputs

Since the production functions satisfy constant returns to scale, we can write:

$$P^{j}G_{r}^{j} = \frac{WL_{r}\left(j\right)g}{l}$$

If we substitute the optimal $L_r(j)$, which is $L_r(j) = \left(\frac{W}{l}\right)^{l-1} \left(\frac{P^j}{g}\right)^g \frac{Y_r(j)}{A(j)}$ and can be written by using Equations 2.7 and 2.27 as $L_r(j) = \frac{(\eta-1)lP_{r,r}(j)Y_r(j)}{\eta W}$, into this expression, we obtain:

$$P^{j}G_{r}^{j} = \frac{(\eta - 1) P_{r,r}(j) Y_{r}(j) g}{\eta}$$

By using Equations 3.4 and 3.3, we can write:

$$G_r^j = Y_r(j) S_r(j) \tag{3.12}$$

where

$$S_{r}(j) = \frac{(\eta - 1) g \prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i}(m)^{-1} \right)^{1 - \eta} \right)^{\frac{\omega^{s}(m)}{1 - \eta}} \right)^{(\mathbf{I} - \boldsymbol{\omega})_{js}^{-1} l}}{\eta A_{r}(j)}$$

3.5. Labor Market Equilibrium and Closing the Model

From the production function of the production firms, we know that:

$$L_{r}(j) = \frac{Y_{r}(j)}{A_{r}(j)}$$
(3.13)

By using the individual optimality condition (Equation 2.3) and Equation 2.4, we can write:

$$N(h) = Z - \frac{C_r(h)P_r}{W}$$
(3.14)

We can write the total supply of workers in the country as:

$$\sum_{r} H_r N(h) = Z \sum_{r} H_r - \frac{\sum_{r} C_r P_r}{W}$$
(3.15)

In equilibrium, total demand should be equal to total supply:

$$\sum_{r} \sum_{j} L_{r}(j) + \sum_{r} \sum_{t} L_{r}(t) = \sum_{r} H_{r}N(h)$$
(3.16)

where $L_r(t)$ is the labor demanded by each transportation firm. This can be rewritten by using $t_{i,r}(j) = \frac{(D_{i,r})^{\delta(j)} - 1}{D_{i,r}}$ together with Equations 3.13 and 3.7 as:

$$\left\{\begin{array}{c} \sum_{r} \sum_{j} \frac{Y_{r}(j)}{A_{r}(j)} \\ + \sum_{r} \sum_{j} \sum_{i} F_{i,r}(j) \left(M_{irj}C_{i} + \sum_{m} V_{irjm}G_{i}^{m}\right)\end{array}\right\} = Z \sum_{r} H_{r} - \frac{\sum_{r} C_{r}P_{r}}{W}$$
(3.17)

where

$$F_{i,r}(j) = \frac{l(t) P_{r,r}(j) \left((D_{i,r})^{\delta(j)} - 1 \right)}{W}$$

$$= \left((D_{i,r})^{\delta(j)} - 1 \right) \frac{l(t)}{l} \left(\frac{\eta}{\eta - 1} \right)^{\frac{1}{t}} \frac{(1 - l)^{\frac{l-1}{t}}}{A_r(j)}$$

$$\times \prod_s \left(\prod_m \left(\sum_i \left(A_i(m)^{-1} \right)^{1 - \eta} \right)^{\frac{\omega^s(m)}{1 - \eta}} \right)^{(\mathbf{I} - \boldsymbol{\omega})_{js}^{-1}g}$$
(3.18)

Now, by using Equations 3.8, 3.9 and 3.12, we can write:

$$C_{r} = \frac{G_{r}^{j} - \sum_{i} \sum_{m} G_{i}^{m} V_{i,r}^{m}(j) S_{r}(j)}{\sum_{i} v_{i,r} M_{i,r}(j) S_{r}(j)}$$
(3.19)

where $v_{i,r} = \frac{C_i}{C_r}$ can be found by Equation 3.8. By combining Equations 3.17 and 3.19, we obtain:

$$G_r^j = \sum_i \sum_m \Upsilon_{imrj} G_i^m + \Lambda_{rj}$$
(3.20)

where

$$\Upsilon_{imrj} = \frac{\frac{V_{i,r}^{m}(j)S_{r}(j)\left(\sum_{i} v_{i,r} \sum_{r} \sum_{j} F_{i,r}(j)M_{irj} + \sum_{i} \frac{v_{i,r}P_{i}}{W}\right)}{\sum_{i} v_{i,r}M_{i,r}(j)S_{r}(j)} - \frac{1}{A_{i}(m)S_{i}(m)} - \sum_{r} \sum_{j} F_{i,r}\left(j\right)V_{irjm}}{\frac{\sum_{i} v_{i,r} \sum_{r} \sum_{j} F_{i,r}(j)M_{irj} + \sum_{i} \frac{v_{i,r}P_{i}}{W}}{\sum_{i} v_{i,r}M_{i,r}(j)S_{r}(j)}}$$

and

$$\Lambda_{rj} = \frac{Z \sum_{k} H_k \sum_{i} v_{i,r} M_{i,r}(j) S_r(j)}{\sum_{i} v_{i,r} \sum_{r} \sum_{j} F_{i,r}(j) M_{irj} + \sum_{i} \frac{v_{i,r} P_i}{W}}$$

where $\frac{P_i}{W}$ can be found by Equation 3.7.

Proposition 2. An expression for G_r^j can be found in terms of exogenous variables.

P roof. See Appendix A. \blacksquare

As we show in Appendix A, G_r^j 's can be solved as follows:

$$\mathbf{G} = \left(\Upsilon - \mathbf{I}\right)^{-1} \Lambda \tag{3.21}$$

Note that we can write Equation 3.19 as:

$$C_{r} = \frac{\widetilde{G}_{r}^{j} - \sum_{i} \sum_{m} \widetilde{G}_{i}^{m} V_{i,r}^{m}(j) S_{r}(j)}{\sum_{i} \upsilon_{i,r} M_{i,r}(j) S_{r}(j)}$$
(3.22)

where G_k^m corresponds to the expression of G_k^m in terms of exogenous variables. Thus, we have written C_r in terms of exogenous variables. Other C_i 's can be found by using Equation 3.8. Since we have solved for other endogenous variables in terms of G_i^j 's and C_i 's up to now, we can easily obtain closed form expressions for them by substituting \tilde{G}_k^m 's and Equation 3.22 into them.

3.6. Distribution of Labor Across Production and Transportation

By using Equations 3.9 and 3.17, the ratio of production workers to transportation workers in region r can be written as:

$$\frac{\sum_{j} L_{r}(j)}{\sum_{t} L_{r}(t)} = \frac{\sum_{j} \sum_{i} A_{r}(j)^{-1} \varsigma_{i,r,j}}{\sum_{j} \sum_{i} F_{i,r}(j) \varsigma_{i,r,j}}$$
(3.23)

where

$$\varsigma_{i,r,j} = \left(M_{i,r}\left(j\right)C_{i} + \sum_{m} V_{i,r}^{m}\left(j\right)G_{i}^{m} \right)$$

where C_i and G_i^m can be found in terms of exogenous variables by Equations 3.21 and 3.22.

Aggregation across regions gives us this ratio at national level:

$$\frac{\sum_{r}\sum_{j}L_{r}\left(j\right)}{\sum_{r}\sum_{t}L_{r}\left(t\right)} = \frac{\sum_{r}\sum_{j}\sum_{i}A_{r}\left(j\right)^{-1}\varsigma_{i,r,j}}{\sum_{r}\sum_{j}\sum_{i}F_{i,r}\left(j\right)\varsigma_{i,r,j}}$$
(3.24)

which basically says, according to Equation 3.18, that the relative ratio of the transportation workers increases as the distance (i.e., the transportation cost) across regions gets higher (i.e., as the regions get dispersed), *ceteris paribus*. Moreover, as the labor share in transportation (i.e., l(t)) increases, or as the labor share in production sector decreases, obviously, the relative ratio of the transportation workers also increases, ceteris paribus. **Remark 8.** The relative ratio of the transportation workers increases as the distance (i.e., the transportation cost) across regions gets higher (i.e., as the regions get dispersed), or as the labor share in transportation (i.e., l(t)) increases, or as the labor share in production sector decreases, ceteris paribus.

3.7. Bilateral Trade

According to Equation 2.28, after using our assumption $\varepsilon = 1$, we have:

$$C_{i,r}(j) + \sum_{m} G_{i,r}^{m}(j) = \frac{\theta_{i,r}\beta_{i}(j)(P_{i}(j))^{\eta-1}P_{i}C_{i}}{P_{r,r}(j)^{\eta}(1+\tau_{i,r}(j))^{\eta}} + \frac{\sum_{m}\omega^{m}(j)P^{m}G_{i}^{m}}{P_{r,r}(j)(1+\tau_{i,r}(j))}$$
(3.25)

which, according to Equations 3.3, 3.4, 3.5 and 3.6, is equal to:

$$C_{i,r}(j) + \sum_{m} G_{i,r}^{m}(j) = M_{i,r}(j) C_{i} + \sum_{m} V_{i,r}^{m}(j) G_{i}^{m}$$
(3.26)

where $M_{i,r}(j)$ is given by Equation 3.10, $V_{i,r}^{m}(j)$ is given by Equation 3.11, C_{i} is given by Equation 3.22, and G_{i}^{m} is given by Equation 3.21. Equation 3.26 tells, according to Equations 3.21 and 3.22, that that the value of bilateral trade between any two regions depends on the geographic location of all regions, production technology of the exporter region together with the technology of other regions, taste parameters of all regions, and good specific transportation technologies.

Claim 3. The ratio of imports of a region across other two regions directly depends on the production technology ratio of the exporters and inversely depends on the distance ratio of the exporters .

P roof. See Appendix A. ■

Equation 3.26 implies that the ratio of imports of region r across regions a and b is given by:

$$\frac{C_{r,a}(j) + \sum_{m} G_{r,a}^{m}(j)}{C_{r,b}(j) + \sum_{m} G_{r,b}^{m}(j)} = \frac{M_{r,a}(j) C_{r} + \sum_{m} V_{r,a}^{m}(j) G_{r}^{m}}{M_{r,b}(j) C_{r} + \sum_{m} V_{r,b}^{m}(j) G_{r}^{m}}$$
(3.27)

It follows that the ratio of the value of imports of region r across regions a and b is given by:

$$\frac{X_{r,a}(j)}{X_{r,b}(j)} = \frac{P_{a,a}(j)\left(C_{r,a}(j) + \sum_{m} G_{r,a}^{m}(j)\right)}{P_{b,b}(j)\left(C_{r,b}(j) + \sum_{m} G_{r,b}^{m}(j)\right)} = \frac{\frac{\theta_{r,a}A_{a}(j)^{n-1}}{(D_{r,a})^{\delta(j)\eta}} + \frac{\Omega_{r}(j)}{(D_{r,a})^{\delta(j)\vartheta_{r}}(j)}}{\frac{\theta_{r,b}A_{b}(j)^{\eta-1}}{(D_{r,b})^{\delta(j)\eta}} + \frac{\Omega_{r}(j)}{(D_{r,b})^{\delta(j)\vartheta_{r}}(j)}}$$
(3.28)

Equation 3.28 says that a region imports more goods (measured in values) from the higher technology regions and less goods from the more distant regions.

Remark 9. A region imports more goods (measured in values) from the higher technology regions and less goods from the more distant regions.

4. Empirical Test

We use the analytical solution of the model to test its empirical power. The model is tested by using four disaggregate data sets (at the state level) obtained within the U.S., namely consumption, production, trade, and price level, for the year 2002. As a methodology, first, by using the highly accepted parameters used in the literature, we calculate the predicted values of the model for consumption, production, trade, and price levels; and then we test (compare) them with the data. The data are introduced in the subsections, and their details are described in Appendix B.¹³

¹³We use MATLAB Version 7.1.0.246 (R14) Service Pack 3 in our analysis. The data and the codes are available upon request.

4.1. Parametrization

According to Equation 3.26, we need to find values for the following exogenous variables, some of which are at the regional (state) level: η , $\delta(j)$, $A_i(j)$, $\omega^m(j)$, H_i , $D_{i,r}(j)$, $\theta_{i,r}$, $\beta_i(j)$, g, l, l(t) and Z for all i, r, j, m. We borrow the value of $\eta = 5$ from the excellent survey by Anderson and van Wincoop (2004); and we set the value of $\delta(j) = 0.3$ for all j which is the value in the literature surveyed by Hummels (2001), and Anderson and van Wincoop (2004). We find $A_i(j)$'s as the value added per hour of labor supplied by using the region/industry specific data from the U.S. Census Bureau for the year 2002. We find $\omega^m(j)$'s, g, l and l(t) by using the annual input-output tables for the year 2002 provided by the Bureau of Economic Analysis. We obtain the population level of each state, H_i , from the U.S. Census Bureau. We calculate the great circle distances between the capital cities of each state for $D_{i,r}(j)$'s. We assume that $\theta_{i,r} = 1$ for all i and r. Since $\varepsilon = 1$, $\beta_i(j)$'s give us the shares of consumption; thus, we use the consumption shares obtained from the Commodity Flow Survey for them. Finally, the value of Z is set equal to 8760, which is the total amount of hours in a year.

4.2. Predictions of Consumption

We use the closed form version of Equation 2.26, together with total retail sales data obtained from the U.S. Census Bureau at the state level, to test for the consumption predictions of the model. The correlation coefficient between the retail sales data and the prediction of our model for the value of regional consumption is found to be around 0.9967. The goodness of fit for consumption is depicted in Figure 4.1. As is evident, our model is highly successful in predicting the state level consumptions. Since we have normalized the consumption of California to 1 (for both the data and the model) to control for the scale effects in Figure 4.1, we can find the relevant R^2 values simply by calculating the residual sum of squares and the total sum of squares. The corresponding R^2 value in Figure 4.1 is around 0.9960.¹⁴

[Insert Figure 4.1]

To further test the model, we also check the geographical implications of it. In particular, we investigate the explanatory power of the model in terms of bilateral consumption ratios across regions (the states of the U.S.), by considering the total distance of regions to all other regions (i.e. remoteness). Figure 4.2 shows the actual and predicted values for the relation between the log bilateral ratio of consumption and the log bilateral ratio of total distance to all other regions. The correlation coefficient between the actual and predicted values is around 0.9937. Since we make a normalization by considering the bilateral ratios across states, we can find the relevant R^2 values simply by calculating the residual sum of squares and the total sum of squares. The corresponding R^2 value for Figure 4.2 is around 0.9862.

[Insert Figure 4.2]

4.3. Predictions of Production

We use the closed form version of Equation 2.25, together with the gross state product (GSP) data obtained from the Bureau of Economic Analysis, to test for the production side of the model. The goodness of fit for production is given in Figure 4.3. As is evident, the model has successful predictions also for state level productions. We find that the correlation coefficient between the GSP data and the prediction of our model for the total value of regional production is found to be around 0.6977. Since we have normalized the production of California to 1 (for both the data and the model) to control for the scale effects in Figure 4.3, the R^2 value is simply calculated by considering the residual sum of squares and the total sum of squares as 0.4877 for Figure 4.3.

[Insert Figure 4.3]

 $^{^{14}}$ When we regress the (unnormalized) actual values on the (unnormalized) predicted values including a constant to control for the scale effects, we obtain an R^2 value of 0.9934.

We then investigate the explanatory power of the model in terms of bilateral production ratios across the states of the U.S. by considering total distance of the state to all other regions (i.e. remoteness). Figure 4.4 shows the actual and predicted values for this relation. The correlation coefficient between the actual and predicted values is around 0.6775. Since we make a normalization by considering the bilateral ratios across states, we can find the relevant R^2 values simply by calculating the residual sum of squares and the total sum of squares. The corresponding R^2 value for Figure 4.4 is around 0.4464.

[Insert Figure 4.4]

4.4. Predictions of Trade

We use Equation 3.26, together with the Commodity Flow Survey (CFS), which consists of bilateral interstate trade data within the U.S. at the state level, to test for the trade implications of our model. We consider three different trade measures in our analysis, namely trade volume, total exports, and total imports. The goodness of fit for trade volume is given in Figure 4.5. As is evident, the model has successful predictions also for state level trade volumes. We find that the correlation coefficient between the CFS trade data and the prediction of our model for state level trade volume is found to be around 0.6295. The R^2 value calculated by considering the residual sum of squares and the total sum of squares is 0.5496 for Figure 4.5.¹⁵

[Insert Figure 4.5]

The goodness of fit for total exports is given in Figure 4.6. As is evident, the model has successful predictions also for state level exports. We find that the correlation coefficient between the CFS trade data and the prediction of our model for state level trade volume is found to be around 0.6710. The R^2 value calculated by considering the residual sum of squares and the total sum of squares is 0.5634 in Figure 4.6.

[Insert Figure 4.6]

The goodness of fit for total imports is given in Figure 4.7. As is evident, the model has successful predictions also for state level imports. We find that the correlation coefficient between the CFS trade data and the prediction of our model for state level trade volume is found to be around 0.5550. The R^2 value calculated by considering the residual sum of squares and the total sum of squares is 0.4882 in Figure 4.7.

[Insert Figure 4.7]

When we move to Figures 4.8, 4.9 and 4.10, we see the actual and predicted values for the relation between the log bilateral ratio of total distance to all other regions and the log bilateral ratio of trade volume, total exports and total imports, respectively. The corresponding correlation coefficients between the actual and predicted values are around 0.7077, 0.7983 and 0.5995 for Figures 4.8, 4.9 and 4.10, respectively. Finally, the corresponding R^2 values for Figures 4.8, 4.9 and 4.10 are around 0.3210, respectively.

[Insert Figures 4.8 - 4.10]

4.5. Predictions of Price Level

We use Equation 3.6, together with the ACCRA Cost-of-Living Index Data, to test for the price level implications of our model. The goodness of fit for price levels is given in Figure 4.11. As is evident, the model has successful predictions also for state level trade volumes. We find that the correlation coefficient between the ACCRA Costof-Living Index data and the prediction of our model for state level price levels is found to be around 0.3194. The R^2 value calculated by considering the residual sum of squares and the total sum of squares is 0.9584 in Figure 4.5.

 $^{^{15}}$ When we make the distinction between *including* and *excluding* zero trade observations, we find that the correlation coefficient between the CFS data *including* the zero trade observations and the prediction of the model is around 0.7004, which corresponds to an R^2 value of 0.4906.

[Insert Figure 4.11]

When we move to Figure 4.12, we see the actual and predicted values for the log bilateral ratio of price levels. The corresponding correlation coefficients between the actual and predicted values are around 0.3552. The corresponding R^2 value in Figure 4.12 is around 0.7334.

[Insert Figure 4.12]

We can say that our empirical results are very promising even though we have restricted ourselves to the case in which $\delta(j) = \delta$ for all j and $\theta_{i,r} = 1$ for all i and r. The sensitivity of our results is supported by the fact that we have used four different data sets in our analysis.

5. Simulation

By considering hypothetical changes in different technology types, we perform counterfactual exercises in this section. In particular, we simulate our model on the U.S. economy, by using the parameters described in the previous section, to search for the effects of technology changes on the regional output, national output, price dispersion across regions, the distribution of labor across production and transportation, and regional population levels. We consider changes in four different technology types in our analysis: 1) National production technology, 2) National transportation technology, 3) Regional production technology, 4) Sectoral production technology. In technical terms, we perform a counterfactual analysis and attempt to find what happens when the considered technology level is reduced by 100% or increases by 100%, *ceteris paribus*.

5.1. National Production Technology

We start our analysis by analyzing the effects of a change in the national production technology (i.e., equal changes in $A_i(j)$'s for all *i* and *j*) on the national output. This effect is shown in Figure 5.1. Note that we have percentage changes in the axes where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1means a 100% decrease. As is evident, higher (lower) national production technology leads to higher (lower) national output. In particular, if we double the level of national technology, the national output increases by 5 times. Figure 5.1 also captures the catch-up effect which states that poorer economies tend to grow faster than richer economies. We don't show the results here - to save space - but, according to our simulation, the output levels in all regions and all sectors are affected exactly equally.¹⁶

[Insert Figure 5.1]

When we move to Figure 5.2, we can see the effect of a change in the national production technology on the cost-of-living index dispersion across regions (i.e., the standard deviation of log absolute P_i 's). As is evident, the price dispersion increases as the level of technology increases: if we double the national production technology, the price dispersion increases by 20%. In other words, even though there is no regional or sectoral technology change, the economy can create such price dispersions through technology changes at national level. The reasons for this dispersion can be understood better when we consider Remark 4 above.¹⁷

[Insert Figures 5.2 - 5.3]

The effect of a change in the national production technology on the ratio between the production and transportation workers is given in Figure 5.3. As is evident, the ratio gets higher along with the national production technology. If we double the national production technology, the ratio increases by 10 times. This relation, which is due to Equations 3.18 and 3.24, actually shows us the increased (decreased) relative demand for labor after a technology increase (decrease) because of the increased (decreased) demand for national output analyzed in Figure 5.1.

 $^{^{16}}$ These results are available upon request. They can also be obtained by using our published Matlab codes.

¹⁷Although we don't show the results here, the price dispersion at the good level (i.e., the standard deviation of log absolute $P_i(j)$'s) doesn't change with respect to a change in the national technology level. Notice that this difference between the standard deviations of P_i 's and $P_i(j)$'s is due to $\beta_i(j)$'s.

[Insert Figure 5.4]

When we move to Figure 5.4, we can see the effect of a change in the national production technology on the population dispersion across regions (i.e., the standard deviation of log absolute H_i 's). As is evident, the population dispersion decreases as the level of technology increases: if we double the national production technology, the population dispersion decreases by around 1%. In other words, even though there is no regional or sectoral technology change, the economy can create such migrations through technology changes at the national level. The reasons for this dispersion can be understood better when we consider Figure 5.5 below.

[Insert Figure 5.5]

As is evident in Figure 5.5, the national production technology changes affect the population of different regions in different ways. As an example, while some of highly populated states, such as California and New York, are negatively affected by a national production technology increase, some low populated states, such as Delaware, Montana, and Maine, are positively affected. These examples give more insight related to Figure 5.4. Nevertheless, we know from Equation 2.23 that these population differences are mostly due to cost-of-living indices. But, what is the motivation behind the relation between prices and population changes? Figure 5.6 provides information about this issue.

[Insert Figure 5.6]

In particular, Figure 5.6 shows the relation between log bilateral price ratios (i.e., log bilateral P_i ratios) and log bilateral ratios of population changes after an 100% increase in the national production technology. As is evident, the higher the initial price ratios, the lower the ratio of population changes. But what is the exact relation between these two? We answer this question by regressing initial price ratios on the ratio of population changes. We find that the relevant coefficient is -0.08, which suggests on average that if the initial percentage deviation in terms of prices is 100% between any two locations, then the percentage deviation in terms of population changes after a 100% increase in the national production technology is going to be 8% less.¹⁸

5.2. National Transportation Technology

This subsection depicts the effects of a change in the national transportation technology (i.e., equal changes in elasticities of distance $\delta(j)$'s for all j). We start with Figure 5.7 which shows the effect of a change in the national transportation technology on the national output. As is evident, the national output decreases (increases) as the transportation costs (i.e., $\delta(j)$'s for all j) increase (decrease). The interesting point of Figure 5.7. is that as the transportation costs approach zero, the national output increases by more than 10 times. If we think of our transportation costs as regional barriers for a second, our result suggests that decreasing regional barriers (which correspond to national borders in international trade context) leads to welfare gains in a multiplicative manner. Although we don't show the results here (to save space), this welfare gain is true for all the regions in the economy.¹⁹

[Insert Figure 5.7]

When we move to Figure 5.8, we see the effect of a change in the national transportation technology on the costof-living index dispersion across regions. As is evident, price dispersion increases (decreases) as the transportation costs get higher (lower). If we double the transportation costs, the cost-of-living index increases by 80%. As transportation costs approach zero, the cost-of-living index dispersion decreases by 20%. Thus, transportation costs are not significant sources of the cost-of-living index dispersion. This result is consistent with the international finance literature that *partly* explains the price dispersions through trade costs. Similarly, Figure 5.9 shows the effect of a change in the national transportation technology on the price dispersion at the commodity level (i.e., the standard deviation of $P_i(j)$'s). Compared to Figure 5.8, the dispersion in Figure 5.9 shows a different pattern, both in terms of second derivatives and in terms of magnitudes. This difference has important implications on applied research based on deviations from law-of-one-price (LOP) or purchasing-power-parity (PPP) comparison analysis. In particular, an applied researcher should be aware of the distinction between aggregate and disaggregate price levels according to our model.

¹⁸The *Rbar sqd*. for this regression is 0.67.

¹⁹The magnitude of the welfare gain in each region differs very slightly. These region specific results are available upon request, or they can be obtained by our published Matlab codes.

[Insert Figures 5.8 – 5.9]

We show the effect of a change in the national transportation technology on the ratio between the production and transportation workers in Figure 5.10.

[Insert Figure 5.10]

As is evident in Figure 5.10, as the transportation costs get higher (i.e., as the transportation technology gets lower) the ratio between production and transportation workers gets lower which says that relatively more labor is needed in the transportation sector. In particular, if we double the transportation costs, the ratio decreases by 50%. However, as transportation costs approach zero, the ratio increases by 5 times. Thus, the labor force allocated to production is significantly affected by transportation costs. This also helps us understand the magnitude of the effect of transportation costs on the national output.

[Insert Figure 5.11]

When we move to Figure 5.11, we can see the effect of a change in the national transportation technology on the population dispersion across regions (i.e., the standard deviation of log absolute H_i 's). As is evident, the population dispersion decreases as the level of technology increases (i.e., as the transportation costs, $\delta(j)$'s for all j, get lower): if we double the transportation costs, the price dispersion decreases by around 1%. In other words, even though there is no regional or sectoral technology change, the economy can create such migrations through national transportation technology. Figure 5.12 shows how the national transportation technology changes affect the population of individual states.

[Insert Figure 5.12]

5.3. Regional Production Technology

In this subsection, we show the results of a change in regional production technologies (i.e., equal technology changes in all sectors of a region that correspond to changes in $A_i(j)$'s all j's for a specific i). We start with Figure 5.13 that shows the effect of a technological change at the regional (state) level on the national output level. As is evident, region specific technology changes have very small effects on the national output compared to the effects of national technological changes. Nevertheless, a technological change in larger states such as California and New York has a bigger effect on the national output compared to the changes in smaller states.

[Insert Figure 5.13]

When we move to Figure 5.14, we see the effects of a change in regional production technologies on the costof-living index dispersion across regions. As is evident, while a technology change in some regions increases the price dispersion, a technology change in others decreases it. This result is true also for the price dispersion at the commodity level given in Figure 5.15. Thus, our results support the view that geography matters for price dispersion.

[Insert Figures 5.14 - 5.15]

Although we don't show it here (to save space), the effects of a change in regional production technologies on the ratio between production and transportation workers is very similar (with slight differences) to the effect of a change in national production technologies.²⁰

²⁰These results are available upon request. They can also be obtained by using our published Matlab codes.

5.4. Sectoral Production Technology

In this subsection, we show the results of a change in sectoral production technologies (i.e., technology changes in a specific industry in all regions which correspond to changes in $A_i(j)$'s all *i*'s for a specific *j*). We start with Figure 5.16 that shows the effect of a technological change at the sectoral level on the national output. As is evident, different sectoral technology changes have different effects on the national output level. In particular, while some sectors such as food-beverage and gasoline have high effects on the national output, some others have a lesser effect on it.

[Insert Figure 5.16]

When we move to Figure 5.17, we see the effects of a change in sectoral production technologies on the cost-ofliving index dispersion across regions. As is evident, while a technology change in some sectors increases the price dispersion, a technology change in some others decreases it.²¹ In particular, a technological increase in non-durable goods mostly increases the price dispersion while a technological increase in durable goods mostly decreases the price dispersion.

[Insert Figure 5.17]

Finally, the effects of a change in sectoral production technologies on the ratio between production and transportation workers is given in Figure 5.18. As we see, while some sectors such as gasoline, coal - petroleum, chemical products have a higher effect on the labor ratio, some other have a lesser effect on it.

[Insert Figure 5.18]

6. Conclusions

We have introduced a general equilibrium trade model that considers the distributions of both consumption and production at the disaggregate level to analyze the effects of different technology types on the national and regional variables of the U.S. The model relates the trade balance of a state to the geographical location of all regions, income levels of all regions, production levels of all regions, price levels of all regions, as well as the good specific transportation costs and region/good specific technology levels. Beyond the gravity models that mostly focus on the aggregate trade flow, our model focuses on trade at the disaggregate level. In particular, we show that the gravity models of Anderson (1979) and Anderson and van Wincoop (2003) are special cases of our model.

When we solve our model analytically, we go beyond the gravity models and find the main determinants of trade of a region as the geographical location of all regions, population of all regions, taste differences across regions, and good specific production/transportation technologies. We have also presented the implications of our model on bilateral ratios of price levels, consumption levels, production levels, bilateral trade volumes, and population levels across regions. In particular, we have shown that the relative production of a region (compared to other regions) increases with its technology level and decreases with its distance to other regions. Similarly, the relative consumption of a region increases with its population level and its distance to other regions. Moreover, a region imports more goods from the higher technology regions and fewer goods from the more distant regions. The relative ratio of the transportation workers increases as the distance across regions gets higher (i.e., as the regions get dispersed), ceteris paribus.

When we test the model empirically, we find that it has high explanatory power on the U.S. economy. Finally, we have simulated the model on the U.S. economy and have shown the effects of a change in technology at the national, regional and sectoral levels. The results have several insights related to output levels, price dispersions, the labor market, and the regional population levels. In particular, it is shown that if the national production technology level is doubled, the output increases by 5 times, the price dispersion increases by 20%, the population dispersion decreases by 1%, and the ratio of the production labor force to the transportation labor force increases by more than 10 times. As the transportation costs approach zero, the national output increases by more than 10 times, the price dispersion decreases by 20%, the population dispersion increases by 1%, and the ratio of the production dispersion increases by 1%, and the ratio of the production dispersion increases by 1%, and the ratio of the population dispersion increases by 1%, and the ratio of the production dispersion increases by 1%, and the ratio of the production dispersion increases by 1%, and the ratio of the production labor force to the transportation of the production labor force to the transport technology that the production labor force increases by 5 times.

²¹Although we don't show the results here, the price dispersion at the commodity level doesn't change when the sectoral technologies change. This is mostly due to our parameter assumption that $\theta_{i,r} = 1$ for all *i* and *r*.

have fewer effects on the macroeconomic variables compared to the effects of nationwide technology changes. An interesting extension of our paper is to analyze international trade patterns by moving our model to an international level. We are currently working on this project.

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Appendix A - Proofs Proof of Claim 1. By using Equations 2.7, 2.10, 2.11 and 2.27, together with $\varepsilon = 1$, we can write:

$$P^{j}(m) \equiv \frac{\eta}{\eta - 1} \left[\frac{W}{l}\right]^{l} \left[\frac{P^{m}}{g}\right]^{g} \left(\sum_{i} A_{i}(m)^{\eta - 1}\right)^{\frac{1}{1 - \eta}}$$

and

$$P^{j} \equiv \left(\prod_{m} \left(P^{m}\right)^{g\omega^{j}(m)}\right) \frac{\eta g^{-g}}{\eta - 1} \left[\frac{W}{l}\right]^{l} \left(\prod_{m} \left(\sum_{i} \left(A_{i}\left(m\right)^{-1}\right)^{1 - \eta}\right)^{\frac{\omega^{j}(m)}{1 - \eta}}\right)^{\frac{\omega^{j}(m)}{1 - \eta}}\right)$$

If we take the logs of both sides, we obtain:

$$p^{j} \equiv \sum_{m} g\omega^{j}(m) p^{m} + k_{j}$$

where

$$k_{j} = \log\left(\frac{\eta g^{-g}}{\eta - 1} \left[\frac{W}{l}\right]^{l} \left(\prod_{m} \left(\sum_{i} \left(A_{i}\left(m\right)^{-1}\right)^{1 - \eta}\right)^{\frac{\omega^{j}\left(m\right)}{1 - \eta}}\right)\right)\right)$$

and $p^j = \log P^j$. In matrix form, we can write:

$$\mathbf{p} = \boldsymbol{\omega} \mathbf{p} + \mathbf{k}$$

where

$$\mathbf{p} = \begin{pmatrix} p^{1} \\ \vdots \\ p^{J} \end{pmatrix}, \quad \boldsymbol{\omega} = \begin{pmatrix} g\omega^{1}(1) & g\omega^{1}(J) \\ \vdots & \ddots & \vdots \\ g\omega^{J}(1) & g\omega^{J}(J) \end{pmatrix}, \quad \mathbf{k} = \begin{pmatrix} k_{1} \\ \vdots \\ k_{J} \end{pmatrix}$$

This system can be solved as:

$$\mathbf{p} = (\mathbf{I} - \boldsymbol{\omega})^{-1} \mathbf{k}$$

1

where **I** represents the identity matrix and j'th row of $(\mathbf{I} - \boldsymbol{\omega})^{-1} \mathbf{k}$ corresponds to p^{j} . Thus, P^{j} 's can easily be found by

$$P^{j} = \exp\left(\left(\mathbf{I} - \boldsymbol{\omega}\right)_{j}^{-1} k_{j}\right)$$
$$= \prod_{s} \left(K_{s}\right)^{\left(\mathbf{I} - \boldsymbol{\omega}\right)_{js}^{-1}}$$

where $K_m = \exp(k_m)$; $(\mathbf{I} - \boldsymbol{\omega})_j^{-1}$ is the *j*'th row of $(\mathbf{I} - \boldsymbol{\omega})^{-1}$; and $(\mathbf{I} - \boldsymbol{\omega})_{jm}^{-1}$ is the *j*'th row of *m*'th column in $(\mathbf{I} - \boldsymbol{\omega})^{-1}$.Since

$$K_{s} = \left(\frac{\eta g^{-g}}{\eta - 1} \left[\frac{W}{l}\right]^{l} \left(\prod_{m} \left(\sum_{i} \left(A_{i}\left(m\right)^{-1}\right)^{1 - \eta}\right)^{\frac{\omega^{s}\left(m\right)}{1 - \eta}}\right)\right)$$

we can obtain:

$$P^{j} = \prod_{s} \left(\left(\frac{\eta g^{-g}}{\eta - 1} \left[\frac{W}{l} \right]^{l} \left(\prod_{m} \left(\sum_{i} \left(A_{i} \left(m \right)^{-1} \right)^{1 - \eta} \right)^{\frac{\omega^{s}(m)}{1 - \eta}} \right) \right) \right) \right)^{(\mathbf{I} - \boldsymbol{\omega})_{js}^{-1}}$$

which can be rewritten as:

$$P^{j} = \frac{W}{l} \left(\frac{\eta}{\eta - 1}\right)^{\frac{1}{l}} (1 - l)^{\frac{l-1}{l}} \prod_{s} \left(\prod_{m} \left(\sum_{i} \left(A_{i}(m)^{-1}\right)^{1-\eta}\right)^{\frac{\omega^{s}(m)}{1-\eta}}\right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}}\right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}}$$

since $\sum_{s} (\mathbf{I} - \boldsymbol{\omega})_{js}^{-1} = l^{-1}$; $\sum_{s} \boldsymbol{\omega}^{s}(m) = 1$; and g + l = 1.²² **Proof of Proposition 2.** In matrix form, Equation 3.20 can be written as:

 $\mathbf{G} = \Upsilon \mathbf{G} + \Lambda$

and

where

This system can be solved as follows:

$$\mathbf{G} = \left(\Upsilon - \mathbf{I}\right)^{-1} \Lambda$$

where row $(((k-1) \times J) + m)$ of **G**, say \widetilde{G}_k^m , corresponds to the expression of G_k^m in terms of exogenous variables.

Proof of Claim 3. Equation 3.26 implies that the ratio of imports of region r across regions a and b is given by:

$$\frac{C_{r,a}(j) + \sum_{m} G_{r,a}^{m}(j)}{C_{r,b}(j) + \sum_{m} G_{r,b}^{m}(j)} = \frac{M_{r,a}(j) C_{r} + \sum_{m} V_{r,a}^{m}(j) G_{r}^{m}}{M_{r,b}(j) C_{r} + \sum_{m} V_{r,b}^{m}(j) G_{r}^{m}}$$

which, according to Equations 3.10 and 3.11, depends on the production technology ratio of the exporters directly and depends on the distance ratio of the exporters inversely. To show this, first, we can write $M_{r,a}(j) C_r$ and $M_{r,b}(j) C_r$ as:

$$M_{r,a}(j) C_{r} = \frac{\theta_{r,a} A_{a}(j)^{\eta}}{\left(\left(D_{r,a} \right)^{\delta(j)} \right)^{\eta}} \vartheta_{r}(j)$$

and

$$M_{r,b}(j) C_{r} = \frac{\theta_{r,b}A_{b}(j)^{\eta}}{\left((D_{r,b})^{\delta(j)} \right)^{\eta}} \vartheta_{r}(j)$$

where

$$\vartheta_{r}\left(j\right) = \frac{C_{r}\beta_{r}\left(j\right)\prod_{j}\left(\prod_{s}\left(\prod_{m}\left(\sum_{k}\left(A_{k}\left(m\right)^{-1}\right)^{1-\eta}\right)^{\frac{\omega^{s}\left(m\right)}{1-\eta}}\right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}g}\right)^{\beta_{r}\left(j\right)}}{\times\left(\sum_{m}\theta_{r,m}\left(\frac{\left(D_{r,m}\right)^{\delta\left(j\right)}}{A_{m}\left(j\right)}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}}\right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}g}}{\prod_{s}\left(\prod_{m}\left(\sum_{k}\left(A_{k}\left(m\right)^{-1}\right)^{1-\eta}\right)^{\frac{\omega^{s}\left(m\right)}{1-\eta}}\right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}^{-1}g}}\times\left(\sum_{m}\theta_{r,m}\left(\frac{\left(D_{r,m}\right)^{\delta\left(j\right)}}{A_{m}\left(j\right)}\right)^{1-\eta}}\right)^{(\mathbf{I}-\boldsymbol{\omega})_{js}}g}$$

 $^{22}\sum_{s} (\mathbf{I} - \boldsymbol{\omega})_{js}^{-1} = l^{-1}$, because the sum of each row of $(\mathbf{I} - \boldsymbol{\omega})$ is equal to (1 - g), and thus, $\sum_{s} (\mathbf{I} - \boldsymbol{\omega})_{js}^{-1} = (1 - g)^{-1}$.

where C_r is again given by Equations 3.22. Similarly, we can write $\sum_m V_{r,a}^m(j) G_r^m$ and $\sum_m V_{r,b}^m(j) G_r^m$ as:

$$\sum_{m} V_{r,a}^{m}(j) G_{r}^{m} = \frac{A_{a}(j)}{\left((D_{r,a})^{\delta(j)} \right)} \Omega_{r}(j)$$

 $\quad \text{and} \quad$

$$\sum_{m} V_{r,b}^{m}\left(j\right) G_{r}^{m} = \frac{A_{b}\left(j\right)}{\left(\left(D_{r,b}\right)^{\delta\left(j\right)}\right)} \Omega_{r}\left(j\right)$$

where

$$\Omega_{r}(j) = \sum_{m} G_{r}^{m} \omega^{m}(j) \prod_{s} \left(\prod_{j} \left(\sum_{i} \left(A_{i}(j)^{-1} \right)^{1-\eta} \right)^{\frac{\omega^{s}(j)}{1-\eta}} \right)^{(\mathbf{I}-\boldsymbol{\omega})_{ms}^{-1}l}$$

where G_r^m is again given by Equation 3.21. Thus, we can rewrite Equation 3.27 as:

$$\frac{C_{r,a}\left(j\right) + \sum_{m} G_{r,a}^{m}\left(j\right)}{C_{r,b}\left(j\right) + \sum_{m} G_{r,b}^{m}\left(j\right)} = \frac{\frac{\theta_{r,a}A_{a}(j)^{\eta}}{(D_{r,a})^{\delta(j)\eta}} + \frac{A_{a}(j)}{(D_{r,a})^{\delta(j)}}\frac{\Omega_{r}(j)}{\vartheta_{r}(j)}}{\frac{\theta_{r,b}A_{b}(j)^{\eta}}{(D_{r,b})^{\delta(j)\eta}} + \frac{A_{b}(j)}{(D_{r,b})^{\delta(j)}}\frac{\Omega_{r}(j)}{\vartheta_{r}(j)}}$$

Hence, we have shown our claim. \blacksquare

Appendix B - Data

For the value of consumption in each state, we use the retail trade data for the year 2002, of which North American Industry Classification System (NAICS) code is 44-45. For the value of production in each state, we use gross state product (GSP) obtained from the Bureau of Economic Analysis for the year 2002. For the bilateral trade analysis, we use the state-level Commodity Flow Survey (CFS) data obtained from the Bureau of Transportation Statistics for the United States for the year 2002. In particular, we use the interstate trade data for the 2-digit Standard Classification of Transported Goods (SCTG) commodities of which codes and names are given in Table A.1 together with the corresponding NAICS code that is used to find the region/sector specific technology levels. The source for the crosswalk between NAICS and SCTG is National Transportation Library of the Bureau of Transportation Statistics.

Because of the data availability, we include all the states of the United States except for Alaska, District of Columbia and Hawaii. In order to obtain the technology levels, we first use an approximate crosswalk between 3-digit North American Industry Classification System (NAICS) and 2-digit SCTG obtained from the National Transportation Library of the Bureau of Transportation Statistics. This crosswalk is given by Table A.1.. After that, we use $A_i(j) = \log \left(\frac{V_i(j)}{P_i L_i(j)}\right)$ as our proxy for the technology levels, where $V_i(j)$ is the industry/region specific value added; P_i is the cost-of-living index for state *i* borrowed from Berry et al. (2003); and $L_i(j)$ is the industry/region specific hours of labor supplied by the production workers. For the value added of each NAICS industry in each state, we use the state level U.S. Census Bureau data for the relevant industries in 2002.

For the distance measures, we calculate the great circle distance between states by using the latitudes and the longitudes of the capital cities of each state published by U.S. Census Bureau. Note that we don't use the average distance measures given by CFS, because those measures are available only for the realized trades across states. Since we include zero observations (i.e., no trade across states) into our analysis, we use the great circle distance measures that are not included in CFS.

The cost-of-living index of Berry et al. (2003) has been used in calculating the technology levels above, because they represent a composite price index including both traded and non-traded goods. However, since our model considers only traded goods, we need a more accurate measure of cost-of-living index, which mostly covers the prices of traded goods. In this sense, for cost-of-living index in each region, we use the price index for traded goods (i.e., grocery index) of the ACCRA cost-of-living index for the year 2002. Since ACCRA cost-of-living index is represented at city level within the U.S., we consider the cost-of-living index in the capital city of each state as a state level measure. This is also consistent with using the location of capital cities in calculating the great circle distances across states.

Definition	SCTG	NAICS
Live Animal	1	$111, 112^{**}$
Food - Beverage	$2, 3, 4, 5, 6, 7, 8, 9^{st}$	$311, 312^{**}$
Mining	$10, 11, 12, 13, 14, 15^{*}$	212^{***}
Gasoline - Fuel Oil	$17, 18^{*}$	324
Coal - Petroleum	19	$324, 325^{**}$
Chemical Products	$20, 21, 22, 23^*$	325
Plastics and Rubber	24	326
Forestry - Fishing	25	113
Wood	26	321
Paper	$27, 28^{*}$	322
Printing	29	323
Textile	30	$313, 314^{**}$
Nonmetallic Minerals	31	327
Base Metal	32	$331, 324^{**}$
Fabricated Metal	33	332
Machinery	34	333
Electronic	35	$334, 335^{**}$
Motor Vehicles	36	3361
Transportation	37	3364
Computer	38	334
Furniture	39	337
Miscellaneous	40	339
Waste - Scrap	41	$313, 331^{**}$

 Table A.1 - Goods Used in Empirical Analysis

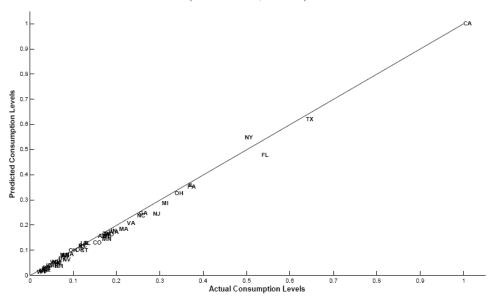
Notes: * stands for the sum of the relevant SCTG industries used to obtain bilateral trade measures. ** means that an average of the relevant NAICS industries used to obtain technology levels. *** means that there is no corresponding production data for that specific NAICS industry in the U.S. Census Bureau; thus, we assume that the technology levels are the same across states for those industries.

FIGURES

<		Goods (j)		
Good 1 Variety 1	•	•		Good J Variety 1
•				
Good 1 Variety R	•	•	·	Good J Variety R

Figure 2.1 - Feasible Consumption Set

Figure 4.1 - Goodness of Fit for Consumption (Normalized, CA=1)



Notes: The 45-degree line has been plotted for reference.

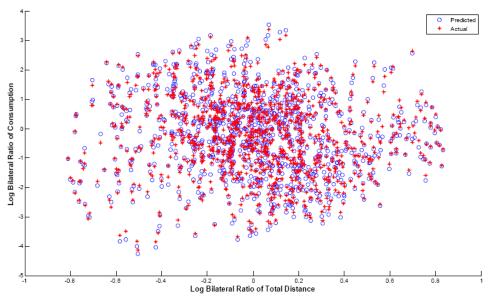
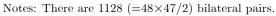


Figure 4.2 - Consumption and Total Distance



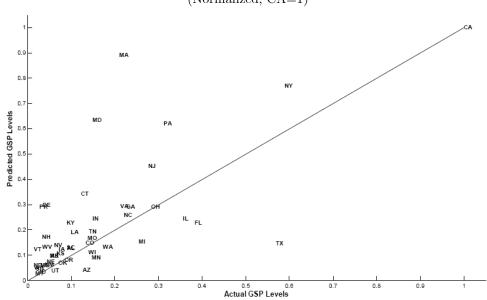


Figure 4.3 - Goodness of Fit for Production (Normalized, CA=1)

Notes: The 45-degree line has been plotted for reference.

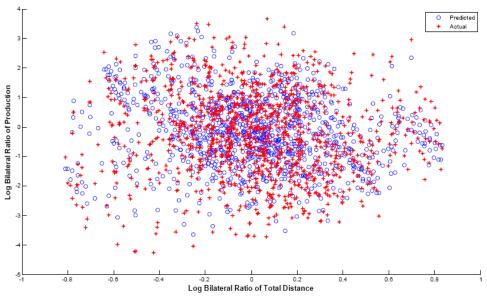


Figure 4.4 - Production and Total Distance

Notes: There are $1128 \ (=48 \times 47/2)$ bilateral pairs.

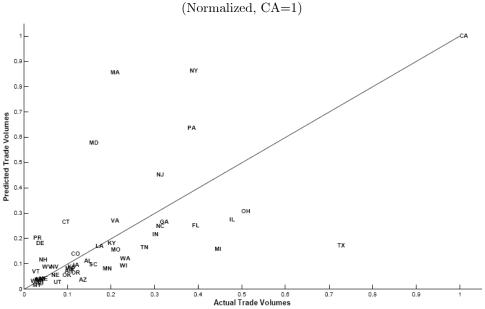


Figure 4.5 - Goodness of Fit for Trade Volume (Normalized CA=1)

Notes: The 45-degree line has been plotted for reference.

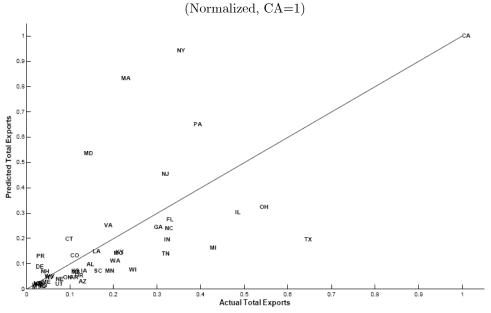
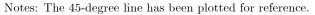


Figure 4.6 - Goodness of Fit for Total Exports



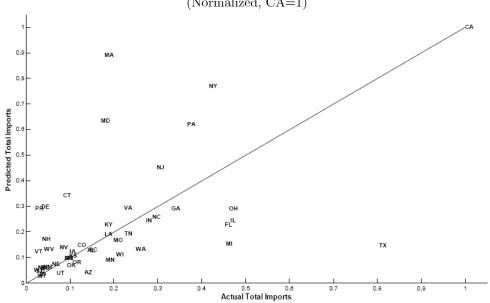


Figure 4.7 - Goodness of Fit for Total Imports (Normalized, CA=1)

Notes: The 45-degree line has been plotted for reference.

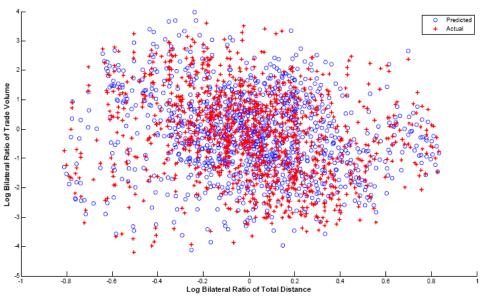
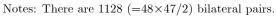
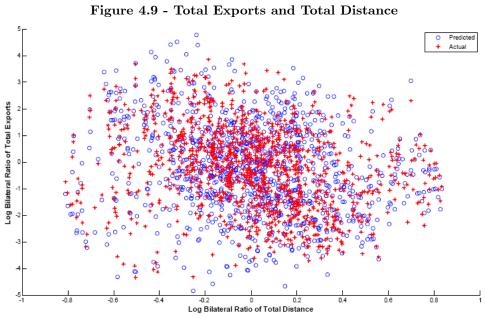
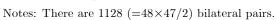


Figure 4.8 - Trade Volume and Total Distance







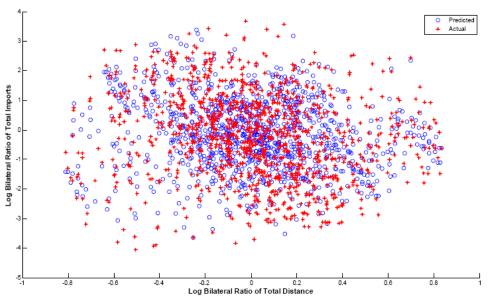
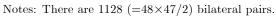
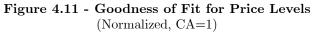
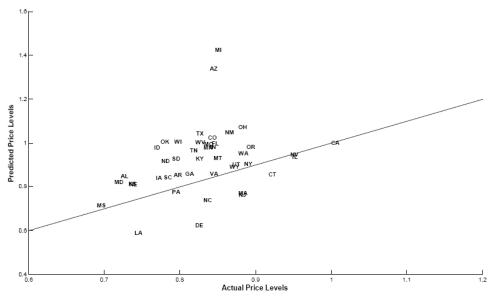


Figure 4.10 - Total Imports and Total Distance







Notes: The 45-degree line has been plotted for reference.

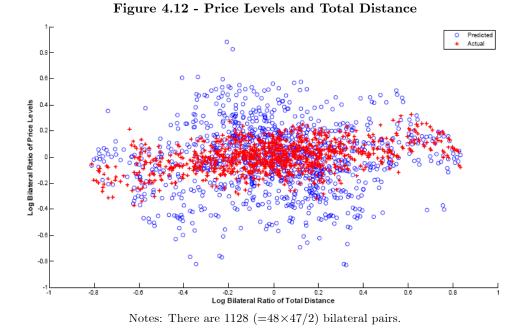
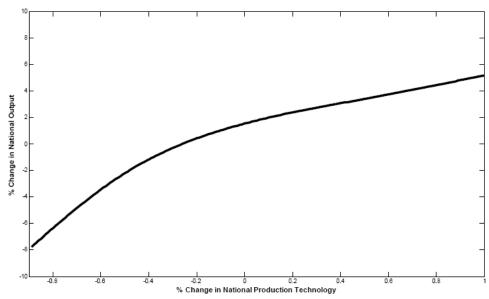
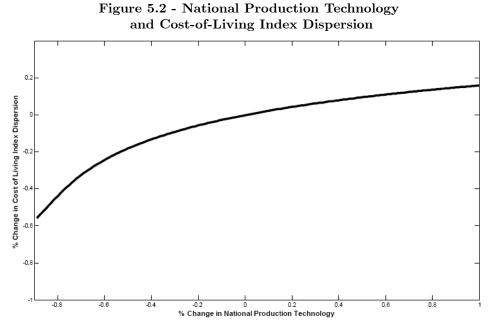




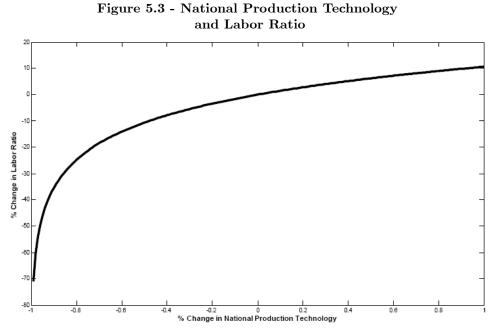
Figure 5.1 - National Production Technology and National Output



Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.



Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.



Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.

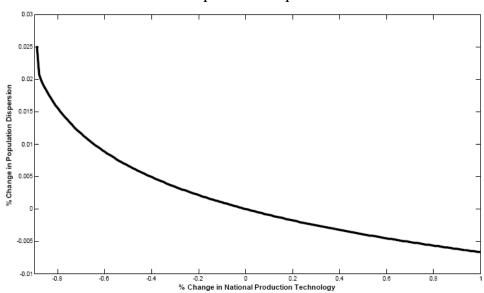


Figure 5.4 - National Production Technology and Population Dispersion

Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.

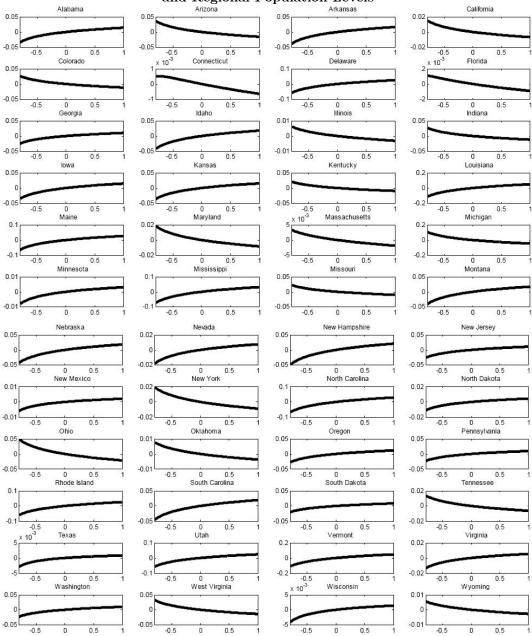


Figure 5.5 - National Production Technology and Regional Population Levels

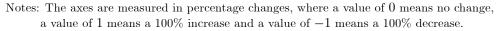
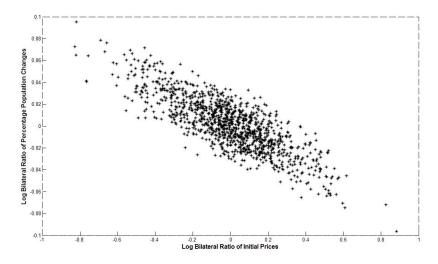
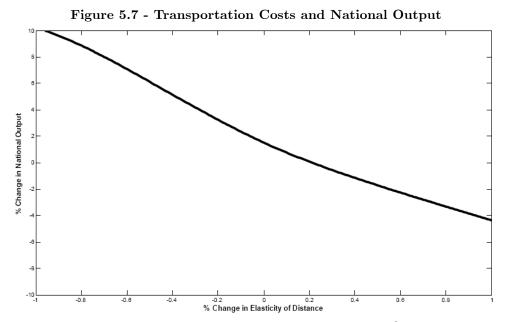


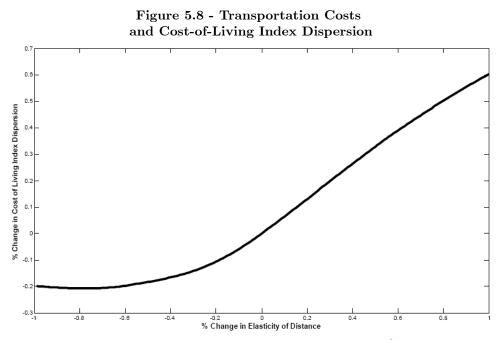
Figure 5.6 - Initial Prices and Population Changes After a 100% Increase in National Production Technology



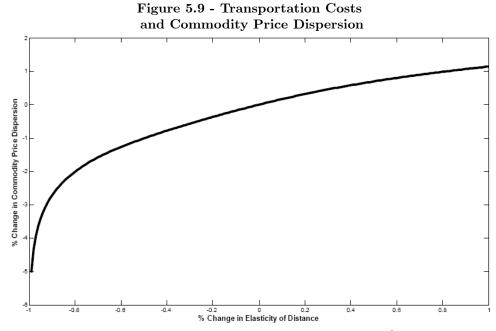
Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.



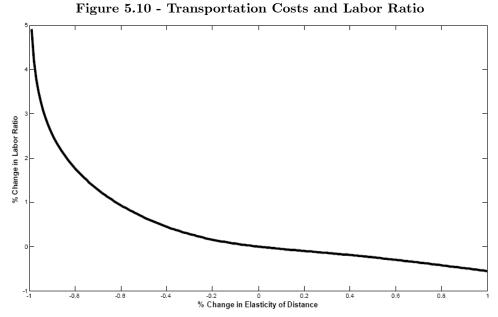
Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.



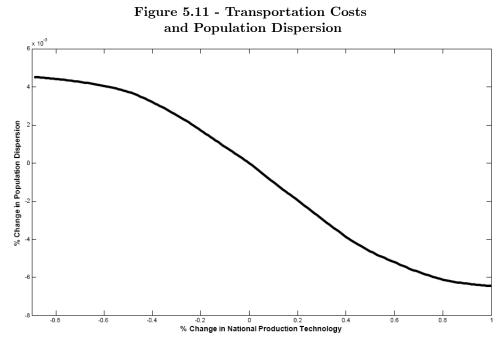
Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.



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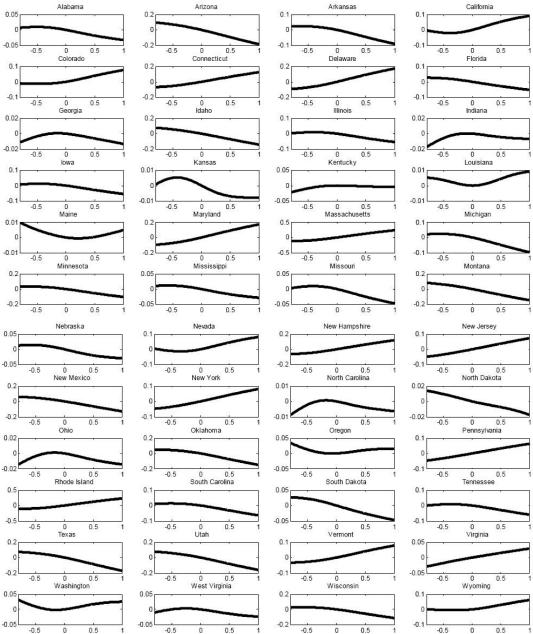


Figure 5.12 - National Production Technology and Regional Population Levels

Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.

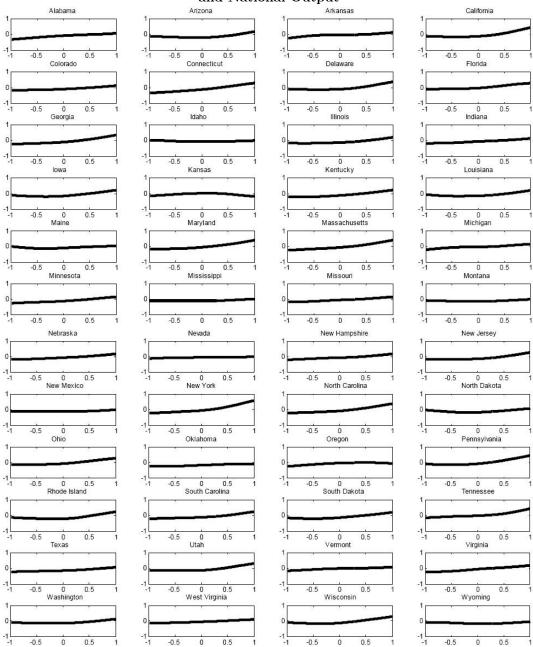


Figure 5.13 - Regional Production Technology and National Output

Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.

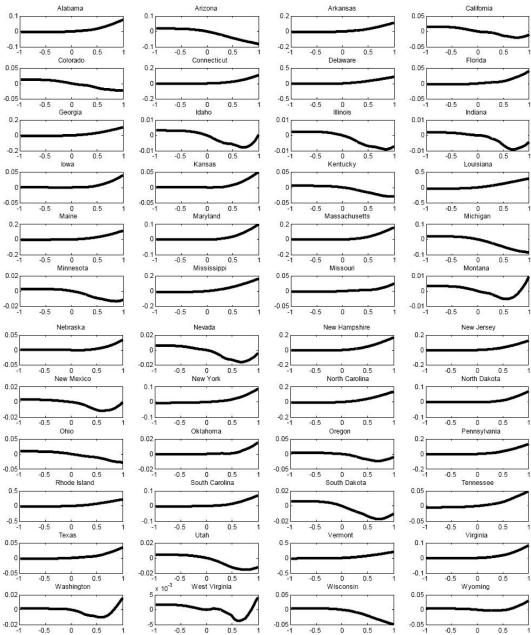


Figure 5.14 - Regional Production Technology and Cost-of-Living Index Dispersion

Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.

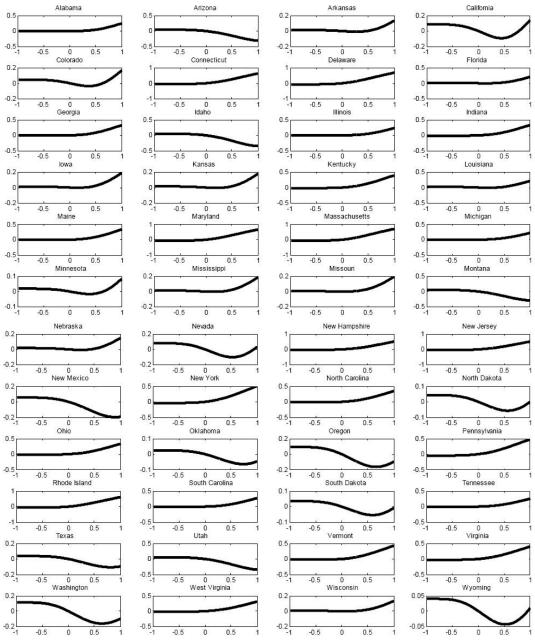


Figure 5.15 - Regional Production Technology and Commodity Price Dispersion

Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.

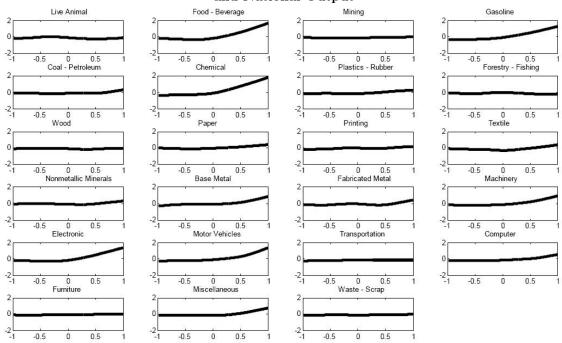


Figure 5.16 - Sectoral Production Technology and National Output

Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.

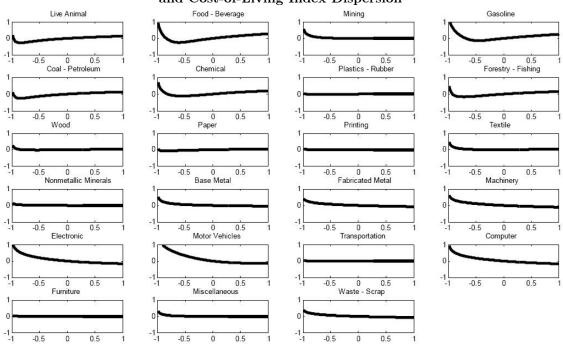


Figure 5.17 - Sectoral Production Technology and Cost-of-Living Index Dispersion

Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.

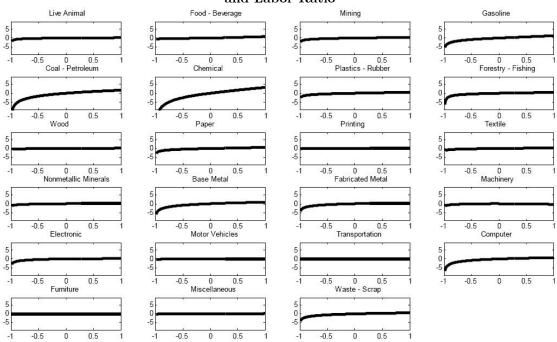


Figure 5.18 - Sectoral Production Technology and Labor Ratio

Notes: The axes are measured in percentage changes, where a value of 0 means no change, a value of 1 means a 100% increase and a value of -1 means a 100% decrease.