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# Skill Shortages, Labor Reallocation, and Growth

von

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**Abstract**

This paper explores the relationship between growth and unemployment. Knowledge formation is the source of growth, which includes the two dimensions technologies and skills. Both are connected through a technology-skill complementarity which may have limiting effects on the reallocation of labor and technology implementation in manufacturing. The reallocation of labor becomes necessary as growth leads to continuous job creation and job destruction. The ratio of job destruction to job creation identifies three regimes, two of which are associated with unemployment either due to restricted labor demand or due to skill shortages. While in the regime with full employment the model confirms the standard result that knowledge formation has positive effects on growth, the outcome is much more ambiguous if we consider a possible technology-skill mismatch and unemployment.

*Keywords:* endogenous growth, knowledge formation, unemployment, skill mismatch
*JEL-classification:* E24, J63, O33

# 1 Introduction

The general belief in economics is that knowledge formation has positive effects on both growth and unemployment. It is argued that the expansion of knowledge stimulates growth and manifests in a higher individual productivity and therefore results in higher wages or a lower unemployment risk.[1] In this paper we address the question of whether knowledge still keeps its positive role if we jointly consider growth and unemployment. Possible limitations come from the consideration of a skill-technological complementarity and the unequal expansion of collective and individual knowledge in the form of technologies and skills.

Several ideas have been offered to account for the impact of growth on employment and the other way around. The very early contributions by Harrod (1939) and Domar (1946) analyzed the link under the assumption of a bounded factor substitution. The more recent literature (see Aricó, 2003, for an overview) focuses on imperfections and frictions in the labor market such as search costs (Pissarides, 1990, and Aghion and Howitt, 1994), efficiency wages (van Schaik and de Groot, 1998, and Meckl, 2001), and unions (Lingens, 2003). Another part of the literature explores employment effects which arise from growth via technological change. King and Welling (1995) and in particular Acemoglu (1997 and 1999) analyze technology choices, skill supply and unemployment in search equilibrium. Şener (2000) considers the case of innovation-based growth and skill-biased technological change, in which unemployment of low-skilled workers results from the emergence of innovative technologies. The idea of this paper is to combine skill formation with both elements of technological change: technological development by research and technology implementation in manufacturing. Technology development and technology implementation are interrelated and a shortage of skilled labor restricts both. This implies that technological change and skill formation compose a twin-engine of growth as it is formulated by Lloyd-Ellis and Roberts (2002).[2] In addition to their results, we show that skill shortages may lead not only to growth constraints but also to involuntary unemployment.

The paper revisits the idea that technological change is accompanied by creative

[1] See, for example, Grilliches (1997) for an overview, Benhabib and Spiegel (1994), and Barro and Lee (1993) for the knowledge-growth relationship, and OECD (1992 chap.2) for the link between knowledge formation and employment.

[2] In further related literature, Chari and Hopenhayn (1998) model technology-specific human capital and analyze changes in technology diffusion with exogenous technological progress. Young (1993) investigates how the speed of technology implementation depends on the degree of learning-by-doing in the economy. Stokey (1991) assumes heterogeneous labor with a skill-technology complementarity in order to study the link between education decisions and growth.

destruction, which causes a continuous reallocation of labor between firms that close and those that newly enter the market (Aghion and Howitt, 1994). We add to this that workers are heterogenous in respect of skills and that skill shortages are a friction in the process of reallocating labor. The use of innovations, that is new technologies, requires workers endowed with new skills. If technologies get updated faster than skills upgrade, this leads to unemployment as firms demand more skilled labor than it is supplied but the opposite is true for non-skilled labor. This implies that innovation-based growth is a misleading strategy to fight unemployment as joblessness results from skill shortages caused by too many innovations.

As we consider a negative feedback of unemployment on growth, we find mixed results of how changes in knowledge formation affect growth. Knowledge formation can be divided into the technology and the skill channel. Which one leads to more growth depends on whether skill shortages result in a consequent decline in employment or not. While the model confirms the standard results of positive effects on growth from improvements in knowledge formation if workers are fully employed, the outcome is much more ambiguous if we assume skill shortages. For example, we find that growth may drop as soon as resources are reallocated towards R&D, which contrasts with the standard innovation models. In this case, a shift from the development towards the implementation of technologies, for example through subsidies to the technology using sector, provides superior results in terms of growth and employment.

This paper is organized as follows: Section 2 introduces the model with a description of the technology formation and the skill formation which leads to equilibrium labor allocation between the two sectors research and development (R&D) and manufacturing. Furthermore, it is analyzed how labor reallocation with job creation and job destruction can result in equilibrium unemployment. Section 3 discusses the consequences of knowledge formation under the assumption of a technology-skill complementarity, considering changes in employment and growth. Finally, Section 4 concludes the paper.

# 2 The model

The model considers the joint formation of technologies and skills and analyzes how this affects the matching of labor demand and supply under the assumption of a technology-skill complementarity. From this we will derive the consequences for growth and employment. We construct a two-sector model with R&D and manufacturing, in which growth is driven by innovations and may be restricted by a shortage of skilled labor. Unemployment occurs as a symptom of this growth restriction.

We consider an economy populated by a mass $L$ of infinite living individuals. They are endowed with one unit of labor which they supply inelastically. Accordingly, $L$ is the total labor supply. All individuals share the same linear intertemporal preferences. The expected utility $\Psi$ of individual $i$ is generated by consuming the amount $Y_i$ of the final good during an infinite time horizon, with $t$ denoting real time units:

$$\Psi(Y_i) = \int_0^\infty Y_{i,t} e^{-rt} dt, \tag{1}$$

We discount future consumption at rate $r$ which is the individual time preference equal to the constant interest rate. Workers are either employed or unemployed and they are paid only in the first case. Unemployment is stochastic but non-permanent. In an economy with a frictionless credit market, workers can insure themselves against unemployment in order to smooth out their consumption path. Furthermore, labor income in the R&D sector is stochastic as well. This implies that equation (1) measures expected instead of actual utility.[3]

The subsequent analysis is twofold. First, we develop the equilibrium labor allocation between R&D and manufacturing. To do so, we set up the growth framework which follows the literature on growth via endogenous technological progress based on Aghion and Howitt (1992). Then, we introduce skill formation and the reallocation of labor with job creation and job destruction. From this it follows how skill shortages change equilibrium labor allocation and on which conditions unemployment occurs.

## 2.1 Technology Formation

We start the modeling of endogenous innovation under the assumption of homogenous labor and full employment in the sectors R&D and manufacturing. In contrast to this, the subsequent analysis in section 2.2 shows that unemployment can occur if labor is heterogenous with regard to skills. The remainder of this section derives

[3] The utility function which is linear in consumption implies that individuals are risk neutral concerning investment decision in R&D. We follow here the Aghion and Howitt (1992) framework, which is restrictive in this point because the choice of (1) includes that optimal household behavior is neglected to a large extent. Wälde (1999) shows that many of the market failures disappear, which apply to the basic model, if risk averse households diversify their investment portfolio. However, the outcome of unemployment, the focus of this paper, is not subject to the formulation of the utility function.

the returns from the production of the final good in manufacturing and the intermediate good in R&D respectively. We then establish the equilibrium intersectoral labor allocation which excludes arbitrage in the expected income between R&D and manufacturing. The equilibrium labor allocation finally defines the relative size of the two sectors which, in turn, determines current output and the rate of growth.

### 2.1.1 Technological Progress

Technological progress is the only source of equilibrium growth. R&D forms new technologies and shifts the frontier technology $\tau_t^{\max}$. When a research unit develops an innovation this adds a new technology $\tau$ to the current number of available technologies in the interval $[1, \tau_t^{\max}]$. Hence, the span of the interval increases over time. Technological progress evolves productivity gains and the embodied productivity level $A_\tau$ increases with any subsequent technology by a factor $\lambda$:

$$A_\tau = \lambda A_{\tau-1}. \tag{2}$$

The number of R&D units which undertake research is $L_R$. Each unit has a Poisson-distributed arrival rate $\varepsilon$ of being the next innovator. This implies that the productivity parameter is expected to increase by a factor $\lambda^{\varepsilon L_R}$ during the time unit $t$. Accordingly, technological progress $g_{A,t}$ shifts the productivity of the frontier technology over time according to:

$$g_{A,t} = \varepsilon L_{R,t} \ln(\lambda). \tag{3}$$

Technological progress is endogenous as the size of the R&D sector, $L_R$, will result from the equilibrium labor allocation between R&D and manufacturing. The equilibrium is defined as no-arbitrage between the expected income earned by the supply of the final good and the intermediate good respectively. The alternative incomes are defined in the following two sections.

### 2.1.2 The Final Good Production

Firms in the manufacturing sector demand technologies and use them to produce the homogenous final good in a set of different vintages. The technology $\tau$ defines the vintage and, hence, $\tau$ denotes both the technology and the related vintage. As technologies are different, relative productivity $a_\tau = A_\tau / A_{\tau_t^{\max}}$ varies among the vintages. Relative productivity is equal to unity in $\tau_t^{\max}$ and lower in all other vintages.

Once the R&D sector supplies a new technology, a new vintage $\tau_t^{\max}$ in manufacturing is created. The new vintage chooses the current maximum technology but no updating is possible afterwards. The fixed vintage technology implies a relative productivity loss, namely $a_\tau$ declines, as soon as a new technology with a higher productivity is implemented in manufacturing. Suppose furthermore that there exists a maximum distance to the current maximum productivity $A_{\tau_t^{\max}}$ which characterizes the minimum relative productivity $a_{\tau_t^{\min}}$.[4] Consequently, the gradual increase in $A_{\tau_t^{\max}}$ according to the move in $\tau_t^{\max}$ leads to final technological obsolescence of vintages below $\tau_t^{\min}$. Hence, although there is a wider range of available technologies, only those with at least minimum productivity are used in manufacturing. While the interval of vintages is fixed to $\left[\tau_t^{\min}, \tau_t^{\max}\right]$, each vintage starts as $\tau^{\max}$ at $t$ and ends as $\tau^{\min}$ some periods after $t$. Accordingly, the manufacturing sector faces a continuous structural change with the emergence and disappearance of vintages.

Two steps are necessary to manufacture the final good $Y$. First, labor transforms an intermediate good into a useful input for production.[5] The transformation of the intermediate good follows a simple linear technology in which one employee transforms a fraction $1/\gamma$ of one unit of the intermediate good $x_\tau$. This connects the two inputs in manufacturing simply as follows:

$$L_{M,\tau} = \gamma x_\tau. \tag{4}$$

In the second step, the final good is created from the intermediate goods at a decreasing rate of return $\alpha$ and at a productivity level $A_\tau$. Furthermore, we assume that some of the output is needed to cover overhead costs[6] $\delta_\tau$, which are constant in technology terms with $\delta_\tau = \delta A_\tau$. The vintage production function therefore yields:

$$Y_\tau = \begin{cases} A_\tau x_\tau^\alpha - \delta_\tau & \text{if } A_\tau \geq a_{\tau_t^{\min}} A_{\tau^{\max}} \\ 0 & \text{otherwise} \end{cases} . \tag{5}$$

Only vintages which fulfill $A_\tau \geq a_{\tau_t^{\min}} A_{\tau^{\max}}$ contribute to the production of total output $Y_t = \sum_{\tau_t^{\min}}^{\tau_t^{\max}} Y_\tau$ as they have a productivity level which represents a technology above $\tau_t^{\min}$.

4 We come back to this technology limitation in section 2.2 when we specify the reallocation of labor, which becomes necessary due to the technological obsolescence of vintages and the disappearance of the firms which have a technology below $\tau^{\min}$.

5 Such as software is only beneficial with the corresponding user.

6 With overhead costs it is not possible to reduce inputs to an infinitesimal number to maintain production. Instead, a market exit occurs as soon as revenues are lower than overhead costs.

The demand for the intermediate good and labor are the result of profit maximization in manufacturing. As long as a vintage exists it earns a flow of profits $\pi_{M,\tau}$. The costs of one unit of the intermediate good are composed of the corresponding price $p_\tau$ paid to the intermediate good supplier and, in addition to this, the complementary labor costs as a factor $\gamma$ of the wage rate $w_\tau$. Revenues, which follow from the production function, face these production costs, and the consequent profit equation is as follows:

$$\pi_{M,\tau} = A_\tau x_\tau^\alpha - \delta_\tau - (p_\tau + \gamma w_\tau) x_\tau. \tag{6}$$

The maximization of (6) over $x_\tau$ yields as a first-order condition the demand for the intermediate good,

$$x_\tau = \left( \alpha a_\tau \frac{A_{\tau_t^{\max}}}{p_\tau + \gamma w_\tau} \right)^{\frac{1}{1-\alpha}}, \tag{7}$$

from which follows the demand for labor

$$L_{M,\tau} = \gamma \left( \alpha a_\tau \frac{A_{\tau_t^{\max}}}{p_\tau + \gamma w_\tau} \right)^{\frac{1}{1-\alpha}}. \tag{8}$$

How much of the intermediate good and of labor is demanded, varies with relative productivity. Highly productive technologies imply c.p. a high demand for the inputs.

Wages are set subject to average productivity in manufacturing. In contrast to the technology differences, labor is homogenous and we therefore assume that all manufacturing workers obtain the same wage $w_\tau$. The wage rate is fixed in such a way that the wage equals a share $\beta$ of the revenues of vintage $\widetilde{\tau}$ which has the average relative productivity $\widetilde{a}$. This yields:

$$w_{\tau,t} = \beta \widetilde{a} A_{\tau_t^{\max}} \left( \frac{L_{M,\widetilde{\tau}}}{\gamma} \right)^\alpha. \tag{9}$$

Average relative productivity and labor input are constant. Therefore, wages increase over time with $A_{\tau_t^{\max}}$ at the rate $g_A$. Higher employment leads to higher revenues and increases the wage rate. Taking account of the decrease of labor demand with the wage rate, we can conclude that an equilibrium exists with the corresponding equilibrium wage rate and employment. It is necessary to generate profits, at least with the minimum employment of one worker, which restricts the share of revenues which might go to the workers to $\beta < 1 - \left[ \left( p_\tau + \delta_\tau \right) / \widetilde{a} A_{\tau_t^{\max}} \right]$. Higher wages due to a higher $\beta$ would not leave enough revenues to cover the costs for the intermediate good and the overhead costs.

While manufacturing workers earn a wage rate $w_\tau$, the manufacturing firms have zero profits over lifetime. Market entry with the emergence of a new vintage is associated with the implementation of the current leading technology. Suppose that this is accompanied by the payment of fixed costs of $F_\tau$. The manufacturing sector is competitive so that the flow of profits over life time only covers implementation costs. Define $\bar{t}$ as the moment of market entry and let $T$ denote the point in time when the firm closes. Then the zero-profit condition implies that $\int_{t=\bar{t}}^{t=T} e^{-r(t-\bar{t})} \pi_{M,\tau,t} dt = F_\tau$.

### 2.1.3 R&D and the Intermediate Good Production

R&D units produce the intermediate good and develop innovations as superior technologies, which provide the intermediate good with a higher productivity level $A_\tau$. Various R&D units compete in the development of the next innovation. As soon as an innovator appears with a new maximum technology $\tau_t^{\max}$, the corresponding R&D unit sells the innovation in form of intermediate goods to the manufacturing sector. The flow of profits, earned from selling the intermediate good, determines the value of an innovation. This value then yields the return to labor of an R&D unit, which is the alternative income to the wage rate in manufacturing.

The R&D units produce the intermediate good at $c_\tau = cA_\tau$ constant marginal costs, which are proportional to the technology level. As soon as an innovation arises the previous technology becomes common knowledge and different firms compete in the supply of the intermediate good so that they set price equal to marginal costs $c_\tau$. Hence, no profits arise for these intermediate good suppliers.

However, only the innovator has the knowledge about the leading technology $\tau_t^{\max}$. Hence, there is no competition in the supply of technology $\tau_t^{\max}$ and its supplier earns monopolistic profits. The innovator replaces the previous monopoly and then sets the profit-maximizing price and output. Profits of the R&D unit therefore arise from serving the vintage with the highest technology level $\tau_t^{\max}$:

$$\pi_{R,t} = \left[ p_{\tau_t^{\max}}(x) - c_{\tau_t^{\max}} \right] x_{\tau_t^{\max}} \tag{10}$$

The monopoly chooses the profit-maximizing quantity of output and sets the corresponding $p_{\tau_t^{\max}}$. The R&D unit faces the inverse demand function of the manufacturing vintage $\tau_t^{\max}$, which results from the demand function (7):

$$p_{\tau_t^{\max}} = \alpha A_{\tau_t^{\max}} x_{\tau_t^{\max}}^{\alpha-1} - \gamma w_\tau. \tag{11}$$

With this expression for $p_{\tau_t^{\max}}$, the first-order condition of the maximization program $\max \pi_{R,\tau_t^{\max}}$ produces the profit-maximizing quantity of the intermediate good

$$x_{\tau_t^{\max}} = \left( \frac{\alpha^2}{\gamma \omega_{\tau_t^{\max}} + c} \right)^{\frac{1}{1-\alpha}}, \tag{12}$$

where $\omega_\tau = w_\tau / A_\tau$ denotes the productivity-adjusted wage. The corresponding price of the intermediate good can then be written as:

$$p_{\tau_t^{\max}} = \frac{c_{\tau_t^{\max}}}{\alpha} + \left( \frac{1}{\alpha} - 1 \right) \gamma w_\tau. \tag{13}$$

The monopolist takes a mark-up to marginal costs which is twofold. First, $1/\alpha$ represents the usual mark-up according to the price elasticity of demand. The second term represents the fact that labor and the intermediate good are used in a fixed ratio. High wages reduce the demand for the intermediate good. However, according to the inverse demand function, a small quantity of output corresponds to a high price.

Price and output determine how much the R&D unit earns as long as it can realize the monopolistic profits. However, the value of an innovation is less than the infinite flow of profits. Competitors undertake R&D and will therefore replace the incumbent at some stage. This means that the flow of profits immediately stops as soon as the next innovation has been developed. This emerges stochastically at probability $\varepsilon$ per labor unit in a number of $L_R$ R&D units which employ one worker each. Thus, the expected present value $V_\tau$ of an innovation takes account of the flow of profits and the probability of a total loss of the asset value. This leads to the following asset equation

$$rV_\tau = \pi_{R,\tau} - \varepsilon L_R V_\tau, \tag{14}$$

which implies that the investment in R&D must bring the same expected returns as the investment in an alternative asset whose return is the constant interest rate $r$. Rearranging the asset equation, together with the profit equation (10) and the values

$p_{\tau_t^{\max}}$ and $x_{\tau_t^{\max}}$, then yields the expression for the expected value of innovation $\tau$:

$$V_{\tau,t} = A_{\tau_t^{\max}} \left( \frac{1}{\alpha} - 1 \right) (\gamma\omega + c) \frac{x_{\tau_t^{\max}}}{r + \varepsilon L_{R,\tau}}. \tag{15}$$

Competition in form of a high $\varepsilon L_{R,\tau}$ lowers the value of an innovation as the expected time span is short, in which monopolistic profits arise. Moreover, the value of an innovation is a function of time as it depends on the technology level $A_{\tau_t^{\max}}$ which increases over time. From this it follows that $V_\tau$ increases proportional to $A_{\tau_t^{\max}}$ at the rate of technological progress.

### 2.1.4 Equilibrium Intersectoral Labor Allocation

Labor can be employed either in manufacturing or in R&D and workers are free to move from one to the other. Moves stop in equilibrium when both alternatives offer the same expected income. This income identity then yields the equilibrium intersectoral labor allocation.

While manufacturing workers earn the wage $w_\tau$, research workers receive no income unless their firm innovates. Innovation is stochastic and research workers get an expected income of $\varepsilon V_{\tau+1}$ as a worker develops the next innovation $\tau + 1$ at the probability $\varepsilon$. As long as the income identity

$$w_\tau = \varepsilon V_{\tau+1} \tag{16}$$

holds, no sector attracts workers with the prospect of a higher income.

The income identity can be written also as an expression of the parameters of the model and the relative labor shares of the two sectors. This finally determines the equilibrium intersectoral labor allocation. We substitute $V_{\tau+1}$ as it arises in the income identity (16) by the value of the innovation according to (15), but with the future technology $\tau + 1$ instead of the current one $\tau$. Solving for $L_R$ then gives us the size of the R&D sector $L_R = x_{\tau_t^{\max}} \lambda \left[(1/\alpha) - 1\right] \left[\gamma + \left(c_{\tau_t^{\max}}/w_\tau\right)\right] - (r/\varepsilon)$. The input of the intermediate good can be replaced by labor, as $x_{\tau_t^{\max}} = L_{M,\tau_t^{\max}}/\gamma$. One can finally show that the ratio of labor input in vintage $\tau_t^{\max}$ to the remaining employment in manufacturing is a constant $1/\eta$ (see appendix), which yields $L_{M,\tau_t^{\max}} = L_M/\left[\eta + 1\right]$. Therefore, $x_{\tau_t^{\max}}$ is equal to $L_M/\left[\gamma(1+\eta)\right]$ and we can then write the income identity as:

$$\textbf{\textit{AE}:} \qquad L_R = L_M \frac{\lambda}{\gamma (1+\eta)} \left( \frac{1}{\alpha} - 1 \right) \left( \gamma + \frac{c}{\omega} \right) - \frac{r}{\varepsilon}, \tag{17}$$

$AE$ is the no-arbitrage equation, in which the equilibrium employment shares of R&D and manufacturing even out the income alternatives. Deviations from $AE$ lead to adjustments in the income levels. To see this, suppose that $L_R$ is higher and $L_M$ is lower than no-arbitrage according to $AE$ implies. In this case the competition in R&D is particularly high and reduces the value of an innovation. Furthermore, the demand for intermediate goods is low because the complementary labor input in manufacturing is low. The consequently reduced profits from selling the intermediate good additionally reduce the value of an innovation. Accordingly, workers would move towards the manufacturing sector as it offers a higher income than R&D. This reduces $L_R$ and increases $L_M$. The process continues until the income differences disappear and the employment ratio corresponds to the one which follows from $AE$. If the adjustments takes place immediately, as it is assumed in this type of models[7], the economy jumps to its equilibrium and remains there afterwards.

The actual size of $L_R$ and $L_M$ follows from the magnitude of the total labor supply $L$. The labor market identity $L = L_M + L_R$ implies that labor is fully employed in manufacturing and R&D. This in $AE$ yields the equilibrium number of employees in manufacturing:

$$L_M^* = \frac{L + \frac{r}{\varepsilon}}{1 + \frac{\lambda}{\gamma(1+\eta)}\left(\frac{1}{\alpha} - 1\right)\left(\gamma + \frac{c_\tau}{w_\tau}\right)} \tag{18}$$

The equilibrium number of researcher follows straightforward from subtracting (18) from $L$. Parameters which tend to increase the expected profits from innovation, such as the size of innovations $\lambda$ and the innovation probability $\varepsilon$, lead to a higher share of labor in R&D and less employment in manufacturing. Since R&D is the engine of growth, economies with a comparable high share of labor in R&D grow faster.

## 2.2 Labor Reallocation and the Skill Shortage

Technology formation, as it is modeled in the previous section, causes a turnover of vintages and a reallocation of labor which results from continuous job creation and job destruction. As soon as an innovator creates a new leading technology this introduces a new vintage in manufacturing which generates new labor demand. In contrast to this, all other vintages experience a decline in their relative productivity

7 We refer here to the standard growth models with endogenous innovation in the tradition of Grossman and Helpman (1991) and Aghion and Howitt (1992).

and, hence, they reduce their labor demand. This process has not been analyzed explicitly so far as we assumed that the reallocation of labor from old vintages to the new one is frictionless. We change this assumption in the following analysis and assume instead that only skilled labor is able to match[8] labor demand in the technologically leading vintage. These skills are formed in a process of learning-by-using in the manufacturing sector. In case of skill shortages, the reallocation of labor causes unemployment as this scenario leads to a job destruction which exceeds job creation.

### 2.2.1 Technology Obsolescence and Job Destruction

Technological progress creates new vintages in manufacturing and results in the disappearance of old ones. Between their emergence and their disappearance, vintages face a gradual technological obsolescence until they finally fall below a minimum technological level. Technological obsolescence leads also to job destruction in the corresponding vintages as older vintages are less productive and labor demand declines with a fall in relative productivity. Furthermore, the labor demand gets zero with the final technological obsolescence.

A minimum level of relative productivity characterizes the final technological obsolescence. Remember that each vintage represents a particular productivity level $A_\tau$. The range of vintages is between $\tau_t^{\min}$ and $\tau_t^{\max}$, which have different productivity levels running from minimum relative productivity $a_{\tau_t^{\min}} = A_{\tau_t^{\min}}/A_{\tau_t^{\max}}$ up to the maximum $a_{\tau_t^{\max}} = 1$. The minimum level $A_{\tau_t^{\min}}$ is defined as the one that corresponds to revenues which yield zero profits, i.e. $\pi_{M,\tau} = 0$. In other words, vintages with a level above $A_{\tau_t^{\min}}$ generate profits which are used to cover implementation costs, but no vintage produces with a level below $A_{\tau_t^{\min}}$ as it would imply selling the final good at a loss. The profits of manufacturing are revenues $A_\tau x_{\tau,t}^\alpha$ minus overhead costs $\delta_\tau$ and variable costs $(c_\tau + \gamma w_\tau)\, x_{\tau,t}$. Variable costs consist of the price of the intermediate good, equal to its marginal costs of production ($p_\tau = c_\tau$), and complementary wage costs. We then find that zero profits correspond to a minimum

[8] With respect to the link between innovation and the matching of jobs and workers, the model is related to the analysis by Aghion and Howitt (1994). However, the introduction of the technology-skill complementarity changes their result that the level of unemployment is unaffected by the frequency of innovations.

productivity level[9] of

$$A_{\tau_t^{\min}} = \left(\frac{\delta_\tau}{1-\alpha}\right)^{1-\alpha} \left(\frac{c_\tau + \gamma w_\tau^{\max}}{\alpha}\right)^{\alpha}. \tag{19}$$

Revenues are constant but we defined wages in such a way that they increase at the rate of technological progress over time. This means that revenues exceed costs in the beginning but profits decline with the subsequent rise in labor costs. Therefore, there is a maximum wage $w_\tau^{\max}$ a vintage with technology $A_\tau$ is able to pay and which corresponds to $\pi_{M,\tau} = 0$:

$$w_\tau^{\max} = (A_\tau)^{\frac{1}{\alpha}} \frac{\alpha}{\gamma} \left(\frac{\delta}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \frac{c_\tau}{\gamma}. \tag{20}$$

Production is given up as soon as productivity is below the minimum level which can be interpreted as the final technological obsolescence. Before this happens, each emergence of an innovation increases the wage rate and lowers profits so that the technology gradually approaches its final obsolescence. Let $\Gamma$ denote the number of innovations after which the vintage disappears. Suppose that vintage $\tau$ started with a technology adjusted wage rate $\bar{\omega} = w_{\tau,t}/A_\tau$, then the maximum wage is given by $w_\tau^{\max}/A_\tau = \bar{\omega}e^{\lambda\Gamma}$. (Remember that each new technology raises the maximum productivity, and consequently wages, by a factor $\lambda$.) This yields that the span of innovations a vintage can survive is:

$$\Gamma = \frac{\ln\left(\frac{w_\tau^{\max}}{A_\tau}\right) - \ln\bar{\omega}}{\lambda}. \tag{21}$$

The rate of technological obsolescence is high if the technology span $\Gamma$ is small. In this case only few innovations are enough until relative productivity reaches its minimum level.

The degree of vintage obsolescence is one measure of job destruction. As soon as a new $\tau_t^{\max}$ arises, the vintage with the lowest productivity $\tau_t^{\min}$ disappears and cuts all prior labor demand. Recall from the description of the manufacturing sector that labor demand according to (8) is $L_{M,\tau} = \gamma\left[\alpha a_\tau A_{\tau_t^{\max}}/\left(c_{\tau,t} + \gamma w_{\tau,t}\right)\right]^{1/(1-\alpha)}$.

[9] $A_{\tau_t^{\min}}$ would be zero without overhead costs. However, the elimination of the threshold technology implies an infinite lifetime of vintages and excludes the final technological obsolescence.

The wage of $\tau_t^{\min}$ is equal to $w_\tau^{\max}$. This implies that vintage $\tau_t^{\min}$ employs

$$L_{M,\tau_t^{\min}} = \gamma \left( \frac{\alpha}{c_{\tau_t^{\min}} + \gamma \bar{\omega} e^{\lambda \Gamma}} \right)^{\frac{1}{1-\alpha}} \tag{22}$$

workers. The subsequent technological obsolescence stops production in that vintage and causes job destruction whose extent is equal to $L_{M,\tau_t^{\min}}$. One can see that a high rate of technological obsolescence in terms of a small $\Gamma$ results in job destruction of large extent. Vintages disappear with a comparable high productivity, and which, therefore, still had a considerable employment share.

It is not only the disappearance of $\tau_t^{\min}$ which causes job destruction but other vintages reduce labor input at the same time. Innovations increase $A_{\tau_t^{\max}}$ but this means that relative productivity declines in all other vintages. We can see from the equation of $L_{M,\tau}$ that labor demand is a function of relative productivity. Only a share of the workers smaller than one remains employed in vintage $\tau$ as soon as the next innovation reduces relative productivity from $a_\tau$ to $a_\tau/\lambda$. Let $\phi$ denote the ratio of employment before to employment after the innovation. Then divide the expression for reduced labor demand $L_{M,\tau}(\alpha_\tau/\lambda)$ by $L_{M,\tau}(a_\tau)$ to obtain the share of workers that remains employed:

$$\phi = \left( \frac{c + w_{\tau,t}}{c + \lambda w_{\tau,t}} \right)^{\frac{1}{1-\alpha}}. \tag{23}$$

Consequently, as soon as the next innovation emerges, a share $1 - \phi$ of the current number of manufacturing workers lose their jobs due to a gradual technological obsolescence.

The total extent of job destruction is the result of the gradual and the final technological obsolescence. Final technological obsolescence destroys $L_{M,\tau_t^{\min}}$ jobs. Furthermore, a share $1 - \phi$ of the current jobs in manufacturing disappears. The number of current employments in manufacturing yields from total labor supply minus unemployed minus workers in the R&D sector. With $u$ as the unemployment rate this is $L_M = (1 - u_t) L - L_R$. Finally, innovations emerge at the probability $\varepsilon L_R$. Therefore, technology obsolescence causes the expected, or average, flow into unemployment of:

$$U^+ = \varepsilon L_R \left\{ (1 - \phi) \left[ (1 - u_t) L - L_R \right] + L_{M,\tau_t^{\min}} \right\} \tag{24}$$

### 2.2.2 Job Creation and Skill formation

The emergence of a new vintage in the manufacturing sector creates new jobs. However, the new jobs relate to the innovative technology and we therefore assume that the employees must be skilled to match the new jobs.[10] This assumption causes frictions in the process of reallocating labor from obsolete vintages to the new one.

The new vintage $\tau_t^{\max}$ arises in manufacturing as soon as the corresponding innovative technology is developed. This vintage aims to employ workers in order to start production. Therefore it opens vacancies and, thereby, generates new labor demand. How many vacancies are created follows from the input demand and monopolistic profit-maximization specified in the description of the intermediate good production. The supply of the intermediate good which embodies the technology update has been shown in (12). Labor demand is a fraction $\gamma$ of this number. Therefore, the amount of vacancies yields:

$$L_{M,\tau_t^{\max}} = \gamma \left[ \frac{\alpha^2}{\gamma\omega + c} \right]^{\frac{1}{1-\alpha}}. \tag{25}$$

This number is equal to job creation if the all vacancies can be filled. However, some vacancies remain unfilled if labor demand does not face an equal supply of skilled labor. So skill shortages can be a restriction to job creation.

We define skills as the ability to use the highest technological standard in manufacturing. Skills are formed in a stochastic process of learning-by-using in manufacturing firms. The probability that workers get skilled is higher in the technologically leading vintage than in the other ones. Those workers who are employed in $\tau_t^{\max}$ have a probability of $\mu < 1$ to acquire skills and therefore to be enabled to fill a vacancy in the vintage which follows next. Suppose that other workers have also the chance to get skilled but at lower probability $\rho\mu$ with ($\rho < 1$) as they do not use the current leading technology. The share of manufacturing workers employed in $\tau_t^{\max}$ is $1/(1+\eta)$. This yields that $\mu L_M/(1+\eta)$ workers can expect to obtain skill updates when they work in $\tau_t^{\max}$ and $\rho\mu L_M/\left[1 - 1/(1+\eta)\right]$ in the remaining vintages. The total number of skilled workers therefore is:

$$D = \mu L_M \left( \rho + \frac{1-\rho}{1+\eta} \right). \tag{26}$$

Skill formation obviously increases with the efficiency of learning-by-using, namely

[10] Hollanders and ter Weel (2002) provide empirical evidence for the necessary skill upgrade with the introduction of new technologies..

$\mu$ and $\rho$. Moreover, the labor share of the leading vintage, $1/(1+\eta)$, has also a positive impact on skill formation as the efficiency of learning about technologies is highest in high-technology firms.

Job creation is unrestricted if $L_{M,\tau_t^{\max}} > D$ and limited through the skill shortages otherwise. Hence, job creation is the minimum of either the number of vacancies or the supply of skilled labor. A constant share of workers get skilled in manufacturing. However, in absolute numbers the supply of skilled labor increases with the scale of the manufacturing sector. Hence, the labor allocation between the sectors has an effect on whether skill shortages restrict job creation or not. The next section combines job creation with job destruction and shows how a mismatch between the two results in unemployment.

### 2.2.3 Equilibrium Labor Allocation with Unemployment

The reallocation of labor with job creation and job destruction characterizes the labor market. The labor flows generated by the reallocation are into and out of employment and must be equal in equilibrium. As the size of R&D and manufacturing in terms of sectoral employment $L_R$ and $L_M$ affect the extent of job destruction and job creation, see (24) and (26), we must find an equilibrium intersectoral labor allocation which evens out differences in the job creation and job destruction. Possibly, the balance between the two flows is accompanied by unemployment.

The continuous reallocation of labor arises between the vintages and between unemployed and the non-leading vintages. Workers dismissed from employment in manufacturing are excluded from entering the R&D sector. Furthermore, as unemployed cannot become skilled, they lack the abilities to match jobs in the newly created vintages. However, skilled workers from other vintages will fill the vacancies. The jobs left by the upgraded workers can then be filled with non-skilled unemployed. Hence, the minimum of either vacancies or the supply of skilled workers yields the job creation for the non-skilled even if new jobs are created only for skilled workers.

The different possibilities to relate job creation to job destruction result in three regimes, two of which produce unemployment. Recall for the following analysis job destruction $U^+$ according (24), job creation in form of fully filled vacancies $L_{M,\tau_t^{\max}}$ according to (25), and job creation with skill shortages $D$ according to (26). Each of them may be the limitation to the reallocation of labor. Depending on whether the value of $U^+$, $L_{M,\tau_t^{\max}}$ or $D$ is the smallest, we obtain a regime with perfect labor reallocation, restricted labor demand, or restricted skill formation.

*(I) Perfect labor reallocation:* $U^+|_{u=0} \leq L_{M,\tau_t^{\max}}, D$

In this regime job destruction is in any case lower than job creation in terms of its two indicators $L_{M,\tau_t^{\max}}$ and $D$. Job destruction is at its maximum if all workers in the labor force are in jobs, denoted with $U^+|_{u=0}$. This is because dismissals due to the gradual technological obsolescence affect a fixed proportion of the employment in manufacturing which is maximum in case of full employment. From reversal conclusion follows that full employment is guaranteed only if job creation can compensate for the maximum extent of job destruction, $U^+|_{u=0}$. We do not consider other frictions in the job-worker matching, such as time consuming search of job seekers. Accordingly, if $U^+|_{u=0}$ is small enough, the number of destroyed jobs faces an equal number of new vacancies and dismissed workers immediately re-enter new jobs. No unemployment occurs and job creation is restricted by the labor supply generated through job destruction. The ratio of $L_{M,\tau_t^{\max}}$ to $D$ has no effect on employment but on the labor allocation between the sectors. Skill formation does not cause any limits to innovation if the labor supply of skilled workers exceeds the number of vacancies, namely $D > L_{M,\tau_t^{\max}}$. In this case, the equilibrium corresponds to the one described in section 2.1.4.[11]

*(II) Restricted labor demand:* $L_{M,\tau_t^{\max}} < U^+|_{u=0}, D$

In this regime job creation in the form of new labor demand, corresponding to the vacancies offered by the new vintage in manufacturing, is lower than the supply of skilled labor and lower than maximum job destruction[12] of $U^+|_{u=0}$. Hence, this scenario is incompatible with full employment as in this case job destruction would exceed job creation. Due to a relative lack of vacancies, job creation can be equal to job destruction only if it is lower than maximum. As job destruction is proportional to employment, set $U^+ = L_{M,\tau_t^{\max}}$ to see that equilibrium flows of job destruction and job creation correspond to the occurrence of unemployment

$$u_t L = L - L_R - \frac{1}{1-\phi}\left[\frac{L_M}{(1+\eta)\,\varepsilon L_R} - \gamma x_{\tau_t^{\min}}\right]. \tag{27}$$

[11] However, some vacancies in the leading vintage remain unfilled if $D < L_{M,\tau_t^{\max}}$, which restricts the implementation of technologies in manufacturing. Otherwise, skilled labor is partly employed in non-leading vintages if $D > L_{M,\tau_t^{\max}}$. There is no effect on employment, but growth is restricted in the first case because the leading technology is not fully exploited.

[12] In this and the next regime only the ratio of job destruction to the actual job creation, i.e. the minimum of either $D$ or $L_{M,\tau_t^{\max}}$, has a crucial impact on unemployment. Whether the alternative parameter of job creation is larger than $U^+|_{u=0}$, or the other way around, is insignificant in this respect.

High rates of labor reallocation in terms of $1-\phi$ and $\gamma x_{\tau_t^{\min}}$ and a high innovation rate in terms of $\varepsilon L_R$ indicate high job destruction. A small share of workers in the leading manufacturing vintage $L_M/[1+\eta]$ is a sign of little job creation. Both high job destruction and low job creation increase the extent of equilibrium unemployment.

*(III) Restricted skill formation:* $D < U^+|_{u=0}, L_{M,\tau_t^{\max}}$

Finally, in this regime skill shortages restrict job creation which consequently is lower than the number of vacancies and lower than maximum job destruction of $U^+|_{u=0}$. Again, unemployment evens out job creation and job destruction because the flow into unemployment is lower in absolute numbers if the employment base is reduced. We set $U^+ = D$ and solve for the number of unemployed, which yields:

$$u_t L = L - L_R - \frac{1}{1-\phi}\left[\frac{\mu L_M}{\varepsilon L_R}\left(\rho + \frac{1-\rho}{1+\eta}\right) - \gamma x_{\tau_t^{\min}}\right]. \tag{28}$$

As before, extensive labor reallocation and a high innovation rate are indicators for a high job destruction which tends to increase unemployment. However, now job creation is low, and unemployment high, in case of unproductive skill formation in terms of low probabilities of acquiring skills, $\mu$ and $\rho$, and a small proportion of workers in the leading vintage which imparts skills in the most efficient way.

Which of the three regimes occurs depends on the parameter values and the consequent ratios:

$$D \gtreqless L_{M,\tau_t^{\max}} \qquad \text{if} \quad \mu \gtreqless \frac{1}{\eta\rho+1}$$

$$D \gtreqless U^+|_{u=0} \qquad \text{if} \quad \mu \gtreqless \frac{\varepsilon L_R}{\eta\rho+1}\left(1-\phi+\frac{L_{M\tau_t^{\min}}}{L_M}\right)$$

$$L_{M,\tau_t^{\max}} \gtreqless U^+|_{u=0} \quad \text{if} \quad \frac{1}{1+\eta} \gtreqless \varepsilon L_R\left(1-\phi+\frac{L_{M,\tau_t^{\min}}}{L_M}\right)$$

It can be seen from the corresponding equations that, for example, a low innovation probability $\varepsilon$ implies low job destruction which may result in regime (I). Another example would be that a high ratio of wages to productivity, $\omega$, reduces new labor demand from the technologically leading vintage.[13] Consequently, with higher wages fewer vacancies are open and low job creation may result in regime (II). Regime (III) occurs, for example, if skill formation is of small scale because learning rates

[13] The effect of $\omega$ becomes apparent if you substitute the expression for vacancies $L_M/(1+\eta)$ with (25). The new labor demand is low if $\omega$ is high.

according to $\mu$ and $\rho$ are low.

All three regimes produce a balance between job creation and job destruction, but in the regimes (II) and (III) the equilibrium is accompanied by unemployment. The corresponding equations (27) and (28) show that unemployment is subject to the employment ratio between R&D and manufacturing, $L_M/L_R$. Remember that this ratio has already been derived as the first condition for the equilibrium intersectoral labor allocation where no-arbitrage between R&D and manufacturing resulted in the $AE$ equation. However, skill shortages have an effect on the income identity which underlies $AE$. Hence, to find the equilibrium allocation of labor with skill shortages and unemployment, it is necessary to perform the following two steps: We first derive the alternative no-arbitrage equation taking account of restricted job creation, and we then add to this the equations for the equilibrium reallocation of labor.

Skill shortages have effects not only on employment in manufacturing but on the innovation intensity as well. Research firms are forward-looking and form expectations on their future returns. These returns are lower in case of skill shortages. The link between the innovation intensity and the skill shortages is the size of manufacturing in terms of sectoral employment. Some vacancies remain unfilled and employment in the technologically leading vintage in manufacturing sector is restricted according to $D < L_{M,\tau_t^{\max}}$ because only skilled labor is employable. As a consequence of the fixed input ratio $\gamma$ between labor and the intermediate good, the R&D innovator sells less of the intermediate good to the manufacturing sector. With skill shortages, the actual market size for an innovator is $D/\gamma$ instead of $L_{M,\tau_t^{\max}}/\gamma$. This implies that the amount produced deviates negatively from the profit-maximizing output of the monopolistic R&D innovator. The condition of equal income opportunities in both sectors becomes unbalanced due to the reduction in expected profits from performing research. The repeated exercise from section 2.1, which is the derivation of the no-arbitrage equation with equality between the wage rate and the expected value of an innovation, yields the alternative $AE$ equation with restricted innovation intensity. Hence, the $AE$ equation is divided in two, depending on whether skill shortages reduce the returns from innovation or not:

$$\textbf{AE:}\quad \begin{cases} L_R = L_M \frac{\lambda}{\gamma(1+\eta)} \left(\frac{1}{\alpha} - 1\right)\left(\gamma + \frac{c}{\omega}\right) - \frac{r}{\varepsilon} & \text{if } L_{M,\tau_t^{\max}} < D \\ L_R = \frac{\alpha}{\omega}\left[L_M \frac{\mu}{\gamma}\left(\rho + \frac{1-\rho}{1+\eta}\right)\right]^{\alpha} - \left(\mu + \frac{c}{\omega\gamma}\right) L_M \left(\rho + \frac{1-\rho}{1+\eta}\right) - \frac{r}{\varepsilon} & \text{if } D < L_{M,\tau_t^{\max}}. \end{cases} \tag{29}$$

The first equation repeats $AE$ of section 2.1. The second equation identifies those employment shares of R&D and manufacturing which result in the income identity between the sectors in case of skill shortages. The first equation corresponds to regime (II) and the second equation to (III), whereas regime (I) can be assigned to both. Restrictions in the innovation intensity due to skill shortages lead to lower income from R&D. Therefore, the R&D sector attracts a smaller share of the working force if $D < L_{M,\tau_t^{\max}}$ relative to the case with unbounded demand for the intermediate good.

In addition to no-arbitrage, equilibrium allocation of labor demands even flows into and out of unemployment. To formulate this condition we consider the labor market identity with unemployment, $L = L_R + L_M + u_t L$, and use the right hand side of (27) and (28) instead of $u_t L$. This results in:

$$\boldsymbol{EE}: \quad \begin{cases} L_R = \dfrac{L_M}{\varepsilon(1+\eta)\left[\phi L_M + \gamma x_{\tau_t^{\min}}\right]} & \text{if } L_{M,\tau_t^{\max}} < D \\ L_R = \dfrac{L_M}{\varepsilon\left[\phi L_M + \gamma x_{\tau_t^{\min}}\right]}\mu\left(\rho + \frac{1-\rho}{1+\eta}\right) & \text{if } D < L_{M,\tau_t^{\max}}. \end{cases} \tag{30}$$

These are the employment equations $EE$ showing those employment shares of R&D and manufacturing which yield the identity between job creation and job destruction. The first equation represents regime (II) with vacancies as the restriction, while the second equation corresponds to regime (III) and the case of skill shortages. As there is no unemployment $EE$ can be omitted in regime (I). Job destruction exceeds job creation as soon as $L_R$ is above the value that $EE$ implies for a given $L_M$. Unemployment would increase as a consequence. The argument is the other way around, and unemployment decreases, if $L_R$ is below the corresponding equilibrium value.

In the two regimes with unemployment, the equilibrium size of the two sectors follows from no-arbitrage according to $AE$, as in the case with perfect reallocation of labor, but additionally taking account of $EE$. Figure 1 gives a graphical illustration of the equilibrium in the $(L_M, L_R)$-space. The properties of the equations are the same for the alternative formulations of $AE$ and $EE$. The locus of $AE$ starts right from the origin[14] and slopes upwards. This indicates in both alternative settings

[14] R&D does not occur until a minimum number of workers are employed in manufacturing. This property can be attributed to the necessity of a minimum demand for new products in order to set costly research. This familiar outcome of innovation models is discussed, for example, by Garcia-Castrillo and Sanso (2002).

that a manufacturing sector of large scale increases the returns from innovations in R&D because of a broad market for the intermediate good. High profits then attract more R&D units. Consequently, $L_R$ increases with $L_M$. The locus of $EE$ is strictly concave (see Appendix) and may correspond to either regime (II) or (III). If vacancies are the restriction as in regime (II), the upward slope indicates that job destruction, which corresponds to $L_R$, can be higher in equilibrium in case of high job creation which then is as a certain fraction of $L_M$. Skill shortages restrict the innovation intensity in regime (III). Hence, in this case the upward slope of $AE$ is the sign of the positive relativity of technology formation to skill formation, which take place in R&D and manufacturing respectively. The intersection of the loci $AE$ and $EE$ establish equilibrium labor allocation with unemployment, from which we obtain the size of $L_M$ and $L_R$. The intersection of the two curves may be right or left from the labor market line with full employment $L = L_R + L_M$. The intersection is right from the labor market line (point $A$) if there are no effects of skill supply on employment. However, this point is located outwards of the employment space and is therefore not attainable. Point $B$ will be realized instead. This represents the case without frictions in regime (I) equal to the result in section 2.1.4. However, labor supply is not fully engaged if $EE$ intersects with $AE$ left from the labor market line, such as in $C$. The distance between point $C$ and the labor market line yields the dimension of unemployment. Less labor is employed in both sectors in comparison to point $B$. This implies less output and a lower growth path because the innovation rate $g_A = \varepsilon L_R \ln(\lambda)$ reduces with a low $L_R$.

Some short considerations show the stability of the equilibrium. Firstly, the $AE$ locus is stable. The space above the line corresponds to a relative disadvantage in income from R&D. The consequent movement towards manufacturing causes a downward adjustment of $L_R$ and the employment combination approaches the $AE$ locus. Suppose furthermore that we start in a point above the $EE$ curve. In this case job destruction exceeds job creation and unemployment increases. This means an adjustment downward and to the left, for example along $AE$ from $B$ to $C$. From the two stable loci follows that any equilibrium labor allocation, such as point $A$, is also stable as long as the change of positions between the sectors can be easily made.

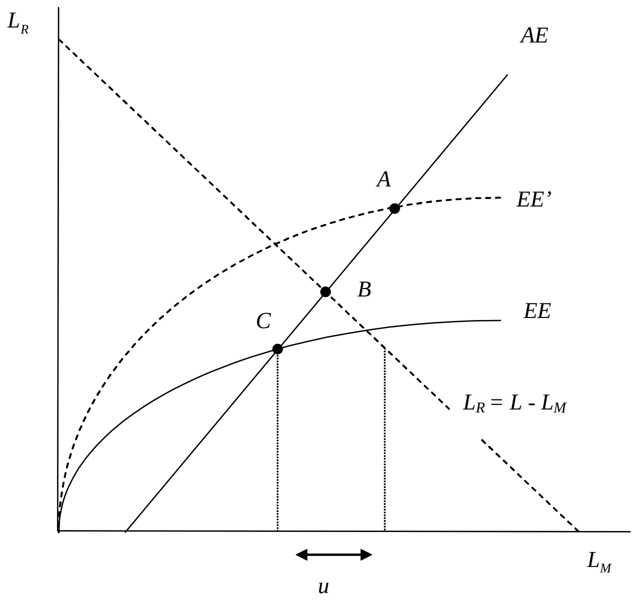


**Figure 1:** Equilibrium labor allocation

# 3 Consequences of Knowledge Formation

Early endogenous growth models (for example, Romer, 1990, Grossman and Helpman, 1991, and Aghion and Howitt, 1992) can be interpreted in the way that it should be a policy concern to support private innovation efforts. This is a result of the properties of the growth equation, which is linear in the scale of inputs to R&D. Hence, any policy that reallocates employment towards R&D also increases total growth. The subsequent literature formulated some doubts on this. Based on Jones (1995a, 1995b), non-scale growth models eliminate the scale effect of R&D through the introduction of a counteracting factor such as increasing difficulties in research over time (see Dinopoulos and Thompson, 1998, for an overview). Others, as Young (1993) and Lloyd-Ellis and Roberts (2002), formulate limits to R&D from the necessity of a parallel development of another dimension of knowledge, such as skills.[15] The model presented here is in line with this part of the literature. However, a scale effect of the labor force enters the model through the fact that more innovators in a large economy lead to more job destruction, an indicator for unemployment. On the other hand, the scale effect on growth disappears in case of skill shortages

[15] The literature, which criticizes the so-called scale effect apparent in the first innovation models, includes models which produce policy invariance. An example would be Arnold (1998), who takes R&D and human capital accumulation as substitutes. Consequently, subsidies to R&D reduce the accumulation of human capital and vice versa.

because $AE$ and $EE$ determine not only the labor allocation but, additionally, total employment and growth as they fix the size of the R&D sector.

The model considers a technology-skill complementarity[16] in the form that it is necessary to employ skilled labor in order to implement innovative technologies in manufacturing. From this it follows that a partial policy focus on R&D will not be fully beneficial as long as skills are insufficiently considered. The three regimes, with their different ratios between job destruction, vacancies, and skill supply, include a variety of effects of knowledge formation on growth and employment. However, we focus here on the two most differentiated cases of the regimes (I) and (III): Perfect reallocation of labor with an abundance of skilled labor, and unemployment due to skill shortages. Knowledge formation can take the channels skill upgrades and technology updates. Exogenous variations may change the intensity of knowledge formation, for example if a government intends to positively influence growth via a set of policy measures. We discuss the examples subsidies to the R&D sector, subsidies to the technologically leading vintage in manufacturing, and the exogenous increase in the efficiency parameters of R&D and learning-by-using. See Table 1 for a summary of the results and the underlying shifts in $AE$ and $EE$.

*Raising the Poisson rate of knowledge formation*

Knowledge formation in terms of technologies and skills has been modeled as a stochastic process. It might be the case that the Poisson parameters of innovation, $\varepsilon$, and skill formation, $\mu$ and $\rho$, change over time (for example because the general level of education increases, which improves the productivity of research and training). Raising the Poisson rates means in the case of $\varepsilon$ that innovations arise at shorter intervals. In the case of $\mu$ and $\rho$ it implies that more workers acquire skills. Equilibrium labor allocation reacts to these changes.

Considering $AE$, we find that the increase in $\varepsilon$ implies more researchers because the time shortens in which research is in vain and produces no revenues. This effect raises the expected profits and consequently R&D output and growth increase in regime (I). However, in case of skill shortages the high attractiveness of R&D faces a limit to technology implementation in manufacturing. The relative gain of the R&D sector raises job destruction and results in more unemployment in regime (III).

[16] According to Goldin and Katz (1998) the technology-skill complementarity emerged early in the twentieth century as new technologies, such as the assembly line, made pure physical labor input less valuable. Empirical evidence for the technology-skill complementarity in the recent past is given, for example, by Machin and van Reenen (1998), who find a significant link between skill upgrading and R&D intensity in seven OECD countries between 1973 and 1989.

Skill formation diminishes with the decline in employment and restricts further job creation. Growth may increase or decrease on these conditions because two opposing effects arise: Fewer researchers perform R&D, but each researcher develops more innovations within a certain period.

Improvements in the skill formation, namely a higher $\mu$ or $\rho$, initially reduce skill shortages in scenario (III), but they have only ambiguous effects on employment and growth. The extra supply of skilled labor means that less limitation to job creation leaves fewer vacancies unfilled. The consequent increase in employment in the high-tech manufacturing raises the revenues of R&D firms from selling technologies to the manufacturing sector. Hence, the R&D sector attracts a higher share of the labor force. This, in turn, tends to increase job destruction. As a result, job creation and job destruction increase at the same time. This yields that the total effect on unemployment and growth is ambiguous and depends on which change is bigger. In contrast to this, it is obvious that the extension of the skill supply has no effect on employment and growth if skilled labor is not in short supply, such as in regime (I). In this case the abundance of skills increases further, and skilled workers are more frequently employed in the part of manufacturing which uses only prior technologies.

*R&D subsidies*

Subsidies to R&D could be used as an instrument of innovation policy which aims to promote growth. Hence, we assume here that the government pays an amount $\sigma$ to all R&D units as long as they perform research. Suppose furthermore that the subsidies are financed through raising a lump-sum tax $\theta$ paid by the entire labor force so that no distortions occur. From this it follows that each R&D unit gets an extra income of $\sigma - \theta$. No-arbitrage between the sectors, which is the first equilibrium condition, demands the identity between expected incomes in manufacturing and R&D. According to the identity in (16), wages $w_\tau$ must be equal to the value of the next innovation $V_{\tau+1}$, weighted at the Poisson arrival rate $\varepsilon$. The extra benefits due to the subsidies change this condition to $w_\tau - \theta = \varepsilon V_{\tau+1} + (\sigma - \theta)$. The consequent $AE$ changes slightly due to the introduction of subsidies to:

$$\textbf{AE}' : \quad \begin{array}{ll} L_R = L_M \frac{\lambda}{\gamma(1+\eta)} \left(\frac{1}{\alpha} - 1\right) \left[\gamma + \frac{c}{\omega - \sigma}\right] - \frac{r}{\varepsilon} & \text{if } L_{M,\tau_t^{\max}} < D \\ L_R = \frac{\alpha}{\omega-\sigma} \left[L_M \frac{\mu}{\gamma} \left(\rho + \frac{1-\rho}{1+\eta}\right)\right]^\alpha & \\ \qquad -L_M \left(\mu + \frac{c}{(\omega-\sigma)\gamma}\right) \left(\rho + \frac{1-\rho}{1+\eta}\right) - \frac{r}{\varepsilon} & \text{if } D < L_{M,\tau_t^{\max}}. \end{array} \tag{31}$$

In both equations subsidies lead to an relative increase in the R&D employment. This is the result of the extra income which attracts a higher share of the labor force to R&D.

As regards growth, subsidies to R&D yield mixed results depending on the response of employment. There is no direct impact of subsidies on job creation or job destruction so that $EE$ remains unchanged. However, via changes in labor allocation according to $AE'$ subsidies have indirect effects on job creation and job destruction. The allocation effect of subsidies tends to increase the number of researchers in regime (I) and the subsequent rise in innovations leads to more future growth. However, in regime (III) the effect of more innovations and less manufacturing is a further increase in the current skill shortages, with a rise in job destruction and a reduction in job creation. The consequent decline in employment produces negative effects on growth. An extension of technology formation can be realized only at the expense of a reduction in skill formation. Hence, we find that, in case of skill shortages, the result of subsidies to R&D is counterproductive. Fewer instead of more innovations are developed because manufacturing firms succeed less in technology implementation and the demand for technology updates drops. The effect of subsidies is that R&D expands in relative terms but it decreases in absolute ones.

*Subsidies to the technologically leading manufacturing vintage*

An alternative strategy of innovation policy might be to subsidize the use of an innovation instead of its development. This refers to a focus on technological diffusion. To see whether this affects growth differently to direct payments to R&D, we analyze the case in which the government subsidizes the technologically leading vintage in manufacturing $\tau_t^{\max}$. For this purpose, the state budget covers a fraction $\sigma$ of the price that vintage $\tau_t^{\max}$ has to pay to the R&D sector. As a consequence, demand and price of the intermediate good change to $x_{\tau_t^{\max}} = \alpha^2 / \left[\gamma\omega + (1-\sigma)c\right]$ and $p_{\tau_t^{\max}} = \left[(1/\alpha) - 1\right]\left[\gamma/(1-\sigma)\right] w_\tau + (c_\tau/\alpha)$ according to the demand function of the manufacturing firm and the consequent price setting of the R&D unit (see section 2.1.3). However, the demand does not change if there are skill shortages. In this case the use of intermediate goods $x_{\tau_t^{\max}}$ is a fixed proportion of the labor input in $\tau_t^{\max}$ which is restricted to the supply of skilled labor $D$. Otherwise, demand *and* producer price increase with the introduction of subsidies. Both effects raise profits of the innovator in R&D. It follows that the R&D sector attracts a higher share of labor. Moreover, the costs of the intermediate good for vintage $\tau_t^{\max}$ are only $(1-\sigma)p_{\tau_t^{\max}}$ and, although the price is higher, the actual costs are below the level without subsidies. Remember that vintages other than $\tau_t^{\max}$ do not demand the technology update and, therefore, their costs remain the same as before. This means that subsidies cause a relative cost advantage of vintage $\tau_t^{\max}$, which consequently increases its contribution to total manufacturing output. This leads to a higher share of workers employed in $\tau_t^{\max}$. Let $1/(1+\widetilde{\eta})$ denote the new raised share, with $\widetilde{\eta} < \eta$. Again, subsidies should be financed though a lump-sum tax, which must be equal to $\theta = \left(p_{\tau_t^{\max}} x_{\tau_t^{\max}}\right)/L$. As $\theta$ appears on both sides of the income identity, the only effect on the no-arbitrage condition comes from $1+\widetilde{\eta}$ and $\sigma$:

$$\mathbf{AE´´:} \quad \begin{array}{ll} L_R = L_M \frac{\lambda}{\gamma(1+\widetilde{\eta})}\left(\frac{1}{\alpha}-1\right)\left(\frac{\gamma}{1-\sigma}+\frac{c_\tau}{w_\tau}\right)-\frac{r}{\varepsilon} & \text{if } L_{M,\tau_t^{\max}} < D \\ L_R = \frac{\alpha}{\omega}\left[L_M \frac{\mu}{\gamma}\left(\rho+\frac{1-\rho}{1+\widetilde{\eta}}\right)\right]^{\alpha} - \left[\mu+\frac{c}{\omega\gamma}\right] L_M \left(\rho+\frac{1-\rho}{1+\widetilde{\eta}}\right) & \text{if } D < L_{M,\tau_t^{\max}}. \end{array} \tag{32}$$

As $1+\widetilde{\eta}$ and $1-\sigma$ in the denominator of $AE´´$ get smaller the more subsidies are paid, we can conclude that the number of researchers $L_R$ increases, such as in the case of direct subsidies to R&D. Consequently, the frictionless regime (I) implies a positive effect on growth but subsidies tend to reduce employment and growth in regime (III), in which job destruction increases further as a result of skill shortages.

However, subsidies to manufacturing additionally affect job creation according to *EE*. We know that subsidies to vintage $\tau_t^{\max}$ increase its labor share compared to other manufacturing vintages. This effect works against the shortage of skilled labor if skill formation is particular high in the current technologically leading vintage. On this condition more workers acquire skills and can be employed in the next $\tau_t^{\max}$. Some of the restriction to job creation disappears. The *EE*-curve shifts and more job creation implies higher employment and more researchers, which opposes the effect from *AE*. In the end, more workers in R&D and in $\tau_t^{\max}$ due to subsidies increase both job destruction and job creation. Hence, the total effect on growth and employment is not clearly cut in regime (III).

Table 1 summarizes the effects of knowledge formation. In contrast to the standard innovation models, the reallocation of resources from manufacturing to R&D can result in a decline in employment and less growth if technology formation increases relative to skill formation. We see the use of technologies as a process of learning-by-using. Hence, the use of technologies improves the supply of skills, whereas the development of innovations permanently demands new skills. In case of skill shortages, subsidizing the use instead of the development of technologies

**Table 1:** The effects of knowledge formation

| | *AE* | *EE* | *Full employment* | *Skill shortages* | |
|---|---|---|---|---|---|
| | | | *growth* | *growth* | *employment* |
| *Increase in* $\varepsilon$ | a | 0 | + | $\sim$ | - |
| *Increase in* $\mu, \rho$ | 0/a* | a | 0 | $\sim$ | $\sim$ |
| *Subsidies to R&D* | a | 0 | + | - | - |
| *Subsidies to manufacturing* | a | a | + | $\sim$ | $\sim$ |

* 'a ' in case of skill shortages, '0 ' otherwise

a = above the initial locus, + = increase, - = decrease, $\sim$= ambiguous effect, 0 = no effect;

provides superior results in terms of employment and growth. While the model confirms the standard results of positive effects on growth from knowledge formation in the full employment case, the outcome is much more ambiguous if we consider skill shortages under the assumption of a technology-skill complementarity.[17]

# 4 Conclusion

We have presented a two-sector model of growth with knowledge formation and continuous reallocation of labor. Technology updates and skill upgrades were the two dimensions of knowledge formation. To what extent labor is reallocated depends on the ratio of technology formation to skill formation. Frictions in terms of skill shortages restrict the reallocation and lead to unemployment as a consequence of technological change.

We can interpret the occurrence of unemployment due to technology formation along the lines of the Neo-Schumpetrian literature on growth which explains the emergence of new technologies and their obsolescence over time. New technologies create new jobs but their obsolescence leads to job destruction as backward technologies are associated with less labor demand. Hence, we take into account that labor has to be reallocated from old employments to new ones in a growing economy with technological change. The considered restriction in the reallocation is that workers can be non-skilled in old jobs but they have to be skilled in the new ones

[17] This outcome matches the contradictory empirical results concerning the effect of growth on unemployment. Davis and Haltiwanger (1992), for example, find that growth reduces unemployment, whereas the results by Tronti and Tanda (1998) and by Caballero (1993) give evidence for the opposite effect.

connected to the leading technologies. In case of skill shortages, firms have difficulties in finding the skilled workers who can fill all new vacancies, and some of the unemployed cannot find jobs for which non-skilled labor is sufficient. In our model, unemployment is a side-effect of growth as technological change is the cause of the reallocation of labor. This implies that innovation-based growth is a misleading strategy to fight unemployment as joblessness results from a skill shortage caused by too many innovations. However, the economy grows without negative effects on the labor market as long as the skill formation is high enough.

The analysis of how changes in knowledge formation affect growth yields mixed results. Only in scenarios with an abundance of skilled workers we obtained the result that more innovations unambiguously lead to more growth. Otherwise, the relative lack of skilled labor is a limitation to the technology implementation of the industries which produce the consumption goods. However, the restricted use of technologies is just as an obstacle to growth as the restricted development of innovations.

# Appendix

**The employment share of vintage $\tau_t^{\max}$:**

Labor demand of vintage $\tau$ yields from (8)

$$L_{M,\tau} = \gamma a^{\frac{1}{1-\alpha}} \left( \frac{\alpha A_{\tau_t^{\max}}}{c_\tau + \gamma w_\tau} \right)^{\frac{1}{1-\alpha}} = \gamma \alpha \left( \frac{1}{c + \gamma \frac{w_\tau}{a}} \right)^{\frac{1}{1-\alpha}}$$

The sum of labor demand over vintages from the one with minimum technology to $\tau_t^{\max} - 1$ is:

$$L_M - L_{M,\tau_t^{\max}} = \gamma \alpha^{\frac{1}{1-\alpha}} \sum_{a^{\min}}^{1/\lambda} \left( \frac{1}{c + \gamma \frac{w_\tau}{a}} \right)^{\frac{1}{1-\alpha}}$$

Labor demand of the technologically leading vintage $\tau_t^{\max}$ is given by (12). Let $\eta$ denote the ratio of labor demand of the non-leading vintages to the leading one:

$$\frac{L_M - L_{M,\tau_t^{\max}}}{L_{M,\tau_t^{\max}}} = \frac{\sum_{a^{\min}}^{1/\lambda} \left( c + \gamma \frac{w_\tau}{a} \right)^{\frac{1}{\alpha-1}}}{\left( c + \gamma w_\tau \right)^{\frac{1}{\alpha-1}}} = \eta$$

From this it follows that:

$$L_{M,\tau_t^{\max}} = \frac{L_M}{1+\eta}$$

**Properties of *EE*:**

The first and second derivations of $EE$ yield:

$$\frac{\partial L_R}{\partial L_M} = \Omega \left[ \frac{1}{(1-\phi)\, L_M + \gamma x_{\tau_t^{\min}}} - \frac{(1-\phi)\, L_M}{\left[(1-\phi)\, L_M + \gamma x_{\tau_t^{\min}}\right]^2} \right]$$

$$=> \quad \frac{\partial L_R}{\partial L_M} > 0 \;\text{ as }\; \gamma x_{\tau_t^{\min}} > 0$$

$$\frac{\partial^2 L_R}{\partial L_M^2} = \Omega \left[ \frac{2\,(1-\phi)\, L_M}{\left[(1-\phi)\, L_M + \gamma x_{\tau_t^{\min}}\right]^3} - \frac{2}{\left[(1-\phi)\, L_M + \gamma x_{\tau_t^{\min}}\right]^2} \right]$$

$$=> \quad \frac{\partial^2 L_R}{\partial L_M^2} < 0 \;\text{ as }\; \gamma x_{\tau_t^{\min}} > 0$$

With: $\Omega = \frac{\mu}{\varepsilon}\left(\rho + \frac{1-\rho}{1+\eta}\right)$ if $D < L_{M,\tau_t^{\max}}$, and $\Omega = \frac{1}{\varepsilon(1+\eta)}$ if $L_{M,\tau_t^{\max}} < D$.
Hence, $EE$ is strictly convex.

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