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by

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Abstract

Is tax competition good for economic growth? The paper addresses this question by means of a simple model of endogenous growth. There are many small jurisdictions in a large federation and individual governments benevolently maximise the welfare of immobile residents. Investment is costly: Quadratic installation and de-installation costs limit the mobility of capital. The paper looks at optimal taxation and long-run growth. In particular, the effects of variations in the cost parameter on economic growth and taxation are considered. It is shown that balanced endogenous growth paths do not always exist, that, if they exist, the economic growth rate is positively related to the mobility of capital, that the impact of the mobility prameter on the tax rate is ambiguous and that the tax rate may go to zero even if mobility costs are strictly positive.

1 The Issue

Fiscal federalism and competition have been important issues in public economics in the past two decades. Static models have shown that there is a tendency towards underprovision of public-sector services emerging from fiscal externalities when the tax base is mobile and the use of non-distorting taxes is restricted. See Wilson (1999) for an overview. This paper attempts to extend this literature to an economic-growth context. In particular, we attempt to answer two questions. Does increased competition for a mobile tax base lead to lower tax rates? And does it enhance economic growth?

The model we use to address these questions is an endogenous-growth version of Wildasin's (2003) steady-state gowth model. Wildasin models tax competition in space (i.e. across jurisdictions) and in time (i.e. across periods) by introducing a convex investment cost function known from the economic-growth literature, e.g. Hayashi (1982) and Blanchard/Fischer (1989, ch. 2.4), into a tax-competition model. Such an investment cost function does not only penalise the relocation of capital from one jurisdiction to another but also makes it more costly to relocate capital quickly than slowly. Thus, a jump in the capital stock as a response to a change in tax rates is not feasible. As regards economic growth, Wildasin's model is traditional in that it assumes decreasing returns to scale in the augmentable factor such that the growth path in the long run approaches a static equilibrium in which economic growth has come to an end. Although such a model provides valuable insights into the impact of tax competition on the growth dynamics along the transition path towards the equilibrium, the concept of a static long-run equilibrium is not satisfying. We extend the Wildasin's model to an endogenous-growth framework. Regarding the results of the analysis, this is not an innocent change of some minor model assumptions but it produces qualitatively different outcomes. We show that, in contrast to Wildasin, the impact of capital mobility on the tax base is ambiguous: depending on the parameters of the model, more mobility may induce higher tax rates. In addition, it is seen that tax-competition growth equilibria do not always exist, in particular if the investment cost is extremely high. Thirdly, we are able to show that, surprisingly, the tax rate may go to zero even if the mobility of the tax base is limited. Our final result is more conventional: more competition for a mobile tax base is good for economic growth.

Besides Wildasin, (2003), the literature on tax competition and growth is still small. Other papers on growth and tax competition include Lejour/Verbon (1997), Razin/Yuen (1999), Rauscher (2005), Brueckner, (2006), and Hatfield (2006). Lejour/Verbon (1997) look at a two-country model of economic growth. Besides the conventional fiscal externality leading to too-low taxes they identify a growth externality going into the opposite direction. Low taxes in one country increase the growth rate in the rest of the world. If this effect dominates the standard fiscal externality due to competition for a mobile tax base, uncoordinated taxes will be too high. This contrasts the finding of the

standard static tax-competition models that taxes tend to be too low. Their result depends on the ad-hoc assumption of a taste for investing abroad on the side of investors that balances mobility costs. This preference for diversity makes sure that there is cross-border investment in their two-country model of fiscal competition. Köthenburger/Lockwood (2007) argue that risk-diversification in the presence of technological shocks leads to similar results. Investors are diversified in their portfolio as they wish to avoid a too high variance of investment returns. Razin/Yuen (1999) look at a more general model that also includes human-capital accumulation and endogenous population growth. They come to the conclusion that optimum taxes should be residence-based, capital taxes should be abolished along a balanced growth path, and taxes will be shifted from the mobile to the immobile factor of production if the source principle is applied in a world of tax-competing jurisdictions. Their results extend those derived by Judd (1985) and are in accordance with the standard economic intuition. The underlying assumption is that the government's set of tax instruments is large enough such that distortion-free taxation becomes feasible. Rauscher (2005) uses an ad-hoc model of limited interjurisdictional capital mobility and comes to the conclusion that the effects of increased mobility are ambiguous. A central parameter in this context is the elasticity of intertemporal substitution, which does not only affect the magnitude of the economic growth rate, but also the signs of the comparative static effects. Similar results will be derived below, albeit with a micro-founded, non ad-hoc, model of limited tax-base mobility. [1]

Closely related to our paper is Hatfield (2006). He addresses the question whether countries that are organised as federations grow faster than centralised countries and looks at the polar cases of either perfect integration or decentralisation. The findings are that decentralised governments choose growth-maximising tax rates, but a centralised government does not. The reason is that the central government is not forced by capital tax competition to offer the most attractive investment environment in order to attract capital. Instead, it balances initial consumption and long-term growth.

Unlike some of the aforementioned papers, we will not deal with the issue of centralisation versus decentralisation. We rather concentrate on the questions of whether deeper integration within a federation, modelled by declining investment costs, enhances economic growth and whether it reduces competitive tax rates. As in most other models of tax competition, we look at a federation consisting of a large number of very small jurisdictions that have no power to affect economic variables determined on the federal

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[1] Brueckner (2006) looks at another possible link between fiscal federalism and growth, namely the idea that local governments are better able to tailor their services to the needs of local people. In his model, old and young people are segregated into different jurisdictions. With decentralisation, regionally differentiated supplies of public services are possible whereas centralisation is assumed to result in uniform provision of public goods. The incentives to save (to consume when old) are then lower with centralisation and hence decentralisation leads to higher growth rates.

level. In our analysis, the only variable determined on the federal level will be the interest rate determining the remuneration of capital. Given this interest rate, governments choose their policies, consisting of a bundle of taxes and the provision of public goods. The set of policy instruments at hand is restricted insofar as capital owners cannot be taxed lump-sum. Therefore, distorting taxes become desirable. Our modifications of Wildasin's (2003) model are the following ones. To get long-term endogenous growth, we assume constant returns to scale with respect to the augmentable factor(s). The simplest way of doing this would be a simple Rebelo-type (1991) *AK* model. The problem with this model, however, is that factor rewards exceed output if, realistically, a second private factor, e.g. labour, is assumed to exist. We do make this assumption and in order to avoid problems with excessive factor rewards, we model technology such that the marginal productivity of private capital is diminishing. To make long-run growth possible nevertheless, we introduce a third factor of production which is augmentable and does not earn a factor income. This input is a flow of services provided by the government and financed through taxes like in Barro (1990).[2] Besides this input, the government, like in Wildasin (2003), provides a public consumption good which in our model for simplicity is assumed to be a perfect substitute for private consumption and is consumed by workers only. Two types of taxes are used: a source-based capital-income tax and a lump-sum tax on labour. The existence of the lump-sum tax results in an optimal provision of the public input. Underprovision like in Zodrow/Miezkowski (1986) is excluded. With undistorted provision of public inputs, however, the only variable affected by the distortion arisingfrom the mobility of capital as a tax base is the public consumption good – like in Wildasin's paper.

Another related paper is Turnovsky (1996), who introduced convex investment costs into the Barro (1990) model. While our modelling of endogenous growth is close to Turnovsky (1996), we extend the model by considering the implications of tax competition for the choice of public policy as in Wildasin (2003) and by an endogenous determination of the equilibrium interest rate. A simplification we employ is to assume that workers do not save and that capital owners do not vote. This helps us to avoid some algebraic complications arising in other two-stages optimisation models in which less restrictive assumptions are made.[3]

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[2] Mainly because the public input is not modelled as a stock variable, there are no transitional dynamics for the evolution of output and physical capital. Models of public policy and growth that address the importance of modelling public capital as a stock variable include Futagami et al. (1993) and Turnovsky (1997).

[3] Optimising governments use the private-sector first-order conditions as constraints. This implies that second-order derivatives show up in the governments' first-order conditions. See Rauscher (2005), for an example. In models with benevolent governments and redistributive taxation, these terms cancel out if workers do not save.

The model is solved as follows. In the first step of the analysis, economic agents in the private sector maximise their utility given the interest rate and the economic policies announced by the government. In the next step, governments decide about policies taking as given the interest rate and the first-order conditions of the private sector. Finally, the interest rate itself is determined. The next section of this paper will present the assumptions of the model regarding production technology and the frictions that limit the mobility of capital. Sections 3 and 4 will look at the behaviour of the private sector and of the government, respectively. Section 5 closes the model by determining the interest rate and derives the central result by investigating the impact of capital mobility on the long-run economic-growth path. Section 6 summarises.

2 Definition of Variables and Characterisation of Technology

Let us consider a federation consisting of a continuum of infinitely small identical jurisdictions, also labelled 'regions', on the unit interval. There is perfect competition in all markets and single jurisdictions do not have any market power vis-à-vis the rest of the federation. The private sector takes prices and policies announced by regional governments as given. Regional governments take variables determined on the federal level as given. As is always the case in models of tax competition, there is a distinction between ex ante objectives and ex post outcomes of actions taken to achieve the objectives. Ex ante, jurisdictions may be willing to use policy instruments to affect the allocation of mobile tax bases. Ex post, however, it turns out that all jurisdictions have acted in the same way and that the interjurisdictional allocation of the tax base is unaffected despite the efforts taken in the first place.

There are three types of agents in this model: workers, governments, and entrepreneurs, who also own the physical and financial capital in the economy. In order to save on notation, we do not distinguish between entrepreneurs and capital owners but assume that there is a homogenous group of capitalist producers.

- Workers are immobile across jurisdictions and inelastically supply one unit of labour per person in the perfectly competitive labour market of their home region at the current wage rate, which they take as exogenously given. Workers do not save and, thus, do not own physical capital or other assets.

- Governments charge taxes and provide a productive input. They are benevolent and maximise the utility of immobile residents. This includes the possibility of income redistribution.

- Capitalist producers own capital, hire labour, produce, save, and consume the unsaved share of their incomes. Saving yields an interest rate, which is determined on the federal capital market and which they take as exogenously given. If they want to transform their financial assets and invest in a particular jurisdiction, they face installation costs. If they want do withdraw physical capital, they have to

bear de-installation costs. With these costs, federal financial assets and local physical capital are imperfectly malleable and, thus, capital is imperfectly mobile.

As all jurisdictions are identical, let us consider a representative jurisdiction. There are three factors of production: capital, labour, and a publicly provided input, denoted $K(t)$, $L(t)$, and $G(t)$, respectively, where t denotes time. For the sake of a simpler notation, the time argument will be omitted when this does not generate ambiguities. Output, $Q(t)$, is produced by means of the three factors where marginal productivities are positive and declining. Moreover, we assume that the production function, $\Phi(.,.,.)$, is linearly homogenous in (K,G) and in (K,L). An example is the Cobb-Douglas function

$$Q = \Phi(K, G, L) = K^{1-\alpha} G^\alpha L^\alpha \tag{1}$$

with $0 < \alpha < 1$. The size of the labour force is normalised to one. Each worker inelastically supplies one unit of labour, i.e. $L=1$. Thus, (1) can be rewritten

$$Q = F(K, G) \equiv \Phi(K, G, 1) \tag{1a}$$

where $F(.,.)$ is a neoclassical constant-returns-to-scale production function measuring output per employee. A worker's income is the wage rate, $w(t)$, which is determined on the regional labour market. Moreover, let us introduce a production function in intensity terms,

$$f(g) \equiv F(1, g) \quad \text{where} \quad g \equiv G/K \tag{1b}$$

with $f'(g)>0$ and $f''(g)<0$, primes denoting derivatives of univariate functions. Regarding the marginal productivities we have

$$\Phi_K = F_K = f - gf', \tag{2a}$$

$$\Phi_G = F_G = f', \tag{2b}$$

$$\Phi_L = F - KF_K = Kgf', \tag{2c}$$

where subscripts denote partial derivatives and arguments of functions have been omitted for convenience.

Regarding the other two factors of production, we assume:

- Capital. $K(t)$ is the quantity of a composite capital good consisting of physical capital, human capital, and knowledge capital. Initially, each jurisdiction is endowed with $K(0)=K_0$. Capital depreciates at a constant exogenous rate m. Let $I(t)$ be the rate of gross investment as a share of the capital stock. Then capital accumulation evolves according to

$$\dot{K} = (I - m)K, \tag{3}$$

a dot above a variable denoting its derivative with respect to time. Capital is mobile, albeit at a finite speed. As already mentioned, there is a capital market on the federal level, yielding an interest rate $r(t)$, which is exogenous to individual

capital owners and to governments of individual jurisdictions, but endogenously determined by demand and supply on the federal level. Assets and physical capital are imperfectly malleable. Transforming financial capital into physical capital and vice versa is costly. We follow Wildasin (2003) in the specification of the installation cost function. Installation costs are defined as

$$c(I)K \quad \text{with} \quad c(0)=0 \quad \text{and} \quad c''(.) > 0.$$

The installation cost per unit depends on the rate of investment as a share of capital, i.e. on the speed of gross accumulation. This function also covers the possibility of de-installation costs for $I<0$. For the derivation of explicit results in the forthcoming sections of the paper we assume a quadratic shape of c such that

$$c(I)K = \frac{b}{2}I^2K \,, \tag{4}$$

i.e. $c'(I)=bI$ and $c''(I)=b$, where the constant positive parameter b measures the barriers to mobility. $b=0$ represents perfect mobility and malleability. If b goes to infinity, capital becomes absolutely immobile. For the interpretation of some of the results to be derived in the following sections, it is useful to introduce the absolute rate of investment, J. Using $I=J/K$ in equation (4) yields

$$c(I)K = c\left(\frac{J}{K}\right)K = \frac{b}{2}\frac{J^2}{K} \,. \tag{4'}$$

- The public-sector input. The government provides a productive input at a rate $G(t)$. This may be interpreted as physical infrastructure such as roads and ports, but also institutional infrastructure including the legal framework in which economic transactions take place. For the sake of simplicity, we treat this good as a flow variable which is provided anew in each period. Interjurisdictional spillovers are excluded. The provision of the public input is financed by taxes. There are two types of fiscal instruments, a source tax on capital, the tax rate being θ,[4] and a redistributive lump-sum transfer going to the immobile factor of production. This transfer is special case of a publicly supplied consumption good to be consumed by workers only. We assume that the government chooses a constant tax rate and allocates a constant share of the budget, 1-s, to redistribution. Thus,

$$G = s\theta K \,, \tag{5}$$

where $s > 0$ ($s > 1$ implies lump-sum taxation of immobile residents). The underlying assumption that the budget is balanced in each period seems to be restictive, but real-world governments are indeed subject to within-period budget constraints. A prominent example is the European Growth and Stability Pact, which

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[4] Other papers like Judd (1985, 1999) and Lejour/Verbon (1997) introduce taxes on capital income rather than on capital itself. But as long as taxation is linear, the two instruments are equivalent.

restricts the policy makers' discretion to borrow. Equation (5) is a possibility of introducing such a restriction in a simplified way. The equation can be rewritten:

$$g = s\theta .$$ (5')

Equation (5') implies that g is constant and using this in (2a) and (2b) yields:

Lemma 1

All first derivatives of the production function F(.,.) are constant over time.

This concludes the exposition of the model.

3 Saving, Investment and Production in the Private Sector

As workers in this model do nothing besides inelastically supplying labour, capitalist producers drive the dynamics of the economy. They decide about consumption and about investment in either the local capital stock K or in financial assets A. These decisions depend on the tax rates and the provision of public inputs set by the local government. Other important determinants are the interest rate in the federation-wide capital market and installation (and de-installation) costs for investment in the local capital stock.

Capitalists hire labour, they save, and they invest. Moreover, unlike workers, capital owners are mobile and can choose to live where they want. If they are not satisfied with their domicile, they can vote with their feet like in Tiebout (1956) and move to another jurisdiction that offers better conditions. In contrast to the Tiebout model, mobile capitalists in our model do not demand local public goods. Thus, they are not willing to pay taxes to contribute to such goods. They will settle in the jurisdictions that tax them at the lowest rates. Real-world examples are Monaco and the Swiss cantons Zug, Schwyz, and Nidwalden, which levy very low taxes and attract millionaires from other parts of the country and from the rest of the world.[5] In a competitive world with many identical jurisdictions, there is a race to the bottom such that capitalists ultimately do not pay any taxes anywhere. Hence, capital income can only be taxed at source. The perfect mobility of capitalists has another important implication for the model. Since capitalists vote with their feet, they are not interested in participating in the political process. They do not show up at the ballot box and, thus, their interests are not taken into account by the policy maker.

The representative capitalist producer has two sources of income. On the one hand, she retains the share of output not being paid as wages to workers. On the other hand she has an interest income from her stock of saved assets, $A(t)$. There is a perfect asset

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[5] According to a report in the "Neue Zürcher Zeitung" from September 23, 2005, 13 percent of the ca. 3300 citizens of the the village of Walchwil in Zug are millionaires, and other villages in Zug, Schwyz, and Nidwalden report similar, though slightly lower, percentages.

market in the federation such that all assets yield the same rate of interest, $r(t)$, to their bearers. There are two possibilities to spend the income. It can be consumed or it can be saved. Moreover, savings (assets) can be transformed into physical capital, however only at a cost, the cost function being defined by (4). The rate of accumulation of assets is output minus the wage payments going to workers minus tax payments minus consumption minus investment into physical capital minus costs of investing into physical capital plus interest income from assets accumulated in the past. In algebraic terms:

$$\dot{A} = \Phi(K, G, L) - wL - \theta K - C - IK - c(I)K + rA. \tag{6}$$

Since all jurisdictions are identical, there will be no lending and borrowing ex post, i.e. $A=0$. In particular, $A(0)=0$. Ex ante, however, capitalists consider the possibility of borrowing and lending according to (6). Ponzi games are excluded, i.e. the present value of assets in the long run must be non-negative

$$\lim_{t \to \infty} e^{-rt} A \geq 0.$$

A representative capitalist producer maximises the present value of her utility. Utility is derived from consumption, $C(t)$, only and is of the constant-elasticity-of-substitution type with σ being the elasticity of intertemporal substitution. The discount rate, δ, is positive and constant and the time horizon is infinite. Thus, the individual's objective is to maximise

$$\int_0^\infty e^{-\delta t} u(C) dt \qquad \text{with} \qquad u(C) = \frac{C^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}$$

subject to (3), (6), the initial endowments, K_0 and A_0, the tax rate θ, and the public expenditure, $G(t)$, the latter two having been announced by the government. Note that an individual capitalist-producer does not take the government's budget constraint, (5), into account. The decision maker's control variables are $C(t)$ and $L(t)$. The corresponding Hamiltonian is

$$H = u(C) + \lambda(\Phi(K, G, L) - wL - \theta K - C - IK - c(I)K + rA) + \mu(I - m)K,$$

where $\lambda(t)$ and $\mu(t)$ are the shadow prices, or co-state variables, of financial and physical capital, respectively. The canonical equations are

$$\dot{\lambda} = (\delta - r)\lambda, \tag{7a}$$

$$\dot{\mu} = (\delta + m - I)\mu - (\Phi_K - \theta - I - c)\lambda, \tag{7b}$$

where subscripts denote partial derivatives and Φ_K will be replaced by F_K in the remainder of the investigation. See equation (2a). Complementary slackness at infinity requires

$$\lim_{t \to \infty} e^{-\delta t} \lambda A = 0,$$

$$\lim_{t \to \infty} e^{-\delta t} \mu K = 0,$$

and, hats above variables denoting growth rates and using (2) to substitute for $\hat{K}$, these conditions imply that

$$\hat{\lambda} + \hat{A} < \delta \qquad \text{for} \qquad t \to \infty, \tag{8a}$$

$$\hat{\mu} + I - m < \delta \qquad \text{for} \qquad t \to \infty. \tag{8b}$$

First-order conditions are

$$w = \Phi_L, \tag{9a}$$

$$u' = \lambda, \tag{9b}$$

and

$$\mu = (1 + c')\lambda. \tag{9c}$$

Condition (8a) is a standard labour-demand equation and from (9b) we can derive the standard Ramsey-type growth equation with $\hat{C}$ as the growth rate of consumption

$$\hat{C} = \sigma(r - \delta). \tag{10}$$

Equation (9c) states that there is a wedge between the shadow prices of financial capital on the federation level and local physical capital. Plausibly, this wedge depends on the marginal cost of installation. From (9c), one can derive a condition that links the rates of returns in the two markets for capital. Taking time derivatives of the shadow prices, inserting (7a) and (7b), and using (9c) again to eliminate λ/μ, we have

$$\dot{I} = \frac{1}{c''}\left((1 + c')r - (F_K - m - \theta - c) - (I - m)c'\right). \tag{11}$$

The condition for a steady state, i.e. for $\dot{I} = 0$, is

$$F_K - m - \theta - c + (I - m - r)c' = r. \tag{12}$$

This is a capital-market indifference condition. On the right-hand side, the interest rate indicates the opportunity cost of investing into physical capital in terms of foregone interest. The left-hand side measures the net benefit from investing into physical capital. The first term is the gross productivity from which the rate of depreciation and the tax rate are subtracted. Without mobility cost, this would constitute the net productivity of capital after taxes. With mobility cost, however, two additional terms emerge. The first one is c. As defined in this model (see equation 4), mobility costs are proportional to capital. Thus, additional capital raises installation costs, the marginal effect being just c. The final term on the left-hand side may be interpreted as an intertemporal benefit from a larger capital stock. If $I>m$, the capital stock grows and this implies lower future installation costs per unit of newly installed capital, J. See equation (4'). This intertemporal cost-saving effect of investment is counterweighted by another intertemporal effect: Investing an additional unit of capital has the opportunity cost of not earning the interest r in the future. Hence a higher interest rate r lowers the (future) benefits of investing in real capital as the raise the opportunity costs of investment.

In the derivation of the optimal rate of investment, we follow Turnovsky (1996). Using the quadratic shape of the investment cost function, (4), we can rewrite (11) such that

$$\dot{I} = \frac{1}{2}\left(-I^2 + 2(r+m)I - \frac{2}{b}\left(F_K - m - \theta - r\right)\right). \tag{11'}$$
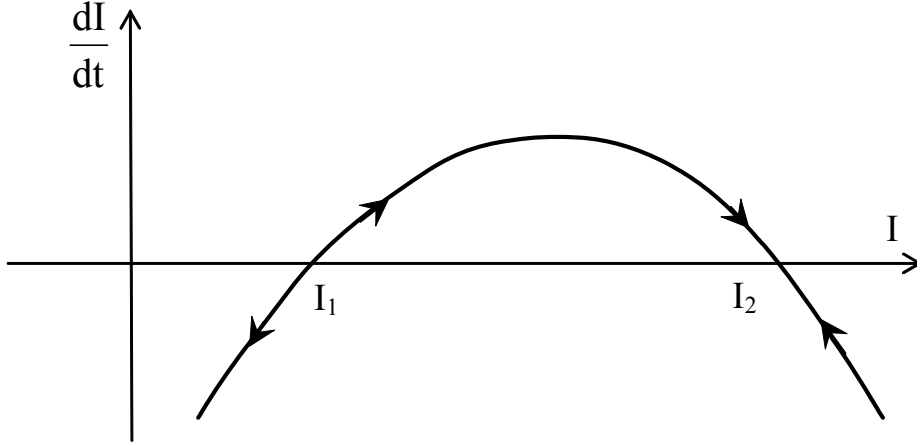

Figure 1: Investment Dynamics

This is a quadratic differential equation that can be represented as a hump-shaped curve in a phase diagram with a stable and an unstable equilibrium. See Figure 1. The condition for a steady state with $\dot{I} = 0$ is

$$I_{1,2} = r + m \pm \sqrt{(r+m)^2 - \frac{2}{b}\left(F_K - m - \theta - r\right)}, \tag{13}$$

where the smaller value, I_1, corresponds to the unstable equilibrium in Figure 1 and the larger one, I_2, to the stable equilibrium. An imaginary solution would imply a fluctuating path of capital accumulation. One can show that I_2 as well as an imaginary solution would violate the transversality condition.[6] Noting that I_1 is an instable solution of (11'), it follows that there are no transitional dynamics. This implies

Proposition 1

The optimal rate of investment is constant along the optimal trajectory, with

$$I_1 = r + m - \sqrt{(r+m)^2 - \frac{2}{b}\left(F_K - m - \theta - r\right)}. \tag{13'}$$

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[6]   Note that I is constant in the steady state. Thus (9c) implies $\hat{\mu} = \hat{\lambda}$. Using (7a), we then have $\hat{\mu} = \delta - r$. Using this in (8b) yields the condition $I < r+m$. This is violated by I_2 and by any imaginary solution to (13) because its real part would be $r+m$.

Constancy follows from Lemma 1, which states that F_K is constant over time. Condition (13') shows that the optimum rate of investment, as expected, is increasing in the marginal productivity of capital and decreasing in the depreciation rate, the interest rate, and the tax rate. The impact of the cost parameter b is ambiguous. It is negative if $I_1>0$ and positive if $I_1<0$. As deviations from zero in both directions are penalised by high installation and de-installation costs, this is also plausible.

Finally, to fully characterise the savings behaviour of the private sector, the initial level of consumption needs to be determined. Using (1a), (9a), (2c), (3), (4), and (10) in (6) yields

$$\dot{A} = \left(F_K - \theta - I_1 - \frac{b}{2}I_1^2 \right)K(0)e^{(I_1-m)t} - C(0)e^{\sigma(r-\delta)t} + rA .$$

In an intertemporal steady-state equilibrium with identical jurisdictions, there is no lending and borrowing, i.e. $A=0=\dot{A}$ for all t. This implies equal growth rates of the capital stock and of consumption,

$$I_1 - m = \sigma(r - \delta), \tag{14}$$

and the starting value of C is:

$$C(0) = \left(F_K - \theta - I_1 - \frac{b}{2}I_1^2 \right)K(0) . \tag{15}$$

Equation (14) determines, together with (13'), the equilibrium interest rate as an implicit function of the parameters of the model and of the tax rate. Note that the assumption of identical jurisdictions is central to determine the equilibrium in the federation-wide capital market. Identical jurisdictions imply that agents cannot differ in the sense that one agent is a borrower and another is a lender. Equation (15) states that consumption is positively related to initial capital endowment and capital productivity and negatively related to the tax rate, the rate of investment, and the installation cost.

Given this characterisation of private-sector behaviour, the next step is to analyse government policies that influence private saving and investment decisions. In the next section this is done for a small open economy. Afterwards, the equilibrium in which many tax-competing governments interact will be addressed.

4. Government Behaviour and Taxation for a Given Rate of Interest

The government maximises the welfare of immobile residents. Immobile residents are workers. Their wage rate is $gf'K$ and their transfer income $(1-s)\theta K$. See equations (9a), (2c), and (5). The government takes the interest rate as exogenously given as the jurisdiction is small. In particular, it does not consider condition (14), which determines the equilibrium interest rate when, ex post, all governments have chosen the same tax

policies. Let us assume that workers have the same preference parameters as the capitalists. Thus, the government's objective is to maximise

$$W^L = \frac{\sigma}{\sigma-1} \int_0^\infty \left(\left[(gf' + (1-s)\theta)K_0 e^{(I_1-m)t} \right]^{1-\frac{1}{\sigma}} - 1 \right) e^{-\delta t} dt .$$ (16)

The condition for the objective function, (16), to be finite is

$$\left(1 - \tfrac{1}{\sigma}\right)\left(I_1 - m\right) < \delta$$ (17)

and it is assumed for the remainder of the investigation that the parameters of the model are such that the condition is satisfied. Maximising (16) with respect to the government's policy parameters, θ and s, yields

Proposition 2

The optimal tax-and-expenditure policy of the government for a given interest rate is characterised by

$$f' = 1 ,$$ (18)

$$\theta = b(r + m - I_1)\left(\delta - \left(1 - \tfrac{1}{\sigma}\right)\left(I_1 - m\right)\right) > 0$$ (19)

where I_1 is determined by (13').

Equations (18) and (19) are derived in the appendix. Equation (18) states that the marginal productivity of government expenditure is unity. This is a standard result in tax-competition models with lump-sum tax instruments. See, e.g., Zodrow/Mieszkowski (1986, p. 363). The underlying intuition to explain this result in our model is the following one. In a first step, capital is taxed and the tax revenue is added to labour income. Out of this gross income, workers pay a lump-sum tax that is used to finance the publicly provided good. Thus, the cost of producing one unit of G is exactly one unit of GDP. Since $f' = \Phi_G$ is the marginal productivity of G, $f' = 1$ is nothing else but the rule that the marginal productivity of a factor should equal the marginal cost of employing it.

Equation (19) determines the optimum tax rate. This tax rate is positive since $(r + m - I_1) > 0$ due to Proposition 1 (see also Footnote 5) and $\left(\delta - \left(1 - \tfrac{1}{\sigma}\right)\left(I_1 - m\right)\right) > 0$ due to assumption (17).[7] For $b \rightarrow \infty$, θ goes to infinity.

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[7] Note that Turnovsky (1996, p. 58) derived a zero-tax rate. The difference between his and our model is the possibility of non-distorting taxation of capital owners. In Turnovsky's model of a small open economy, non-distorting taxation via consumption taxes is feasible whereas in our model capitalist producers are footloose and their incomes can only be taxed at source.

5. Government Behaviour and Taxation in the Equilibrium

In the equilibrium, the interest rate is determined by equation (14). This implies that the transversality condition, (17), can be rewritten such that

$$(1-\sigma)r + \sigma\delta > 0 . \tag{20}$$

This is the transversality condition known from models of economic growth in small economies, where the interest rate is exogenous. If $\sigma > 1$, the interest rate must not be too large. Otherwise, the economic growth rate would be so large that the increase in utility flows would dominate the effect of discounting.[8] If $\sigma < 1$, the interest rate must not be too negative. Otherwise, the fast rate of economic decline would make the welfare integral go to minus infinity. In order to rule out a non-converging welfare integral, we make

Assumption 1

The parameters of the model are such that condition (20) is satisfied.

Using (14) in the tax-rate formula, (19) and determining the interest rate via (13'), yields

Proposition 3

In the tax-competition equilibrium, the tax rate and the interest rate are determined by

$$\theta = b\left((1-\sigma)r + \sigma\delta\right)^2 \tag{21}$$

and the interest rate is determined by

$$b = \frac{2(F_K - r - m)}{\left((1-\sigma)r + \sigma\delta\right)^2 + (r+m)^2} . \tag{22}$$

Equation (21) follows directly from (19) and (22) is obtained by using first (23) in (13') and then (14) to eliminate I_1. In what follows, the right-hand side of equation (22) will be referred to as the $b(r)$ function. From (21) we have

Corollary 1

For $b > 0$, the equilibrium tax rate is positive. It goes to zero if $b \to 0$ or if $(1-\sigma)r + \sigma\delta \to 0$.

That perfect capital mobility leads to zero taxation, is a standard result. A perfectly mobile tax base should not be taxed. The other case is more surprising. Even if $b > 0$, i.e. if the tax base is imperfectly mobile, taxation may not be warranted. The underlying intuition is that the transversality condition, (20) is violated if $(1-\sigma)r + \sigma\delta$ happens to be zero. In this case, the welfare integral would go to infinity. If a tax is introduced, however small it may be, the growth rate of the economy would be reduced and the welfare integral would become finite. Thus, in this limiting case the marginal welfare

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[8]   This becomes more obvious if the condition is rewritten such that $(1-1/\sigma)r < \delta$, where the left-hand term is the growth rate of utility, which must not exceed the discount rate.

cost of taxation would be infinity and the tax rate should therefore be zero. If $(1-\sigma)r+\sigma\delta$ is slightly greater that zero, the tax rate is very small. Finally, note that $s\to\infty$ if $\theta\to 0$, i.e. the government relies on lump-sum taxation of immobile residents to finance the public input if the source tax on capital goes to zero.
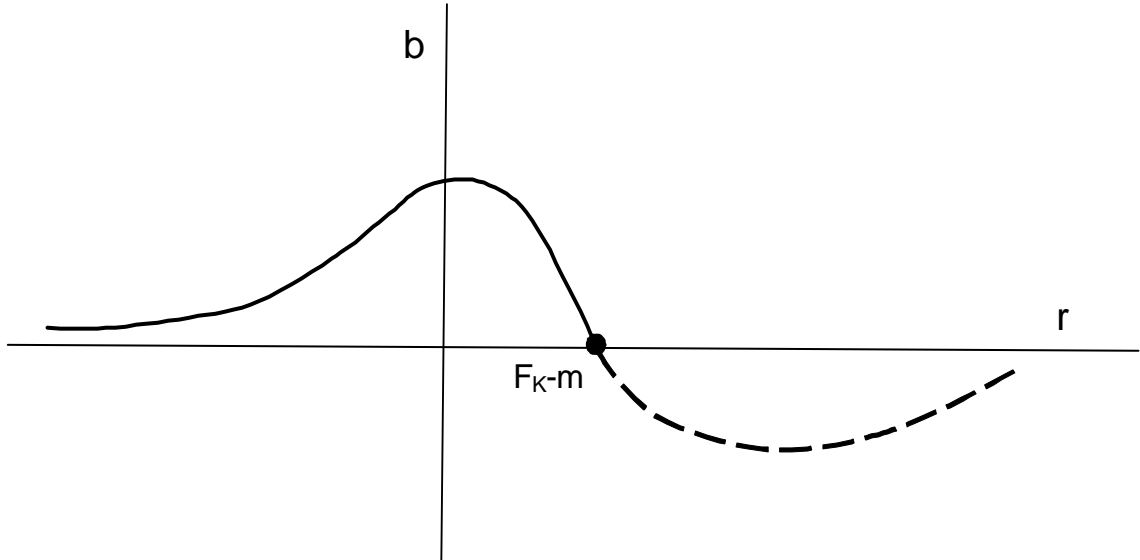

Figure 2: Equilibrium interest rate and capital mobility

Equation (22) determines the interest rate and, thus, the growth potential of the economy. The properties of the $b(r)$ function are explored in the appendix and they are depicted in Figure 2. The dashed part of the curve is irrelevant as b would be negative there. If $b=0$, $r=F_K-m$, i.e. the interest rate equals the net marginal product of capital, which is a standard and straightforward result. If $b>0$, $r<F_K-m$. The maximum of the curve is attained for

$$r^{maxb} = F_K - m - \sqrt{\frac{F_K^2 + \left((1-\sigma)(F_K-m)+\sigma\delta\right)^2}{1+(1-\sigma)^2}}, \tag{23}$$

which follows from eq. (A7) in the appendix. For many realistic parameter con-stellations, (23) implies a negative interest rate, indicating negative growth and possibly dis-investment at the maximum feasible level of b. An interest rate exceeding the discount rate, leading to positive growth, is possible, but very unlikely. Note that

$$r^{maxb} = -m \quad \text{and} \quad b^{max} = \infty \quad \text{for} \quad \sigma = \frac{m}{m+\delta} \ .$$

For all other cases, the b^{max} is finite, implying that an equilibrium interest rate does not exist if b exceeds a certain threshold value. The reason is that for large values of b a small country's government would choose an extensively high tax rate. An individual government neglects the impact of its tax policy on the interest rate. If all countries do this, the asset market equilibrium collapses – unless one introduces an exogenous upper

limit to taxation. Moreover, Figure 2 shows that for each $b>0$, there are two values of r satisfying (23a). However, the lower of the two values of r is irrelevant here. Assume that $b=0$. In this case r should equal $F_K - m$ and not $-\infty$. Increasing b generates the decreasing segment of the curve. Hence, as the growth rate of the economy is determined by the interest rate via Ramsey's rule (equation (10), lowering capital mobility or increasing the degree of disintegration in the federation results in slower economic growth.

Before we proceed, we restrict the parameters of the model such that positive growth rates are possible:

Assumption 2

$$F_K - m - \delta > 0 .$$

F_K is determined via $f'=1$ (equation (18)) and is constant. In the absence of installation costs, the growth rate would be $\sigma(F_K - m - \delta)$ like in Ramsey's (1928) model, the difference being that F_K does not decline along the accumulation path since we are in an endogenous-growth framework

Given the properties of the $b(r)$ function, one can now determine the effects of changes in b on taxation. Differentiation of (22) yields

$$\frac{d\theta}{db} = \left((1-\sigma)r + \sigma\delta\right)^2 + 2(1-\sigma)\left((1-\sigma)r + \sigma\delta\right)\left[\frac{db}{dr}\right]^{-1} \tag{24}$$

where db/dr is the slope of the part of the function located to the right its maximum and it is negative. Equation (24) implies that the impact of b on θ is not necessarily positive. Let us distinguish the cases $\sigma>1$ and $\sigma<1$. In the limiting case $\sigma=1$, matters are simple because $\theta=\beta\delta^2$ and the tax rate is linear in b.

Case A: $\sigma>1$

From (24) it follows that the impact of b on the tax rate is positive as long as the transversality condition, (21), is satisfied. If $(1-\sigma)r + \sigma\delta = 0$, then $\theta=0$ even if $b \neq 0$. Using $(1-\sigma)r + \sigma\delta = 0$ in (23), the corresponding value of b, b_0, turns out to be

$$b_0 = \frac{(1-\sigma)\left((1-\sigma)(F_K - m) + \sigma\delta\right)}{\left((1-\sigma)m + \sigma\delta\right)^2} , \tag{25}$$

which may be positive or negative depending on whether σ is larger or less than $\frac{F_K - m}{F_K - m - \delta}$, respectively. Figure 3 depicts $\theta(b)$ for the two cases. In the right-hand diagram, where σ is relatively small, matters are simple: the tax rate is increasing in b. If, however, σ is large, the function is S shaped. As long as $b<b_0$, the transversality condition is not satisfied since the rate of economic growth is so large that the welfare integral does not converge in spite of discounting. This part of the curve is depicted as a

dotted line. If $b>b_0$, the transversality condition is met and the tax rate is again monotonically increasing in b. $b=b_0$ is the limiting case referred to in Corollary 1. The tax rate goes to zero even though the tax base is imperfectly mobile.
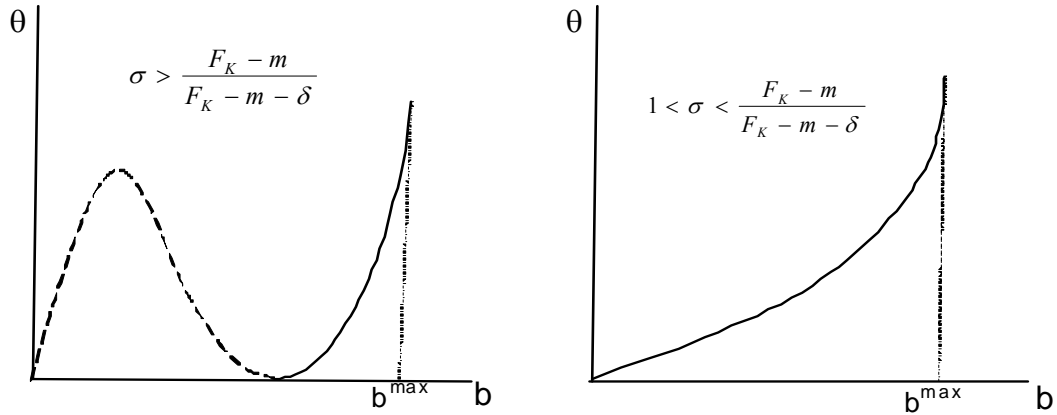

Figure 3. The tax rate as a function of b for $\sigma>1$

Case B: $\sigma<1$

From (24) it can be seen that the impact of b on the tax rate is ambiguous even if the transversality condition is satisfied. Like before, b_0 is determined, by equation (25). If Assumption 2 is fulfilled, $b_0>0$. For $\sigma \to m/(m+\delta)$, b_0 goes to infinity. In this case, using (23) in the tax-rate formula gives

$$\theta = \frac{2(F_K - m - r)}{1 + (1 + m/\delta)^2} \quad \text{if} \quad \sigma = \frac{m}{m+\delta}.$$

r ranges from $F_K - m$ for $b=0$ to $-m$ for $b\to\infty$ and is monotonously decreasing in b. This implies that θ is monotonously increasing in b and goes to a value less than F_K for $b\to\infty$. Monoticity does not hold for the other cases in which σ is less or larger than this critical value. Let us distinguish these two sub-cases. The shapes of the curves are derived in the appendix.

Sub-case B1: $0 < \sigma < \dfrac{m}{m+\delta}$

The tax rate as a function of b is S shaped. Initially the tax rate is increasing in b. Above a certain threshold level of b, the curve bends back and the tax rate declines until it becomes zero at for the value of b at which the transversality condition starts to be violated. For values of b larger than b_0, the transversality condition continues to be violated until b attains its maximum level, b^{max}. Again the part, of the function along which the condition is not met, is illustrated by a dotted line. The underlying intuition for the S shape is rather straightforward. The initial increase in the tax rate is intuitive. As the tax rate increases, the interest rate declines and at eventually the growth rate

becomes negative. With $\sigma < 1$, negative growth implies utility flows that are negative and become larger in absolute value. These losses in utility can be reduced by lower taxes. If, in the extreme, the transversality condition is close to be violated, an increase in the tax rate by a small amount would turn a finite negative welfare integral into an infinite one. Such taxes are avoided and this explains why the tax rate goes to zero as $(1-\sigma)r + \sigma\delta \to 0$.

Sub-case B2: $\dfrac{m}{m+\delta} < \sigma < 1$

In the appendix, it is shown that $\theta=0$ is not possible for $b>0$ and that the slope of the $\theta(b)$ function is positive for $b=0$ and goes to $-\infty$ for $b \to b^{max}$. There is a segment of the curve along which the tax rate is decreasing in openness.
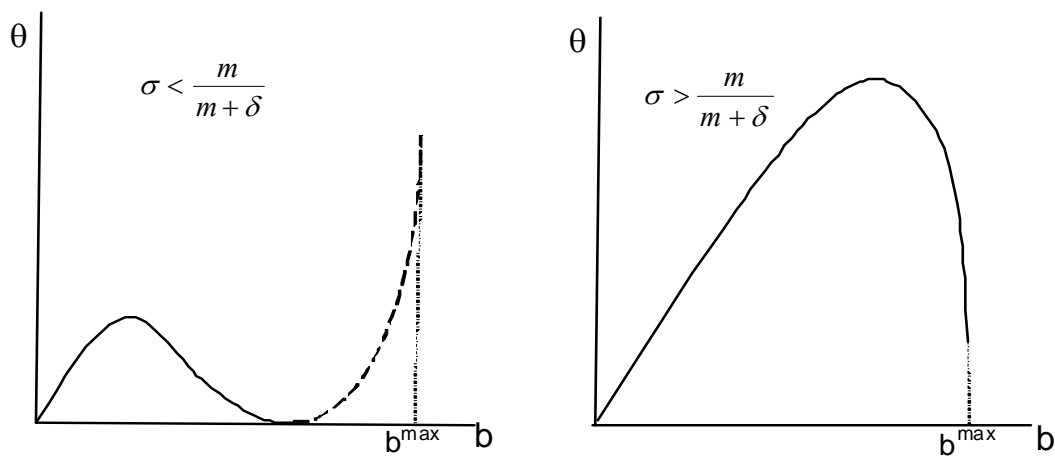

Figure 4. The tax rate as a function of b for $\sigma<1$

Summarising, we have the following results.

- If the elasticity of substitution exceeds 1, the tax rate is monotonically increasing in the cost parameter, b. For particularly large values of σ, the rate of taxation may go to zero although installation costs are still positive. The intuition behind this result is that welfare, i.e. the present value of future utility flows, is so large that small increases in the tax rate would lead to drastic welfare losses.

- If the elasticity of substitution is less than 1, the tax rate is a backward-bending function of b (with the notable exception of $\sigma=m/(m+\delta)$), where the tax rate is monotonously increasing in b. Decreasing taxes are possible for large values of b, for which the economic growth rate is likely to be negative. If $\sigma<1$, the welfare integral is negative and with negative growth rates it becomes large in absolute value. Increases in b then lead to large welfare losses which can be offset by lower taxes. If σ is particularly small, the tax rate may even go to zero because the welfare integral goes to minus infinity at a certain threshold value of

b. In this situation, small increases in the tax rate have dramatically negative consequences for welfare and the optimum tax rate, therefore, goes to zero.

6. Final Remarks

The paper has addressed tax competition in a general-equilibrium endogenous growth model. The starting point was Wildasin's (2003) model with quadratic installation costs. We are able to show that not all results carry over from the traditional-growth to the endogenous-growth framework. In particular, we established the following new results

1. A tax-competition equilibrium does not always exist. If installation costs and capital depreciation are large and the rate of discount and the elasticity of intertemporal substitution are small, an equilibrium does not exist since the tax rate becomes very high and capitalists want to reduce their capital stock at a rate that is incompatible with the smooth consumption path implied by the low rate of discount and the small elasticity of intertemporal substitution. A way out of this the problem of non-existence would be the introduction of an exogenous upper limit to taxation.

2. Tax rates may go to zero even if installation costs are positive. This result is counter-intuitive at a first glance, but it can be explained by the fact that the present value of welfare may go to plus or minus infinity in an endogenous-growth framework. This implies that even small tax rates can induce dramatic welfare losses.

3. The impact of installation cost on the capital tax rate is ambiguous. Tax rates may be reduced when the mobility of the tax base is reduced. This result differs from the one derived by Wildasin (2003) for a growth model that approaches a no-growth steady state in the long run.

4. Many of the interesting results of this paper can be established only for variants of the model with an elasticity of substitution <u>not</u> equalling 1. This indicates that the assumption of logarithmic utility made in many papers on economic growth and taxation is by no means an innocent one. Since the empirical evidence suggests that σ is significantly smaller than one (see Hall (1988) and Guvenen (2006)), the assumption $\sigma = 1$ is hardly defendable on empirical grounds, too.

A result that is in line with what has been established by others, e.g. Hatfield (2006), is that increased factor mobility enhances economic growth. This result is challenged in the recent paper by Köthenbürger/Lockwood (2007), who, however, rely on stochastics and use a portfolio-diversification argument. In deterministic models, the general result seems to be that tax competition is pro-growth.

A caveat we would like to mention is that our results have been derived by varying the costs of installing new or de-installing old capital. These costs definitely constitute a real-resources loss to the economy and no one should be surprised if the economy is

worse off once these costs are increased. However, the economic growth rate is not affected by this resource loss. Assume that the installation cost is a pure distortion, which does not involve a direct resource loss. An example would be a payment made by an individual investor which, at the end of the day, is rebated lump-sum. The only effect this modification would have on the results of the model would be a change in the level of consumption in equation (15). The growth rate would remain unchanged.

As a possible agenda of future research, research could aim at looking for modifications of the model leading to a negative impact of tax competition on growth. Moreover, one could try to compare tax competition to a coordinated tax policy in the endogenous-growth framework. Given the algebraic complexities of our model, however, we conjecture that this will be possible only for specific assumptions about the parameters of the model, in particular a logarithmic utility function, which implies an elasticity of intertemporal substitution of unity.

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Appendix

Derivation of (18).

Assuming that the integral in (16) is finite, the growth rate of the integrand must be negative. Then the integral can be rewritten:

$$W^L = \frac{\left[(gf'+(1-s)\theta)K_0\right]^{1-\frac{1}{\sigma}}}{\left(1-\frac{1}{\sigma}\right)\left(\delta-\left(1-\frac{1}{\sigma}\right)(I_1-m)\right)} - \frac{1}{\delta\left(1-\frac{1}{\sigma}\right)}.$$

Taking first derivatives with respect to s and θ and noting that $g=\sigma\theta$ (equation (5')), we have

$$\frac{\left[(gf'+(1-s)\theta)K_0\right]^{-\frac{1}{\sigma}}K_0\theta(gf''+f'-1)}{\delta-\left(1-\frac{1}{\sigma}\right)(I_1-m)} + \frac{\left[(gf'+(1-s)\theta)K_0\right]^{1-\frac{1}{\sigma}}\frac{\partial I_1}{\partial s}}{\left(\delta-\left(1-\frac{1}{\sigma}\right)(I_1-m)\right)^2} = 0 \quad (A1)$$

$$\frac{\left[(gf'+(1-s)\theta)K_0\right]^{-\frac{1}{\sigma}}K_0(s(gf''+f'-1)+1)}{\delta-\left(1-\frac{1}{\sigma}\right)(I_1-m)} + \frac{\left[(gf'+(1-s)\theta)K_0\right]^{1-\frac{1}{\sigma}}\frac{\partial I_1}{\partial\theta}}{\left(\delta-\left(1-\frac{1}{\sigma}\right)(I_1-m)\right)^2} = 0 \,(A2)$$

Combining (A1) and (A2) yields

$$\frac{\theta(gf''+f'-1)}{s(gf''+f'-1)+1} = \frac{\dfrac{\partial I_1}{\partial s}}{\dfrac{\partial I_1}{\partial\theta}} \qquad (A3)$$

To determine $\dfrac{\partial I_1}{\partial\theta}$ and $\dfrac{\partial I_1}{\partial s}$, substitute (2a) for F_K in (13') and note that $g=s\theta$ according to (5'). Then taking derivatives and re-substituting from (13') to eliminate the square-root term in the denominator yields

$$\frac{\partial I_1}{\partial s} = \frac{-\theta gf''}{b(r+m-I_1)} \qquad (A4)$$

and

$$\frac{\partial I_1}{\partial\theta} = \frac{-sgf''-1}{b(r+m-I_1)}, \qquad (A5)$$

Using (A4) and (A5) in (A3) yields

$$\frac{\theta(gf''+f'-1)}{s(gf''+f'-1)+1} = \frac{\theta gf''}{sgf''s+1}. \qquad (A6)$$

Simple calculus then leads to $f'=1$. Thus, condition (18) has been derived.

Derivation of (19).

Rearranging terms in (A2) and using $f'=1$ yields

$$\left(sgf''+1\right)\left(\delta-\left(1-\tfrac{1}{\sigma}\right)\left(I_1-m\right)\right)=-\left[\left(gf'+(1-s)\theta\right)_0\right]\frac{\partial I_1}{\partial \theta}$$

Use (A5) to substitute for $\dfrac{\partial I_1}{\partial \theta}$. Noting that $sgf''+1$ cancels out, we have:

$$b\left(\delta-\left(1-\tfrac{1}{\sigma}\right)\left(I_1-m\right)\right)=\frac{g+(1-s)\theta}{r+m-I_1}$$

From $g=s\theta$, (19) then follows immediately.

Properties of (22).

From (23a), we have

$$r=F_K-m \quad \Leftrightarrow \quad b=0, \quad r<F_K-m \quad \Leftrightarrow \quad b>0$$

$$r\to+\infty \quad \Rightarrow \quad b\to-0, \quad r\to-\infty \quad \Rightarrow \quad b\to+0$$

Taking the derivative in (24a), noting that $dI_1/dr=\sigma$ yields

$$\frac{db}{dr}=-2\frac{\left((1-\sigma)r+\sigma\delta\right)^2+(r+m)^2+2(F_K-m-r)\left[(1-\sigma)\left((1-\sigma)r+\sigma\delta\right)+r+m\right]}{\left[\left((1-\sigma)r+\sigma\delta\right)^2+(r+m)^2\right]^2} \quad \text{(A7)}$$

For $db/dr=0$, we get a quadratic equation in r, which implies that $b(r)$ has two extrema, a maximum for $r<F_K-m$ and a minimum for $r>F_K-m$.

Properties of $\theta(b)$ for $\sigma<1$ (Case B).

The general properties of $\theta(b)$ cannot be determined algebraically. E.g. it can be shown that $d\theta/db=0$ is a highly complex polynomial of degree 3 which cannot be solved in general. Therefore, we restrict the investigation to particular values of b, namely 0, b_0, and b^{max}.

- Let $b=0$. Then $\theta=0$ and $d\theta/db>0$.

- Let $b\to b^{max}$. If $(1-\sigma)r+\sigma\delta>0$ and $b\to b^{max}$, then $d\theta/db\to-\infty$. If $(1-\sigma)r+\sigma\delta<0$ and, then $d\theta/db\to-\infty$.

- Let $b=b_0$. This is the value of b for which $(1-\sigma)r+\sigma\delta=0$. Using this in (A7) yields

$$\frac{db}{dr}=\frac{-2(r+m)^2-4(F_K-m-r)(r+m)}{(r+m)^4}=\frac{-2(r+m)(2F_K-m-r)}{(r+m)^4}$$

Since $2F_K-m-r$ is always positive, this implies that $db/dr<0$ if $r+m>0$ and $db/dr>0$ if $r+m<0$. In the former case, we are on the decreasing segment of the $b(r)$ curve depicted in Figure 2, in the latter case on the increasing segment, which cannot be an equilibrium. From $r+m>0$ and $(1-\sigma)r+\sigma\delta=0$, we have that $\sigma<m/(m+\delta)$. Thus, in

order to have a zero-tax equilibrium at a positive level of installation cost, the elasticity of intertemporal substitution must be less than this critical value. For larger values of σ, such an equilibrium is not feasible.