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Dynamic Tax Competition and Public-Sector Modernisation

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This paper addresses the question whether increased mobility of capital enhances public-sector modernisation. Public-sector modernisation is modelled as the accumulation of knowledge (or another accumulated production factor) that serves as an input in the government's production of a consumption good. The public-sector provides a direct transfer to households. The tax competition model in the background is a dynamic model in which capital flight induced by taxation is a process that takes time. The speed with which firms can relocate capital to other jurisdictions is taken as a measure of the degree of capital mobility. The main result of the paper is a contradiction of the idea that the competitive pressure caused by increased capital mobility enhances public sector modernisation.

Keywords: public-sector modernisation, dynamic tax competition, imperfect capital mobility

JEL-Classification: H11, H77, H54, O40

1 Introduction

Competition for mobile resources can lead to an inefficiently low provision of public goods and can therefore be harmful for the wellbeing of nations. This is the major lesson to be drawn from the tax-competition literature surveyed by Wilson (1999). However, there are circumstances under which the underprovision result does not hold. For example, if the public sector's activities involve the waste of ressources - caused by rent-seeking politicians or by an inefficiently operating bureaucracy - tax competition can be beneficial as it promotes public-sector modernisation. See, for example, Edwards & Keen (1996), Rauscher (2000) and Keen & Kotsogiannis (2003) for models in which the public sector is seen as such a Leviathan that needs to be tamed.

This paper takes a different look at the efficiency of the public sector. Improvements in public-sector efficiency are modelled as reducing its reliance on tax revenue for the provision of public services. In the model presented below, there is no wasteful behavior of the public bureaucracy that needs to be repelled. The public sector's main task in this model is to

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provide a redistributive transfer to households. But this is not as simple as raising tax revenue and giving it away to a group of the society that has been selected by the political process. Instead, I model redistribution as a production process where current revenue is only one input. The efficiency of the public sector's production technology depends on the availability of "public sector efficiency" that is modelled similar to an input. "Public sector efficiency" is the result of past investment of the public sector and could be IT technology, knowledge or some other stock that is necessary to fulfill the task of providing a transfer to a target group. As public sector efficiency is modelled as a stock, a local government has control over its evolution. It decides about the investment in the efficiency of the production process, given the revenue it raises.

When public-sector modernisation is seen as an investment activity as in this paper, higher capital mobility is not helpful to improve public-sector efficiency. When capital is mobile, firms are able to shift capital to other jurisdictions. This loss of the tax base is the major disadvantage of capital taxation in open economies. Of course, the severity of this negative effect depends on the degree of capital mobility. Capital mobility in this paper weakens the ability of local governments to raise tax revenue that is needed for beneficial tasks such as the provision of redistribution and public-sector modernisation. The exact meaning of this statement will be developed below.

The aim of the paper is not only to express some doubt concering the "Leviathan wisdom" (Keen & Kotsogiannis, 2003) about the benefits of tax competition when the public sector has a "grabbing hand".[1] Another aim is to present the use of a particular dynamic modelling framework based on Wildasin (2003). It allows the introduction of dynamics in an analytically solvable model of interjurisdictional competition. Furthermore it allows to characterise different degrees of capital mobility.[2] Both the accumulation of public-sector knowledge and the re-location of capital in response to a change in capital taxation are dynamic processes and therefore a dynamic framework seems to be appropriate.

Wildasin (2003) presents a dynamic version of the "canonical" tax-competition model and analyses the dynamic reaction of the local capital stock to a change in capital taxation. Whether the reaction of firms is immediate or not depends on the convexity of an adjustment-cost function that is common in macroeconomic models. Wildasin's dynamic model provides some support for static models of tax competition. The long-term effects in the dynamic model are similar to those known from static models. However, the decision whether to tax capital or not, and at which rate, differs under imperfect capital-mobility. When it takes time for capital to flee a jurisdiction, the trade-off between capital-taxation and the loss of tax base is altered. The adjustment speed in Wildasin (2003) can serve as a reasonable measure of the degree of capital mobility. This allows to consider not only the polar cases of autarky compared with perfect integration but also the more realistic intermediate cases.

[1] There are a number of arguments both in favor and against the view that tax competition might be favourable for the efficiency of the public sector. In Wilson (2005), for example, the mobility of the tax base is unambiguously welfare-improving because the Leviathan in his model is identified as a bureaucracy that has an interest to strengthen the tax base when capital is more mobile. Cai & Treisman (2005) are more pessimistic as they argue that poorly endowed governments do not engage in tax competition at all as they anticipate that they are not able to attract capital. Hence, tax competition cannot be a discipline device for these governments. In Cai & Treisman (2004), competition between jurisdiction weakens the central government that could otherwise eliminate the disadvantages of interjurisdictional competition.

[2] There are surprisingly few models in the tax competition literature that deal with imperfect factor mobility, with Lee (1997) being one of the exceptions. Public sector modernisation, growth and tax competition is also analysed in Rauscher (2005), albeit in an endogenous growth model and with an alternative modelling of imperfect capital mobility. He finds that the effect of increased capital mobility on growth and on the behaviour of a Leviathan is ambiguous.

Modelling public-sector modernisation as an dynamic investment problem as in this paper introduces another consideration a government needs to take into account when deciding about the optimal tax structure. Investing in the stock of efficiency means that future redistribution becomes more efficient. To discuss intertemporal considerations like this, it is necessary to use a dynamic model as it is presented in this paper.

The plan of the paper is as follows. Section 2 presents the setup of the model, including the dynamic approach to capital mobility and tax competition. Section 3 analyses the decision problem the government faces in this dynamic environment. The following result are derived: the optimal input mix in the production of redistribution, for the comparative dynamics of public sector efficiency and of local capital, for the optimal tax rate given a certain degree of capital mobility, and for the relationship between capital mobility and public sector efficiency. Section 4 provides some numerical examples and section 5 concludes the paper.

2 The model

2.1 Households

Let us consider a federation with many small jurisdictions. A single jurisdiction cannot influence decisions in other jurisdictions. Each jurisdiction is inhabited by an immobile representative household that has no market power. The household's budget constraint is

$$C(t) = w(t) + G(t) + A(t)r - S(t), \tag{1}$$

where $C(t)$ is consumption of a numeraire good, $w(t)$ is the wage rate, $G(t)$ is a redistributive transfer via a consumption good provided by the government and t is an index of time. Labour is inelastically supplied in a perfectly competitive labour-market and the number of households is normalized to one. There is an international capital market where a stock of accumulated savings $A(t)$ earns a return of r, expressed in terms of the numéraire good. $A(t)$ is a stock of financial capital. All agents take the interest rate r as given and I assume that $r \geq 0$. Capital is supplied by an integrated world capital market consisting of the accumulated savings of all countries. The federation is a small country compared to the rest of the world. Therefore, $A(t)$ represents the part of the world capital stock that is held by domestic residents. To simplify the model, I assume that domestic households hold only shares of foreign firms and that local firms are exclusively owned by foreigners.[3] $S(t)$ represents current savings.

2.2 Government

The role of the government is to provide a redistributive transfer $G(t)$. This is more complex than simply to redistribute tax revenue from the foreign-owned firms to the worker-household: $G(t)$ is *produced* with a Cobb-Douglas-Technology

$$G(t) = R(t)^{\gamma} H(t)^{1-\gamma} \quad \text{with } 0 < \gamma < 1 \,, \tag{2}$$

where $H(t)$ is a stock of knowledge and $R(t)$ a flow of revenues devoted to the production of redistribution. Technology (2) plays a central role in this model. $G(t)$ is a lump-sum transfer

[3] Alternatively I could introduce foreign and domestic shares. I will discuss this and other assumptions in the conclusion.

to households and can also be labeled redistribution.[4] The level of $G(t)$ a government can provide does not only depend on the tax revenue $R(t)$ devoted to redistribution, but also on the stock of $H(t)$.

In an ideal world, a government that provides redistribution simply diverts revenue taken from one group of citizens to another group of citizens that has been defined as a target group. One Euro raised implies that there is one Euro that can be passed on to the target group. This would be a perfectly efficient redistribution technology. The idea behind technology (2) is that in reality, redistribution is a complex procedure that can be modelled as a production process. Redistribution involves flow inputs of labour, for example public servants that process applications for social assistance. And it also involves stock inputs, for example IT-technology or knowledge about good practices to provide redistribution according to a politically defined goal. A government that produces redistribution with a technology like (2) has - similar to a private firm that produces goods - to decide about an optimal input mix between stocks and flows. It needs to decide about an investment strategy, where investment is broadly defined and includes training of public servants or the establishment of rules and procedures in the bureaucracy.

$G(t)$ and labour income are perfect substitutes. One could see technology (2) also as a way to model situations where the public sector provides a private good that is a close substitute to private consumption. Examples are public education or public health care. They are rival in consumption and can also be provided privately. Another way to interpret (2) is to see it as the technology of tax collection. $H(t)$ would then be the effectiveness of the public sector to raise tax revenue.

Public sector efficiency in this paper is measured by the level of $H(t)$. It can be interpreted as the knowledge of the public sector about efficient technologies to transform tax revenue into a transfer. Without any knowledge, i.e. $H(t) = 0$, the public sector is a black hole in which tax revenues vanish without any benefits for the citizens. A high level of $H(t)$ can be seen as an indicator of a very efficient public sector where at least the first units of tax revenue generate very high marginal benefits for the household. The development of $H(t)$ over time reflects the development of public sector efficiency. I assume throughout the paper that parameters are such that the efficiency of the public sector is not above the efficiency frontier ($G \leqslant R$).[5]

Tax revenue is used to provide a redistributive transfer and to modernise the public sector. It is assumed that the government cannot tax the immobile factor labour, but imposes a source tax τ per unit on the local capital stock $k(t)$. To keep the model simple, public debt

[4] A referee pointed out that redistribution in this model is not across consumers but from foreign capital owners to local households.

[5] The usage of an input as a measure of efficiency is a bit unusual. Think of a general technology $G = f(R)$, where G is one output, R is an input. Productivity is then measured as G/R. A more productive production unit is also more efficient. In this paper, where R is tax revenue and G a transfer, the efficiency border is naturally given by $R = G$. Hence, the productivity of the public sector and its distance from the efficiency border are related in a simple way. The reason for efficiency differences is usually that different technologies are used. The idea behind (2) is that the state of technology behind the transformation of R into G is under the control of the public sector. The state of technology is $H^{1-\gamma}$ and modelled similar to an input. The assumption that parameters are such that the public does not produce above its efficiency frontier means that I assume that the level of H is always low enough to ensure that $G(t) < R(t)$. Efficiency is *not* defined as the best possible way to use R *and* H for the production of G.

is ruled out[6] and therefore, the government's budget constraint is

$$\tau k(t) = R(t) + M(t) \ , \tag{3}$$

where $M(t)$ is the investment in public efficiency or the modernisation effort at time t. Current public expenditure is financed by current capital tax revenue. Public sector efficiency develops according to

$$\dot{H}(t) = M(t) = \tau k(t) - R(t) \quad \text{with } H(0) = H_0 \geq 0 \text{ given} \ . \tag{4}$$

Initially, the public sector's efficiency is H_0. If $H_0 = 0$, modernisation, i.e. accumulation of $H(t)$, is a prerequisite for redistribution. It is assumed, again for the sake of simplicity, that the public sector does not forget technologies and procedures to transform tax revenue into redistribution it has previously known. Thus, there is no depreciation in (4).

2.3 Private firms and the dynamics of taxation and capital accumulation

In the local jurisdiction there are many identical firms. The representative firm takes prices and decisions by the government as given. This firm produces with a constant-returns-to-scale production function $F(k(t), L)$. The decision to hire labour and capital is dominated by the aim to maximise the current value of future cash flows. Labour is normalised to one such that the production function can be written as $f(k(t))$. The wage rate is determined in a competitive labour market. It depends on the current capital stock only and is given by

$$w(t) = f(k(t)) - f'(k(t))k(t) \ . \tag{5}$$

The development of the local capital stock $k(t)$ depends on the investment rate $i(t)$ chosen by the representative firm. The local capital stock evolves according to

$$\dot{k}(t) = (i(t) - \delta)\, k(t) \ \text{ with } k(0) = k_0 \geq 0 \text{ given,} \tag{6}$$

where δ is the depreciation rate.

The alternative investment opportunity for the local firm is to invest in financial capital A which bears an interest at rate r. The interest rate is assumed to be exogenous to the jurisdiction. Hence, the objective of the firm is to maximize the present value of profits with r being the discount rate. The cash flow at time t is

$$\pi(t) = f(k(t)) - c\,(i(t))\, k(t) - \tau k(t) - i(t)k(t) - w(t) \ \ , \tag{7}$$

with $c(i)$ being an investment cost function. The objective of the firm is

$$\max \ \Pi = \int_0^\infty \pi(t) e^{-rt} dt \ \ , \tag{8}$$

subject to (6). Profits and therefore investment depend on the local tax rate on capital, τ, which is assumed to be constant. From a technical point of view, τ is a parameter in the firm's optimization problem. Another important determinant of the firm's decision about the local capital stock is the investment cost function $c(i)$. It is assumed to be convex and its

[6] The assumption that the public sector has no access to the international capital market is meant to reflect borrowing constraints governments face. An example is the Europan Growth and Stability Pact or the common practice of states in the U.S. to impose constitutional bans on borrowing. I see the assumption of no access at all as a simple way to incorporate such restrictions into the model.

curvature will turn out to be crucial for the speed of a firm's reaction to a variation in the capital tax rate. To simplify the notation, assume the following quadratic specification of investment costs:

$$c\,(i(t))\,k(t) = \frac{b}{2}i(t)^2k(t), \tag{9}$$

where $c(0) = 0$, $c' = bi(t)$, $c'' = b$. Parameter b can be interpreted as a measure for the mobility of capital. The lower b, the cheaper it is to adjust the capital stock. The total costs of investing one unit of capital are $i(t)\,(1 + c\,(i(t))\,k(t)) = i(t)\left(1 + \frac{b}{2}i(t)^2k(t)\right)$.

Employing the Maximum Principle, and making use of the functional form for the investment cost function, the process of capital accumulation of the firm can be described as follows:[7]

$$\dot{k}(t) = \left(\frac{\lambda(t)-1}{b} - \delta\right)k(t) \tag{10a}$$

$$\dot{\lambda}(t) = -f'(k(t)) + \tau + \lambda(t)\,(r+\delta) + \frac{(\lambda-1)^2}{2b} \tag{10b}$$

$$k(0) = k_0 > 0 \text{ is given;} \quad \lim_{t\to\infty}\left(\lambda(t)e^{-rt}k(t)\right) = 0\ . \tag{10c}$$

λ is the costate variable associated with capital. The first-order condition for investment, $\lambda(t) = 1 + c'\,(i(t))$, has been used to express investment as a function of λ and the investment cost parameter b. Investment is $i(t) = \frac{\lambda(t)-1}{b}$. The boundary conditions in (10c) are standard.

(10) is a system of ordinary differential equations. Its solution $\{k(t,\tau), \lambda(t,\tau)\}$ depends on the parameter τ. The additional argument τ will be omitted if this does not cause any confusion in the remainder of the paper. Given all parameters and the initial value of $k_0 = k(0)$, the local capital stock grows towards a steady state $\{K_{SS}(\tau), \lambda_{SS}(\tau)\}$, which is defined by

$$\delta b + 1 = \lambda_{SS} \tag{11a}$$

$$\tau + (\delta b + 1)\,(r+\delta) + \frac{\delta^2 b}{2} = f'(k_{SS})\ . \tag{11b}$$

The setup of the private sector is very similar toWildasin (2003), where the focus is on the comparative-dynamic response of the local capital stock to taxation. A key insight of Wildasin (2003) is that when capital is modelled as a stock variable, the elastic reaction of the local capital stock to a change of the capital tax rate evolves over time. Wildasin finds that a convex investment cost function implies that the tax base capital is not perfectly elastic as it is assumed in most static models of tax competition. The scenario in Wildasin (2003) and also in this paper is the following: At time $t = 0$, the government sets a time-invariant tax rate τ which comes as a surprise to the private sector.[8] Furthermore, it is assumed that the government can commit itself to its policy announcements. Raising the capital tax rate drives capital out of the jurisdiction as local firms have an incentive to disinvest. When this adjustment of the local capital stock in response to capital taxation is immediate, the mobility of (physical) capital is perfect.[9] If capital is not perfectly mobile, firms face

[7] See Becker (2007) for a step-by-step derivation.

[8] One could argue that real-world governments decide about a path of tax rates. While this might be true, a model with a time-invariant tax rate generates dynamics that can be analysed by making use of the Peano Theorem about the comparative dynamic response of a dynamic system to a variation of a parameter. This will be shown below.

[9] There are no constraints on the mobility of financial capital. The adjustment of the stock of financial capital is free of cost. See, for example, Persson & Tabellini (1992) or Gordon & Bovenberg (1996) for models where capital mobility costs are associated with the investment abroad. In the present model, investment is either local (in physical capital k) or it is investment in the federation-wide market for financial capital.

the following trade off: When the new tax rate is higher than initially, local capital is less profitable compared with the external rate of return r. Therefore, they find it profitable to de-accumulate until the net return of local capital equals the external rate of return. On the other hand, de-accumulation (a negative rate of investment) raises adjustment costs. If the investment-cost function is convex, quick adjustments of the capital stock are more expensive per unit than slow adjustments.

The immobility of capital is an important determinant of the optimal tax policy. Assume for simplicity that firms are owned only by foreigners. The inhabitants of the local jurisdiction are workers. The local government tries to raise tax revenue that it can then redistribute to workers. Tax revenue is a prerequisite for redistribution. Furthermore, in this model, tax revenue is also needed to improve the efficiency of the public sector. On the other hand, capital taxation implies a lower local capital stock, lower marginal productivity of labour and therefore lower wages. If the adjustment of the capital stock in response to capital taxation is not immediate, the benefits of taxation occur immediately but the disadvantages need time to take effect. In a dynamic model, this is an elegant way to model imperfect capital mobility.

Trading off the present values of benefits and costs can result in positive capital tax rates even if non-distorting lump-sum taxes are available, as has been shown in Wildasin (2003). The reason is that the tax burden is shifted away from worker-households and towards foreign owners of the capital stock temporarily. While the economy adjusts, the government extracts quasi-rents from foreign capital owner.

A key result in Wildasin (2003, eqs. (5), (6)) that will be derived for the specific functional forms in this paper later on, is

$$\frac{dk(t)}{d\tau} = \frac{1}{f''(k)}\left(1 - e^{\rho_2 t}\right) , \tag{12}$$

with $\rho_2 < 0$ representing the speed of adjustment.[10]

This is the comparative-dynamic response of the local capital stock to a change in the capital tax rate τ. When time goes to infinity, the response of the local capital stock to a change in the tax rate is the inverse of the second derivative of the production function with respect to capital, a result that is familiar from static models of tax competition. See, for example, Zodrow & Mieszkowski (1986, eq. 5). For $0 < t < \infty$, the comparative-dynamic response to a change in the tax rate is less, depending on the magnitude of ρ_2. For the derivation of (12), it has been assumed that the starting value for capital, k_0 happens to be a steady state value. The need for this assumption will be explained later on, but let us introduce some notation: Initially, the capital stock has a value of k_0 that is a steady state. There might be an historically given tax rate on capital τ such that the initial steady state can be denoted by $\{k_{SS}(\tau), \lambda_{SS}(\tau)\}$, where λ is the costate variable associated with capital. The local government decides in time $t = 0$ about a new tax rate on capital. The new tax rate will be denoted by $\tilde{\tau}$. If $\tilde{\tau} \neq \tau$, the local capital stock will adjust from the initial steady state to another one, denoted by $\left\{\tilde{k}_{SS}(\tilde{\tau}), \tilde{\lambda}_{SS}(\tilde{\tau})\right\}$.

Further on, the parameter b in the adjustment cost function is used as a measure for capital mobility as it measures the speed with which the response of the local capital stock to a change in the tax rate becomes effective. More convex investment cost functions imply higher values of b and values of ρ_2 that are smaller (shown in detail later on). The investment cost function can be seen as a punishment of quick adjustments. High investment or de-investment implies higher than proportional investment costs and therefore it is profitable to

[10] At time $t = 0$, $\frac{dk(t)}{d\tau} = 0$. For $t \to \infty$, $\frac{dk(t)}{d\tau} = \frac{1}{f''(k)}$. The absolute value of ρ_2 determines the speed of the transition of $\frac{dk(t)}{d\tau}$ from 0 to $\frac{1}{f''(k)}$.

adjust the capital stock only gradually, in a process that takes time. A more pronounced punishment is represented by a greater value of b and a value of ρ_2 that is smaller. If $\rho_2 \to 0$, capital is immobile. If $\rho_2 \to -\infty$, capital is perfectly mobile. When adjustment costs per unit of investment are strongly convex, firms are reluctant to adjust the local capital stock immediately as this would imply very high adjustment costs.[11]

The structure of the model in this paper is such that the comparative dynamics of the local capital stock are similar to (12). The speed of capital-stock adjustment will be crucial for the results in this paper, where the government uses the revenue of capital taxation for redistribution and for public sector modernisation. The solid line in Figure 1 shows the adjustment of the local capital stock in response to a change in the tax rate on capital from τ to $\tilde{\tau}$ with $\tau < \tilde{\tau}$ and a convex investment cost function ($b > 0$).

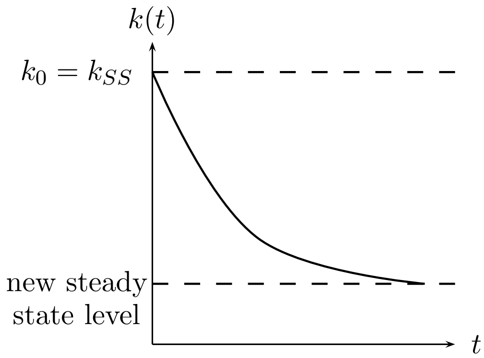

Figure 1: Comparative-dynamic response of the capital stock for $\tau < \tilde{\tau}$ and $b > 0$

A local government solves the problem of optimally exploiting (foreign) capital owners. Capital owners can defend themselves against exploitation because they can adjust the local capital stock. This in turn needs to be considered by a local government as the wage rate of its inhabitants depends on the capital stock. The other decision the local government needs to make is how to split the revenue extracted from foreigners between modernisation and redistribution.

Local governments are engaged in tax competition as they have to consider that the tax base capital depends on their decisions. Raising the tax on local capital triggers an outflow of capital. Many model of tax competition deal with the Nash equilibrium in a federation where tax rates are too low a long as the positive externality of capital outflows is not corrected. In this model, the local jurisdiction is assumed to be small such that the federation-wide interest rate does not depend on the decisions of the local government. Hence, there is no externality by definition. Nevertheless, local jurisdictions have an incentive to set tax rates that attract capital, a scenario that is commonly seen as one of tax competition.

This completes the setup of the model.

[11] A potential problem with Wildasin's approach is that this adjustment cost function can be applied to closed economies as well. One might argue that adjustment costs should therefore not be used as a microfoundation for imperfect capital mobility that instead could to be caused by barriers to cross-border capital flows, see for example Persson & Tabellini (1992) or Lejour & Verbon (1997). In this paper, it is assumed that it is costless to borrow and invest in the international capital market. Nevertheless, capital is not perfectly mobile in the sense that the process of capital flight is time-consuming.

Another possibility is to model the investment decision of the firm with investment in a second capital stock abroad as the alternative investment opportunity. Investment abroad then can be associated with a cost function similar to an adjustment cost function but representing the cost of overcoming barriers to capital. The resulting response of a firm would be very similar to the one illustrated in Figure 1.

3 The intensity of tax competition and public-sector modernisation

3.1 The problem

The government's policy instruments are τ, $M(t)$ and $R(t)$. It needs to take the technology (2), the budget constraint (3) and the equation of motion for public-sector efficiency (4) and the evolution of the local capital stock into account. The objective is to maximise the lifetime income of the jurisdiction's citizens. The optimisation problem therefore is

$$\max_{\tau, R(t)} \int_0^\infty \left(w(t) + R(t)^\gamma H(t)^{1-\gamma} \right) e^{-rt} dt \,, \tag{13}$$

subject to (4) and (10a)-(10c). The present value of the household's income from saving over time, $\int_0^\infty (A(t)r - S(t))\, e^{-rt} dt$, is not part of the objective function. The world interest rate serves as discount factor, as households can borrow and lend on the international capital market. The government controls the supply of redistribution and (indirectly, via the capital stock) also the wage rate. Whatever the paths of $w(t)$ and $G(t)$ are, households will adjust their savings to maximise their lifetime utility of consumption. In this model, the government has no interest to intervene in the intertemporal consumption decision but maximises the income that is distributed over time to the household. Using this objective function instead of the discounted lifetime utility from consumption will simplify the algebra considerably and has the additional advantage that the government's decisions do not rely on the knowledge of the household's utility function.

The assumption that the income from assets and the saving decision of households is not part of the objective function may sound a bit unusual. In many growth models, it is the discounted utility of households that is maximised. In a model with an externally given interest rate, the best possible lifetime utility from consumption is achieved with the highest possible lifetime income. The path of consumption is irrelevant. A benevolent government has no reason to influence the time path of consumption as it has no influence on the external interest rate by assumption. A benevolent government tries to achieve the highest possible lifetime-income for its citizens and it can do so by maximising the lifetime-income from redistribution and work as it is assumed in (13). In a model without a fixed interest rate, this would be different. A government could try manipulate the time path of interest rate and investment activity in order to achieve a higher life-time utility, for example if saving and investment generates external effects not taken into account by households. This is not possible in this model where any investment project is evaluated against the alternative investment in assets bearing a fixed interest rate determined on the world market.

The assumption of an externally given and fixed interest rate is critical in this model. Note that with a utility function that implies consumption smoothing additional transfers to households will be used for both current consumption and savings. There is an additional supply of savings in the international capital market. The assumption of small jurisdictions means that the governments' decision to manipulate households income will not change supply and demand conditions in the international capital market substantially and that the market clearing interest rate is unchanged. The model is solved from the perspective of a single jurisdiction, given external conditions on the world capital market. I do not solve for the equilibrium of a system of identical jurisdictions. If jurisdictions were symmetric, all jurisdictions would set the same tax rate simultaneously, implying that the relative attractiveness of jurisdictions would not change and *ex post*, capital would not flee the country when the tax rate is raised. In a symmetric model, this would imply that the market-clearing interest rate would be lowered by the simultaneous decision to change capital taxation. The assumption that jurisdictions are small means that the decision of the individual government have no influence on the demand and supply conditions in the international capital market, neither ex ante nor ex post.[12]

3.2 Public sector efficiency, the dynamics of the local capital stock and optimal taxation

The current-value Hamiltonian $\mathcal{H}$ for the government's decision problem is

$$\mathcal{H} = w(t) + R(t)^{\gamma} H(t)^{1-\gamma} + \mu(t)\left(\tau k(t) - R(t)\right). \tag{14}$$

Hestenes' Theorem[13] states that the following conditions hold for an optimal policy $\{\tau, R(t)\}$:

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial R(t)} &= \gamma R(t)^{\gamma-1} H(t)^{1-\gamma} - \mu(t) = 0 \,, && \text{(15a)}\\
\dot{\mu(t)} &= \mu(t) r - \frac{\partial \mathcal{H}}{\partial H(t)} = \mu(t) r - (1-\gamma)\left(\frac{R(t)}{H(t)}\right)^{\gamma} , && \text{(15b)}\\
\frac{d}{d\tau}\int_0^{\infty} w(t) e^{-rt} dt &= -\frac{d}{d\tau}\int_0^{\infty} \Big(R(t)^{\gamma} H(t)^{1-\gamma} + \mu(t)\left(\tau k(t) - R(t)\right)\Big) e^{-rt} dt \,, && \text{(15c)}\\
H(0) &= H_0 > 0 \text{ is given;} \quad \lim_{t\to\infty} \Big(\mu(t) e^{-rt} H(t)\Big) = 0 \,, && \text{(15d)}
\end{aligned}$$

together with (10) and (4).

Before the determination of the optimal tax rate from (15c), I solve for the dynamics of the model for a given tax rate τ. It is necessary to find closed-form solutions for the comparative-dynamic response of the state variables to a change of the tax rate before the integrals in (15c) can be calculated. Hence, the next step is to analyse the dynamic system that is described by (15a), (15b) and (15d).

From (15a), it follows that

$$R(t) = \left(\frac{\mu(t)}{\gamma}\right)^{\frac{1}{\gamma-1}} H(t). \tag{16}$$

This is the optimal flow of revenues devoted to the production of $G(t)$. Using (16) in (4) and (15b) results in the following non-linear and non-homogeneous system of differential equations:

$$\begin{aligned}
\dot{H(t)} &= -\left(\frac{\mu(t)}{\gamma}\right)^{\frac{1}{\gamma-1}} H(t) + \tau k(t) \,, && \text{(17a)}\\
\dot{\mu(t)} &= r\mu(t) - (1-\gamma)\left(\frac{\mu(t)}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} , && \text{(17b)}
\end{aligned}$$

[12] The endogeneity of the interest rate is considered, for example, by Makris (2005). He considers whether decentralised decision making implies too low or too high capital tax rates and what the coordinated tax rate should be. Another paper where the interest rate effect of tax competition between indentical jurisdictions is considered is Becker & Rauscher (2007).

[13] As τ is time-invariant, it is not a control-variable in the maximisation of (14) but a "control parameter". For this kind of policy variables, Hestenes' Theorem applies, see Takayama (1985, ch. 8.C). For the same reason, the equations of motion for $k(t)$ and $\lambda(t)$ (10) are not incorporated in the Hamiltonian. They are constraints for the local government's optimization problem, but for any given parameter value of τ, the expression $\tau k(t)$ could in principle be substituted by the solution of (10) that is independent of the choice of the control variable $R(t)$.

where $k(t)$ is given by (10). Equation (17b) is can be reduced to the linear form and therefore easily solved.[14] Applying the appropriate border condition $\lim_{t\to\infty}\left(\mu(t)e^{-rt}H(t)\right) = 0$ reveals[15] that $\mu(t)$ is a constant:

$$\mu(t) = \gamma\left(\tfrac{1-\gamma}{r\gamma}\right)^{1-\gamma} \quad . \tag{18}$$

Together with (16), this yields

Proposition 1 (constant optimal input mix) *The optimal mix $R(t)/H(t)$ for the provision of the publicly provided consumption good $G(t)$ is*

$$\frac{R(t)}{H(t)} = \frac{r\gamma}{1-\gamma} \ . \tag{19}$$

and constant over time. There are no transitional dynamics for the input mix.

It is a standard result that for a given price, the input mix is constant (linear exapnsion path). In this model, the relative price of the inputs R and H is also constant, both in a steady state and in a transition phase. The local government always chooses some optimal input mix that reflects a trade-off between the future and the presence. A high return of households' savings in the international capital market makes investment in future benefits, i.e. in public-sector efficiency, relatively unattractive. This implies a relatively low level of $H(t)$ in the optimal input mix. A high output elasticity of current tax revenue $R(t)$ has a similar effect.

Using (18) in (17a) simplifies the equation of motion for public sector efficiency a lot:

$$\dot{H}(t) = -\frac{r\gamma}{1-\gamma}H(t) + \tau k(t) \ . \tag{17a'}$$

The remaining problem is that the path for $k(t)$ is unknown. Hence, the dynamic system that describes the evolution of public sector efficiency consist of three differential equations for $H(t)$, $k(t)$ and the costate variable for capital, $\lambda(t)$. It has non-constant coefficients. There exists a steady state for this system consisting of (17a'), (10a) and (10b) when time goes to infinity. One possible solution method would be to linearise around this steady state. The disadvantage of this approach would be that the solution would be valid only locally, in the neighbourhood of the steady state. To avoid this limitation and to achieve results that are global, the method used in Boadway (1979), and also in Wildasin (2003), is employed. The idea of this method can be described as follows: Assume that the system is initially in a steady state. The system of "variational equations" which is derived by differentiation of the system of equations (17) with respect to the policy parameter τ then has *constant* coefficients. It is then possible to solve the resulting system and to calculate the response of the state variable $H(t)$ to a variation of the policy parameter τ.[16]

[14](17b) is an Bernoulli Equation, see Chiang (1984, p. 491). Thanks to an anonymous referee for pointing this out.

[15]See the appendix on page 24 for the details.

[16]This method employs the Peano-Theorem from the theory of differential equations that deals with the behaviour of a system of differential equations when a parameter – here the time-invariant tax rate – is changed, see for a textbook discussion Caputo (2005, ch.11). The assumption of an initial steady state is necessary to achieve the property of constant coefficients in the system of differential equations that describe the evolution of the state and costate variables in response to the change of a parameter. Becker (2007) contains a more detailed discussion of the Peano-Theorem in the comparative-dynamic analysis of fiscal competition models.

The system of variational equations that needs to be solved is

$$\begin{bmatrix} \frac{d\dot{k(t)}}{d\tau} \\ \frac{d\dot{\lambda(t)}}{d\tau} \\ \frac{d\dot{H(t)}}{d\tau} \end{bmatrix} = \begin{bmatrix} \frac{\lambda(t,\tau)-1}{b} - \delta & \frac{k(t,\tau)}{b} & 0 \\ -f''(k(t,\tau)) & r+\delta+\frac{\lambda(t,\tau)-1}{b} & 0 \\ \tau & 0 & \frac{-r\gamma}{1-\gamma} \end{bmatrix} \begin{bmatrix} \frac{dk(t)}{d\tau} \\ \frac{d\lambda(t)}{d\tau} \\ \frac{dH(t)}{d\tau} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ k(t,\tau) \end{bmatrix}. \tag{20}$$

To see the effect of a change in capital taxation on the efficiency $H(t)$ of the public sector and on the local capital stock $k(t)$, system (20) has to be solved for $dH(t)/d\tau$ and $dk(t)/d\tau$. Note that I do not try to find a closed-form solution for $H(t)$ or $k(t)$.

The next step to transform (20) in a dynamic system that can be solved with standard methods is to make use of the assumption that the system has been initially in a steady state. (20) describes how the dynamic system $\{k(t,\tau), \lambda(t,\tau), H(t,\tau)\}$ for a given parameter value τ reacts to a variation of the parameter.In this case to a change of τ to $\tilde{\tau}$. The remaining problem is that the coefficients of the dynamic system are not constant in general and the closed-form solution for $k(t,\tau)$, $\lambda(t,\tau)$ and $H(t,\tau)$ that corresponds to the initial value of the parameter τ is unknown. This is where the assumption of an initial steady state is useful. If the initial situation happens to be steady state, then $\{k(t,\tau), \lambda(t,\tau), H(t,\tau)\}$ is constant by assumption.

Assumption 1 (initial steady state) *The capital stock $k(t)$, its costate variable $\lambda(t)$ and public sector efficiency $H(t)$ are in a steady state in time $t = 0$. Their initial steady state values are denoted as k_{SS}, λ_{SS} and H_{SS}, respectively.*

Assumption 1 together with equation (11) allows to rewrite (20) as:

$$\begin{bmatrix} \frac{d\dot{k(t)}}{d\tau} \\ \frac{d\dot{\lambda(t)}}{d\tau} \\ \frac{d\dot{H(t)}}{d\tau} \end{bmatrix} = \begin{bmatrix} 0 & \frac{k_{SS}(\tau)}{b} & 0 \\ -f''(k_{SS}(\tau)) & r+2\delta & 0 \\ \tau & 0 & \frac{-r\gamma}{1-\gamma} \end{bmatrix} \begin{bmatrix} \frac{dk(t)}{d\tau} \\ \frac{d\lambda(t)}{d\tau} \\ \frac{dH(t)}{d\tau} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ k_{SS}(\tau) \end{bmatrix}. \tag{21}$$

(21) is a dynamic system with constant coefficients that can be solved with standard methods. Note that $\frac{d\dot{k(t)}}{d\tau}$ and $\frac{d\dot{\lambda(t)}}{d\tau}$ are independent from $\frac{d\dot{H(t)}}{d\tau}$ due to the structure of the model where the representative firm is connected with the local government only through the parameter τ.

By setting $\frac{d\dot{k(t)}}{d\tau} = \frac{d\dot{\lambda(t)}}{d\tau} = \frac{d\dot{H(t)}}{d\tau} = 0$, one can find the long-term solution of (21):

Proposition 2 (long-term effect) *In the long run ($\lim_{t\to\infty}$), the effects of a change of the capital tax rate on the local capital stock $k(t)$, its co-state variable and on public sector efficiency H are:*

$$\lim_{t\to\infty} \frac{dk(t)}{d\tau} = \frac{1}{f''(k_{SS})} , \tag{22a}$$

$$\lim_{t\to\infty} \frac{d\lambda(t)}{d\tau} = 0 , \tag{22b}$$

$$\lim_{t\to\infty} \frac{dH(t)}{d\tau} = \frac{1-\gamma}{r\gamma}\left(\tau \frac{1}{f''(k_{SS})} + k_{SS}\right) . \tag{22c}$$

(22a) is well known from static models of tax competition, see, for example, Zodrow & Mieszkowski (1986, eq. 5). It states that the tax base capital shrinks when the tax rate is raised. (22b) stems from the fact that the steady state of the co-state variable for capital is $\delta b + 1$ and therefore independent of τ, see equation (11a). (22c) states that the loss or gain of revenue that results by a change in the capital tax rate has a stronger effect on public sector efficiency the more important the role of the stock-variable $H(t)$ is for redistribution.[17]

The long-term effects of a change in the capital tax rate on public sector efficiency depends only on parameters of the production technology for redistribution, on the initial tax rate τ, and the corresponding initial capital stock k_{SS}. Investment in public-sector efficiency is costly from the perspective of the household as it is foregone current income through redistribution. The investment opportunity alternative to public sector modernisation from the households point of view is to save and earn a return r. Higher rates of interest in the international capital market therefore let a government choose a lower stock of capital in the long run. Furthermore, a high output-elasticity of $H(t)$ in the public production technology (2) makes investment in public-sector efficiency attractive and vice versa. Once the steady state level of public-sector efficiency is reached, tax revenue is entirely used as a flow input in the production of redistribution.

The eigenvalues corresponding to the Jacobi matrix of coefficients in (21) are

$$\begin{bmatrix} \rho_1 = \frac{br+2\,b\delta+\sqrt{b(br^2+4\,br\delta+4\,b\delta^2-4\,f''(k_{SS})\,k_{SS})}}{2b} > 0 \\ \rho_2 = \frac{br+2\,b\delta-\sqrt{b(br^2+4\,br\delta+4\,b\delta^2-4\,f''(k_{SS})\,k_{SS})}}{2b} < 0 \\ \rho_3 = \frac{-r\gamma}{1-\gamma} < 0 \end{bmatrix} . \tag{23}$$

There are two negative eigenvalues and one positive eigenvalue and the system therefore has the property of saddle-point stability. Applying the appropriate border conditions[18] yields

Proposition 3 (comparative dynamics) *1. The comparative-dynamic response of the local capital stock k and public sector efficiency H is*

$$\frac{dk(t)}{d\tau} = \frac{1}{f''(k_{SS}(\tau))}\left(1-e^{\rho_2 t}\right) < 0 \;\forall\, t > 0\,, \tag{24a}$$

$$\frac{dH(t)}{d\tau} = \frac{1-\gamma}{r\gamma}\left(\tau\frac{1}{f''(k_{SS})}+k_{SS}\right)\left(1-e^{\rho_3 t}\right)\,. \tag{24b}$$

2. The sign of $\frac{dH(t)}{d\tau}$ depends on the initial tax rate τ and technological parameters: If $\tau\frac{1}{f''(k_{SS})}+k_{SS} > 0$ (i.e. if the initial tax rate τ is low enough), then $\frac{dH(t)}{d\tau} > 0 \;\forall\, t > 0$ and vice versa. In the former case, raising the tax rate implies higher public sector efficiency $H(t)\;\forall\, t > 0$.

3. The initial tax rate τ, the initial capital stock k_{SS} and the initial level of public sector efficiency H_{SS} are not independent from each other as it has been assumed that the economy is in a steady state initially (Assumption 1). The initial tax rate determines the initial steady state of capital (equation (11b)). The initial value of public sector efficiency, H_{SS}, is then implicitely given by the initial revenue τk_{SS} and by setting $\dot{H}(t) = 0$ in (17a').

[17]The long-term effect on revenues is calculated as $\lim_{t\to\infty}\frac{d(\tau k)}{d\tau} = \lim_{t\to\infty}\tau\frac{dk(t)}{d\tau} + k(t)$. This can be seen as an approximation.

[18]The border conditions are $\frac{dk(0)}{d\tau} = 0$, $\lim_{t\to\infty}\frac{d\lambda(t)}{d\tau} = 0$ and $\frac{dH(0)}{d\tau} = 0$. See Oniki (1973) and Becker (2007) for a more detailed discussion about border conditions when applying the Peano-Theorem.

(24a) replicates the result for the comparative-dynamic response of the local capital stock to a change in the tax rate that has already been introduced in equation (12) on page 7. Note that ρ_2, ρ_3 do not contain endogenous variables. The speed of adjustment of $k(t)$ and $H(t)$ depends on parameters and initial values only, see (23). Capital mobility, measured by b, has no direct influence on the comparative dynamics of public sector efficiency as long as the initial capital stock k_{SS} is held constant. But – as will be shown below – it has an impact on the optimal tax rate that is chosen by local governments.

The capital tax rate chosen by the local government can in principle be both positive or negative, depending on how the local government decides in the trade-off between raising capital tax revenue for modernization and redistribution versus dispelling capital and reducing labour productivity. I will discuss the necessary conditions for a positive tax rate in more detail below.

The tax rate τ in this model is time-invariant. Technically spoken, the government sets a parameter to find the maximum of the objective function. Using the results from Proposition 3 in (15c), the optimal tax rate when capital is imperfectly mobile ($0 < |\rho_2| < \infty$ or $b > 0$) can be calculated as

$$\tilde{\tau} = \frac{k_{SS}\, f''(k_{SS})\ \left(\rho_2 r(-\rho_3)^{-\gamma} + (2-\gamma(3-\gamma))\,(r-\rho_2)\right)}{\left(\rho_2(2-\gamma) - r(1-\gamma)\right)(1-\gamma)} \quad . \tag{25}$$

See the appendix for the derivation of (25). The tax rate is labeled $\tilde{\tau}$ to make clear that this the optimal tax rate. The tax rate that is historically given is labeled τ (without a tilde). Interestingly, the tax rate $\tilde{\tau}$ does not depend on the initial value of public sector efficiency H_0. A local government with a low level of public sector efficiency does not set a higher tax rate than other governments. The initial capital stock $K_0 = k_{SS}$ is crucial for the question whether capital should be taxed (further).

The optimal tax rate τ depends on the size of the integrals in (15c). If the time needed for the transition from the initial to the new steady state changes, the size of those integrals representing the welfare effects of taxation changes and, accordingly, the optimal tax rate.

The government in this model can tax capital in order to provide redistribution to households. In addition to the trade-off between capital flight and redistribution, it has to take into account that the amount of redistribution depends on its modernisation effort. Hence the complicated role of the technical parameter γ in the tax rate formula.

The optimal tax rate is positive if the following assumption is met:

Assumption 2 *The parameter space is restricted to values that fulfill the following condition:*

$$-r\rho_3^{-\gamma} + \gamma(\gamma-3) + 2 > 0\ . \tag{26}$$

Figure 2 illustrates it for the range $0 < \gamma < 1 <$*. Interest rates below the solid line ensure that the assumption is met.*

I assume in the remainder of the paper that parameters fulfil Assumption 2. It is a sufficient but not a necessary condition to ensure positive tax rates. If it fails, positive tax rates are still possible if capital is not very mobile.

Note that

$$\lim_{\gamma\to 1} \tilde{\tau} = \frac{k_{SS} f''(k_{SS}) r}{\rho_2} > 0\ . \tag{27}$$

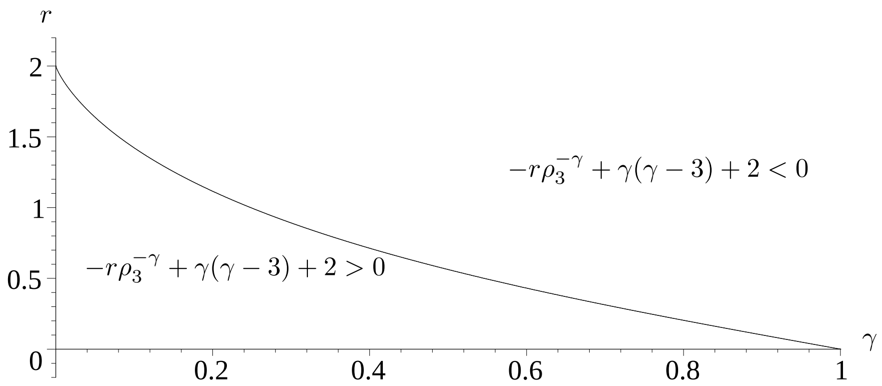

Figure 2: Condition (26) in the $\{r, \gamma\}$-Space.

With γ approaching 1, the relevance of the capital good $H(t)$ for the production of the capital good vanishes. The tax rate established in (27) is identical to the tax rate in WILDASIN (2003, eq. (9)). In Wildasin's paper, the government redistributes tax revenue directly to households. The tax rate (25) in comparison to the one in (27) reflects the additional considerations because of the need to accumulate knowledge in the public sector.

Even in the presence of perfect capital mobility ($b \to 0$ or $\rho_2 \to -\infty$), there is an incentive to tax capital:

$$\lim_{\rho_2 \to -\infty} \tilde{\tau} = \frac{-k_{SS} f''(k_{SS}) \left(-r(-\rho_3)^{-\gamma} + \gamma(\gamma-3) + 2\right)}{2 - 3\gamma + \gamma^2} \quad . \tag{28}$$

A condition for the tax rate being positive in the special case of $\lim_{\rho_2 \to -\infty}$ is again given by (26). The government does not only have to consider the trade-off between the immediate benefits of redistribution versus lower wages later on. In addition, taxation allows to invest in public sector modernisation and to achieve a higher level of redistribution in the economy. The possibility to use tax revenue to generate benefits that accrue in all following moments of time is another motivation to tax capital even if capital flight is possible immediately. It is this additional incentive which lets the optimal tax rate be positive even in the presence of perfect capital mobility.

Higher capital mobility leads governments to choose lower tax rates. This can be seen by differentiation of (25) with respect to b while holding the initial capital stock k_{SS} constant:

$$\begin{aligned}\frac{d\tilde{\tau}}{db}_{|k_{SS}\text{ constant}} &= \frac{d\tilde{\tau}}{d\rho_2}_{|k_{SS}\text{ constant}} \frac{d\rho_2}{db} \\ &= \frac{r\left(2-\gamma+r\left(-\rho_3\right)^{-\gamma}\right)\left(k_{SS}f''(k_{SS})\right)^2}{b\left(-r+2\,\rho_2+r\gamma-\gamma\,\rho_2\right)^2\sqrt{b\left(br^2+4\,br\delta+4\,b\delta^2-4\,f''(k_{SS})k_{SS}\right)}} > 0\end{aligned}$$

Lower values of b imply smaller values of ρ_2 such that $d\rho_2 < 0$ – as long as the initial capital stock is not altered by the change in b.[19] The result in (29) is straightforward: higher capital mobility makes it easier (less costly) for foreign capital owners to flee jurisdictions that try to redistribute from foreigners to local households. The capital stock, labour productivity and wages shrink faster. Shrinking wages are the main reason for local governments to be reluctant to use capital taxation for the purpose of providing a transfer to households.

[19]The statement that increasing capital mobility (lower values of b) implies lower a tax rate is only valid if the initial capital stock is the same. The initial capital stock depends on b itself, see (11b). Holding k_{SS} constant while b varies can be achieved by a variation of the initial tax rate τ.

The optimal tax rate τ is always positive as long as Assumption 2 holds. But it is not necessarily greater than the initial tax rate. Consider two extreme situations. If the initial tax rate τ equals zero, the initial capital stock is relatively high, the tax rate $\tilde{\tau}$ is greater than the initial tax rate and there will be an outflow of capital from time $t = 0$ on. If the initial tax rate is almost a confiscatory rate, the initial steady state value for the local capital stock is close to zero and the tax rate $\tilde{\tau}$ is below the initial tax rate. In this situation, there will be an inflow of capital. The initial situation is historically given. Both cases – $\tilde{\tau} > \tau$ with an outflow of capital and $\tilde{\tau} < \tau$ with an inflow – are possible. The local government – as long as capital mobility is imperfect ($\rho_2 > -\infty$) – doesn't always have an interest to extract rents from foreign capital owners. In situations where the historically given capital stock is very low, the opportunity costs of loosing even more capital due to a higher tax rate than initially are too high.

The dependence of the optimal tax rate on the historically given capital stock has an important consequence: The decision about capital taxation in time $t = 0$ that is characterized in the present model is a one-shot decision, with full commitment, to set a constant capital tax rate. It has been assumed that capital-owners do not anticipate the decision. (25) is an open-loop strategy. The fact that it is not history independent is important if one considers the closed-loop strategy where the tax authority can decide again about the optimal tax rate in all subsequent periods. In general, a tax authority that has the possibility to revise its decision in the future will find that the capital stock is lower than initially and hence will choose another tax rate. (25) is therefore not time-consistent.

Proposition 4 summarizes the results on the optimal tax rate on capital:

Proposition 4 (optimal capital taxation) *The optimal tax rate $\tilde{\tau}$ in (25) has the following properties:*

1. *It is positive if Assumption 2 if fulfilled.*
2. *If Assumption 2 is not fulfilled, values of ρ_2 that are close enough to zero (high enough values of b) ensure a positive tax rate.*
3. *Even in the presence of perfect capital mobility ($b \to 0$ or $\rho_2 \to -\infty$), $\tilde{\tau}$ is positive as long as Assumption 2 holds.*
4. *$\tilde{\tau}$ and capital mobility are negatively related: higher capital mobility leads governments to choose lower tax rates, given that the initial value of the local capital stock is the same.*
5. *$\tilde{\tau}$ can be greater or smaller than the initial tax rate τ. If the initial tax rate τ is small enough, $\tilde{\tau} > \tau$ and there will be an outflow of capital.*
6. *τ is not time-consistent.*[20]

3.3 Capital mobility, optimal taxation and public-sector modernisation

Does increased mobility of capital enhance public-sector modernisation? Is tax competition conducive to an efficient public sector? This is the central question in this paper. Proposition 4 states that higher capital mobility lets local governments choose lower tax rates. There is no direct effect of capital mobility on the accumulation of public sector efficiency. The decision about the taxation of capital and the decision about the use of the resulting revenue are separate decisions. It follows from Proposition 3 that the direction of the change of the

[20] Note that 5. and 6. apply also to the model in Wildasin (2003).

path of public sector efficiency caused by a change in the tax rate on capital depends on the long-term consequences of that tax rate change on revenue. The second part of Proposition 3 already states that the path of public sector efficiency is below the initial path if the tax rate, $\tilde{\tau}$, is smaller than the old, τ, and if revenue shrinks because of that change in capital taxation. It is therefore necessary to know under which circumstances the local government lowers the capital tax rate and whether this is revenue enhancing or not.

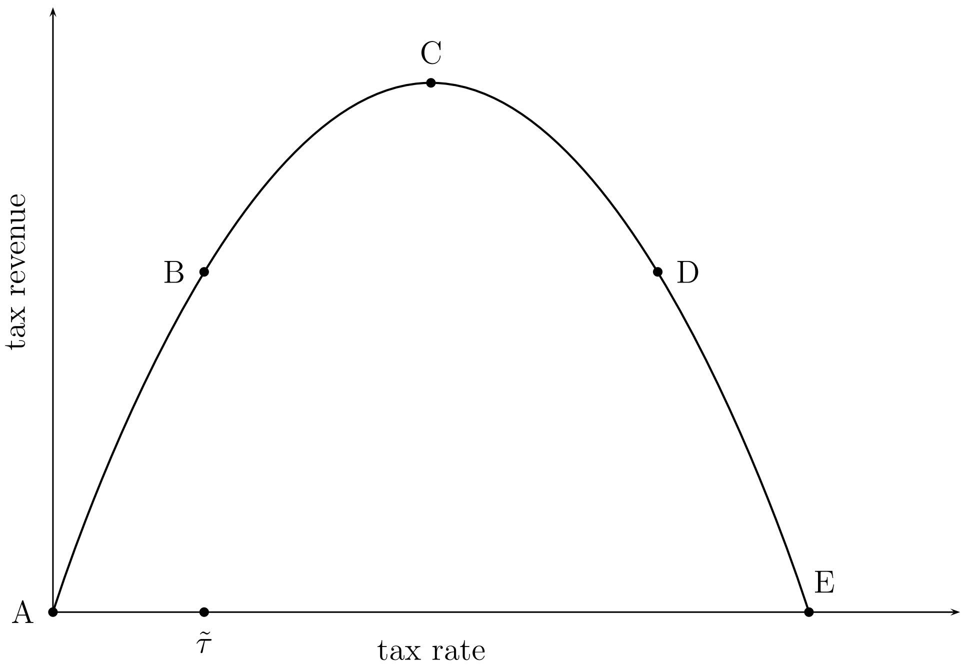

Figure 3: stylized Laffer-Curve

Figure 3 shows a stylized Laffer curve.[21] If the tax rate is zero (point A), the local government doesn't raise any revenue. If the tax rate is confiscatory[22], there is no capital accumulation in the economy and the revenue is again zero (point E). There is a revenue-maximising tax rate corresponding to the peak of that stylized Laffer curve (point C). The local government in this model does not choose the revenue-maximising tax rate. Revenue is beneficial for worker-households as it can be used for redistribution both directly or indirectly through public sector modernisation. But raising the tax rate comes at the cost of lower wages. In equation (15c), if the left hand side would be zero, the local government would choose the revenue-maximising tax rate. But in this model, the optimal tax rate is always somewhere below, for example at point B in Figure 3. The position of point B depends on the degree of capital mobility. The more mobile capital is, the more rapid is the adjustment of the capital stock and the more important is the wage effect on life-time income that is caused by a change in capital taxation. In Wildasin (2003), the optimal tax rate is zero when capital is perfectly mobile. In this model, point B is always somewhere to the right of the origin.

The revenue consequences of the tax rate chosen in time $t = 0$ can be categorized depending on the initial position of the local government on the Laffer Curve. Take point B as the optimal

[21] The exact shape of the Laffer curve depends on the functional form of the private production function $f(k)$. The Laffer curve is single-peaked, for example, if the production function is $f(k) = k^{\alpha}$ with $\alpha < 1$.

[22] As capital taxation is not a taxation of capital income in this model, the tax rate has no natural upper bound at 100 %. A confiscatory rate is the tax rate for which capital accumulation is not profitable in the long run. This is the case for $\tau \to \infty$.

position on the Laffer curve. If the initial tax rate is greater than the one corresponding to B, the local government decides to lower it, $\tilde{\tau} < \tau$, and there is an inflow of capital during the adjustment to the new steady state. The revenue-consequences of this change of the capital tax rate can be either negative (for a initial position somewhere between B and D) or positive (for a initial position somewhere between D and E). If the initial tax rate is below the one corresponding with B, the local government decides to raise it, $\tilde{\tau} > \tau$ and there is an outflow of capital and revenue rises (initial position somewhere between A and B). This leads to:

Proposition 5 (capital mobility and public sector modernisation)
Whether capital mobility enhances public sector modernisation or not, for $t \to \infty$, depends on the initial position of the local government on a Laffer curve like the one in Figure 3. The local government will always choose a capital tax rate $\tilde{\tau}$ that is below the revenue-maximising rate. Group local governments according to their initial tax policy as follows:

- *(low-tax) Local governments with an initial tax rate below $\tilde{\tau}$ choose a higher tax rate on capital and face an outflow of capital. Revenue for $t \to \infty$ rises. Public sector efficiency is higher than initially.*
- *(high-tax) Local governments with an initial tax rate that is higher than $\tilde{\tau}$ and less revenue than that corresponding with $\tilde{\tau}$ face an inflow of capital and an increase of revenue in the long run. Public sector efficiency is higher than initially.*
- *(intermediate-tax) Local governments with an initial tax rate that is higher than $\tilde{\tau}$ and higher revenue than that corresponding with $\tilde{\tau}$ face an inflow of capital and less revenue in the long run. Public sector efficiency is lower than initially.*

Higher capital mobility lets a local government choose a lower tax rate $\tilde{\tau}$. The range of tax rates that correspond with being a low-tax government is smaller when capital mobility is higher. In this sense, higher capital mobility causes lower public-sector efficiency. Whether this statement is true for a specific local government depends on its initial position on the Laffer-curve.

Whether increased mobility of capital enhances public-sector modernization or not depends on the ability of local governments to increase revenue that is then used to support local households either through the modernization of the public sector or by direct redistribution. But their problem is not simply to choose the revenue-maximising tax rate on a Laffer curve: To maximise the life-time income of households, they have to consider the time path of revenues and of the local capital stock as well.

The model in this paper is such that the relationship between public sector efficiency and capital mobility narrows down to the question whether revenue shrinks or increases when the local government raises capital taxes. It is only necessary to know the long-term consequences on revenue, as can be seen from equation (24b) in Proposition 3. The revenue plotted in Figure 3 as a Laffer curve is the revenue in a steady state. The impact of capital mobility on the efficiency of the public sector depends on the existence of a Laffer effect. In a situation where increased capital mobility lets local governments choose lower tax rate, is this revenue enhancing or not?[23] However, whether a local government raises its capital taxes or not

[23] It is of course difficult to decide in which part of the Laffer curve for the taxation of capital a specific country or jurisdiction is. See Blinder (1981) for an example of an "guesstimation" of such a question. In general, it can be said that it is more likely to be on the wrong side of the Laffer curve if the activity taxed can be substituted by other activities easily. In the case of capital taxation, the integration of capital markets make the substitution of real investment in the local capital stock by investment in financial assets easier. In this model, this is captured by the fact that point B in Figure 3 moves to the left when capital mobility is increased.

depends on the initial situation: The higher the initial tax rate is, the more likely it is that a local government sets a lower tax rate than initially.[24]

Mankiw & Weinzierl (2006) show for a closed economy and several variants of the neoclassical growth model that tax cuts can be partly self-financing, depending on the assumptions about parameters. The main difference in this paper compared to Mankiw & Weinzierl (2006) is that the adjustment path of the local capital stock is not determined by local conditions only. In this paper, the adjustment path of the local capital stock depends on the externally interest rate r and on the intensity of tax competition ρ_2. In the long run, the local capital stock is determined by the equilibrium condition 11 and does not depend on local capital taxation as in Mankiw & Weinzierl (2006). To what extent a tax cut is partly self-financing in the long run through its positive effect on the local capital stock can then be found out by a simple back-of-the-envelope calculation, once a functional form for the production function and the initial values for the capital tax rate are known. According to Proposition 5, it is also sufficient to associate initial tax rates with one of the three groups to make a qualitative statement.[25]

Market integration in the form of higher capital mobility hampers public-sector modernisation. The reason is that higher capital mobility implies that a smaller range of initial positions on the Laffer curve correspond to increased revenues and higher public sector efficiency in the long run. The intuition for this result is that public-sector modernisation is modelled as an investment activity that needs to be funded by tax revenues. Higher capital mobility discourages taxation and tax revenues that are needed to pursue public-sector modernisation.

Before the results are summarized in the conclusion, the next section presents several numerical examples to illustrate the model.

4 A numerical example

This section provides a numerical example to illustrate the results derived so far.[26] In this numerical example, with given parameters and functional forms, it is possible to find a closed-form solution for the path of the local capital stock and to provide plots of tax rates, revenue and public sector efficiency.

Throughout the section, I assume $f(k) = k^{0.3}$, $r = 0.1$ and $\delta = 0.1$. Assumption 2 is fulfilled. This allows to calculate the initial capital stock that corresponds to a tax rate τ or $\tilde{\tau}$ (equation (11b)), if necessary. From equation (25), it is possible to calculate the tax rate $\tilde{\tau}$ that corresponds to some initial τ.

Figure 4a shows the optimal tax rate for all possible values of γ and different values of

[24] In many static models of tax competition, capital tax rates of a closed economy is compared against those of an open economy. In a standard tax competition model where capital taxation gives rise to a positive externality, comparing those polar cases leads to the clear result that capital tax rates too low in the presence of tax competition. In this model, I do not compare those polar cases. Parameter b ranges from perfect capital mobility ($b = 0$) to a closed economy ($b \to \infty$) and covers all intermediate cases. In this model, the sign of $d\tilde{\tau}/db$ is non-ambiguous. But whether a local government raises its tax rate in capital or lowers it depends on the characteristics of an initial steady state.

[25] Agell & Persson (2001) analyse Laffer effects in a simple AK model of endogenous growth. The key message of their paper is that in a dynamic context, it is very important to have a proper definition of what they call a dynamic Laffer effect. Note that in the preceding analysis, where growth is not endogenous and the central question is whether capital mobility has an influence on the level of public sector efficiency, I am not looking for a dynamic Laffer effect. It is only necessary to know the revenue effect of a tax variation in the long run.

[26] I have used Maple 9.5 for Mac OS X to derive the plots in this section. The workfile is available on request.

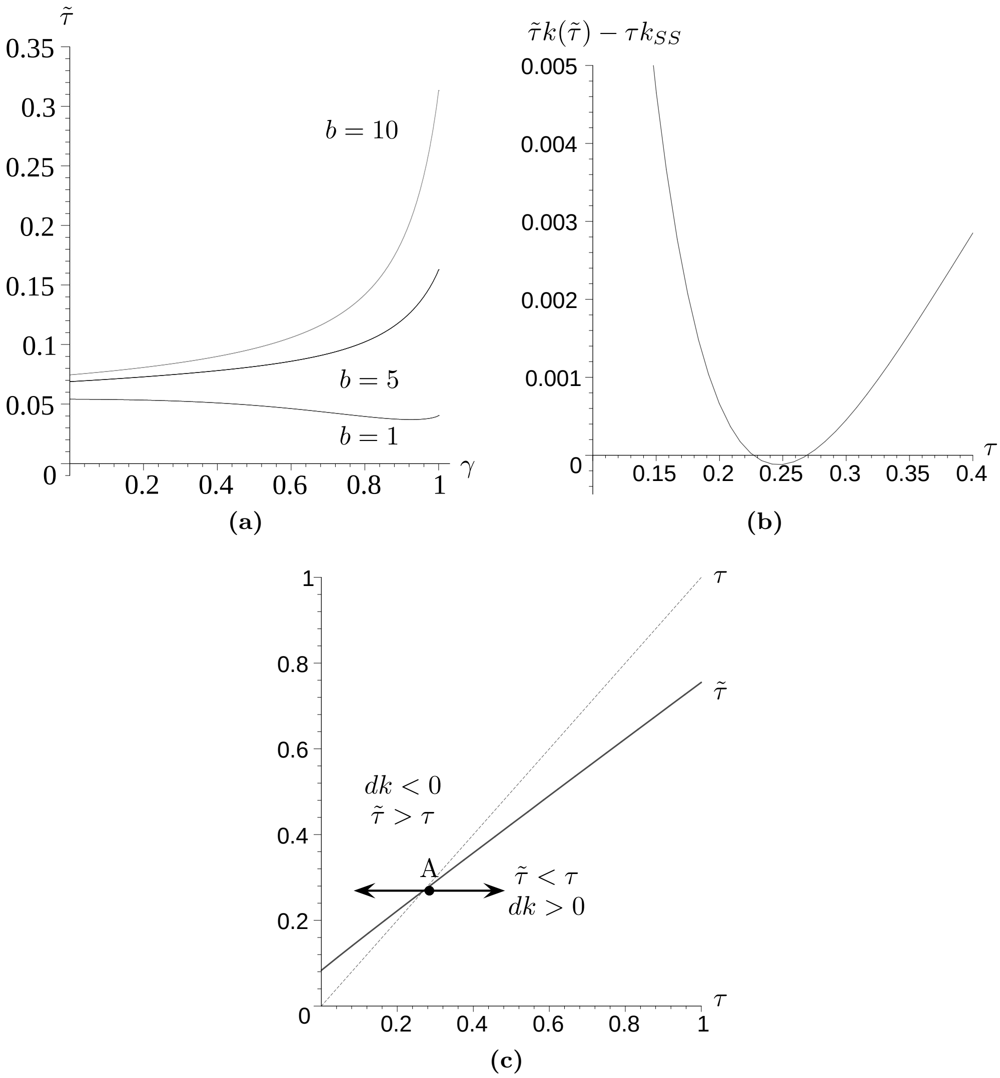

Figure 4: Numerical example (tax rate, revenue) (a) Tax rate (25) for different values of the capital mobility parameter b and an initial capital stock $k_{SS} = 10$. (b) Difference between the revenue in the initial and the new situation $b = 1$). (c) New tax rate $\tilde{\tau}$ as a function of the initial tax rate τ $(b = 1)$

b. The initial capital stock is assumed to be $k_{SS} = 10$. Note that fixing the initial capital stock k_{SS} ensures that higher values of b correspond with values of ρ_2 that are closer to zero. Higher values of b correspond with less capital mobility. The initial tax rate differs in the three cases illustrated in Figure 4a such that equation (11b) holds. In Figure 4a, higher capital mobility (lower values of b) implies lower tax rates. See Proposition 4.

Figure 4b plots the difference between the revenue raised in the initial steady state and the steady state that corresponds to the new tax rate that is chosen in time $t = 0$. Parameter b is set to 1. The initial situation is characterised by a tax rate between zero and 0.4. The initial capital stock then varies according to (11b). For small and high values of the initial tax rate, there is a clear revenue gain. These are tax rate of local governments in either the "low-tax" or the "high-tax" group in Proposition 5. The negative values are the revenue losses of local governments in the "intermediate-tax" group.

Figure 4c compares the initial tax rate $0 < \tau < 1$ with the new tax rate chosen, $\tilde{\tau}$. Again, Parameter b is set to 1 and the initial capital stock varies. It can be seen that $\tilde{\tau} > \tau$ for small values of the tax rate. This implies a shrinking capital stock. There is a crucial initial tax rate where this changes and the local government sets a tax rate the triggers a capital inflow from time $t = 0$ on.

Figure 5 illustrates the evolution of tax revenue and public sector efficiency for different values of the capital-mobility parameter b when the initial capital stock is set to $k_{SS} = 1$ in all three scenarios. The initial tax rate is different in the three scenarios shown. Figure 5a shows the path of tax revenue. In all three scenarios, the new tax rate $\tilde{\tau}$ is lower than the initial tax rate. Revenue is higher than initially but then shrinks with the capital stock - see table 1 for the numbers. The initial values of public sector efficiency differs. In all three cases, public sector efficiency rises over time (Figure 5b).

b	τ	τk_{SS}	H_{SS}	$\tilde{\tau}$
$b = 1.0$	0.08	0.08	6.75	0.22
$b = 0.5$	0.09	0.09	7.88	0.21
$b = 0.1$	0.10	0.10	8.78	0.20

Table 1: Values of some key parameters and variables used for Figure 5. All numbers rounded to two digits.

In Figure 6, the initial value of public sector efficiency is set to zero. The initial tax rate is zero as well. The initial capital stock differs and shrinks as the new tax rate $\tilde{\tau}$ is positive. Revenue shrinks over time (Figure 6a) but is greater than initially. The higher capital mobility is (the lower b), the faster is the modernisation of the public sector (Figure 6b).

5 Concluding Remarks

This paper tries to answer the question whether the competitive pressure induced by capital tax competition enhance the efficiency of the public sector. Public-sector efficiency is modelled as a stock variable and public-sector modernisation is seen as an investment activity that is financed by tax revenues. The model in this paper assumes that the jurisdiction under consideration is in an initial steady state. The main result is that the presence of tax competition hampers public-sector modernisation.

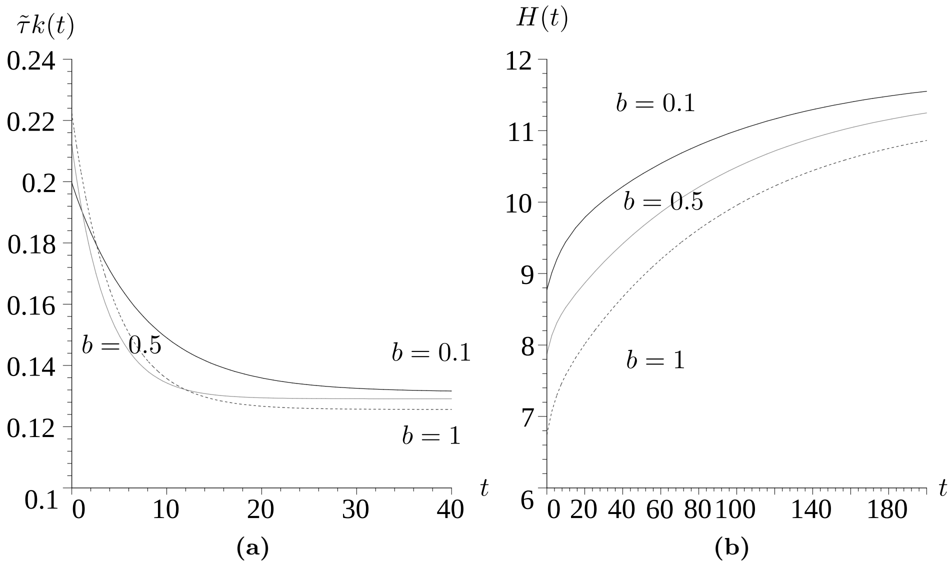

Figure 5: Revenue and public-sector efficiency for different values of the capital mobility parameter b and a common initial capital stock. (a) Revenue $\tilde{\tau}k(t)$ (b) Public-sector efficiency $H(t)$

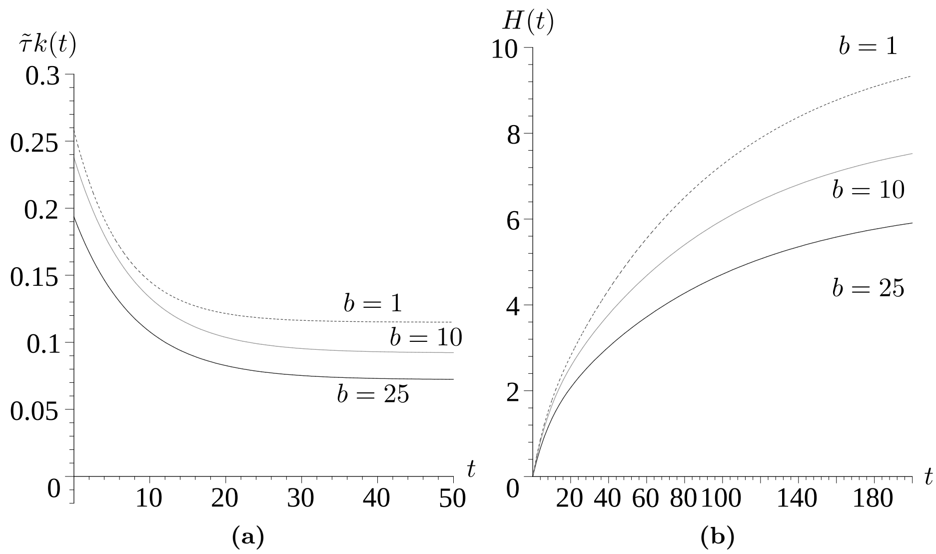

Figure 6: Revenue and public-sector efficiency for different values of the capital mobility parameter b and a common initial value of public-sector efficiency. (a) Revenue $\tilde{\tau}k(t)$ (b) Public-sector efficiency $H(t)$

Public-sector efficiency depends on revenue that is either invested in modernisation or used directly as a flow input in the production of redistribution. Higher capital mobility implies lower tax rates. Whether this is revenue- and efficiency-enhancing or not depends on the situation the local jurisdiction is initially. But a smaller range of initial tax rates is compatible with a revenue-gain when capital mobility is higher.

A key assumption of the model is that private agents are caught by surprise by the government's decision to tax capital and that it can commit itself to this policy announcement. This is not very realistic. As long as the capital stock in the jurisdiction has some positive level, there is an incentive to deviate from the announcement in time $t = 0$. Rational investors would foresee that the announcement of a tax policy in time $t = 0$ is not time-consistent. With rational expectations, a credible policy announcement must not depend on initial conditions as in this model, see Taylor (2000, sect. 2.1). One could see this as a major disadvantage of the model presented above. Still, it is a dynamic model as it describes local jurisdictions as growing economies. Furthermore, transitional dynamics matter: the speed of adjustment of the local capital stock to the new steady state is crucial for optimal policy of the local government. But the model is not dynamic in the sense that forward-looking decisions like investment in a capital stock and forward-looking tax-policy without commitment is analysed simultaneously, as it is done, for example, in Hassler et al. (2005). A model of tax competition between growing jurisdictions where local governments cannot commitment themselves to their policy announcements is missing in the literature.

Another simplifying assumption is that households do not own parts of the local capital stock. Optimal policy is therefore the optimal exploitation of foreigners in the favour of local households. Households do not suffer from the fact that the net return of local capital is lower than the world interest rate during the transition phase after an surprising rise in capital taxation. The only reason not to expropriate foreign capital owners is that wages depends on the availability of capital. This is not a very realistic setup. If the local capital stock is partly owned by local households, the incentives for the local governments are different. The greater the share of local ownership is, the lower is the tax rate, revenues and public-sector efficiency in the long run. But as long as a part of the local capital stock is owned by foreigners, the local government tries to optimally exploit them. An alternative ad-hoc modelling strategy would be to introduce a parameter that represents the share of the local capital stock that is owned by foreign households. The higher the share of domestic ownership, the lower the optimal tax rate would be. This is done in Wildasin (2003). As the qualitative results do not change, I abandoned domestic ownership of the capital stock altogether. It might be interesting to think about the endogenous evolution of the ownership structure of the local capital stock in a tax competition model, but this is beyond the scope of this paper.

The assumption of foreign ownership of the capital stock could have abandoned alltogether with a different structure of the model: Assume that there are two types of households in the economy. Worker households without access to the international capital market and capitalists that own capital. In this alternative model, assume further that the local government does not care about the welfare of capitalists as they do not vote. They express their will in the political process only by deciding where to invest in physical capital. In a model like this, the problem of the local government would be to optimally exploit capitalists regardless of their place of residence. Redistribution would be from "footloose" capitalists to local worker-households. The only group with access to the international capital market were firms or capital-owners. This alternative model would lead to similar results than those derived above.[27]

[27] A referee pointed out that there are indeed examples of taxes on foreign owned firms. For example, Argentina forced foreign firm to convert assets at the pre-devaluation rate in 2002. Since governments are sovereign

Public services benefit only households in this model. But of course the public sector provides also infrastructure and other public inputs that enter the private sector's production function. In a dynamic context, public inputs can be a potential source of sustained growth. Becker & Rauscher (2007) analyse a growth model with a balanced growth path. Imperfect capital mobility is modelled in a way similar to this paper. They analyse a symmetric equilibrium of tax-competing jurisdictions, but without an application to public-sector modernisation. Whereas in Becker & Rauscher (2007) public inputs are a flow variable, Futagami et al. (1993) analyse a model in which a public capital stock serves as an input in private production. Extending their analysis by a tax competition scenario and a public sector that can operate at various levels of efficiency seems to be interesting. It might be necessary to use numerical methods to tackle such a model, though.

The major lesson that can be drawn from this paper is that interjurisdictional tax competition might not be useful for the goal of an efficient public sector. Efficiency in this paper is the ability of the public sector to transform tax revenue into a transfer. A more efficient sector is one with a higher level of a accumulated knowledge or some other stock that improves the productivity of this transformation. If public-sector efficiency is plagued by egoistic politicians or lazy bureaucrats that waste resources, tax competition among jurisdictions might serve as a disciplining device. But when low efficiency is caused by underinvestment, as in this paper, competition and efficiency are no complements.

Acknowledgements

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Appendix

Equation (17b) as a Bernouilli-Equation

A Bernoulli-Equation is a non-linear differential equation that can be reduced to a linear form. Following Chiang (1984, p. 491), assume that the equation in question is

$$\dot{y(t)} + Ry(t) = Ty^m.$$

Define $z = y^{1-m}$. The equation can then be written

$$\dot{z(t)} = (m-1)\, Ry(t) + T.$$

The general solution of this transformed equation is $z(t) = \frac{T}{R(m-1)} + e^{R(m-1)t}C_1$, where C_1 is an arbitrary constant. Substitution of z then leads to

$$y(t) = \left(\frac{T}{R} + e^{R(m-1)t}C_1\right)^{\frac{1}{1-m}}.$$

these firms had difficulties in claiming their losses back even if the Argentine government broke international treaties.

Applying this formula to (17b) with $m = \frac{\gamma}{\gamma-1}$, $T = -(1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}$ and $R = -r$ then gives as the general solution:

$$\mu(t) = \left(\frac{1-\gamma}{r}\gamma^{\frac{\gamma}{1-\gamma}} + e^{\frac{r}{1-\gamma}t}C_1\right)^{1-\gamma}.$$

Together with the appropriate border- condition, this leads to the following equation to determine C_1:

$$\lim_{t->\infty}\left(\left(\frac{1-\gamma}{r}\gamma^{\frac{\gamma}{1-\gamma}} + e^{\frac{r}{1-\gamma}t}C_1\right)^{\frac{1}{1-m}}\right)H(t) = 0$$

because of $\frac{r}{1-\gamma} > 0$, C_1 is equal to 0 and (18) follows.

Calculation of the tax rate

Start with equation (15c):

$$\frac{d}{d\tau}\int_0^\infty w(t)e^{-rt}dt = -\frac{d}{d\tau}\int_0^\infty \left(R(t)^\gamma H(t)^{1-\gamma} + \mu(t)\left(\tau k(t) - R(t)\right)\right)e^{-rt}dt$$

The wage rate $w(t)$ depends on the current capital stock $k(t)$ but not on the tax rate τ directly. The impact of taxation on the wage rate therefore is

$$\frac{dw(t)}{d\tau} = \frac{dw(t)}{dk(t)}\frac{dk(t)}{d\tau} = -f''(k_{SS})k_{SS}\frac{dk(t)}{d\tau} = -k_{SS}\left(1 - e^{\rho_2 t}\right).$$

The LHS of (15c) then can be written

$$-k_{SS}\int_0^\infty \left(1 - e^{\rho_2 t}\right)e^{-rt}dt = -k_{SS}\int_0^\infty e^{-rt}dt - k_{SS}\int_0^\infty -e^{(\rho_2 - r)t}dt.$$

Calculation of these integrals[28] then gives for the LHS of (15c):

$$-k_{SS}\left(\frac{1}{r} - \frac{1}{(r-\rho_2})\right) = \frac{k_{SS}\rho_2}{r(r-\rho_2)}.$$

Substitution of (16), (18) and rearranging then gives

$$\begin{aligned}\frac{k_{SS}\rho_2}{r(r-\rho_2)} &= -\frac{d}{d\tau}\int_0^\infty \left(\gamma r\left(\frac{1-\gamma}{r\gamma}\right)^{1-\gamma}H(t) + \gamma\left(\frac{1-\gamma}{r\gamma}\right)^{1-\gamma}\tau k(t)\right)e^{-rt}dt \\ &= -\gamma\left(\frac{1-\gamma}{r\gamma}\right)^{1-\gamma}\int_0^\infty \left(r\frac{dH(t)}{d\tau} + \frac{d\left(\tau k(t)\right)}{d\tau}\right)e^{-rt}dt\end{aligned}$$

Note first that

$$\int_0^\infty r\frac{dH(t)}{d\tau}e^{-rt} = \frac{1-\gamma}{r\gamma}\left(\frac{\tau}{f''(k_{SS})} + k_{SS}\right)\frac{-\rho_3}{r-\rho_3}.$$

Furthermore,

$$\frac{d(\tau k(t))}{d\tau} = \tau\frac{dk(t)}{d\tau} + k_{SS}.$$

This allows to calculate that

$$\int_0^\infty \left(\left(\tau\frac{dk(t)}{d\tau} + k_{SS}\right)\right)e^{-rt}dt =$$

[28]Note that $\int_0^\infty e^{-xt}dt, x>0 \leftrightarrow \frac{1}{-x}e^{-xt}\Big]_0^\infty = 0 - \frac{1}{-x}e^0 = \frac{1}{x}$.

$$\left(\frac{k_{SS}}{r}+\frac{\tau}{f''(k_S)}\frac{-\rho_2}{r(r-\rho_2)}\right).$$

Collecting terms, the following remains. It needs to be solved for τ:

$$\frac{\rho_2 k_{SS}}{r\,(r-\rho_2)} =$$
$$-\gamma\left(\frac{1-\gamma}{r\gamma}\right)^{1-\gamma}\left(\frac{1-\gamma}{r\gamma}\left(\frac{\tau}{f''(k_{SS})}+k_{SS}\right)\frac{-\rho_3}{r-\rho_3}+\frac{k_{SS}}{r}+\frac{\tau}{f''(k_S)}\frac{-\rho_2}{r(r-\rho_2)}\right)$$

This results in:

$$\tau=\frac{k_{SS}\,f''(k_{SS})\left(\rho_2 r\left(-\frac{-1+\gamma}{r\gamma}\right)^{\gamma}+2\,r-3\,r\gamma-2\,\rho_2+3\,\gamma\,\rho_2+\gamma^2 r-\gamma^2\rho_2\right)}{(r-2\,\rho_2-r\gamma+\gamma\,\rho_2)\,(-1+\gamma)}$$

With $\left(-\frac{-1+\gamma}{r\gamma}\right)^{\gamma}=\left(\frac{1-\gamma}{r\gamma}\right)^{\gamma}=(-\rho_3)^{-\gamma}$:

$$\tau=\frac{\overbrace{k_{SS}\,f''(k_{SS})}^{<0}\left(\overbrace{\rho_2 r(-\rho_3)^{-\gamma}+2\,r-3\,r\gamma-2\,\rho_2+3\,\gamma\,\rho_2+\gamma^2 r-\gamma^2\rho_2}^{???}\right)}{\underbrace{(\rho_2(2-\gamma)-r(1-\gamma))\,(1-\gamma)}_{<0}}$$

From this, the optimal tax rate (25) follows as

$$\tau=\frac{k_{SS}\,f''(k_{SS})\,\left(\rho_2 r(-\rho_3)^{-\gamma}+(2-\gamma(3-\gamma))\,(r-\rho_2)\right)}{(\rho_2(2-\gamma)-r(1-\gamma))\,(1-\gamma)}\ .$$

Condition for τ being positive

The condition for τ being positive is:

$$\rho_2 r\left(\frac{1-\gamma}{r\gamma}\right)^{\gamma}+(2-\gamma(3-\gamma))\,(r-\rho_2)<0$$

or, with assumption 2, as $(2-\gamma(3-\gamma))>0$ for $0<\gamma<1$:

$$\rho_2<\frac{(2-\gamma(3-\gamma))\,r}{r(-\rho_3)^{-\gamma}+2-\gamma(3-\gamma)}\ .$$

which is always fulfilled for $0<\gamma<1$ and positive interest rates.

Calculation of (29)

This has been done with Maple 9.5 for Mac OS X:

$$\frac{d\tau}{d\rho_2}=-\left(-\gamma+r\left(-\frac{-1+\gamma}{r\gamma}\right)^{\gamma}+2\right)rk_{SS}\,f''(k_{SS})\,(-r+2\,\rho_2+r\gamma-\gamma\,\rho_2)^{-2}$$

(29) follows after some simplifications.

Concavity check

The necessary conditions for optimal policy $\{\tau,R(t)\}$ discussed in the main text can be shown to be sufficient for an optimal policy when some concavity-conditions are met. See Takayama (1985, Theorem 8.C.5).

What needs to be checked is whether the functions $w(t) + R(t)^{\gamma} H(t)^{1-\gamma}$ and $\tau k(t) - R(t)$ are concave in $R(t)$ and $H(t)$. For the first of these, the quadratic form is

$$\begin{bmatrix} -\frac{(1-\gamma)}{\gamma} R(t)^{\gamma} H(t)^{-\gamma-1} & \gamma(1-\gamma) R(t)^{\gamma-1} H(t)^{-\gamma} \\ \gamma(1-\gamma) R(t)^{\gamma-1} H(t)^{-\gamma} & \gamma(\gamma-1) R(t)^{\gamma-2} H(t)^{1-\gamma} \end{bmatrix}.$$

For the second, the quadratic form is $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, as it is independent of $H(t)$ and linear in $R(t)$. Both of them are negative-semidefinite and therefore, the sufficiency conditions are met.

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