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The effect of the status quo tie−breaking rule on prize winning

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Abstract

This paper considers a contest in which two identical players participate to compete for an indivisible prize. It is shown that, in the presence of incomplete information, the player favored by the status quo tie−breaking rule may be less likely to get the prize than his competitor, even though, under the coin toss tie−breaking rule, the two players participate with the same positive probability and hence are equally likely to get the prize.

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1. Introduction

This paper considers a situation in which two identical individuals play a participation game, competing for an indivisible prize. The two players place the same known valuation on the prize. Participation is costly, as it requires a player to input time and effort, while non-participation is not. The paper considers two information structures of players' participation costs. In the first, the two players' costs of participation are known and the same. In the second, each player's cost of participation is his private information; the other player knows about only the distribution of his competitor's cost of participation. Each player is risk-neutral, with his utility or payoff being the difference between the expected benefit of the prize and cost of participation.

The rule of the game is as follows. If only one player participates, then the player wins the prize. If the two players take the same action, i.e., they both participate or do not participate, then the winner is determined by a tie-breaking rule. The paper considers two commonly used tie-breaking rules: coin toss and the status quo.

When a tie is broken by a coin toss, the participation game is symmetric. Under this tie-breaking rule, participation by a player raises his chance of winning the prize by 50 percent, no matter whether or not the other player participates. Consequently, each player participates if and only if his cost of participation does not exceed half of the value of the prize. Given symmetric nature of the game, the two players have the same chance to get the prize, and their expected payoffs are the same.

While the coin toss tie-breaking rule is fair, the status quo tie-breaking rule is unfair but favors one player, say player 1. Under this rule, if players 1 and 2 tie, i.e., they take the same action, player 1 wins the prize. One such example is as follows. Two candidates run for an office, with candidate 1 being the incumbent, and candidate 2 the challenger. When the two candidates tie, the incumbent stays in the office. The status quo rule can be found in practice: The oldest candidate wins in tied elections in France. Another example is that, two athletes, one the former champion, the other a challenger, compete for a championship. If they score the same, the incumbent keeps his title. Under the status quo rule in favor of player 1, one would intuitively think that player 1 is more likely to get the prize, and his expected payoff higher than that of player 2.

Unfortunately, this intuition is not completely correct. For participation games of complete information in which the two players' costs of participation are known and the same, it can be easily shown that the game admits only mixed-strategy equilibrium under which the challenger may participate with a higher probability than the incumbent. As a result, it is possible that the former is more likely than the latter to get the prize. It can, nevertheless, be shown that this possibility arises only if neither of the two players participates under the coin toss tie-breaking rule. Differently put, for participation games of complete information, if both players participate under the coin toss rule, then under the status quo tie-breaking rule, the incumbent is more likely than the challenger to get the prize.

The above result does not extend to participation games of incomplete information in which each player's cost of participation is his private information. For such a game, under the coin toss tie-breaking rule, each player participates if and only if his cost of participation does not exceed a common cutoff level, half of the value of the prize. To contrast with the results obtained for complete-information participation games, assume that both players, ex ante, participate with positive probability under the coin toss rule.

Under the status quo tie-breaking rule, each player's participation decision also follows a cutoff rule; the cutoff cost for each player depends on the ex ante probability that the other player participates. There is no presumption that the cutoff level is higher for player 1 than for player 2. As a result, it is possible that, in equilibrium, player 1, the incumbent, is less likely to get the prize than player 2, even though, under the coin toss tie-breaking rule, the two players both participate with the same positive probability (and hence are equally likely to get the prize). In other words, the status quo rule may have an unexpected negative effect on the favored player's getting the prize. Nevertheless, it can be shown that player 1's expected payoff is greater than that of player 2.

The remainder of the paper is structured as follows. Section 2 outlines simple participation games and analyzes the effect of the status quo rule on the relative chance that the two players get the prize, and Section 3 contains the conclusion.

2. The model

Two individuals, 1 and 2, play a participation game, competing for a prize. The action set for each player is binary, {participation, non-participation}. Participation is costly while non-participation is not. The value of the prize to each player is $\theta > 0$, which is common knowledge.

The utility or payoff to each player is the difference between the value of the prize and his cost of participation. If there is only one player participates, then the player wins the prize. If the two players take the same action, i.e., they both participate or do not participate, then the two players tie, and a tie-breaking rule is called. Under a general tie-breaking rule, player 1 gets the prize with probability τ, where $\tau \in [0, 1]$ is exogenously given, and player 2 gets the prize with probability $1 - \tau$. There are two special cases, $\tau = 1/2$ and $\tau = 1$. The first case is referred to as the coin toss rule, and the second as the status quo rule favoring player 1. This paper focuses the analysis on the two cases.

Consider first the case that the two players' costs of participation are the same and known, $c_1 = c_2 = c$. It is assumed that $c < \theta$. Otherwise, there is no possibility that either of the players will participate. Note that, no matter whether or not the other player participates, the net benefit of participation (to that of non-participation) is $\theta/2$. Thus, under the coin toss tie-breaking rule, it is a dominant strategy for each player to participate if $c \leq \theta/2$, and not to participate if $\theta/2 < c < \theta$. In short, under the coin toss tie-breaking rule, the two players in equilibrium take the same action, for all parameter values. Thus, both players get the prize with equal chance and obtain the same expected utility.

Suppose now that the tie-breaking rule is status quo in favor of player 1, the incumbent. It is easy to see that the game has no pure-strategy but only mixed-strategy equilibrium. If the incumbent participates (does not participate), the best response for the challenger is non-participation (participation). Then, the incumbent would not participate (would participate).

It can be easily verified that, in mixed-strategy equilibrium, player 1 participates with probability $p_1 = 1 - c/\theta$, and player 2 participates with probability $p_2 = c/\theta$.

The probability that player 1 gets the prize is $P_1 = p_1 + (1 - p_1)(1 - p_2) = (\theta - c)/\theta + c/\theta\,(1 - c/\theta) = (\theta^2 - c^2)/\theta^2$, and the probability that player 2 gets the prize is $P_2 = (1 - p_1)p_2 = c^2/\theta^2$. It is not always the case that the incumbent is more likely than the challenger to get the prize. Indeed, when $\theta^2 < 2c^2$, $P_1 < P_2$. Note that a necessary condition for this to happen is only when both of the two players do not participate under the coin toss tie-breaking rule.

The expected utility of player 1, the incumbent, is $U_1 = P_1\theta - p_1 c = \theta(\theta^2 - c^2)/\theta^2 - c(\theta - c)/\theta = \theta - c > 0$. And the expected utility of player 2, the challenger, is $U_2 = P_2\theta - p_2 c = \theta c^2/\theta^2 - c\, c/\theta = 0$.

When players' valuation of prize and cost of participation are such that neither of them participates under the coin toss tie-breaking rule, it is possible that, under the status quo rule, the incumbent is less likely to get the prize than the challenger in equilibrium. However, when both players participate under the coin toss rule, it is always the case that the incumbent is always more likely to get the prize than the challenger. And the incumbent's utility is always higher than the challenger's.

In summary, in the presence of complete information, if the two players both participate (and hence are equally likely to get the prize) under the coin toss tie-breaking rule, a player is more likely to get the prize under the favorable status quo tie-breaking rule than the other player, and the favored player's payoff is higher than that of the other player.

Consider now the case where each player's cost of participation is his private information; the other player knows about only his competitor's distribution of cost. Assume for simplicity that the two players' costs of participation are independently and identically distributed. More formally, let $F(c)$ be the distribution function of cost of participation, for $c \in [d, D]$, $0 \leq d < D$. Ex ante, the two players are identical.

Assumption 1. $d < \theta < D$.

This assumption implies that, if each player were existed in isolation, he would participate with a probability lying strictly between 0 and 1, ex ante.

Each player's strategy, s_i, $i = 1, 2$, is a mapping from his cost of participation, $[d, D]$, to his action set, {participation, non-participation}. As a benchmark, consider first the coin toss rule, under which the participation game is symmetric. Recall that the net benefit of participation (to that of non-participation) is $\theta/2$, no matter whether or not the other player participates. Thus, player i participates if and only if his cost of participation does not exceed $\theta/2$. Assume that $\theta/2 > d$. As a result, the two players both participate with the same positive probability, $p_1 = p_2 = F(\theta/2) > 0$. Hence, the two players have the equal chance to get the prize. Given the symmetric structure of the participation game, their expected payoffs are also the same.

Consider now the status quo rule which favors player 1 in case that the two players take the same action. If player 1 participates, he gets the prize for sure under the status quo rule, and his payoff is $\theta - c_1$. If he does not participate, he gets the prize when player 2 does not participate, and his expected payoff is $(1 - p_2)\theta$. Player 1 participates if and only if his expected payoff from participation is no less than that without participation, i.e., $\theta - c_1 \geq (1 - p_2)\theta$. So, player 1 participates if and only if his cost of participation is less than a cutoff level, $p_2\theta$. Thus, the ex ante probability that player 1 participates is $p_1 = F(p_2\theta)$.

If player 2 does not participate, his payoff is 0. If he participates, he gets the prize only when player 1 does not participate, and his expected payoff is $(1 - p_1)\theta - c_2$. Player 2 participates if and only if his expected payoff from participation is no less than that when he does not participate, i.e., $(1 - p_1)\theta - c_2 \geq 0$. Hence, player 2 participates if and only if his cost of participation is no greater than a cutoff level, $(1 - p_1)\theta$. Thus, the ex ante probability that player 2 participates is $p_2 = F((1 - p_1)\theta)$.

Obviously, p_1 depends on p_2, and vice versus. There is no presumption that $p_1 > p_2$. When $1 - p_1 > p_2$, $p_2 > p_1$. These two inequalities imply that $p_1 < 1/2$.

The probability that player 1 gets the prize is $P_1 = p_1 + (1 - p_1)(1 - p_2)$, and the probability that player 2 gets the prize is $P_2 = (1 - p_1)p_2$. Clearly, if $p_1 \geq p_2$, then $P_2 < 1 - p_1 < 1 - p_2 + p_1p_2 = P_1$. However, when $p_1 < p_2$, it is possible that $P_1 < P_2$. And this happens when $p_2(1 - p_1) > 1/2$, which implies that $p_1 < 1/2 < p_2$.

It is not hard to find distribution functions such that $P_1 < P_2$. For example, this happens when $F(\theta/2) > 0$, $F(3\theta/4) = 1/8$, and $F(7\theta/8) = 3/4$. For such a cost distribution, $p_1 = 1/8$, $p_2 = 3/4$, $P_1 = 11/32$, and $P_2 = 21/32$.

One important question is whether player 1 under the favorable status quo rule will do better than player 2 in that he will have a higher expected payoff. It can be shown that the answer to this question is positive. Note that player 1's expected payoff is $U_1 = P_1\theta - \int_d^{p_2 q} c\,dF(c)$, and player 2's expected payoff is $U_2 = P_2\theta - \int_d^{(1-p_1)q} c\,dF(c)$. So, $U_1 - U_2 = (1 - 2p_2 + 2p_1p_2)\theta - \int_{(1-p_1)q}^{p_2 q} c\,dF(c) > (1 - 2p_2 + 2p_1p_2)\theta - (1 - p_1)\theta\int_{(1-p_1)q}^{p_2 q} dF(c) = (1 - 2p_2 + 2p_1p_2)\theta - (1 - p_1)[F(p_2\theta) - F((1 - p_1)\theta)]\theta = (1 - 2p_2 + 2p_1p_2)\theta - (1 - p_1)\theta(p_1 - p_2)\theta = (1 - p_1)(1 - p_2)\theta > 0$.

In the presence of incomplete information, a player under the favorable status quo tie-breaking rule may have less incentive to participate than the other player. In other words, the status quo rule may have a perverse incentive effect on players' incentives to participate. As a consequence, the incumbent may be less likely than the challenger to get the prize, even though, under the coin toss tie-breaking rule, both of the two players participate with the same positive probability. However, the incumbent's expected payoff is always higher than the challenger's.

3. Conclusion

This paper shows that, in the presence of asymmetric information, the status quo tie-breaking rule may have an unexpected negative effect on the favored player's getting the prize, even if, under the coin toss tie-breaking rule, the players participate with the same positive probability (and hence are equally likely to get the prize). However, the favored player's expected payoff is always greater than that of his competitor.

In this paper, only two risk-neutral players are involved to compete for a prize. The analysis can be extended to cases where more than two players are involved and the players are risk averse.