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# Modelling smooth and uneven cross-sectoral growth patterns: an identification problem

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## *Abstract*

This paper shows that the available stylized facts on productivity dynamics, such as persistent cross-sectoral heterogeneity, do not allow to solve an identification problem regarding the impact of common drivers - such as General Purpose Technologies (GPTs) - on economic growth. The evidence of persistently heterogeneous productivity performances is consistent both with a GPT-driven model, and with a model characterized by purely independent and idiosyncratic sectoral dynamics. These results are obtained within a simple theoretical framework, and illustrated with reference to measures of concentration of the sectoral contributions to aggregate total factor productivity growth.

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## 1. Introduction

According to a well established research tradition, technological complementarities and pervasive technological innovations are fundamental engines of long-run economic growth. Such a view is supported by the analysis of a number of important historical examples, such as machine tools, chemicals, electricity, semiconductors, and information and communication technologies (Rosenberg 1976, David 1990, Rosenberg and Trajtenberg 2001, Freeman and Louca 2001). In the recent debate on this topic, General Purpose Technologies (GPTs henceforth) have occupied a salient place, as testified by the influential models in the book edited by Helpman (1998). GPTs are pervasive technologies, i.e. they find application in a wide range of industrial sectors. GPTs are moreover characterized by technological cumulativeness and innovation complementarities.

Under such conditions, growth is supposed to spread evenly across sectors. However, a good deal of empirical research on longitudinal microdata (Nelson 1981, Harberger 1998, Doms and Bartelsman 2000, Peneder 2005, Dosi and Grazzi 2006, Sapio and Thoma 2006) suggests that the growth process is uneven: the evidence of huge and persistent cross-firm and cross-sectoral asymmetries in productivity levels and growth rates appears to be a robust feature of industrialized economies.

Compared with the prediction of a uniform rate of economic progress across sectors, the cited evidence may cast doubts on the empirical relevance of the GPT view: the productivity data look like most of the action behind the sectoral dynamics is purely idiosyncratic. Yet, in order to falsify the GPT model, one has to formally test the hypothesis that the common component of sectoral productivity growth rates is negligible. This in turn requires reliance on statistics which allow to identify common and idiosyncratic components, and to evaluate their respective magnitudes. The question arises as to whether indicators of heterogeneity of productivity growth rates, as such, are able to perform this function.

This paper shows that the available stylized facts on productivity do not allow to solve the aforementioned identification problem. Indeed, persistently heterogeneous productivity performances can be reproduced by *both* a GPT-driven model and by a model characterized by purely idiosyncratic dynamics. These results are obtained within a simple theoretical framework, and illustrated with reference to measures of concentration of sectoral contributions to aggregate total factor productivity growth (TFP). The more concentrated

are TFP growth contributions, the more diverse are sectoral growth paths. Our analysis reveals that such a diversity can as well be the outcome of a GPT dynamics, under the condition that different sectors respond differently to common shocks.

## 2. The model

Suppose the economy is composed of  $n > 1$  industries, and that total output of the generic industry  $i$  at time  $t$ ,  $Q_{it}$ , is given by the following Cobb-Douglas function:

$$Q_{it} = A_{it} K_{it}^{\alpha} L_{it}^{1-\alpha} \quad (1)$$

where  $K_{it}$  and  $L_{it}$  are capital and labour inputs, and  $A_{it}$  is Total Factor Productivity (TFP). Assume that TFP in sector  $i$  is a function of industry specific shocks,  $Z_k$ ,  $k = 1, \dots, i, \dots, n$ , stemming from all industries in the economy:

$$A_{it} = A_{it}(Z_{1t}, Z_{2t}, \dots, Z_{it}, \dots, Z_{nt}) \quad (2)$$

where  $A_{it}$  is continuous and twice differentiable with respect to all its arguments. After some algebra, the growth rate of TFP in sector  $i$  reads:

$$g_{it} = \frac{1}{A_{it}} \frac{dA_{it}}{dt} = \frac{1}{A_{it}} \sum_{j=1}^n \frac{\partial A_{it}}{\partial Z_{jt}} \frac{dZ_{jt}}{dt} = \sum_{j=1}^n \epsilon_{it}^j a_j \quad (3)$$

where  $a_j \equiv \frac{dZ_{jt}}{dt} \frac{1}{Z_{jt}}$ , constant over time for simplicity, and  $\epsilon_{it}^j \equiv \frac{\partial A_{it}}{\partial Z_{jt}} \frac{Z_{jt}}{A_{it}}$  is the elasticity of TFP in industry  $i$  with respect to the shock  $Z_{jt}$ . A low (high) elasticity means that the TFP growth rate in sector  $i$  reacts poorly (greatly) to a shock in sector  $j$ . The time subscript allows for time dynamics in the value of the elasticity term.

The formulation given in (1)-(3) is very general and features two special cases:

- the *purely idiosyncratic* process, in which TFP growth in industry  $i$  depends only on an idiosyncratic shock  $Z_{it}$ :

$$g_{it} = \epsilon_{it}^i a_i \quad (4)$$

- the *pure GPT* process, such that TFP growth in each industry is entirely due to a shock stemming from a given sector  $z$ :

$$g_{it} = \epsilon_{it}^z a_z \tag{5}$$

Linear combinations of these two and other hybrid cases can be envisioned, too.<sup>1</sup>

### 3. Results

All this given, we ask whether asymmetries in aggregate TFP growth contributions can occur even in a pure GPT economy. To do so, we need to investigate upon the conditions behind growth heterogeneities. The following is proposed:

**Proposition 1.** If  $\epsilon_{it}^k = \epsilon_{jt}^k \forall (i, j, k)$ , then  $g_{it} = g_{jt} \forall (i, j) \in \{1, \dots, n\} \times \{1, \dots, n\}$ .

The above proposition establishes a sufficient condition for homogeneity of sectoral TFP growth rates, as given by (3). This sufficient condition states that if all sectors react in the same way to a shock stemming from a given sector, then all their TFPs grow at the same rate. To see why, notice that

$$g_{it} - g_{jt} = \sum_{k=1}^n (\epsilon_{it}^k - \epsilon_{jt}^k) a_k$$

which is null if  $\epsilon_{it}^k - \epsilon_{jt}^k = 0 \forall k$ . Repeating this for all  $(i, j)$  couples yields the above result. As a consequence of this result, for asymmetries in TFP growth rates to emerge, it suffices that elasticities differ for at least one couple of industries. However, the condition stated in Proposition 1 is not necessary: homogeneity may hold even when industries are characterized by different elasticities. The given condition is both necessary and sufficient only in a *pure GPT* economy (Eq. 5): in that case, all sectors absorb shocks from the same source, and inter-sectoral differences can only emerge if elasticities are heterogeneous. Importantly, asymmetries in TFP growth rates are possible regardless of the structure of inter-sectoral linkages

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<sup>1</sup>The two above examples correspond, respectively, to Harberger's (1998) *mushrooms* and *yeast* visions of economic growth.

- whether growth is the outcome of purely idiosyncratic processes, pure GPT processes, or intermediate cases.

As reported in the Introduction, a number of empirical works on longitudinal microdata suggests that the growth process is uneven. Such statistical contributions are based on diverse methodological tools, such as empirical density fit (Dosi and Grazzi 2006, Sapio and Thoma 2006), as well as concentration measures (Harberger 1998, Peneder 2005). The former works suggest that the cross-sectional variance of growth rates is not simply due to noise, but to some deeper mechanisms, closely related to the structure of linkages between firms and sectors. The latter provide evidence of concentration in the sectoral contributions to aggregate productivity gains, by making use of Lorenz curves. While Proposition 1 has a clear theoretical content, we are also interested in shedding light on what the proposed model yields in terms of quantitative measures, e.g. in terms of concentration indexes. In what follows, the model implications are illustrated with a focus on Lorenz curves, as in Harberger (1998) and Peneder (2005), in order to directly address their empirical results in light of our analysis.<sup>2</sup>

Generally speaking, a Lorenz curve depicts the percentage contribution to a variable  $Y$  by individuals (agents, firms, households etc.) who account for a given share of a variable  $X$ . If individuals accounting for  $x\%$  of the variable  $X$  hold  $x\%$  of the variable  $Y$ ,  $\forall x \in [0, 100]$ , then a situation of *equidistribution* occurs. Graphically, in such a case the Lorenz curve coincides with a 45-degree line. Deviations of the Lorenz curve from the equidistribution line denote the presence of *concentration* - relatively large shares of  $Y$  are concentrated in individuals holding a relatively low percentage of  $X$ , so that individual endowments are heterogeneous. When the  $Y$  variable can assume negative values, as with (TFP) growth rates, the ever-increasing Lorenz curve has to be replaced by a quasi-Lorenz curve which can feature a decreasing region, as shown in Figure 1, drawn from Harberger (1998).

In the context of our model, the quasi-Lorenz curve displays the cumulative sum of sectoral Initial Value Added shares on the horizontal axis and, on the vertical axis, the cumulative sum of contributions of individual industries to aggregate Real Cost Reduction, or RCR (Initial Value Added multiplied by TFP growth).<sup>3</sup> Formally, for industry  $i$  at time  $t$  we have, on

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<sup>2</sup>The relevance of the Lorenz curve approach in the mentioned literature is witnessed by the debate in David and Wright (2003).

<sup>3</sup>In line with the empirical literature on cross-sectoral growth patterns (e.g. Harberger

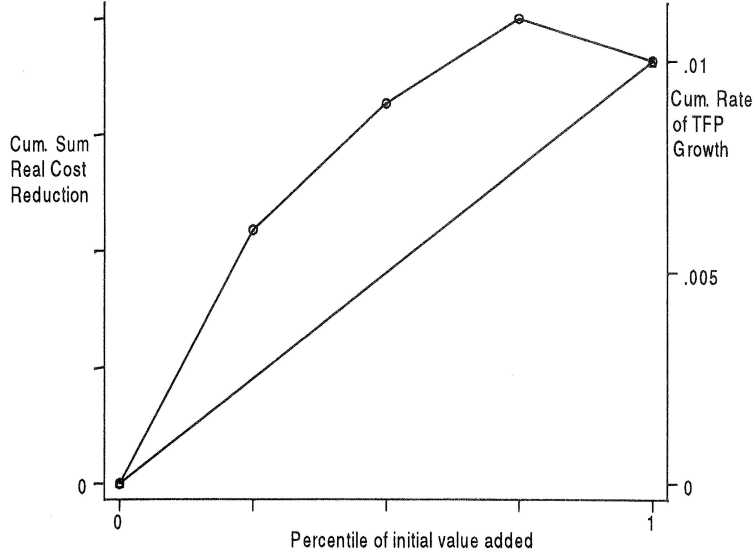


Figure 1: Quasi-Lorenz curve. The horizontal axis displays the percentiles of Initial Value Added (see Eq. 6). The vertical axes report the cumulative sum of Real Cost Reduction (left axis; see Eq. 7), and the Cumulative Rate of TFP Growth (right axis). The straight line is the equidistribution line. Deviations from the equidistribution line denote the presence of concentration. Source: Harberger (1998).

the horizontal axis, the percentiles of Initial Value Added,

$$q_{it} = \frac{\sum_{j=1}^i Q_{jt}}{\sum_{j=1}^n Q_{jt}} \quad (6)$$

and, on the vertical axis,

$$x_{it} = \frac{\sum_{j=1}^i Q_{jt}g_{jt}}{\sum_{j=1}^n Q_{jt}g_{jt}} \quad (7)$$

Within this framework, we propose a necessary and sufficient condition for the evidence of concentration in productivity gains.

**Proposition 2.** Concentration in sectoral contributions to RCR emerges if and only if  $g_{it} \neq g_{jt}$  for at least one couple  $(i, j) \in \{1, \dots, n\} \times \{1, \dots, n\}$ ,  $i \neq j$ .

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1998 and Peneder 2005), RCR is used instead of TFP growth.

Multiplying and dividing (7) by  $\sum_{j=1}^i Q_{jt} \sum_{j=1}^n Q_{jt}$  we obtain:

$$x_{it} = \frac{G_{it}}{G_t} q_{it} \quad (8)$$

where  $G_{it} \equiv \sum_{j=1}^i Q_{jt} g_{jt} / \sum_{j=1}^i Q_{jt}$ , and  $G_t \equiv \sum_{j=1}^n Q_{jt} g_{jt} / \sum_{j=1}^n Q_{jt}$  are, respectively, the weighted average of TFP growth rates over the first  $i$  sectors and the aggregate TFP growth rate. Notice that in (8) the cumulative sum of sectoral contributions to RCR is a linear function of the cumulative shares of Initial Value Added, i.e. the variable on the horizontal axis. The ratio  $G_{it}/G_t$  is thus the “local” slope of the Lorenz-like curve. Concentration is zero if and only if  $|G_{it}/G_t| = 1 \forall i$ .<sup>4</sup> Equidistribution therefore requires:

$$\frac{\sum_{j=1}^i Q_{jt} g_{jt}}{\sum_{j=1}^i Q_{jt}} = \frac{\sum_{j=1}^n Q_{jt} g_{jt}}{\sum_{j=1}^n Q_{jt}} \quad (9)$$

Via induction, one can show that a necessary condition for zero concentration is  $g_{it} = g_{jt} \forall i, j$ . The latter is also sufficient. To see why, suppose  $g_{it} = g_{jt} = g_t, \forall i, j$ , and plug into expressions for  $G_{it}$  and  $G_t$ . This yields  $G_{it} = G_t = g_t, \forall i$ . Hence, we conclude that

$$\frac{G_{it}}{G_t} = 1 \Leftrightarrow g_{it} = g_{jt}, \forall i, j \quad (10)$$

The above implies that concentration in the sectoral contributions to aggregate RCR arises whenever industries are heterogeneous in terms of TFP growth rates. Since  $g_{it} = \sum_{j=1}^n \epsilon_{it}^j a_j$ , the necessary and sufficient condition for concentration in contributions to aggregate RCR is:

$$\sum_{j=1}^n \epsilon_{it}^j a_j \neq \sum_{j=1}^n \epsilon_{kt}^j a_j \quad (11)$$

for at least one couple  $(i, k) \in \{1, \dots, n\} \times \{1, \dots, n\}, i \neq k$ .

Let us now restrict our analysis to the pure GPT process (Eq. 5). In such a case, condition (11) boils down to:

$$\epsilon_{it}^z \neq \epsilon_{kt}^z \quad (12)$$

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<sup>4</sup>Actually, in the quasi-Lorenz curve diagrams, the ratio  $G_i/G$  equals the cumulative rate of TFP growth. Without loss of generality, we normalize this to one.

for at least one couple  $(i, k) \in \{1, \dots, n\} \times \{1, \dots, n\}$ ,  $i \neq k$ . Hence, concentration in the pure GPT case arises if and only if at least two sectors have different elasticities of TFP with respect to the shock stemming from sector  $z$ .

The above results suggest that concentration in the contributions to sectoral RCR is consistent with both the pure GPT and the purely idiosyncratic cases. The GPT model can only produce a smooth cross-sectoral growth pattern if the elasticities of sectoral TFP to shocks from other sectors are similar across sectors. On the contrary, under heterogeneity of elasticities, growth asymmetries can be observed even if a common component drives the productivity growth of all industries.

Heterogeneity in the elasticities of TFP with respect to technology shocks is therefore a key driver of the proposed results. The heterogeneity assumption is supported by a large body of empirical research and case studies (see e.g. David 1990, Helpman 1998, Bresnahan et al. 2002, OECD 2003). These works reveal that the rise and diffusion of a new GPT require firms to engage in a process of organizational change, and to invest additional resources in order to introduce complementary technologies and to educate and train the employees. This process of “co-invention” (Bresnahan and Greenstein 2001) is firm-specific and very uncertain in its outcomes, being influenced by many factors like the availability of human capital, the size of the firm and the complexity of its organization. The characteristics of such a process - as well as its returns - are also likely to display huge cross-sectoral variability, which stems from the differences existing among industries in terms of workers’ skills, organizational structures and in the distribution of firm size.<sup>5</sup>

#### 4. Conclusion

As a conclusion, the evidence of persistently heterogeneous productivity performances in different firms and industries, as such, does not allow to discriminate between the GPT model and an opposite view of growth driven by many independent sectoral components. This is due to an identification problem, related to heterogeneity in the elasticities of sectoral TFPs to shocks from other sectors, which ends up “hiding” the true, underlying source of economic dynamics.

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<sup>5</sup>Bottazzi and Secchi (2003a) have shown that the distribution of firm size can considerably vary across sectors.



Our result has important implications. *First*, the existing evidence is not enough to discard common drivers of growth as negligible, in spite of Harberger's (1998) claims. Cross-sectoral diversities in productivity performances may be the outcome of either idiosyncratic shocks, or idiosyncratic reactions to common drivers, or both. Disentangling these effects is a challenge for future research.

*Second* and relatedly, the validity of the GPT model needs to be evaluated in light of how firms belonging to different sectors absorb both GPTs and the related bodies of knowledge. To this end, the recent applied research on diffusion of GPTs (see Section 3) has highlighted the key role played by factors like the adoption of complementary technologies, human capital, and organizational change, in determining the heterogeneous absorptive capacity (Cohen and Levinthal, 1990) of firms and industries as to GPT shocks. This literature could provide useful hints towards realistic modelling solutions for the contribution of GPTs to productivity growth.

*Third*, the introduction of a GPT in an economy populated by heterogeneous firms and sectors may considerably increase the cross-sectoral volatility of productivity growth rates, in sharp contrast with the traditional view of GPTs as imposing a common pace of productivity improvement to all industries. Any rigorous assessment of the impact of GPTs on the social welfare of an economy should therefore take account of its effects on the cross-sectoral distribution of growth rates. Future research on the nature of the growth process should be inspired by these insights.

## References

- [1] Bartelsman E., and M. Doms (2000), "Understanding Productivity: Lessons from Longitudinal Microdata", *Journal of Economic Literature* 38: 569-594.
- [2] Bottazzi G., and A. Secchi (2003a), "Common Properties and Sectoral Specificities in the Dynamics of U.S. Manufacturing Firms", *Review of Industrial Organization*, 23: 217-232.
- [3] Bresnahan T., E. Brynjolfsson, and L. Hitt (2002), "Information Technology, Workplace Organization and the Demand for Skilled Labour: Firm-Level Evidence", *Quarterly Journal of Economics*, 117: 339-376.
- [4] Bresnahan T., and S. Greenstein (2001), "The Economic Contribution of Information Technology: Towards Comparative and User Studies", *Journal of Evolutionary Economics*, 11: 95-118.

- [5] Cohen W. M., and D. Levinthal (1990), “Absorptive Capacity: A New Perspective on Learning and Innovation”, *Administrative Science Quarterly* 35(1): 128-152.
- [6] David P. A. (1990), “The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox”, *American Economic Review* 80(2): 355-361.
- [7] David P. A. and G. Wright (2003), “General Purpose Technologies and Productivity Surges: Historical Reflections on the Future of the ICT Revolution”, in P. A. David and M. Thomas (eds.), *The Economic Future in Historical Perspective*, Oxford University Press for the British Academy.
- [8] Dosi G., and M. Grazzi (2006), “Technologies as Problem-solving Procedures and Technologies as Input-Output Relations: Some Perspectives on the Theory of Production”, *Industrial and Corporate Change* 15(1): 173-202.
- [9] Freeman C., and F. Louca (2001), *As Time Goes By: From the Industrial Revolutions to the Information Revolution*, Oxford University Press.
- [10] Harberger A.C. (1998), “A Vision of the Growth Process”, *American Economic Review* 88: 1-32.
- [11] Helpman E. (ed.) (1998), *General Purpose Technologies and Economic Growth*. Cambridge, MA: MIT Press.
- [12] Nelson R.R. (1981), “Research on Productivity Growth and Productivity Differences: Dead Ends and New Departures”, *Journal of Economic Literature* 19: 1029-1064.
- [13] OECD (2003), *ICT and Economic Growth: Evidence from OECD Countries, Industries and Firms*, Paris.
- [14] Peneder M. (2005), “Tracing Empirical Trails of Schumpeterian Development”, in U. Cantner, E. Dinopoulos and R.F. Lanzillotti, eds., *Entrepreneurship, the New Economy and Public Policy*. Springer, Berlin-Heidelberg.
- [15] Rosenberg N. (1976), *Perspectives on Technology*. Cambridge: Cambridge University Press.
- [16] Rosenberg N., and M. Trajtenberg (2001), “A General Purpose Technology at Work: The Corliss Steam Engine in the late 19th Century”, NBER working paper n. 8485.
- [17] Sapio S., and G. Thoma (2006), “The Growth of Industrial Sectors: Theoretical Insights and Empirical Evidence from U.S. Manufacturing”, LEM Working Paper 2006-09, Sant’Anna School of Advanced Studies, Pisa.