E C O N O M I C S   B U L L E T I N

Strategic choice of price policy under exogenous switching costs

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Abstract

This paper examines the equilibrium incentive for firms to use behavior-based price discrimination in a duopoly market with exogenous switching costs. We find that if there is a large difference in the existing market shares between two firms, then discriminatory pricing is a unique Nash equilibrium. Otherwise, there are three Nash equilibria: both firms engage in discriminatory pricing, or engage in uniform pricing, or engage in mixed strategies. The respective firms are worse off in the discriminatory equilibrium compared with the others.

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1. Introduction

There are several studies that deal with competitive discrimination with exogenous switching costs (Bester and Petrakis, 1996; Shaffer and Zhang, 2000; Chen, 1997, and Taylor, 2003). However, except for an analysis of spatial pricing policies by Thisse and Vives (1988), to the best of our knowledge, the firms' pricing policies are assumed, and there is little discussion on whether the respective firms do in fact select a strategy of carrying out price discrimination. The purpose of this paper is to examine the equilibrium incentive for firms to use behavior-based price discrimination. Thisse and Vives (1988) shows that under oligopoly competition firms may have unilateral incentives to engage in price discrimination. In direct contrast to this, our novel results show that there are multiple equilibria depending on the existing market share.

The rest of the paper is organized as follows. Section 2 sets up the basic model. Section 3 analyzes the determination of the prices in a mature market where each firm has already established a market share. We conclude in Section 4. Proofs are gathered in the Appendix.

2. The Basic Model

We describe the competition of firms in a market with exogenous switching costs. Suppose that there are two firms, A and B, who produce products A and B, respectively, with constant marginal cost c. Each consumer can consume either a unit of product A or a unit of product B, but not both. Total number of consumers is normalized to one. We assume that the consumer's reservation price is so high that in the equilibrium, all consumers will use one of the products. We consider competition in the period after all consumers have purchased some products. Let α denote the market share of firm A in the previous period, and $1-\alpha$ that of firm B.

During the current period, the two firms sell new products with the same features and quality, and all consumers buy the new products that are now provided. At this time, in cases where given consumers switch to the other firm, the switching cost s must be borne by these consumers. We assume that the switching cost s is uniformly distributed across the consumer population on the interval [0, θ] with density $1/\theta$. To ensure the equilibria (specifically, to satisfy the second-order conditions), it is assumed that $0<\theta<9/2$. We normalize $\theta=1$ without loss of generality.

It is assumed that the products of these respective firms are homogeneous. For this reason, the consumers consider only the price and the switching costs s when selecting goods. There is a cut-off $\bar{s}^{ij}$ such that among the consumers who have bought from firm *i* in the previous period, all consumers with $s>\bar{s}^{ij}$ continue to buy from firm *i*, and all consumers with $s<\bar{s}^{ij}$ switch to buy from firm *j*, where $i, j = A, B$ and $i \neq j$. Then, the number of consumers $\alpha(1-\bar{s}^{AB})$ continue to buy from firm A, and the number of consumers $\alpha\bar{s}^{AB}$ switch from firm A to firm B. Similarly, the number of consumers $(1-\alpha)(1-\bar{s}^{BA})$ continue to buy from firm B, and the number of consumers $(1-\alpha)\bar{s}^{BA}$ switch from firm B to firm A.

The structure of the game is as follows. We consider a two-stage game, with simultaneous moves in each stage. In stage one, the firms decide on their pricing policy. That is, they can exercise price discrimination or uniform pricing. In stage two, knowing each other's pricing policy, the firms choose the price and consumers make their purchase decisions. If a firm chooses a price discrimination policy in stage one, then in stage two it can set different prices of p_0^i for its own previous (original) customers and p_N^i for new customers who switch away from its rival. Otherwise, a firm chooses a uniform price of p^i for all customers, where $i=$A, B. If

$\alpha = 0$ or $\alpha = 1$, then it is not possible for any firm to exercise price discrimination for its previous customers and new customers. Thus, in this paper, we consider a situation in which both firms have their own previous customer, i.e., $0 < \alpha < 1$.

3. The Equilibrium

As a result of the selections of pricing policy by the two firms in stage one, four cases are possible: (DP, DP), (UP, UP), (DP, UP), (UP, DP). Here, "DP" indicates discriminatory pricing and "UP" indicates uniform pricing. Thus, for example, "(DP, UP)" indicates that firm A is using discriminatory pricing and firm B is using uniform pricing.

We examine equilibrium prices in stage two for each of these respective cases. Firstly, we consider the subgame in which both firms carry out discriminatory pricing (DP, DP). The consumer who has bought from firm *i* in the previous period is indifferent between continuing to buy from firm *i* and switching to buy from firm *j*, if her switching costs *s* satisfy the condition $p_0^i = p_N^j + s$. For this reason, the cut-off becomes $\bar{s}^{ij} = (p_0^i - p_N^j)$. There exist some switchers from firm *A* to firm *B* and other switchers who switch from firm *B* to firm *A*. It can be easily seen that in the equilibrium $0 \le p_0^A - p_N^B \le 1$ and $0 \le p_0^B - p_N^A \le 1$. Here, provided that $0 \le p_0^A - p_N^B \le 1$ and $0 \le p_0^B - p_N^A \le 1$, each firm's profit ($\pi^i$) in this period is given by:

$$\pi^A(\text{DP, DP}) = \alpha\{1 - (p_0^A - p_N^B)\}(p_0^A - c) + (1-\alpha)(p_0^B - p_N^A)(p_N^A - c),$$

$$\pi^B(\text{DP, DP}) = (1-\alpha)\{1 - (p_0^B - p_N^A)\}(p_0^B - c) + \alpha(p_0^A - p_N^B)(p_N^B - c).$$

Each firm *i* maximizes its profits π^i with respect to its current prices, taking the market shares, $\alpha, 1-\alpha$ in the previous period, and the other firm's prices as given. From the conditions of profit maximization we can derive the Nash equilibrium.

Secondly, we consider the subgame in which both firms carry out uniform pricing (UP, UP). When $p^A \ge p^B$, switching from firm A to firm B will occur, and the cut-off becomes $\bar{s}^{AB} = (p^A - p^B)$, but switching from firm B to firm A will not occur, therefore, $\bar{s}^{BA} = 0$. The profits of firm A and B (π^A and π^B) in this period are given by:

$$\pi^A(\text{UP, UP}) = \alpha\{1 - (p^A - p^B)\}(p^A - c),$$

$$\pi^B(\text{UP, UP}) = \{(1-\alpha) + \alpha(p^A - p^B)\}(p^B - c).$$

Each firm *i* maximizes its profits π^i with respect to its price, taking the market shares, $\alpha, 1-\alpha$ in the previous period, and the other firm's price as given. From the conditions of profit maximization we can derive the Nash equilibrium.

For the cases of (DP, DP) and (UP, UP) there were similar analyses by Chen (1997). In fact, p^A, p_0^A, p_N^A of our models correspond to $p_{A2}, p_{A2}, p_{A2} - m_A$ of Chen (1997), respectively. So we make the following remark (See Appendix for details).

Remark: Chen (1997). When both firms use discriminatory pricing (DP, DP), there is a unique Nash equilibrium, and the equilibrium prices are:

$$p_0^A* = p_0^B* = c + \frac{2}{3}, \quad p_N^A* = p_N^B* = c + \frac{1}{3}. \tag{1}$$

When both firms use uniform pricing; (UP, UP), the equilibrium prices are:

$$p^A* = c + \frac{1+\alpha}{3\alpha},\quad p^B* = c + \frac{2-\alpha}{3\alpha},\qquad \text{if}\quad p^A \ge p^B \ \ (1/2 \le \alpha < 1),$$
$$p^A* = c + \frac{1+\alpha}{3(1-\alpha)},\quad p^B* = c + \frac{2-\alpha}{3(1-\alpha)},\quad \text{if}\quad p^A < p^B \ \ (0 < \alpha < 1/2). \tag{2}$$

Chen (1997) shows that, for (DP, DP), each firm's equilibrium price is independent of its market share in the previous period. On the other hand, for (UP, UP), each firm's equilibrium price depends on its existing market shares, ($\alpha,\ 1-\alpha$). The results also show that $p^A* \ge p^B*$ if and only if $\alpha \ge 1/2$, that is, the firm with higher market shares sets higher prices and places weight on securing profits from existing customers, while the firm with lower market share sets lower prices and attempts poaching to attract customers from the rival firm.

Thirdly, we consider the asymmetric case of (DP, UP) where firm A carries out price discrimination and firm B uses a policy of uniform pricing. The consumer who has bought from firm A in the previous period (the previous customer of firm A) is indifferent between continuing to buy from firm A and switching to buy from firm B, if her switching costs s satisfy the condition $p_0^A = p^B + s$. Similarly, the previous customer of firm B is indifferent between remaining and switching, if her switching costs s satisfy the condition $p_N^A + s = p^B$. For this reason, when $p_0^A \ge p^B \ge p_N^A$, the cut-offs become $\bar{s}^{AB} = (p_0^A - p^B)$ and $\bar{s}^{BA} = (p^B - p_N^A)$. In this instance, there exist some switchers from firm A to firm B and other switchers who switch from firm B to firm A.

We can see that in the equilibrium it must hold that $p_0^A \ge p^B \ge p_N^A$. In contradiction, if $p^B > p_0^A$, then every previous customer of firm A does not switch to firm B, hence there is an incentive for firm A to raise price p_0^A. Similarly, if $p_N^A > p^B$, then every previous customer of firm B does not switch to firm B, hence there is an incentive for firm B to raise price p^B. Here, if $p_0^A \ge p^B \ge p_N^A$, then each firm's profit (π^A and π^B) in this period is given by, respectively:

$$\pi^A(\text{DP, UP}) = \alpha\{1-(p_0^A - p^B)\}(p_0^A - c) + (1-\alpha)(p^B - p_N^A)(p_N^A - c),$$
$$\pi^B(\text{DP, UP}) = [(1-\alpha)\{1-(p^B - p_N^A)\} + \alpha(p_0^A - p^B)](p^B - c).$$

In stage two of the game, each firm maximizes its profits with respect to prices, taking the previous market shares α and $1-\alpha$, and the other firm's price as given. The following proposition summarizes the equilibrium in the asymmetric case.

Proposition 1 When firm A practices discriminatory pricing and firm B uses uniform pricing (DP, UP), the equilibrium prices are given by:

$$p_0^A* = c + \frac{5-\alpha}{6},\quad p_N^A* = c + \frac{2-\alpha}{6},\quad p^B* = c + \frac{2-\alpha}{3}, \tag{3}$$
$$p_0^A* \ge p^B* \ge p_N^A* \quad \text{for any } \alpha \in (0,1).$$

Similarly, when firm A practices uniform pricing and firm B uses discriminatory pricing (UP, DP), the equilibrium prices are given by:

$$p^A* = c + \frac{1+\alpha}{3},\quad p_0^B* = c + \frac{4+\alpha}{6},\quad p_N^B* = c + \frac{1+\alpha}{6}, \tag{4}$$
$$p_0^B* \ge p^A* \ge p_N^B* \quad \text{for any } \alpha \in (0,1).$$

Proof. From the first-order conditions for profit maximization it is easy to derive the results. The second-order conditions are satisfied (See Appendix for details).

The discriminating firm sets a higher price $(p_0^A*\text{or } p_0^B*)$ for old (existing) customers and a lower price for new customers $(p_N^A*\text{or } p_N^B*)$ than the price $(p^B*\text{or } p^A*)$ set by the non-discriminating firm, irrespective of the market share in the previous period.

When we substitute the equilibrium prices given by equations (1), (2), (3), and (4) into the profit functions, we obtain the equilibrium profits as follows.
In the case of discriminatory pricing:

$$\pi^A*(DP,DP)=\frac{1+3\alpha}{9},\quad \pi^B*(DP,DP)=\frac{4-3\alpha}{9}.$$

In the case of uniform pricing:

$$\pi^A*(UP,UP)=\frac{(1+\alpha)^2}{9\alpha},\quad \pi^B*(UP,UP)=\frac{(2-\alpha)^2}{9\alpha},\ \text{ if }\ p^A\ge p^B\ \ (1/2\le\alpha<1),$$

$$\pi^A*(UP,UP)=\frac{(1+\alpha)^2}{9(1-\alpha)},\quad \pi^B*(UP,UP)=\frac{(2-\alpha)^2}{9(1-\alpha)},\ \text{ if }\ p^A<p^B\ \ (0<\alpha<1/2).$$

In the asymmetric case:

$$\pi^A*(DP,UP)=\frac{(4+17\alpha-5\alpha^2)}{36},\quad \pi^B*(DP,UP)=\frac{(2-\alpha)^2}{9},$$

$$\pi^A*(UP,DP)=\frac{(1+\alpha)^2}{9},\quad \pi^B*(UP,DP)=\frac{(16-7\alpha-5\alpha^2)}{36}.$$

We suppose that the two firms select pricing strategies in stage one to maximize their own profits. Then we derive the following proposition.

Proposition 2 If there is a large difference in market shares, or formally either $0<\alpha<1/5$ or $4/5<\alpha<1$ is satisfied, then (DP, DP) is a unique Nash equilibrium regardless of the value of c.Otherwise (i.e., $1/5\le\alpha\le 4/5$), there are two pure-strategy equilibria (DP, DP) and (UP, UP), and one mixed-strategy equilibrium $p*=\frac{(4-5\alpha)(4+\alpha)}{16-16\alpha-\alpha^2}$ and $q*=\frac{(2-\alpha)(2+5\alpha)}{4+8\alpha-\alpha^2}$, where $p*$ and $q*$ $(\in[0,1])$ are the probabilities of firms A and B exercising price discrimination, respectively.

Proof. Suppose that firm A uses DP with probability p and UP with probability $(1-p)$, and firm B uses DP with probability q and UP with probability $(1-q)$. First, we consider the case in which $p^A\ge p^B$, that is, $\alpha\ge 1/2$. Then the expected profit of firm A is given by:

$$E\pi^A=\frac{(1+\alpha)^2}{9\alpha}\big(1-(1-\alpha)q\big)+p\left(\frac{(1-\alpha)}{36\alpha}\right)\Big[\big(4+8\alpha-\alpha^2\big)q-(2-\alpha)(2+5\alpha)\Big].$$

Firm A chooses the probability p that maximizes its expected profit for a given probability q. The expected profit of firm B is given by:

$$E\pi^B=\frac{(2-\alpha)^2}{9\alpha}\big(1-(1-\alpha)p\big)+q\frac{(1-\alpha)}{36\alpha}\Big[(16-16\alpha-\alpha^2)p-(4-5\alpha)(\alpha+4)\Big]$$

$$=\frac{(2-\alpha)^2}{9\alpha}\big(1-(1-\alpha)p\big)+q\frac{(1-\alpha)}{36\alpha}\Big[(5\alpha-4)(\alpha+4)(1-p)+4\alpha^2p\Big]$$

Firm B chooses the probability q that maximizes its expected profit for a given probability p. Then it is easy to show that the best response functions are given by:

$$p = \begin{cases} 0 & \text{if} \quad q < q^* \\ [0,\ 1] & \text{if} \quad q = q^* \\ 1 & \text{If} \quad q > q^* \end{cases}, \text{ and } q = \begin{cases} 0 & \text{if} \quad p < p^* \\ [0,\ 1] & \text{if} \quad p = p^* \\ 1 & \text{If} \quad p > p^* \end{cases} \text{ if } 1/2 \le \alpha \le 4/5$$

$$q = 1 \text{ for any } p \in [0,\ 1] \text{ if } 4/5 < \alpha < 1,$$

where $p^* = \dfrac{(4-5\alpha)(4+\alpha)}{16-16\alpha-\alpha^2}$ and $q^* = \dfrac{(2-\alpha)(2+5\alpha)}{4+8\alpha-\alpha^2}$. From the best response functions we have a unique pure-strategy equilibrium (DP, DP) if $4/5 < \alpha < 1$, and we have two pure-strategy equilibria (DP, DP) and (UP, UP) and one mixed-strategy if $1/2 \le \alpha \le 4/5$. The same argument can be applied to the case in which $p^A < p^B$, that is, $\alpha < 1/2$. Then we have a unique pure-strategy equilibrium (DP, DP) if $0 < \alpha < 1/5$, and we have two pure-strategy equilibria (DP, DP) and (UP, UP) and one mixed-strategy equilibrium if $1/5 \le \alpha < 1/2$. **QED**.

Interestingly, our results are in direct contrast to the current state of the literature on strategic pricing policies. For example, in a static duopoly model on spatial pricing policies, Thisse and Vives (1988) show that price discrimination is the dominant strategy for both firms.

The intuition behind this proposition is as follows. If one firm engages in DP and tries to poach customers from its competitor by offering them special discounts, then the other firm should respond to retain its customers. In this case, it is more profitable for the other firm to also engage in DP than it is to respond by lowering its price uniformly for all customers under UP. Assuming each firm has existing customers, DP is the best-response strategy for each firm against other firm's DP. Thus, (DP, DP) is one of the equilibria. On the other hand, suppose that one firm engages in UP. If the other firm also employs UP, then it allows firms to relax their price competition, thereby raising prices and earning higher profits on their partially locked-in customers. Usually this is profitable for each firm, and so (UP, UP) is another equilibrium. However, when there is an extremely large difference in the existing market shares, i.e., $0 < \alpha < 1/5$ or $4/5 < \alpha < 1$, the equilibrium outcome is different. Then, a smaller firm has a greater incentive to employ DP and tries to poach customers from its competitor, regardless of the rival firm's pricing policy. That is, DP is a dominant strategy for a smaller firm. In this case, (DP, DP) is a unique equilibrium.

Proposition 3 Regardless of the values of c and α, both firms' equilibrium profits are lower under (DP, DP) than under (UP, UP).

Proof. Comparing the profits, it is easy to prove this proposition (See Appendix for details).

Unlike the monopoly case, competitive price discrimination intensifies competition, and the profits of each firm will be lower than if none of the firms practice price discrimination. This result is similar to results found in the analysis involving coupon-based price discrimination carried out by Bester and Petrakis (1996), the poaching analysis carried out by Chen (1997), and the analysis of loyalty-rewarding pricing schemes by Caminal and Claici (2007). The result comes from the fact that firms differ in their view of which markets are strong and which are weak, i.e., the “best-response asymmetry” termed by Corts (1998). In our model of customers’ switching costs, each firm has a “strong market” of its own partially locked-in customers, preferring to set a higher price in this market than in the “weak market” of rival’s customers. Thus, the two firms have different strong markets. When price discrimination is permitted and some firm offers a lower price to its competitor’s customers, we would expect a rival firm to react by offering a lower price to these customers, with the result that prices for every group may be lower than the uniform price. This escalation of competition may make firms worse off.

4. Implication

We have considered the behavior-based price discrimination in a duopoly market with switching costs. The analysis in this paper has been applied to several areas. To the extent that the firms have information about consumers’ purchase histories, it may be possible for firms to engage in behavior-based price discrimination. We think our results are most applicable to the mobile telephone market. From the beginning, when NTT DoCoMo dominated the market, this market has been characterized by such practices as selling mobile telephones to new customers for one yen (approximately, one cent) and other examples of price discrimination. These efforts have caused extremely severe price competition and they make firms worse off. However, when there is not so large difference of market shares in recent years, such a discriminatory pricing becomes rare. These facts coincide with our propositions.

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Appendix

Proof of Remark

In the case of (DP, DP), the first-order conditions for firm *i* are given by:

$$\frac{\partial \pi^A}{\partial p_0^A} = \alpha(1-2p_0^A + p_N^B + c) = 0, \quad \frac{\partial \pi^A}{\partial p_N^A} = (1-\alpha)(p_0^B - 2p_N^A + c) = 0,$$

$$\frac{\partial \pi^B}{\partial p_0^B} = (1-\alpha)(1-2p_0^B + p_N^A + c) = 0, \quad \frac{\partial \pi^B}{\partial p_N^B} = \alpha(p_0^A - 2p_N^B + c) = 0.$$

Solving the first-order conditions, we can derive the Nash equilibrium prices:

$$p_0^A* = p_0^B* = c + \frac{2}{3}, \quad p_N^A* = p_N^B* = c + \frac{1}{3},$$

where the second-order conditions are satisfied:

$$\frac{\partial^2 \pi^A}{\partial p_0^{A^2}} = -2\alpha < 0, \quad \frac{\partial^2 \pi^A}{\partial p_N^{A^2}} = -2(1-\alpha) < 0,$$

$$\frac{\partial^2 \pi^A}{\partial p_0^{A^2}} \frac{\partial^2 \pi^A}{\partial p_N^{A^2}} - \frac{\partial^2 \pi^A}{\partial p_0^A \partial p_N^A} \frac{\partial^2 \pi^A}{\partial p_N^A \partial p_0^A} = 4\alpha(1-\alpha) > 0.$$

Furthermore, concerning these equilibrium prices, it can be easily shown that $0 \le p_0^A * - p_N^B * \le 1$, and $0 \le p_0^B * - p_N^A * \le 1$.

Next, we consider the case of (UP, UP). When $p^A \ge p^B$, the first-order conditions for firm A and B are given by:

$$\frac{\partial \pi^A}{\partial p^A} = \alpha(1-2p^A + p^B + c) = 0, \quad \frac{\partial \pi^B}{\partial p^B} = 1 - \alpha + \alpha(p^A - 2p^B + c) = 0.$$

Solving the first-order conditions, we obtain:

$$p^A* = c + \frac{(1+\alpha)}{3\alpha}, \quad p^B* = c + \frac{(2-\alpha)}{3\alpha},$$

where the second-order conditions:

$$\frac{\partial^2 \pi^A}{\partial p^{A^2}} = -2\alpha < 0, \quad \frac{\partial^2 \pi^B}{\partial p^{B^2}} = -2\alpha < 0,$$

are satisfied. Here, in order to satisfy $p^A* \ge p^B*$, it must hold that:

$$\frac{(-1+2\alpha)}{3\alpha} \ge 0, \text{ or } \alpha \ge \frac{1}{2}.$$

Therefore, $p^A* \ge p^B*$ if and only if $\alpha \ge 1/2$. Furthermore, concerning these equilibrium prices, it holds that $1-(p^A*-p^B*) \ge 0$, because $1-(p^A*-p^B*) = \frac{(1+\alpha)}{3\alpha} > 0$.

When $p^A < p^B$, the prices can be derived similarly as follows:

$$p^A* = c + \frac{(1+\alpha)}{3(1-\alpha)}, \quad p^B* = c + \frac{(2-\alpha)}{3(1-\alpha)}.$$

Proof of Proposition 1

When firm A practices discriminatory pricing and firm B uses uniform pricing (DP, UP), the first-order conditions for firm *i* are given by:

$$\frac{\partial \pi^A}{\partial p_0^A} = \alpha(1-2p_0^A + p^B + c) = 0, \quad \frac{\partial \pi^A}{\partial p_N^A} = (1-\alpha)(p^B - 2p_N^A + c) = 0,$$

$$\frac{\partial \pi^B}{\partial p^B} = (1-\alpha)(1-2p^B + p_N^A + c) + \alpha(p_0^A - 2p^B + c) = 0.$$

Solving the first-order conditions, we obtain:

$$p_0^A* = c + \frac{(5-\alpha)}{6}, \quad p_N^A* = c + \frac{(2-\alpha)}{6}, \quad p^B* = c + \frac{(2-\alpha)}{3},$$

where the second-order conditions:

$$\frac{\partial^2 \pi^A}{\partial p_0^{A^2}} = -2\alpha < 0, \quad \frac{\partial^2 \pi^A}{\partial p_N^{A^2}} = -2(1-\alpha) < 0,$$

$$\frac{\partial^2 \pi^A}{\partial p_0^{A^2}} \frac{\partial^2 \pi^A}{\partial p_N^{A^2}} - \frac{\partial^2 \pi^A}{\partial p_0^A \partial p_N^A} \frac{\partial^2 \pi^A}{\partial p_N^A \partial p_0{}^A} = 4\alpha(1-\alpha) > 0,$$

$$\frac{\partial^2 \pi^B}{\partial p^{B^2}} = -2 < 0,$$

are satisfied. Concerning the equilibrium prices, it holds that $p_0^A* \geq p^B* \geq p_N^A*$. The relationships between them can easily be established as follows:

$$p_0^A* - p^B* = \frac{(1+\alpha)}{6} \geq 0, \quad p^B* - p_N^A* = \frac{(2-\alpha)}{6} \geq 0.$$

It also holds that $1 \geq (p_0^A* - p^B*)$, and $1 \geq (p^B* - p_N^A*)$.

Furthermore, the equilibrium prices of (UP, DP) case can be derived in a similar way.

Proof of Proposition 3

We can show that:

when $p^A \geq p^B$ or $1/2 \leq \alpha < 1$,

$$\pi^A*(DP,DP) - \pi^A*(UP,UP) = -\frac{(1-\alpha)(1+2\alpha)}{9\alpha} < 0,$$

$$\pi^B*(DP,DP) - \pi^B*(UP,UP) = -\frac{4(1-\alpha)^2}{9\alpha} < 0, \text{ and}$$

when $p^A < p^B$ or $1/2 > \alpha > 0$,

$$\pi^A*(DP,DP) - \pi^A*(UP,UP) = -\frac{4\alpha^2}{9(1-\alpha)} < 0,$$

$$\pi^B*(DP,DP) - \pi^B*(UP,UP) = -\frac{\alpha(3-2\alpha)}{9(1-\alpha)} < 0.$$

Therefore, Proposition 3 holds.