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Some Monte Carlo results for a generalized error component model with heteroskedastic disturbances

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Abstract

This note provides Monte Carlo evidence illustrating that feasible and true GLS estimators of the Baltagi and Griffin (1988) generalized error component model do not have the same sampling behavior. Indeed, while the true GLS estimator is consistent, a feasible GLS estimator need not be, an observation corroborated by the Monte Carlo results.

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1. Introduction

Baltagi and Griffin (1988) suggested a generalization of a standard panel data model given by

$$y_{it} = \alpha + x_{it}'\beta + u_{it}, \qquad u_{it} = \mu_i + v_{it}, \qquad (i = 1,\ldots,N, \;\; t = 1,\ldots,T) \tag{1}$$

In this expression x_{it} and β are $K \times 1$ vectors of regressors and slope coefficients, α is an overall intercept term, and the regression error (u_{it}) consists of a heterogeneity component (μ_i) and a remainder term (v_{it}). The error components μ_i and v_{it} are typically assumed to satisfy $\mu_i \sim (0, \sigma_\mu^2)$ all i, and $v_{it} \sim (0, \sigma_v^2)$ all i and t, with the μ_i's and v_{it}'s all mutually independent. Baltagi and Griffin (1988), however, recognizing that heteroskedasticity may be a common problem when working with panel data, generalized this model by allowing for the possibility that each μ_i has a distinct variance $\sigma_{\mu i}^2$ ($i = 1,\ldots,N$). This model is also discussed by Baltagi (1995, pp. 77-80), Greene (2000, pp. 580-581; 2003, pp. 316-317), and Phillips (2003).

Baltagi and Griffin (1988) proposed feasible generalized least-squares (GLS) estimators based on estimators of $\sigma_{\mu 1}^2,\ldots,\sigma_{\mu N}^2$ and σ_v^2, feasible GLS estimators which they claimed have the same asymptotic distribution as the true GLS estimator, that is, the GLS estimator based on the true variance components $\sigma_{\mu 1}^2,\ldots,\sigma_{\mu N}^2$ and σ_v^2. That claim, however, has been questioned by Greene (2003) and Phillips (2003). The usual asymptotic equivalence between feasible and true GLS breaks down because, regardless of how large N and T are, we have only one draw from the distribution generating μ_i ($i = 1,\ldots,N$), which makes consistent estimation of $\sigma_{\mu i}^2$ impossible. This observation led Greene to point out that the efficiency of GLS estimators based on estimated $\sigma_{\mu i}^2$'s "seems unlikely" (Greene 2003, p. 617), and Phillips (2003) goes even further by providing an example in which a feasible GLS estimator is not even consistent while the true GLS estimator is consistent. This note provides Monte Carlo evidence confirming that, for the Baltagi-Griffin model, one can learn little about the sampling behavior of a feasible GLS estimator by studying the sampling behavior of the true GLS estimator.

2. Monte Carlo Experiments

The finite sample behavior of one of the feasible GLS estimators considered in Baltagi and Griffin (1988) was investigated with Monte Carlo experiments. The sampling design relied on the model in (1) with $K = 1$, $\alpha = 1$, and $\beta = 1$. The explanatory variable x_{it} was generated according to $x_{it} = -3 + 6d_i + \varepsilon_{it}$, where $\varepsilon_{it} \sim IIN(0,1)$ for all i and t, while the d_i's were generated as independent Bernoulli random variables with $\Pr(d_i = 1) = 1/2$. Moreover, for most of the experiments, the regression errors were generated as $u_{it} = \mu_i + v_{it}$, with $v_{it} \sim IIN(0,1)$, $\mu_i = \sigma_{\mu i}\omega_i$, and $\omega_i \sim IIN(0,1)$. For each combination of N and T considered, the parameter values $\sigma_{\mu 1}^2,\ldots,\sigma_{\mu N}^2$ were generated once from a uniform distribution over the interval from 0 to 5 and then held fixed over all samples drawn for that combination of N and T. The values of N and T considered were 5, 10, 50, 100, and 200, for a total of 25 different N, T combinations.

TABLE I: Relative efficiency of slope estimators: the RMSE of within (W) and feasible GLS (FGLS) estimates over the RMSE of true GLS estimates. Each relative efficiency estimate is based on 2500 independent samples.

	$T=5$		$T=10$		$T=50$		$T=100$		$T=200$	
N	W	FGLS	W	FGLS	W	FGLS	W	FGLS	W	FGLS
5	1.30	1.19	1.10	1.17	1.03	1.25	1.03	1.24	1.02	1.26
10	1.47	1.30	1.20	1.33	1.08	1.48	1.02	1.44	1.02	1.52
50	1.52	1.39	1.32	1.48	1.07	1.71	1.03	1.79	1.03	2.04
100	1.55	1.44	1.27	1.47	1.16	1.85	1.04	1.93	1.02	2.11
200	1.53	1.46	1.41	1.50	1.06	1.79	1.06	2.01	1.02	2.16

For estimators of the slope coefficient I considered the within estimator, the true GLS estimator, and a feasible GLS estimator proposed by Baltagi and Griffin (1988). To construct the latter estimator, the variance components had to be estimated. To that end, I used one of the iterative methods recommended by Baltagi and Griffin (1988). The method requires first applying ordinary least-squares (OLS) to obtain OLS residuals ($\hat{u}_{it}$) and then estimating $\sigma_{ui}^2 = \text{var}(u_{it}) = \sigma_{\mu i}^2 + \sigma_v^2$ with $\hat{\sigma}_{ui}^2 = \sum_{t=1}^{T} \hat{u}_{it}^2 /(T-K-1)$. Then the sample residual variance from the within regression (s_v^2) is subtracted from $\hat{\sigma}_{ui}^2$ to obtain $\hat{\sigma}_{\mu i}^2 = \hat{\sigma}_{ui}^2 - s_v^2$, an estimate of the variance component $\sigma_{\mu i}^2$. The estimates s_v^2 and $\hat{\sigma}_{\mu 1}^2,\ldots,\hat{\sigma}_{\mu N}^2$ are then used to construct a feasible GLS estimate of $b = (\alpha, \beta')'$. After the feasible GLS estimate of b is obtained, the procedure is repeated using the feasible GLS residuals to re-estimate the $\sigma_{\mu i}^2$'s, which are used in turn to construct a new feasible GLS estimate, and so on, until the iterative scheme converges. Negative estimated $\sigma_{\mu i}^2$'s were set to zero, as recommended by Baltagi and Griffin (1988).

The sampling behavior of the feasible GLS estimator is illustrated in Table I. Table I reports Monte Carlo relative efficiency estimates for estimators of the slope coefficient. For each combination of N and T two numbers are given. One number is the ratio of the root mean squared error (RMSE) of the within estimates of the slope coefficient over the RMSE of the true GLS estimates. (Relative efficiency estimates for the within estimator appear in the columns headed by "W".) The other figure is the RMSE of the feasible GLS estimates over the RMSE of the true GLS estimates. For each combination of N and T, each pair of relative efficiency estimates is based on 2500 independent samples.

The data in Table I do not provide a compelling case for abandoning the conventional within estimator in favor of the feasible GLS estimator. Only for $T = 5$ is the feasible GLS estimator of the slope coefficient more efficient than the within estimator. Moreover, we can only conjecture as to what the sampling distribution of the feasible GLS estimator might be. For all combinations of N and T, its RMSE is too large relative to the RMSE of the true GLS estimator to warrant the conclusion that its sampling behavior approximates that of the true GLS estimator.

Indeed, for the sample sizes considered, the feasible GLS estimator's efficiency relative to the true GLS estimator tends to deteriorate as N and T increase, with its RMSE increasing to over twice that of the true GLS estimator for large N and T. On the other hand, for every N, the efficiency of the within estimator approaches that of the true GLS estimator as T increases, a result that is consistent with the observation that the within estimator has the same asymptotic (as $T \to \infty$) efficiency as the true GLS estimator of the slope coefficient (see Baltagi and Griffin, 1988).

Moreover, the feasible GLS estimator can be biased and inconsistent in situations where the true GLS estimator is not. In particular, the symmetry of the distribution of μ_i is important for the unbiasedness and consistency of the feasible estimator. To illustrate this numerically, I generated one additional set of 2500 independent samples with $N = 200$ and $T = 200$ while using the same experimental design and estimators used in the other Monte Carlo experiments except for one alteration. Instead of generating the ω_i's as standardized normal random variates, they were generated as standardized lognormal random variables with zero mean and unit variance.[1] Thus, the distribution of $\mu_i = \sigma_{\mu i}\omega_i$ conditional on $\sigma^2_{\mu i}$ had a mean of zero, but it was asymmetric about zero. For this set of Monte Carlo samples, the averages of the true GLS estimates for both the intercept and slope parameters were close to the true value of one. On the other hand, although the average of the slope coefficient feasible GLS estimates was close to the true value of one, the average of the intercept feasible GLS estimates was significantly less than one: it was 0.603 with a standard error of 0.009.

To see why symmetry is important, consider a simple example, which draws on an example used in Phillips (2003). Phillips (2003) points out that, although σ^2_v can be estimated consistently, at best we can expect an estimator of $\sigma^2_{\mu i}$ to actually only estimate μ^2_i consistently rather than $\sigma^2_{\mu i}$. In light of this, consider a "feasible" GLS estimator that is based on σ^2_v and $\mu^2_1, \dots, \mu^2_N$. Also, for the sake of simplicity, let the model be $y_{it} = \alpha + \mu_i + v_{it}$ ($i = 1, \dots, N$, $t = 1, \dots, T$). Then, from results provided in Phillips (2003), the "feasible" GLS estimator of α is

$$\hat{\alpha}_{FGLS} = \alpha + \frac{1}{N}\sum_{i=1}^{N} g_N(\mu^2_i)\left(\mu_i + \bar{v}_i\right) \tag{2}$$

where $g_N(\mu^2_i) = N(\mu^2_i + \sigma^2_v / T)^{-1} / \sum_{j=1}^{N}(\mu^2_j + \sigma^2_v / T)^{-1}$ and $\bar{v}_i = \sum_{t=1}^{T} v_{it} / T$.[2] Clearly $E[g_N(\mu^2_i)\bar{v}_i] = 0$ if μ_j and $\bar{v}_i$ are independent for all j and $E(\bar{v}_i) = 0$. Moreover, $E[g_N(\mu^2_i)\mu_i] = 0$ if the μ_j's are independent and the distribution of μ_i is symmetric about zero. Thus, under these conditions $E(\hat{\alpha}_{FGLS}) = \alpha$. On the other hand, if μ_i is not symmetrically distributed about zero, there is no guarantee that $E[g_N(\mu^2_i)\mu_i] = 0$ and thus there is no guarantee that $E(\hat{\alpha}_{FGLS}) = \alpha$. Moreover, this observation holds for all N and T, and since an estimator of

[1] Specifically, I set $\omega_i = [\exp(z_i) - \exp(1/2)] / \{\exp(1)[\exp(1) - 1]\}^{1/2}$, where z_i is a standard normal variate.

[2] Eq. (2) follows from Eq. (3) in Phillips (2003) upon replacing $\sigma^2_{\mu i}$ in the latter expression with μ^2_i.

$\sigma^2_{\mu i}$ only consistently estimates μ_i^2, at best, we see from the foregoing that we cannot expect a feasible GLS estimator, based on estimated $\sigma^2_{\mu i}$'s, to consistently estimate α when the μ_i's have asymmetric distributions.

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