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Autoregressive Lag Length Selection Criteria in the Presence of ARCH Errors

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Abstract

We study the effects of ARCH errors on the performance of the commonly used lag length selection criteria. The most important finding of this study is that SIC, FPE, HQC and BIC perform considerably well in estimating the true autoregressive lag length, even in the presence of ARCH errors. Thus, we conclude that these criteria are applicable to empirical data such as stock market returns and exchange rate volatility that exhibit ARCH effects.

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1. Introduction

Recently, Başçi and Zaman (1998) has done a simulation study to see the effects of nonnormal errors on various autoregressive (AR) lag length selection criteria. In the past, the performance of these criteria has been studied based on the assumptions that the error terms are normal in nature. Liew (2004) for instance, study the performance of few commonly used selection criteria in the presence of normal errors. Başçi and Zaman (1998) argued that it is important for applied econometricians to understand the behaviour of various criteria under nonnormal errors. They further demonstrate via a simulation study that the performances of some criteria are affected by kurtosis but not skewness. In the spirit of Başçi and Zaman (1998), our main objective is to investigate via a simulation study, the effects autoregressive conditional heteroscedastic (ARCH) (Engle, 1982) errors on the performance of the aforementioned criteria in the estimation of true lag length. Specifically, we are interested to know whether the application of these criteria is still appropriate in the presence of ARCH effects, as it is widely known that many empirical data especially financial variables such as stock price returns and exchange rate volatility are actually better characterized by the ARCH models (Engle 1982; Engle *et al*. 1990; Bollerslev *et al*. 1992; Speight and McMillan 2001; Bautista 2003; Li and Lin 2004 and many more).

We note that the current study differs from the former in threefold. First, rather than studying the general form of nonnormality, we include the specific ARCH errors, which is a common form of nonnormal errors attributed to most economic data sets. Second, besides evaluating the probability of correctly picking up the true lag length, we are also interested to know the probabilities of under- and over-estimating the true lag length, in which the estimated lag length based on the selection criteria is less than and more than the true lag length, respectively. Thirdly, to obtain a clearer picture on the effects of ARCH errors over the homoscedastic errors, we contrast the performance of various criteria under both errors.

The most important finding of this study is that the commonly used selection criteria like SIC, FPE, HQC and BIC perform considerably well in estimating the true autoregressive lag length, even in the presence of ARCH errors. Thus, we conclude that these criteria are applicable to empirical data that exhibits ARCH effects.

2. Methodology

To accomplish our objective discussed in the preceding section we simulate AR (p) process with ARCH (q) errors, which is defined for a given set of data $\{X_1, \ldots, X_T\}$ that is in fact observations of an AR process of lag length p as:

$$X_t = c + \sum_{i=1}^{p} \phi_i X_{t-i} + \varepsilon_t \tag{1}$$

where c is a constant, ϕ_i, i = 1, …, p are autoregressive parameters to be estimated and $\varepsilon_t = z_t \sigma_t$, where z_t is a standard normal variable and

$$\sigma_t = \sqrt{\alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2} \tag{2}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1, \ldots, q$, with q is the order of ARCH errors.

We arbitrary set the true lag length $p = 4$ and generate ϕ_i, $i = 1, 2, 3, 4$ from a uniform distribution in the region (– 0.25, 0.25). The choice of this region allows us to avoid undesired nonstationary process. We consider ARCH (q) errors for $q = 1, 2, 3$, and 4 and generate α_j, $j = 0, 1, 2, 3, 4$ from a uniform distribution in the region (0.0, 1.0). A special case of ARCH errors namely the homoscedastic errors is generated from the standard normal distribution with zero mean and unit variance, for the purpose of comparison. We simulate data sets for various sample sizes: 25, 50, 100, 500, 1000 and 100000. For each combination of sample sizes and types of error, we simulated 1000 independent series for the purpose of lag length estimation. The estimated p can be any integer from 1 to 20.

Following Liew (2004), the following lag length selection criterion are evaluated in this study:

(a) Akaike information criterion,

$$\text{AIC}_p = -\ln(\hat{\sigma}_p^2) + 2p/T; \tag{3}$$

where $\hat{\sigma}_p^2 = (T - p - 1)^{-1} \sum_{t=p}^{T} \hat{\varepsilon}_t^2$;

(b) Schwarz information criterion,

$$\text{SIC}_p = \ln(\hat{\sigma}_p^2) + [p \ln(T)]/T; \tag{4}$$

(c) Hannan-Quinn criterion,

$$\text{HQC}_p = \ln(\hat{\sigma}_p^2) + 2T^{-1} p \ln[\ln(T)]; \tag{5}$$

(d) the final prediction error,

$$\text{FPE}_p = \hat{\sigma}_p^2 (T - p)^{-1} (T + p); \tag{6}$$

(e) Bayesian information criterion,

$$\text{BIC}_p = (T-p) \ln[(T - p)^{-1} T \sigma_p^2] + T[1 + \ln(\sqrt{2\pi})] + p \ln[p^{-1}(\sum_{t=1}^{T} X_t^2 - T\hat{\sigma}_p^2)]; \tag{7}$$

and finally, we include the improved version of AIC, but not studied in Liew (2004), namely,

(f) Akaike's information corrected criterion,

$$\text{AICC}_p = -2T[\ln(\hat{\sigma}_p^2)] + T[1 - (p-2)/T]^{-1} [1 + (p/T)]; \tag{8}$$

Interested readers are referred to Brockwell and Davis (1996) and Başçi and Zaman (1998) and the references therein for more details.

The probability of estimating the true lag length by each of these criteria is determined. We also compute the probabilities of under- and over-parameterization as mentioned in the preceding section.

3. Simulation Results

To conserve space, only the probabilities of various criteria in correctly estimating the true lag length are tabulated in this paper, as in Table 1.

Table 1
Probability of Correctly Estimating the True Lag Length

$T = 25$						$T = 50$					
Criteria	Types of Errors					Criteria	Types of Errors				
	H	A(1)	A(2)	A(3)	A(4)		H	A(1)	A(2)	A(3)	A(4)
AICC	18.8	21.5	19.4	19.8	20.0	AICC	18.1	20.3	17.7	19.0	19.0
AIC	18.8	21.5	19.4	19.8	20.0	AIC	18.1	20.3	17.7	19.0	19.0
SIC	57.7	58.5	59.8	58.7	59.8	SIC	58.0	60.3	59.1	59.2	60.9
FPE	65.4	67.7	67.5	65.6	67.8	FPE	65.4	67.2	67.0	67.3	67.6
HQC	64.5	66.0	68.2	66.0	66.9	HQC	65.7	68.3	66.5	66.8	66.7
BIC	65.7	68.1	69.4	67.3	67.9	BIC	66.6	68.6	68.3	69.1	67.8

$T = 100$						$T = 500$					
Criteria	Types of Errors					Criteria	Types of Errors				
	H	A(1)	A(2)	A(3)	A(4)		H	A(1)	A(2)	A(3)	A(4)
AICC	21.0	20.7	19.4	19.3	21.0	AICC	19.8	19.4	19.2	21.2	19.3
AIC	21.0	20.7	19.4	19.3	21.0	AIC	19.8	19.4	19.2	21.2	19.3
SIC	61.0	57.7	59.5	63.1	60.2	SIC	67.9	65.1	64.1	67.0	65.8
FPE	70.2	67.6	67.5	67.4	69.4	FPE	71.9	70.9	68.0	70.5	71.6
HQC	69.1	66.2	66.6	69.1	67.6	HQC	74.5	71.4	70.8	73.4	73.3
BIC	70.3	67.7	68.5	70.1	70.0	BIC	74.5	72.7	71.0	73.5	74.1

$T = 1000$						$T = 100000$					
Criteria	Types of Errors					Criteria	Types of Errors				
	H	A(1)	A(2)	A(3)	A(4)		H	A(1)	A(2)	A(3)	A(4)
AICC	18.2	20.5	21.1	23.3	21.1	AICC	23.0	24.4	20.5	22.8	24.0
AIC	18.2	20.5	21.1	23.3	21.1	AIC	23.0	24.4	20.5	22.8	24.0
SIC	69.6	65.4	65.6	67.4	69.4	SIC	96.5	91.2	89.6	93.1	92.8
FPE	73.5	71.5	72.0	73.0	74.1	FPE	85.0	86.8	80.8	84.0	82.7
HQC	74.8	71.6	71.8	73.8	74.7	HQC	96.6	90.4	90.0	93.2	88.8
BIC	75.7	72.8	72.8	75.2	74.8	BIC	96.8	92.0	90.0	94.0	91.6

Notes: H denotes homoscedastic errors with $N(0, 1)$ distribution. A(i) stands for ARCH errors of order i for $i = 1,\ldots, 4$.

Table 1 revealed three stylized facts. First, SIC, FPE, HQC and BIC (but not AICC and AIC) perform considerably well in estimating the true autoregressive lag length in all simulated series. Second, the performance of these selection criteria involved improves as the sample size increases. For instance with a sample size of 100000 observations for homoscedastic errors, we find that their performances have achieved a record of 96.5 for SIC, 85.0 (FPE), 96.6 (HQC), 96.8 (BIC), 23.0 (AICC) and 23.0 (AIC). Note that AICC and AIC perform poorly even for extraordinary large sample size.

Table 1 shows that there is no distinct difference in the performance of these criteria in correctly estimating the true lag order p for all processes. For instance, the probability score of BIC in correctly estimating the true order of are 75.7, 72.8, 72.8, 75.2 and 74.8, in that order, for homoscedastic, ARCH (1), ARCH (2), ARCH (3) and ARCH (4) errors (T=1000). This implies that BIC is applicable in AR process regardless of whether the errors are heteroscedastic or homoscedastic in nature. The last statement applies for SIC, FPE and HQC as well.

We observed further that whenever the criteria fail to pick up the true p, the chance of under-estimation by SIC, FPE, HQC and BIC is drastically higher than over-estimation for both the ARCH and homoscedastic errors, whereas the reverse is true for AICC and AIC (full results not shown). For instance, with a sample size of 1000 observations in the case of homoscedastic errors, the probabilities of under-estimation (over-estimation) by SIC, FPE, HQC, BIC, AICC and AIC are, in that order, 30.3 (0.1), 16.5 (10.0), 23.1 (2.1), 21.0 (3.3), 2.8 (79.0) and 2.8 (79.0).

4. Conclusions

Various lag length selection criteria have been proposed based on the normal errors, which may be easily violated in the empirical economic research. One common form of nonnormal errors is the heteroscedastic errors. This study investigates the effects of ARCH errors on the performance of the commonly used lag length selection criteria. The most important finding of this study is that SIC, FPE, HQC and BIC perform considerably well in estimating the true autoregressive lag length, even in the presence of ARCH errors. Thus we conclude that these criteria are applicable to autoregressive process that exhibits ARCH effects.

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