E C O N O M I C S   B U L L E T I N

Direct externalities, specific performance and renegotiation design

M'hand Fares
MOISA and University of Paris 1 (ATOM)

Abstract

This paper examines how the model of De Fraja [Games and Economic Behavior, 1999] can be amended after Che [Games and Economic Behavior, 2000] negative result. We mainly show that the optimal level of investments can be obtained with the specific performance contract considered by De Fraja if the renegotiation process does not follow the Hart–Moore procedure but allocates an extreme bargaining power.

1   Introduction

Incomplete contract theory has addressed the issue of how to design the optimal contractual arrangement to achieve the efficient investments in the presence of contract incompleteness (Hart, 1995). In this theory, incompleteness comes from the parties inability to write complete contingent contract on all future states of nature and ex ante investments because they are too costly to describe or unverifiable by court (see Tirole, 1999 for a survey). After the relevant state of nature is realized, the parties may renegotiate the initial contract to implement the efficient outcome. But such ex post renegotiation may lead to inefficient investments. This is called under-investment result or the "hold-up problem" (Hart-Moore, 1988). The litterature on contract solutions to the hold-up problem (Chung, 1991; Aghion et al., 1994) suggested that the efficient level of investment can be obtained if the renegotiation process of a specific performance contract is designed so that one party has the whole bargaining power. However, Che-Hausch (1996) show that these solutions to the hold-up problem fall through when the investments generate direct externalities, that is when each party's investment affects directly the benefit or cost of the other party. De Fraja (1999) claimed that this result can be overturned if the investments are sequentially undertaken and the renegotiation follows the Hart-Moore (1988) renegotiation procedure[1]. Che (2000) points out an error in this sequential solution since the Hart-Moore renegotiation procedure considered implies that the late mover does not get right incentives to invest. Thus the specific performance contract considered by De Fraja has value only under restrictive conditions (proposition 3). Therefore, Che suggests an alternative contract specifying a menu of prices that can solve the hold-up problem.

In contrast to Che, we show that no alternative contract arrangement is needed to achieve efficiency. The optimal level of investments can be obtained with a specific performance contract if the renegotiation process does not follow the Hart-Moore procedure but allocates an extreme bargaining power (Chung 1991, Aghion *et al.* 1994). This result implies that the restrictive conditions derived by Che on the investment effect are unecessary to implement the efficient result.

The remainder of this note is organized as follows. Section 2 presents the model. Section 3 provides the efficient result. Section 4 contains a further discussion.

---

[1] The procedure can be defined as follows: "if a party prefers the original contract to the exchange of the efficient quantity for the original price, then this party is offered just enough to make him indifferent between accepting this new offer and insisting on legal enforcement of the original contract [outside option]. If on the other hand, neither party perfers the original contract, the efficient output is traded at the original price" (De Fraja, p.29). Since this procedure is a variant of the Outside-Option Principle (Muthoo, 1999) where both outside options can bind, renegotiation design (extreme bargaining power allocation to *one* party such that only her partner's outside option binds) is not possible.

2 Model

2.1 Set-up

Consider a long-term procurement relationship between a buyer (she) and a seller (he), both of whom are risk neutral. The timing of events is the following. At date 0, the buyer undertakes a specific investment $\beta \in [0, \infty)$ with a direct cost denoted $h_B(\beta)$. At date 1, she makes a "take-it-or-leave-it" offer of a contract specifying a quantity $q \in \mathbb{R}_+$ of the good, and a monetary transfer $p \in \mathbb{R}$ from the buyer to the seller. At date 2, the seller makes a relationship-specific investment $\sigma \in [0, \infty)$ with a direct cost denoted $h_S(\sigma)$. At date 3, the state of nature $\omega \in \Omega$ is realized. Renegotiation occurs at date 4, just before trade can take place.

Let $v(q, \omega, \beta, \sigma)$ be the buyer's (gross) benefit of receiving q units of the good when the realized state of the world is ω, and when her level of investment is β and the seller's investment is σ. Similarly, let $c(q, \omega, \beta, \sigma)$ denote the seller's production cost of delivering q units of output when ω, σ and β are the arguments of the cost function. Investments have an indirect externality since the investments affect the probability of trade, but also a direct externality. The benefit (cost) to the buyer (seller) depends directly on the investment made by the seller (buyer). It is assumed that both $v(q, \omega, \beta, \sigma)$ and $-c(q, \omega, \beta, \sigma)$ are increasing in each of their arguments at a decreasing rate. The economically interesting case is one where incentives to invest are sensitive to the expected level of trade: $v_{q\beta}(.) \geq 0$, $c_{q\sigma}(.) \leq 0$; and $v_{q\sigma}(.) \geq 0$, $c_{q\beta}(.) \leq 0$. We next assume that for every ω, β, σ: $v(0, \omega, \beta, \sigma) \equiv c(0, \omega, \beta, \sigma) \equiv 0$, i.e. the benefit and the cost of having no trade are both equal to zero. This assumption formalizes the idea that the cost of the investment cannot be recouped in the absence of trade. Assume also that $h_B(.)$ and $h_S(.)$ are increasing and convex.

2.2 Benchmark

The first best, as a benchmark, is the outcome achieved when ω, β, σ are verifiable and hence contractible. For any given (β, σ), let

$$\{q^*(\omega, \beta, \sigma), \ p^*(\omega, \beta, \sigma) : \omega \in \Omega\}$$

be the complete contingent solution of this first best problem conditional on (β, σ). The efficient investments (β^*, σ^*) can be defined as follows

$$(\beta^*, \sigma^*) \equiv \underset{\beta, \sigma}{\arg\max} \int v(q^*(.), \omega, \beta, \sigma) - c(q^*(.), \omega, \beta, \sigma) dF(\omega) - h_B(\beta) - h_S(\sigma) \qquad (1)$$

The efficient investments maximize the difference between the total expected gain and the direct cost of investments. If both β and σ were verifiable, then the parties would jointly choose (β^*, σ^*) which maximizes the expected net gains from the relationship.

3 An efficient result

3.1 Efficiency and renegotiation design

We assume that, although β and σ as well as the state of the world ω (and so v and c) are perfectly observable by both agents, they are not verifiable by the court. Then, the court cannot enforce outcomes contingent on these variables, and thus parties can write only an incomplete contingent contract $(\widetilde{q}, \widetilde{p})$. Consider the renegotiation stage of this initial incomplete contract where $\omega, \beta, \sigma, \widetilde{q}, \widetilde{p}$ are all fixed, and suppose that

- **<u>Assumption 1</u>**: the buyer has all the bargaining power in the renegotiation game

Proposition 1 : *Given assumption 1, there is an initial contract $(\widetilde{q}, \widetilde{p})$ that implements the first best level of investments.*

Proof. Given assumption 1, the buyer makes a "take-it-or-leave-it" offer (q^*, p^*) in the renegotiation game such that

$$(q^*, p^*) \equiv \underset{q,p}{\arg\max} \ \{v(q, \omega, \beta, \sigma) - p\}$$
$$s.t. : p - c(q, \omega, \beta, \sigma) \geqq \widetilde{p} - c(\widetilde{q}, \omega, \beta, \sigma)$$

the solution is given by $q^* = q^*(\omega, \beta, \sigma)$ satisfying

$$\frac{\partial v(q, \omega, \beta, \sigma)}{\partial q} = \frac{\partial c(q, \omega, \beta, \sigma)}{\partial q}$$

and $p = p^*(\omega, \beta, \sigma, \widetilde{q}, \widetilde{p}) = \widetilde{p} + c(q^*(\omega, \beta, \sigma), \omega, \beta, \sigma) - c(\widetilde{q}, \omega, \beta, \sigma)$. The buyer gets

$$u_B = v(q^*(.), \omega, \beta, \sigma) - c(q^*(.), \omega, \beta, \sigma) - [\widetilde{p} - c(\widetilde{q}, \omega, \beta, \sigma)]$$

thus, the outcome of this designed game is such that the seller gets a constant payoff defined by the initial contract $(u_S = \widetilde{p} - c(\widetilde{q}, \omega, \beta, \sigma))$ and the buyer is residual claimant since she gets $v(q^*(.), \omega, \beta, \sigma) - c(q^*(.), \omega, \beta, \sigma) - u_S$, i.e. the social surplus minus a fixed payoff.

Considering the investment choice, at date 2 the seller chooses $\widetilde{\sigma}$ taking the initial allocation $(\widetilde{q}, \widetilde{p})$ and the buyer's investment β as fixed. The seller's program is then

$$\max_{\sigma} U(\widetilde{q}, \widetilde{p}, \beta, \widetilde{\sigma}(\widetilde{q}, \beta)) - h_S(\widetilde{\sigma}(\widetilde{q}, \beta))$$

where $U(\widetilde{q}, \widetilde{p}, \beta, \widetilde{\sigma}(\widetilde{q}, \beta)) = \int \widetilde{p} - c(\widetilde{q}, \omega, \beta, \widetilde{\sigma}(\widetilde{q}, \beta)) dF(\omega)$. Then $\widetilde{\sigma}$ is implicitly defined by

$$\frac{\partial U(\widetilde{q}, \widetilde{p}, \beta, \widetilde{\sigma}(\widetilde{q}, \beta))}{\partial \sigma} = \int -c_\sigma(\widetilde{q}, \omega, \beta, \widetilde{\sigma}(\widetilde{q}, \beta)) dF(\omega)$$
$$= h'_S(\widetilde{\sigma}(\widetilde{q}, \beta))$$

Note that $\widetilde{\sigma}$ is a function of $\widetilde{q}$ and β. Consider now the two buyer's choices: at date 0, she chooses $\widetilde{\beta}$ knowing the effect of this choice on the contract $(\widetilde{q}, \widetilde{p})$ she will propose to the seller at date 1. Then, the buyer's program can be written as

$$
\begin{cases}
\max\limits_{\beta, \widetilde{q}, \widetilde{p}} \int v(q^*(.), \omega, \beta, \widetilde{\sigma}(\widetilde{q}, \beta)) - p^*(.) dF(\omega) - h_B(\beta) \\
s.t. : U(\widetilde{q}, \widetilde{p}, \beta, \widetilde{\sigma}(\widetilde{q}, \beta)) - h_S(\widetilde{\sigma}(\widetilde{q}, \beta)) \geqq \overline{U}_S
\end{cases}
\tag{2}
$$

where $\overline{U}_S$ is the seller's reservation level of utility. As the constraint is satisfied as an equality at any solution since the buyer can choose $\widetilde{p}$ such that the seller's participation constraint binds, rearranging (2) we obtain

$$
\max\limits_{\beta, \widetilde{q}} \int v(q^*(.), \omega, \beta, \widetilde{\sigma}(.)) - c(q^*(.), \omega, \beta, \widetilde{\sigma}(.)) dF(\omega) - h_S(\widetilde{\sigma}(.)) - h_B(\beta) - \overline{U}_S
\tag{3}
$$

From (1) and (3), the buyer chooses $\beta = \beta^*$ at date 0.

At date 1, the buyer proposes an incentive contract $(\widetilde{q}, \widetilde{p})$ to the seller. As $\widetilde{\sigma}$ continuously changes with its argument, $\widetilde{q}$ can be used as an instrument to implement the preferred choice of σ. Suppose that $\widetilde{q}$ satisfies

$$
\begin{aligned}
\frac{\partial U(\widetilde{q}, \widetilde{p}, \beta^*, \widetilde{\sigma}(\widetilde{q}, \beta))}{\partial \sigma} &= \int -c_\sigma(\widetilde{q}, \omega, \beta^*, \widetilde{\sigma}(\widetilde{q})) dF(\omega) \\
&= \int v_\sigma(q^*(.), \omega, \beta^*, \sigma) - c_\sigma(q^*(.), \omega, \beta^*, \sigma) dF(\omega) \\
&= h'_S(\sigma^*)
\end{aligned}
\tag{4}
$$

$\widetilde{\sigma}(., \beta^*)$ is continuous and increasing because of $c_{\sigma q}(.) < 0$. Note also that $\widetilde{\sigma}(0, \beta^*) = 0$ and $\widetilde{\sigma}(\widetilde{q}, \beta^*) > \sigma^*$ when $\widetilde{q} = \arg\max\limits_{\omega} q^*(\omega, \beta^*, \sigma^*)$. Thus, by the intermediate value theorem there exists a quantity $\widetilde{q}$ such that $\widetilde{\sigma}(\widetilde{q}, \beta^*) = \sigma^*$. Then the first best result (β^*, σ^*) in the investment game is achieved. ∎

Recall that in the Hart-Moore renegotiation procedure considered by De Fraja, both outside options bind alternatively. When $q^* > \widetilde{q}$, the seller's outside options is binding, so she receives the same payoff as if there wer no renegotiation ($u_S = \widetilde{p} - c(\widetilde{q}, \omega, \beta, \sigma)$). But when $q^* < \widetilde{q}$, the buyer's outside option is binding which makes the seller residual claimant only relative to the payoff the buyer gets from the initial contract, $(u_S = [v(q^*(.), \omega, \beta, \sigma) - c(q^*(.), \omega, \beta, \sigma)] - [v(\widetilde{q}, \omega, \beta, \sigma) - \widetilde{p}])$. As shown by Che, this procedure implies that the seller's marginal return is lower than the social marginal return when the seller's investment has positive externality (proposition 1).

In our setting, only the seller's outside option is binding since the buyer has all the bargaining power in the renegotiation game (renegotiation design). Thus, she renegotiates the initial quantity by offering an optimal contract (q^*, p^*) that makes the seller indifferent between trading at the efficient quantity and enforcing the initial contract (specific performance). Therefore, the seller's payoff is

uniquely determined by the initial contract $u_S = \widetilde{p} - c(\widetilde{q}, \omega, \beta, \sigma)$, and thus the externality on the buyer's outside option generates no disincentive effect. Appropriately choosing the initial quantity $\widetilde{q}$ gives the seller efficient incentives to invest. Thus, the seller's side of the hold-up problem is solved. This result is related to Che 's proposition 1 (which proves that De Fraja result cannot hold) and proposition 3 (which states the sufficient conditions to implement the efficient solution). First, equation (4) above shows that the existence of an investment externality ($v_\sigma(.) > 0$) implies no further distorsion since it can be internalized by choosing a higher $\widetilde{q}$ than in the no externality case. That is why the second term holds as an equality, and not an inequality as in proposition 1 of Che. Therefore, contrary to part (i) of proposition 3 of Che, there exists a contract term that yields first best outcome even when the seller's investment has external effect. Moreover, in our setting an intermediate level of $\widetilde{q}$ implements the efficient investment σ^* whatever the seller's investment effect. That is, restrictive conditions defined by parts (ii) and (iii) of proposition 3 are unecessary to achieve the efficient solution.

The buyer's side of the hold-up problem is solved by the combination of sequential decision mechanism and ex ante whole bargaining power attribution, since it allows the buyer to extract the entire ex ante surplus by stting the initial price $\widetilde{p}$ so as to make the seller's ex ante participation constraint binding.

3.2   How to design renegotiation ?

The question is how to contractually design the ex post extreme allocation of bargaining power (assumption 1). Note that the initial quantity $\widetilde{q}$ is not the right mechanism. Suppose that in the renegotiation game the seller rejects any offer from the buyer. Then the buyer cannot credibly threat him to impose the initial trade $(\widetilde{q}, \widetilde{p})$ by demanding the specific performance. Indeed, this requires a small $\widetilde{q}$ whereas seller's efficient investment requires a large $\widetilde{q}$. A second instrument is then necessary to control contractually the allocation of bargaining power. Aghion *et al.* (1994) suggests an explicit well-designed contract, including a financial hostage, which generates a renegotiation game satisfying assumption 1.

4   Discussion

This paper stands in contrast to Che (2000) claim that the sequential mechanism of De Fraja (1999) failed because parties sign a specific performance contract. According to Che (2000), the seller always underinvests with a specific performance contract. Indeed, in the renegotiation game considered by De Fraja even if the buyer's outside option is binding, the seller is not residual claimant at the margin since there is a disincentive effect through the impact the seller's investment externality has on the buyer's outside option. Then, only a menu of prices can give right incentives to invest because the buyer can be made indifferent to no-trade and thus her outside option payoff is a constant independent

of σ. However, making the seller residual claimant in the renegotiation game is not the only way to give him the right incentives to invest. Right incentives can also be provided through the effect the seller's investment has on the value of his outside option. This can be achieved with a specific performance contract if the initial contract is designed such that the seller's outside option: (i) is always binding, i.e. the whole bargaining power is allocated to the buyer, and (ii) is directly determined by the initial contract. (i) implies that there is no disincentive effect since the buyer's outside option never binds and (ii) implies that choosing appropriately the initial quantity will give the seller the right incentives to invest efficiently. Therefore, a fixed quantity contract is still relevant to solve the hold-up problem even when the investments have externalities. Achieving efficiency in De Fraja (1999) specification requires however condition (i) to be fulfilled, that is, the possibility to allocate contractually an extreme bargaining power.

Therefore, the reason why the sequential mechanism of De Fraja (1999) failed is the lack of renegotiation design. But it is not intrinsic to the specific performance contract: it is also needed in the efficient menu of prices suggested by Che, where the whole bargaining power is given to the seller (since only the buyer's outside option binds). Indeed, Che define an extreme bargaining power allocation that makes the buyer's outside option always binding in the menu of prices by choosing a high enough initial price $(p_1 - p_0 > v(q^*, \omega, \beta^*, \sigma^*))$.

Finally, in contrast to De Fraja (1999) we stress the complementarity between his sequential decision mechanism and the renegotiation design mechanism. Considering sequentiality alone (De Fraja 1999), or renegotiation design alone (Aghion *et al.* 1994), the first best can be achieved in a specific performance contract only if there is one-way direct externality. A combination of both mechanisms achieves the first best in a two-sided direct externalities setting. Sequentiality (together with ex ante full bargaining power allocation) solves one side (the buyer' side) of the hold-up problem by making the buyer ex ante residual claimant. Renegotiation design solves the other side (the seller' side) of the hold-up problem since: (i) the buyer's outside option never binds and thus generates no disincentive effect; (ii) the seller's outside option is contractually designed such that he has the right incentives to invest. Therefore, the efficient result with a specific performance contract requires only two behavioral assumptions: (1) sequential investment; (2) full bargaining power allocation to the buyer in the negotiation game (ex ante)and in the renegotiation game (ex post). Given both assumptions, the Che-Haush (1999) result on the vanishing of contracting gains in the presence of investment externalities does not hold.

References

[1] Aghion P., M. Dewatripont, and P. Rey. (1994) "Renegotiation Design with Unverifiable Information" *Econometrica* **62**, 257-282.

[2] Che, Y.-K. (2000) "Can a Contract Solve Hold-Up When Investments Have Externalities? A Comment on De Fraja (1999)" *Games and Economic. Behavior* **33**, 195-205.

[3] Che, Y.-K., and Hausch, D.B. (1999) "Cooperative Investments and the Value of Contracting" *American Economic Review* **89**, 125-147.

[4] Chung, T.-Y. (1991) "Incomplete Contracts, Specific Investments, and Risk Sharing" *Review of Economic Studies* **58**, 1031-1042.

[5] De Fraja, G. (1999) "After You Sir: Sequential Investment as a Solution to the Hold–Up Problem" *Games and Economic Behavior* **26**, 22-39.

[6] Hart, O. (1995) *Firms, Contracts, and Financial Structure*, Oxford University Press : Oxford.

[7] Hart, O. and J. Moore (1988) "Incomplete Contracts and Renegotiation" *Econometrica* **56**, 755-785.

[8] Muthoo A. (1999), *Bargaining Theory with Applications*, Cambridge University Press.

[9] Tirole J. (1999), "Incomplete Contracts: Where do we Stand ?" *Econometrica* **67**, 741-781.