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# Endogenous Market Structure and Fiscal Policy in an Endogenous Growth Model with Public Capital

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## *Abstract*

This paper develops an endogenous growth model with public capital and imperfect competition. In the model, we take into account monopolistic competition of intermediate sector and endogenous determination of the number of firms in the sector by considering the fixed cost to keep production going. We show that the number of firms in intermediate sector is affected by fiscal policy. Furthermore, it is demonstrated in the paper that market structure plays a key role in reducing the growth-maximizing tax rate.

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# 1 Introduction

The literature on endogenous growth and fiscal policy, as developed by Barro (1990) and Futagami et al. (1993), examines the macroeconomic effects of various policies in endogenous growth models with perfect competition and constant returns to scale with respect to private capital and public capital. They show that the growth-maximizing tax rate on output is equal to the output elasticity of public capital.

This paper describes an endogenous growth model with public capital and monopolistic competition in the intermediate goods sector. Our model departs from the precedent standard models by incorporating monopolistic competition in the intermediate goods sector and endogenizing each incumbent firm's price-cost markup.<sup>1</sup> The main results of this paper are summarized as follows. We find that a tax rate increase on revenue in the intermediate goods sector decreases the equilibrium number of firms in the intermediate sector. Furthermore, because a higher tax rate raises industrial concentration and raises equilibrium markups, the positive effect on economic growth is notably smaller than those in conventional models of perfect competition without markups.

This paper is organized as follows. In Section 2, we develop the basic model. Section 3 establishes the unique saddle-path equilibrium of the dynamic economy and examines the macroeconomic effects of fiscal policy. Finally, Section 4 presents some concluding remarks.

## 2 The model

We extend the exogenous growth model of Wu and Zhang (2000) to an endogenous growth model by considering the production technology of Barro (1990) and Futagami et al. (1993).

*Producers.* The economy has numerous monopolistically competitive intermediate goods producers, indexed by  $i$ . Final goods produced in a competitive sector are given as

$$Y = \left[ n^{-1/\epsilon} \int_0^n y_i^{(\epsilon-1)/\epsilon} di \right]^{\epsilon/(\epsilon-1)}, \quad (1)$$

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<sup>1</sup>In exogenous growth models with monopolistic competition, some studies examine the macroeconomic effects of fiscal policy (e.g., Wu and Zhang (2000), Molana and Zhang (2001), and Coto-Martinez (2006)).

where  $Y$  is the output of final goods,  $y_i$  the quantity of the  $i$ th intermediate good used, and  $\epsilon > 1$  represents the elasticity of substitution among intermediate goods. Defining  $\Pi$  as the profit of a final good producer,

$$\Pi = PY - \int_0^n p_i y_i di. \quad (2)$$

In Eq. (2),  $P$  and  $p_i$  respectively denote the prices of the final good and the  $i$ th intermediate good. Solving the profit maximization problem, the demand function for a typical intermediate good is derived as

$$y_i = \left(\frac{P}{p_i}\right)^\epsilon \frac{Y}{n}. \quad (3)$$

The production technology of intermediate goods is given as

$$y_i = F(k_i, g) = k_i^{1-\alpha} g^\alpha, \quad (4)$$

where  $k_i$  is the private capital input,  $g$  the public capital input, and  $0 < \alpha < 1$ . The production function of intermediate goods is assumed to exhibit the constant returns to scale and diminishing returns to each input. The profit function for the  $i$ th intermediate good producer,  $\pi$ , is given as

$$\pi = (1 - \tau)p_i y_i - r k_i - p_i \psi \quad (5)$$

where  $\tau$  is the tax rate on revenue,  $r$  the factor price of private capital and  $\psi > 0$ .<sup>2</sup> In each moment, an amount  $\psi$  of the intermediate good is used up immediately for administration purposes to maintain production, which is independent of how much output is produced. Consequently,  $\psi$  denotes the fixed cost. Then, the first-order condition for profit maximization yields

$$r = (1 - \tau)p \frac{\eta - 1}{\eta} [f(x) - f'(x)x] = (1 - \tau)(1 - \alpha)p \frac{\eta - 1}{\eta} f(x), \quad (6)$$

where  $x \equiv g/k$ ,  $f(x) \equiv F(1, g/k)$ , and  $\eta \equiv -(\partial y / \partial p)(p/y)$ . Using the definition of  $\eta$  and (3), we obtain

$$\eta(n) = \epsilon - \frac{\epsilon - 1}{n}, \quad (7)$$

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<sup>2</sup>For convenience, the firm index  $i$  is omitted hereafter.

where  $\eta'(n) = (\epsilon - 1)/n^2 > 0$ . Let

$$\mu(n) = \frac{\eta(n)}{\eta(n) - 1}, \quad (8)$$

where  $\mu'(n) = -(\eta(n) - 1)\eta(n)/\eta'(n) < 0$ . Then, Eq. (8) corresponds to the firm's price markup over its marginal cost. It is assumed that

$$\psi = \phi\bar{y}, \quad (9)$$

where  $0 < \phi < 1$  and  $\bar{y}$  is the average output of intermediate goods. To produce intermediate goods, the purchase of  $\phi\bar{y}$  units of intermediate goods is necessary.

In a symmetric equilibrium, all firms employ the same input amounts, produce the same quantities and set the same prices:  $k = K/n$ ,  $y = Y/n$ , and  $p = P = 1$  (by normalization). Finally, free entry into the intermediate sector forces profits to be zero:  $\pi = 0$ . Note that the symmetric equilibrium is not socially efficient because imperfection pertains in the intermediate sector. Using the free entry condition  $\pi = 0$  and (9), the number of firms in the intermediate sector is determined as

$$\frac{1 - \tau - \phi}{(1 - \tau)(1 - \alpha)} = \frac{1}{\mu(n)}. \quad (10)$$

Then, Eq. (10) leads to  $n^* = n(\tau)$ . Combining Eq. (6) with Eq. (10), the factor price of private capital is given as

$$r = (1 - \tau - \phi)f(x). \quad (11)$$

*Consumers.* The lifetime utility of a representative consumer is defined as

$$U = \int_0^\infty \frac{C^{1-\sigma} - 1}{1 - \sigma} \exp(-\rho t) dt, \quad (12)$$

where  $C$  represents consumption,  $\sigma > 0$  is the inverse of the intertemporal elasticity of substitution, and  $\rho > 0$  is the subjective discount rate. Because the profit is zero, the total income of consumers is derived solely from private capital income. Then, the budget constraint of consumers is given as

$$\dot{K} = rK - C. \quad (13)$$

Each consumer maximizes his objective function, Eq. (12), subject to his budget constraint, Eq. (13). Solving the optimization problem, we obtain

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\sigma}. \quad (14)$$

*Government.* The government is assumed to invest in public capital and finance its investment with a tax on revenue in the intermediate goods sector. Therefore, the government's budget constraint is given as

$$\dot{g} = \tau n y. \quad (15)$$

### 3 Dynamic general equilibrium

#### 3.1 Existence and stability of the stationary equilibrium

Defining  $z$  as  $C/K$  and using  $x \equiv g/k$ ,  $z \equiv C/K$ , Eq. (11), and Eq. (13) – Eq. (15), the dynamic system of the economy is given as

$$\frac{\dot{x}}{x} = \tau n \frac{f(x)}{x} - (1 - \tau - \phi) f(x) + z, \quad (16)$$

$$\frac{\dot{z}}{z} = (1 - \tau - \phi) \left( \frac{1 - \sigma}{\sigma} \right) f(x) - \frac{\rho}{\sigma} + z. \quad (17)$$

Existence and stability of a stationary equilibrium such as a solution of a dynamic system, is established as follows.

**Proposition 1** *There exists a unique stationary equilibrium that is saddle-point stable.*

(Proof) See Appendix.

Under Proposition 1, the economic growth rate and the number of firms in the intermediate goods sector are determined.<sup>3</sup> From Eq. (6) and Eq. (14), the economic growth rate in the long run is given as

$$\gamma^* = \frac{(1 - \tau - \phi) f(x^*) - \rho}{\sigma}. \quad (18)$$

#### 3.2 Macroeconomic effects of fiscal policy

We now investigate the effects of fiscal policy on the price markup rate, the long-run number of firms in the intermediate sector, and the economic growth rate.

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<sup>3</sup>Superscript \* denotes the stationary value of the endogenous variable.

Differentiating Eq. (10) with respect to  $\tau$ , the effects of a change in the tax rate,  $\tau$ , on the price markup are derived as

$$\frac{\partial \mu^*}{\partial \tau} = \frac{\phi \mu(n^*)}{(1-\tau)(1-\tau-\phi)} > 0, \quad (19)$$

where  $\mu^* = \mu(n^*)$ . Therefore, Eq. (19) leads to

$$\frac{\partial n^*}{\partial \tau} = \frac{\phi \mu(n^*)}{(1-\tau)(1-\tau-\phi)\mu'(n^*)} < 0. \quad (20)$$

The effects of a change in  $\phi$  on the price markup are obtained from Eq. (10):

$$\frac{\partial \mu^*}{\partial \phi} = \frac{\mu(n^*)}{1-\tau-\phi} > 0. \quad (21)$$

Using  $\mu^* = \mu(n^*)$ , Eq. (19) yields

$$\frac{\partial n^*}{\partial \phi} = \frac{\mu(n^*)}{(1-\tau-\phi)\mu'(n^*)} < 0. \quad (22)$$

The above results are summarized as follows.

**Proposition 2** *The price markup,  $\mu^*$ , is increasing in any of  $\tau$  and  $\phi$ . Then, the number of firms in the intermediate sector,  $n^*$ , is decreasing in any of  $\tau$  and  $\phi$ .*

An increase in the tax rate,  $\tau$ , raises the price markup. Then, the smaller price markup engenders the higher the degree of market power. It engenders less intense competition in the intermediate sector. Consequently, the number of firms in the intermediate sector is decreased by a rise in the tax rate,  $\tau$ . This mechanism simulates the effects of a change in the fixed fraction of running costs,  $\phi$ .

From  $\dot{x} = \dot{z} = 0$ , Eq. (16), and Eq. (17), we obtain

$$\frac{\partial x^*}{\partial \tau} = \frac{(1-\theta)n^* + x^*/\sigma}{(1-\alpha)\tau n^*/x^* + (1-\tau-\phi)\alpha/\sigma} > 0, \quad (23)$$

$$\frac{\partial x^*}{\partial \phi} = \frac{x^*/\sigma - n^*\tau\lambda/\phi}{(1-\alpha)\tau n^*/x^* + (1-\tau-\phi)\alpha/\sigma} \stackrel{\leq}{\geq} 0, \quad (24)$$

where  $\theta \equiv -(\tau/n^*)(\partial n^*/\partial \tau)$  and  $\lambda \equiv -(\phi/n^*)(\partial n^*/\partial \phi)$ .<sup>4</sup>

<sup>4</sup>We assume that  $\theta$  is less than unity.

We now examine the effect of a change in the tax rate,  $\tau$ , on the economic growth rate,  $\gamma^*$ . Differentiating Eq. (18) with respect to  $\tau$  yields

$$\frac{\partial \gamma^*}{\partial \tau} = -f(x^*) \left[ 1 - (1 - \tau - \phi) \frac{\alpha}{x^*} \frac{\partial x^*}{\partial \tau} \right] \begin{matrix} \geq \\ \leq \end{matrix} 0. \quad (25)$$

Using Eq. (23) and Eq. (26), we obtain

$$\text{sign} \frac{\partial \gamma^*}{\partial \tau} = \text{sign} \left[ \frac{(1 - \phi)(1 - \theta)\alpha}{1 - \alpha\theta} - \tau \right]. \quad (26)$$

Differentiating Eq. (18) with respect to  $\phi$ , the impact of a change in  $\phi$  on the economic growth rate is given as

$$\frac{\partial \gamma^*}{\partial \phi} = -\frac{n^* \tau f(x^*)}{\sigma \phi x^*} \frac{(1 - \alpha)\phi + (1 - \tau - \phi)\alpha\lambda}{(1 - \alpha)\tau n^*/x^* + (1 - \tau - \phi)\alpha/\sigma} < 0. \quad (27)$$

Therefore, we establish the following proposition.

**Proposition 3** *The growth-maximizing tax rate is given as  $\tau^*$ , such as*

$$\tau^* = \frac{(1 - \phi)(1 - \theta^*)\alpha}{1 - \alpha\theta^*}.$$

*Furthermore, an increase in the fixed rate of running cost,  $\phi$ , reduces the economic growth rate,  $\gamma^*$ : i.e.,  $\partial \gamma^*/\partial \phi < 0$ .*

The economic intuition for the growth effect of a change in the tax rate,  $\tau$ , is as follows. Public investment enhances economic growth through public capital accumulation (positive growth effect). On the other hand, a raised tax rate,  $\tau$ , simultaneously reduces the household's income, which might otherwise be used for private investment. Moreover, a rise in  $\tau$  raises a price markup,  $\mu^*$ , which reduces the household's income. This effect is peculiar to our model with monopolistic competition in the intermediate goods sector. These two negative effects on income have a negative growth effect through a decrease in private investment. Therefore, imperfect competition in the intermediate goods sector weakens the positive growth effect of public investment, and the growth-maximizing tax rate is different from that established by the models with perfect competition in the intermediate goods sector.<sup>5</sup> The explanation for the growth effect of a change in  $\phi$

<sup>5</sup>We consider the relationship between the growth-maximizing tax rate,  $\tau^*$ , and the fixed fraction of running cost,  $\phi$ . When  $\alpha + \phi \leq 1$ ,  $\tau^* < \alpha$  holds. In contrast,  $\tau^* > \alpha$  might hold if  $\alpha + \phi > 1$ . Without imperfection of intermediate goods, the growth-maximizing tax rate is equal to the output elasticity of public capital,  $\alpha$  (e.g., Barro (1990) and Futagami et al. (1993))

is given as follows. A rise in  $\phi$  raises a price markup,  $\mu^*$  (Proposition 2). On the other hand, the effect of a rise in  $\phi$  on public capital accumulation is ambiguous. These effects combine to reduce the economic growth rate,  $\gamma^*$ .

## 4 Concluding remarks

This paper described an endogenous growth model with public capital and monopolistic competition of the intermediate goods sector. The main novelty of the model compared with pertinent literature on endogenous growth and fiscal policy is that it incorporates monopolistic competition of the intermediate goods sector.

Without monopolistic competition in the intermediate goods sector, numerous firms exist, obviating markups. Then, fiscal policy has no influence on market structure, and the growth-maximizing policy is not affected by its market structure.

However, the opposite implications are established in our model with monopolistic competition in the intermediate goods sector. Imperfection in the intermediate goods sector brings about markups and restrictions on the number of firms that enter the intermediate sector. Fiscal policy affects the market structure and its market structure strongly influences macroeconomic effects of fiscal policy. Finally, imperfect competition in the intermediate goods sector weakens the positive growth effects of public investment more than it would weaken growth in a perfectly competitive intermediate goods sector; the growth-maximizing tax rate is different from that established by models with perfect competition in the intermediate goods sector.

Therefore, monopolistic competition in the intermediate goods sector brings about a drastic difference in the macroeconomic effects of fiscal policy in comparison with that in a perfectly competitive intermediate goods sector. Based on our analyses, it is essential for evaluating the macroeconomic effects of fiscal policy to consider imperfection of the intermediate goods sector.



## Appendix

In a stationary equilibrium,  $\dot{x} = \dot{z} = 0$  holds. Equations (16) and (17) yield

$$\tau n \frac{f(x)}{x} - (1 - \tau - \phi)f(x) + z = 0, \quad (28)$$

$$(1 - \tau - \phi) \left( \frac{1 - \sigma}{\sigma} \right) f(x) - \frac{\rho}{\sigma} + z = 0. \quad (29)$$

Using Eq. (28) and Eq. (29), we obtain

$$\tau n \frac{f(x)}{x} - \frac{(1 - \tau - \phi)f(x) - \rho}{\sigma} = 0. \quad (30)$$

Let

$$P(x) \equiv \tau n \frac{f(x)}{x} - \frac{(1 - \tau - \phi)f(x) - \rho}{\sigma}.$$

$P(x)$  has the following properties:  $P(0) = +\infty$ ,  $P(\infty) = -\infty$ , and  $P'(x) < 0$ . These properties shows that Eq. (30) has a unique solution,  $x^*$ . Because  $x^*$  is uniquely determined, Eq. (28) or Eq. (29) leads to  $z^*$ .

Linearizing Eq. (16) and Eq. (17) around the unique stationary equilibrium, the dynamics are approximated using the following linear system:

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} x - x^* \\ z - z^* \end{pmatrix}, \quad (31)$$

where

$$\begin{aligned} J_{11} &= -\frac{\tau n^*(1 - \alpha)f(x^*)}{x^*} - (1 - \tau - \phi)f'(x^*)x^*, \\ J_{12} &= x^*, \\ J_{21} &= (1 - \tau - \phi) \left( \frac{1 - \sigma}{\sigma} \right) f'(x^*)z^*, \\ J_{22} &= z^*. \end{aligned}$$

Then, the determinant of the Jacobian matrix of Eq. (31) is given as

$$\begin{aligned} \det J &= J_{11}J_{22} - J_{12}J_{21} \\ &= -\frac{\tau n^*(1 - \alpha)f(x^*)}{x^*} - (1 - \tau - \phi)f'(x^*)x^*z^* < 0. \end{aligned}$$

This result shows that the stationary equilibrium is a saddle-point.

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