ECONOMICS BULLETIN

A note on: jury size and the free rider problem

Parimal Bag
University of Surrey

Paul Levine
University of Surrey

Christopher Spencer
University of Surrey

Abstract

This note reassesses the basic result in Mukhopadhaya (2003) that, when jurors may acquire costly signals about a defendant's guilt, with a larger jury size the probability of reaching a correct verdict may in fact fall, contrary to the Condorcet Jury Theorem. We show that if the jurors coordinate on any one of a number of (equally plausible) asymmetric equilibria other than the symmetric equilibrium considered by Mukhopadhaya, the probability of accuracy reaches a maximum for a particular jury size and remains unchanged with larger juries, thus mitigating Mukhopadhaya's result somewhat. However, the case for limiting the jury size – a recommendation by Mukhopadhaya – gains additional grounds if one shifts the focus from maximizing the probability of reaching a correct verdict to the maximization of the overall social surplus, measured by the expected benefits of jury decisions less the expected costs of acquiring signals.

Citation: Bag, Parimal, Paul Levine, and Christopher Spencer, (2006) "A note on: jury size and the free rider problem." *Economics Bulletin,* Vol. 4, No. 3 pp. 1−12
Submitted: December 2, 2005. **Accepted:** January 16, 2006.
URL: http://www.economicsbulletin.com/2006/volume4/EB−05D70034A.pdf

1 Introduction

In a recent paper, Mukhopadhaya (2003) makes an interesting observation: when jurors may incur information gathering costs relating to a defendant's possible guilt, the Condorcet Jury Theorem that a larger jury is more likely to reach a correct verdict is not necessarily true. Because the information gathered by an individual juror is a public good, the associated free-rider problem may motivate each juror to invest less in information gathering costs and result in a lower probability of jury accuracy when there are more jurors involved. This argument prompts Mukhopadhaya to counter a recent argument by the judges in the United States that 6-person juries are inferior to 12-person juries.

Underlying Mukhopadhaya's analysis, three assumptions are particularly noteworthy: (1) the optimal jury size should maximize the probability of an accurate verdict; (2) each juror's payoff when no juror pays any attention is $\phi(0) = 0$; and (3) the jurors play a *symmetric* mixed strategy equilibrium in a binary-decision, information gathering game – pay attention or don't pay attention.

The first assumption, motivated by the Condorcet Jury Theorem, turns out to be equivalent to the maximization of expected benefits of accurate verdicts less expected costs of inaccurate verdicts (type I and type II errors), given Mukhopadhaya's assumption that the accurate verdicts yield payoffs equal to 1 and inaccurate verdicts yield zero payoffs. But if such welfaristic interpretation is to be imposed, the implicit social objective in Mukhopadhaya would still be incomplete as it ignores the jurors' information gathering costs. Since information gathering costs are likely to vary with jury size, a priori it is not clear that the jury size that maximizes the probability of an accurate verdict would necessarily maximize the expected social welfare of verdict decisions (net of the information gathering costs). Similarly, it is not clear how social welfare would change with the increase in jury size. We address these issues.

Mukhopadhaya justifies the second assumption by claiming that a positive valued $\phi(0)$ "would bias results toward more free riding, but would not qualitatively change the findings" (see page 30 of Mukhopadhaya's article).[1] This claim is not straightforward: while the Mukhopadhaya-noted increased free riding tendency is definitely true for any

[1] $\phi(0)$ can be positive if the defendant is declared guilty with probability 1/2, when the number of signals indicating guilt equals the number of signals indicating innocence.

given number of jurors, hence each juror pays attention with a lower probability for both small and large juries, whether this necessarily implies relatively greater reduction (due to positive $\phi(0)$) in the probability of reaching a correct verdict for larger juries is unclear; thus, the implication of such an assumption for the optimal jury size question should be properly examined.

The argument in favor of symmetric mixed strategy equilibrium is standard. However, ruling out possible asymmetric mixed strategy equilibria, where some jurors play mixed strategies and others play pure strategies (i.e., pay no attention), would have been more acceptable if one can show that the symmetric equilibrium would Pareto dominate the asymmetric equilibria. We examine this possibility.

Our results are as follows. We start by showing that asymmetric mixed strategy equilibria exist, and the best such equilibrium (in terms of the probability of reaching a correct verdict) for any given jury size is equivalent to a symmetric equilibrium corresponding to a smaller jury. Thus, by varying the jury size and comparing across the *best* asymmetric equilibria, we find that the probability of reaching a correct verdict is maximized for a particular jury size and this probability will remain *unchanged with further increases in jury size*. Thus, Mukhopadhaya's main result about larger juries strictly lowering the probability of accuracy is somewhat mitigated if one focuses on the best asymmetric equilibrium.[2] Importantly, the asymmetric equilibria neither Pareto dominate, nor are Pareto dominated by, the symmetric equilibrium of Mukhopadhaya. Thus, our asymmetric equilibria are no less compelling as a plausible description of equilibrium.

Next, focusing exclusively on the symmetric equilibrium we show two things: (i) with the probability of reaching a correct verdict as the primary social objective (as in Mukhopadhaya), a more plausible assumption of $\phi(0) = \frac{1}{2}$ would add to Mukhopadhaya's argument that the jury size should be restricted; (ii) a broader social objective by considering the information gathering costs would also strengthen Mukhopadhaya's suggestion about the jury size restriction. While both points (i) and (ii) accentuate the basic findings of Mukhopadhaya, it will be shown that arriving at our second conclusion (point (ii)) is not intuitively that obvious, especially for $\phi(0) = \frac{1}{2}$. Thus, while Mukhopadhaya's assumption that $\phi(0) = 0$ turns out not to matter in the core recommendation of jury

[2] This remains true whether $\phi(0) = 0$ (Mukhopadhaya's assumption) or $\phi(0)$ positive.

size restriction, to fully understand the underlying economic reasons it would be better to assume $\phi(0) = \frac{1}{2}$ instead.

The next section presents the model. In section 3, we take another look at the symmetric mixed strategy equilibrium of Mukhopadhaya. In section 4, we analyze asymmetric mixed strategy equilibria and compare with the symmetric equilibrium. Section 5 considers the issue of social efficiency, and section 6 concludes.

2 The Model

Consider the model of jury trial as in Mukhopadhaya (2003), with n risk-neutral jurors. The state of the world is that the defendant is either guilty or nor guilty, $\{G, NG\}$. The outcome of the trial is that the defendant is liable or not liable, $\{L, NL\}$. The uncertainty about the true state is denoted by a common prior probability $p = \frac{1}{2}$ that the true state is NG. Each juror's payoffs over outcomes and states, the same as the society's (or the mechanism designer's) payoffs, are $U(L, G) = U(NL, NG) = 1$ and $U(L, NG) = U(NL, G) = 0$.

During the trial a juror who pays attention receives a private signal S_0 or S_1 about the true state of the world where $Pr[S_1|G] = Pr[S_0|NG] = q \in (\frac{1}{2}, 1]$ is the precision of a signal. There is a fixed cost $c \in (0, 1)$ of paying attention. The jurors choose an alternative by majority voting.[3] In the case of a tie, the decision to convict or acquit is chosen with probability 1/2. With these assumptions sincere and informative voting (i.e., expected utility maximizing voting in accordance with the signal received) is rational (Austin-Smith and Banks, 1996). Jurors who do not pay attention simply abstain from voting and the majority voting rule applies to the actual votes cast.[4]

During the trial a juror who pays attention receives a private signal S_0 or S_1 about the true state of the world where $Pr[S_1|G] = Pr[S_0|NG] = q \in (\frac{1}{2}, 1]$ is the precision of a signal. There is a fixed cost $c \in (0, 1)$ of paying attention. The jurors choose an

[3] We are not interested in the question of optimal *voting rule*; see Persico (2004) for such an analysis.

[4] Abstention as an option in this note differs from Mukhopadhaya's assumption that the uninformed jurors can simply follow the majority voting of informed jurors. Since Mukhopadhaya does not specify the voting rule to be sequential, it is not clear how uninformed jurors can observe informed jurors' votes. Even with sequential voting, the observability problem remains unless one also assumes that all uninformed jurors will vote only after the informed jurors have voted.

alternative based on the majority of realized signals, declaring the defendant liable if the majority receives signal S_1 and not liable if the majority receives S_0; in the event of a tie, the decision to convict or acquit is chosen with probability 1/2.

We will consider two alternative objectives to assess the question of optimal jury size: (1) maximization of the probability of an accurate verdict; (2) maximization of the overall social welfare measured by the expected benefits of jury decisions less the expected costs of acquiring signals. Mukhopadhaya focused exclusively on the first objective.

3 Symmetric Mixed-Strategy Nash Equilibrium

Our analysis in this section will be based mostly on Mukhopadhaya's analysis, with some difference. Let

$$b(m;k,q)=\frac{k!}{m!(k-m)!}q^m(1-q)^{k-m} \tag{1}$$

denote the binomial probability that out of k independent signals $m \leq k$ are correct, given the precision of the signal. Then the probability of a n-person jury reaching a correct decision when $k \geq 1$ jurors pay attention is given by

$$\begin{aligned}\phi(k) &= \sum_{m=[\frac{k}{2}]+1}^{k} b(m;k,q), \quad \text{if k is odd} \\ &= \sum_{m=\frac{k}{2}+1}^{k} b(m;k,q)+\frac{1}{2}b\left(k/2;k,q\right), \text{ if k is even}\end{aligned} \tag{2}$$

where $[x]$ denotes the greatest integer less than x.

Note that (2) applies to $k \geq 1$. When no jurors pay attention and receive no signals Mukhopadhaya assumes that the payoff is zero; i.e., $\phi(0)=0$. An alternative assumption is that they vote according to their prior of $\frac{1}{2}$ so that $\phi(0)=\frac{1}{2}$. Note that $b(0;0,q)=\frac{0!}{0!0!}q^0(1-q)^0=1$ so that with this assumption (2) still applies with $k=0$, treating zero as even.

Let

$$B(k,q)=\phi(k)-\phi(k-1) \tag{3}$$

be the benefit to the kth juror from paying attention when exactly $k-1$ other jurors pay attention gross the cost of paying attention. Then an extremely useful result for the analysis is given in Lemma 1 in Mukhopadhaya, reproduced below:

$$B(k,q) = \begin{cases} 0, & \text{if k is even;} \\ (q-\frac{1}{2})b((k-1)/2;(k-1),q)\,, & \text{if k is odd.} \end{cases} \tag{4}$$

If we assume $\phi(0) = 0$ as in Mukhopadhaya then (4) does not hold for $k = 1$ and $B(1,q) = 1$. However if we assume $\phi(0) = \frac{1}{2}$, then (4) holds for $k = 1$ as well. In what follows we will pursue the consequences of both assumptions regarding $\phi(0)$.

The net benefit to a juror of paying attention when every other juror pays a mixed strategy 'attention' with probability σ and 'no attention' with probability $1-\sigma$ is given by

$$\begin{aligned} \Pi = \Pi(n,\sigma;c,q) &= \sum_{k=1}^{n} b(k-1;n-1,\sigma)B(k,q) - c \\ &= \sum_{k=1,3,\ldots}^{n} b(k-1;n-1,\sigma)B(k,q) - c \quad \text{if n is odd;} \\ & \quad \sum_{k=1,3,\ldots}^{n-1} b(k-1;n-1,\sigma)B(k,q) - c \quad \text{if n is even.} \end{aligned} \tag{5}$$

If we assume $\phi(0) = \frac{1}{2}$ then Π can be written as

$$\Pi(n,\sigma;c,q) = (q-\tfrac{1}{2}) \sum_{m=0}^{[\frac{n-1}{2}]} \frac{(n-1)!(\sigma q)^m(\sigma(1-q))^m(1-\sigma)^{(n-1-2m)}}{(m!)^2(n-1-2m)!} - c. \tag{6}$$

If we follow Mukhopadhaya in assuming that $\phi(0) = 0$ then we need to add an extra term $\frac{(1-\sigma)^{n-1}}{2}$ to the net benefit in (6).

The symmetric mixed strategy Nash equilibrium $\sigma^*(n;c,q)$ is derived by solving

$$\Pi(n,\sigma;c,q) = 0, \tag{7}$$

for $\sigma \in (0,1)$ and the probability of the jury making a correct decision is

$$\Phi = \Phi(n;c,q) = \sum_{j=0}^{n} b(j;n,\sigma^*(n;c,q))\phi(j). \tag{8}$$

Note that if $\phi(0) \neq 0$, then the summation must include a $j = 0$ term, unlike the case considered in Mukhopadhaya.

Figures 1 and 2 show numerical results for σ^* and Φ using MATLAB.[5] Mukhopadhaya reports results for parameter values $c = 0.1$ and $q = 0.65, q = 0.7$ and $q = 0.95$. We

[5] All MATLAB files can be made available on request.

can reproduce his results but in this note we confine ourselves to one intermediate value, $q = 0.75$. Furthermore, we compare results assuming $\phi(0) = 0$, with our alternative assumption $\phi(0) = \frac{1}{2}$. Clearly the assumption that $\phi(0) = 0$ is not innocuous as it heavily penalizes no attention by all jurors and so biases the mixed equilibrium towards paying attention. The result is that whereas with $\phi(0) = 0$ jury accuracy is maximized at a jury size $n = 3$, with $\phi(0) = \frac{1}{2}$ accuracy increases monotonically as n decreases from $n = 12$ to $n = 1$. Using jury accuracy as the measure of social benefit then sees our alterative assumption regarding $\phi(0)$ further undermine the Condorcet Jury Theorem. As we will see in a latter section, it is undermined still further when we rank different jury sizes using *social welfare* as our measure.

4 Asymmetric Mixed-Strategy Nash Equilibria

We now explore asymmetric mixed-strategy equilibria where k out of n jurors play the symmetric mixed-strategy $\sigma^*(k : c, q)$ and the remaining jurors pay no attention with probability 1. The probability of a correct decision is then

$$\Phi(k; c, q) = \sum_{j=0}^{k} b(j; k, \sigma^*(k; c, q))\phi(j), \tag{9}$$

and the net payoff for each attentive juror is $\Phi - c$ and for each non-attentive juror Φ. The condition for this to be an equilibrium is that, the net payoff when the $(k+1)$th juror pays attention must be less than when she does not; i.e.,

$$\Phi = \sum_{j=0}^{k} b(j; k, \sigma^*(k; c, q))\phi(j) > \sum_{j=0}^{k} b(j; k, \sigma^*(k; c, q))\phi(j+1) - c. \tag{10}$$

(10) says that the probability of a correct decision when only k jurors pay attention with probability σ^* must exceed the increased probability when one more juror pays attention net of the cost of doing so. Write this as

$$\Delta\Phi \equiv \sum_{j=0}^{k} b(j; k, \sigma^*(k; c, q))\phi(j+1) - \sum_{j=0}^{k} b(j; k, \sigma^*(k; c, q))\phi(j) < c. \tag{11}$$

Figure 3 shows that this condition is satisfied for the same parameter values as before and for both assumptions regarding $\phi(0)$. We have therefore shown that alongside the symmetric equilibrium (SE), there exist asymmetric mixed-strategy equilibria (AE) for which $k = 2, 3, \ldots$ out of n jurors play a mixed strategy with probability $\sigma^*(k; c, q)$ of paying attention at a fixed c, and the remaining $n - k$ pay no attention with probability 1 and zero cost. Now in an AE, the overall probability of accuracy stops being increasing in jury size as more inattentive jurors are added. Put differently, if one were to focus on the *best* asymmetric equilibrium (best in terms of the overall probability of accuracy) corresponding to each jury size, then the probability of accuracy will be maximized for a particular jury size and will remain stationary for any further jury additions.

Next we turn to a particular equilibrium selection test, that of Pareto ranking, to see if any of the equilibria, SE and AE, is more plausible. For the following analysis, fix the jury size n.

First note that, for the fixed jury size n there are n possible equilibria to choose from, one symmetric equilibrium and $n-1$ asymmetric equilibria with $k = 1, 2, \ldots n-1$ attentive jurors. An AE with $1 < k \leq n-1$ attentive jurors, $AE(k)$ say, is essentially equivalent to a SE with jury size k, $SE(k)$ say. The probability of accuracy and the costs incurred by the jurors are identical in $AE(k)$ and $SE(k)$. Thus figures 1 and 2 for $SE(n)$ also apply to asymmetric equilibria as well.

Now consider the two groups of jurors, non-attentive and attentive. For the former they incur no attention costs and so prefer equilibria that maximize the probability of accuracy, Φ. From figure 2 this occurs when 3 jurors pay attention if $\phi(0) = 0$ and when 1 juror pays attention if $\phi(0) = \frac{1}{2}$. Now consider the attentive jurors. Figure 4 plots their expected utility as the size of the group k increases. Two opposite effects are at work here: as k increases, the probability of accuracy first rises and then falls monotonically with k for the case $\phi(0) = 0$ as we have seen in figure 2 (since the $SE(k)$ is equivalent to $AE(k)$), and falls monotonically for the case $\phi(0) = \frac{1}{2}$. However, this effect that reduces the expected utility of an attentive juror is more than cancelled out by a reduction in his attention costs owing to the free-rider effect. The net result is that the expected utility of an attentive juror *rises* as k increases reaching a maximum at $k = n$, the symmetric equilibrium. We conclude from this that we *cannot* establish Pareto dominance of any of the multiple equilibria.

5 Social Welfare

We now return to the symmetric equilibrium and examine the question of social welfare maximization. Figure 5 plots the expected social cost of a symmetric equilibrium with n jurors defined as $C = n\sigma^* c$. For $\phi(0) = 0$, as jury size decreases the expected social costs fall and as we have seen the expected accuracy of the jury rises. This provides a further argument for limiting the size of a jury. However the same figure shows that this result is sensitive to the assumption regarding $\phi(0)$. If we assume that $\phi(0) = \frac{1}{2}$ then the improvement in accuracy of the jury comes at an increasing social cost as jury size decreases from $n = 12$ to $n = 3$, but a further decrease in size sees social costs falling as before.

Figure 6 assesses the net benefit of reducing jury size by plotting the expected social surplus $\Phi - C$ against jury size. Whereas for $\phi(0) = 0$ the probability of accuracy is maximized at $n = 3$, social surplus is maximized at $n = 1$. Social efficiency considerations therefore adds weight to the case for a smaller jury.

With our alternative assumption $\phi(0) = \frac{1}{2}$ we arrive at the same qualitative conclusion, though social surplus rises by less as jury size decreases. Thus for both assumptions and for this numerical example social surplus rises monotonically as n decreases and suggests that, owing to the free-rider problem, any jury may yield less benefit than a decision arrived by a single judge or magistrate.

6 Conclusions

We have investigated whether the results in Mukhopadhaya are strengthened or weakened by the assumption regarding the payoff $\phi(0)$ when no jurors pay attention. In Mukhopadhaya $\phi(0) = 0$ is assumed, and our first numerical result is that whereas with $\phi(0) = 0$ jury accuracy is maximized at a jury size $n = 3$, with $\phi(0) = \frac{1}{2}$ accuracy increases monotonically as n decreases from $n = 12$ to $n = 1$. Using jury accuracy as the measure of social benefit then sees our alterative assumption regarding $\phi(0)$ strengthen the result of Mukhopadhaya.

Our second numerical result is that using social surplus as the criterion for comparing different jury sizes further strengthens his result in that with both assumptions regarding

$\phi(0)$, social surplus is maximized at a jury size $n = 1$.

Finally, we show the existence of asymmetric equilibria for which the probability of a jury making correct decisions does not decline with jury-size. None of the n possible equilibria (one symmetric and $n - 1$ asymmetric) can be Pareto-ranked. To rule out asymmetric equilibria would therefore be ad hoc and this mitigates the case for restricting the jury size somewhat. Of course one final and obvious reason to limit the jury size, in addition to the free-rider problem and social welfare considerations, that would work for both symmetric and asymmetric equilibria is, if there is some fixed cost of summoning a juror irrespective of whether the particular juror pays attention or not to the judicial proceedings.

References

[1] Austen-Smith, David, and Jeffrey S. Banks. 1996. "Information Aggregation, Rationality, and the Condorcet Jury Theorem," American Political Science Review, vol. 90, 34–45.

[2] Mukhopadhaya, Kaushik. 2003. "Jury Size and the Free Rider Problem," Journal of Law, Economics and Organization, vol. 19, 24–44.

[3] Persico, Nicola. 2004. "Committee Design with Endogenous Information," Review of Economic Studies, vol. 71, 165–191.

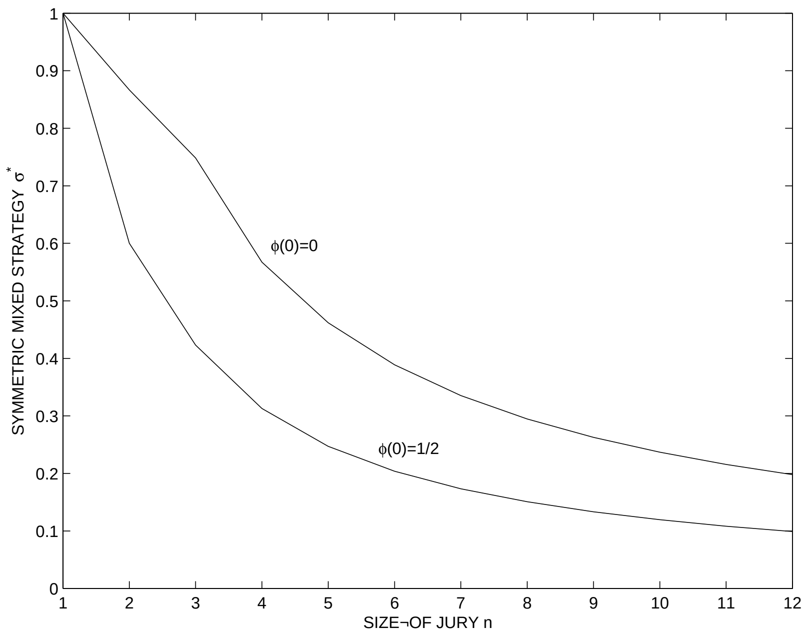

Figure 1: **Symmetric mixed-strategy Nash equilibrium, σ^*, as n increases.** $q = 0.75$, $c = 0.1$. $\phi(0) = 0$ **compared with** $\phi(0) = \frac{1}{2}$.

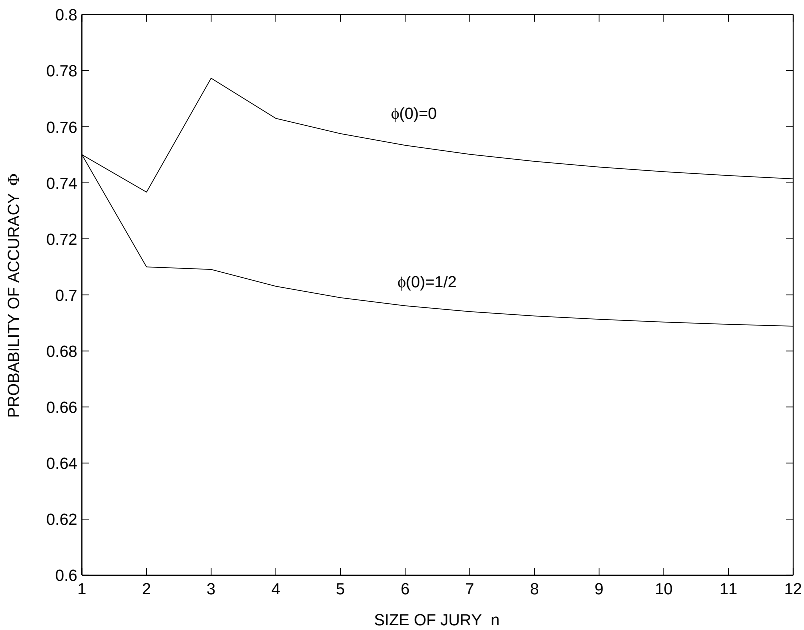

Figure 2: **Probability of jury arriving at a correct decision, Φ, as $n \geq 2$ increases.** $q = 0.75$, $c = 0.1$. $\phi(0) = 0$ **compared with** $\phi(0) = \frac{1}{2}$.

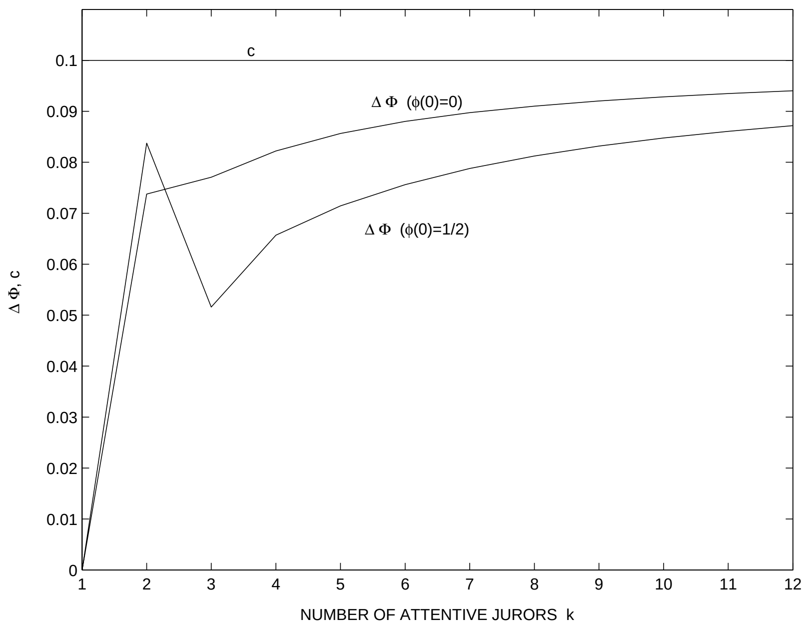

Figure 3: **Condition for an asymmetric mixed-strategy Nash equilibrium with k out of n jurors playing σ^*, and the remaining n-k paying no attention, as $k \geq 1$ increases.** $q = 0.75$, $c = 0.1$. $\phi(0) = 0$ **compared with** $\phi(0) = \frac{1}{2}$.

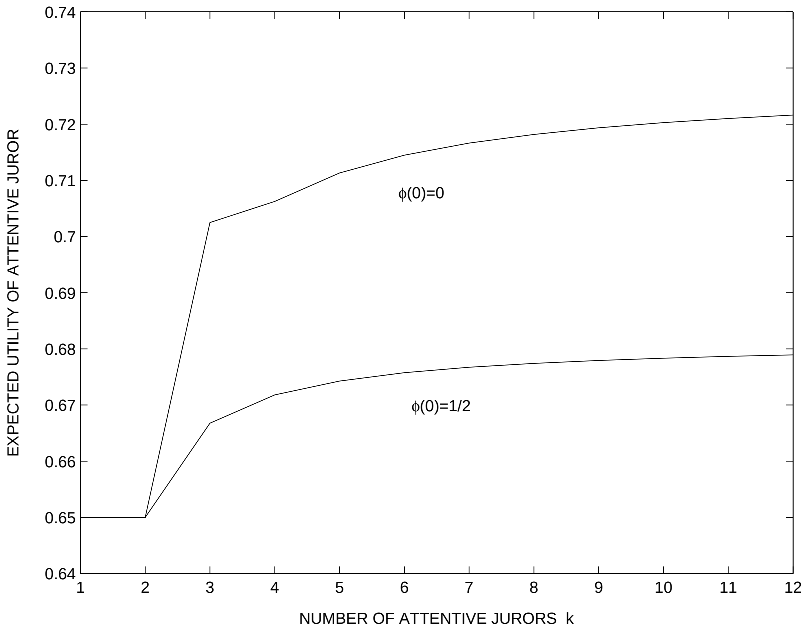

Figure 4: **Expected utility of attentive jurors in an asymmetric equilibrium with k out of n jurors playing σ^*, and the remaining n-k paying no attention, as $k \geq 1$ increases.** $q = 0.75$, $c = 0.1$. $\phi(0) = 0$ **compared with** $\phi(0) = \frac{1}{2}$.

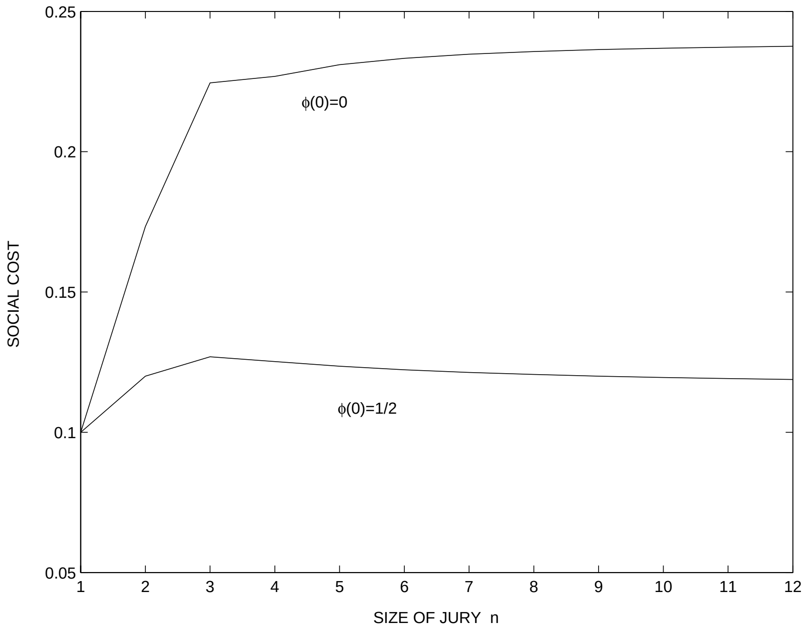

Figure 5: **Expected social cost for symmetric equilibria as $n \geq 1$ increases. $q = 0.75$, $c = 0.1$. $\phi(0) = 0$ compared with $\phi(0) = \frac{1}{2}$.**

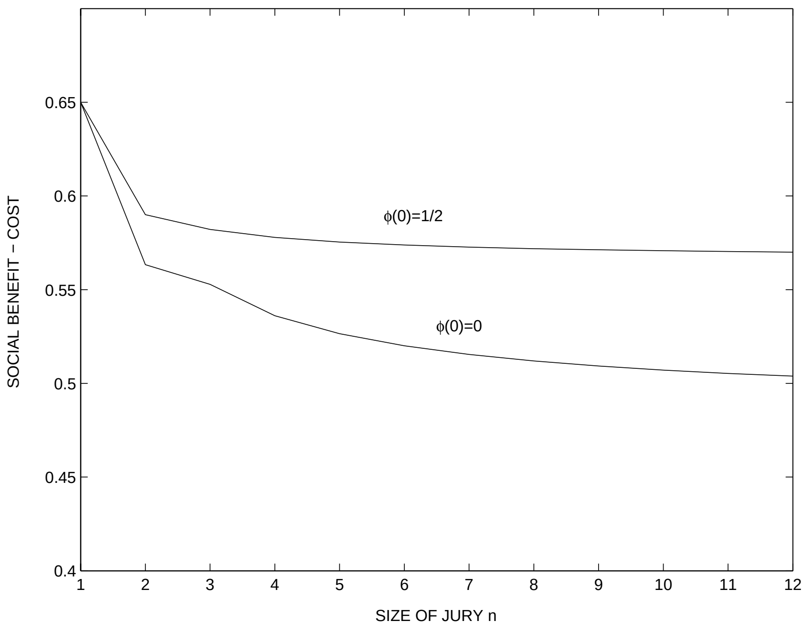

Figure 6: **Benefit-cost measure for symmetric equilibria as $n \geq 1$ increases. $q = 0.75$, $c = 0.1$. $\phi(0) = 0$ compared with $\phi(0) = \frac{1}{2}$.**