E C O N O M I C S   B U L L E T I N

Threshold autoregressive testing procedures and structural change in cointegrating relationships

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Abstract

The finite−sample properties of threshold autoregressive cointegration tests are examined in the presence of structural changes in cointegrating relationships. It is shown that spurious asymmetric cointegration may be exhibited when there is a change in the degree of cointegration between two series.

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1 Introduction

Following the seminal work of Perron (1989), a large literature has emerged considering the properties of alternative unit root tests in the presence of structural change. The recent studies of Gregory and Hansen (1996) and Gregory *et al.* (1996) can be thought of as an extension of this research. In these closely related papers, the finite-sample properties of cointegration tests and tests of structural change are considered in the presence of structural breaks in cointegrated relationships. Using Monte Carlo simulation, Gregory *et al.* (1996) show the rejection frequency of the Engle-Granger (1987) test of the null of no cointegration to decrease substantially when there is a change in the cointegrating relationship between two series. That is, there is a fall in the detection of cointegration when the degree of cointegration changes. In this paper the analysis is extended to examine the properties of threshold autoregressive (TAR) and momentum-threshold autoregressive (MTAR) tests of cointegration when there is a break in the cointegrating relationship between two series. Initially, it is found that both tests exhibit similar behaviour to the Engle-Granger test, with reduction in rejection of the null of no cointegration apparent. However, when the analysis is extended to consider joint testing of the no cointegration and symmetry hypotheses, an interesting finding is uncovered as the presence of a break in the cointegrating relationship leads to *increased* rejection, resulting in the increased detection of spurious asymmetric cointegration.

2 Cointegration tests with asymmetric adjustment

To examine possible cointegration between two series $\{y_t, x_t\}$, the Engle and Granger (1987) two-step approach firstly estimates a static cointegrating regression:[1]

$$y_t = \gamma_0 + \gamma_1 x_t + \epsilon_t \tag{1}$$

[1]In this paper attention will focus upon the 'with intercept' model.

before performing a Dickey-Fuller (1979) (DF) using the derived residual series $\{\widehat{\epsilon}_t\}$:

$$\Delta\widehat{\epsilon}_t = \phi\widehat{\epsilon}_{t-1} + \eta_t \tag{2}$$

The null hypothesis of no cointegration (a unit root in $\{\widehat{\epsilon}_t\}$) is then tested against the alternative of cointegration. However, it is apparent that this is an implicitly symmetric procedure, with reversion to the defined equilibrium or attractor occurring at single rate determined by $\phi < 0$, irrespective of whether convergence occurs from above or below equilibrium.

To allow for the possibility of asymmetric adjustment about a stationary attractor, Enders and Siklos (2001) draw upon the threshold autoregressive methods of Tong (1983,1990). The modified procedure proposed therefore combines the static cointegrating regression of (1) with the following revised version of (2):

$$\Delta\widehat{\epsilon}_t = I_t\rho_1\widehat{\epsilon}_{t-1} + (1 - I_t)\,\rho_2\widehat{\epsilon}_{t-1} + \xi_t \tag{3}$$

where I_t denotes the Heaviside indicator function. To partition $\widehat{\epsilon}_{t-1}$, Enders and Siklos (2001) consider two specifications of I_t based upon $\{\widehat{\epsilon}_t\}$ and $\{\Delta\widehat{\epsilon}_t\}$ leading to threshold autoregressive (TAR) and momentum threshold autoregressive (MTAR) cointegration tests. Under the assumption of TAR adjustment, I_t is given as:

$$I_t = \begin{cases} 1 & \text{if } \widehat{\epsilon}_{t-1} \geq 0 \\ 0 & \text{if } \widehat{\epsilon}_{t-1} < 0 \end{cases} \tag{4}$$

while under MTAR adjustment, I_t is given as:

$$I_t = \begin{cases} 1 & \text{if } \Delta\widehat{\epsilon}_{t-1} \geq 0 \\ 0 & \text{if } \Delta\widehat{\epsilon}_{t-1} < 0 \end{cases} \tag{5}$$

Following the Enders-Siklos approach, the null hypothesis of no cointegration is examined via the joint hypothesis $H_0 : \rho_1 = \rho_2 = 0$ in (3) using specifically derived critical values. Following rejection of the null of no cointegration, a secondary test of symmetry ($H_0 : \rho_1 = \rho_2$) can be applied using a conventional F-statistic to examine any difference in the asymmetric adjustment coefficients. Should this second null also be rejected, asymmetric cointegration is presumed to exist, with reversion to equilibrium occurring at different speeds on either side of the attractor.

3 Simulation analysis

3.1 Monte Carlo design

To examine the properties of the TAR, MTAR and Engle-Granger (EG) tests in the presence of structural change in cointegrating relationships, the following data generation process (DGP) is employed:

$$y_{1,t} = \alpha + \beta_t y_{2,t} + \varepsilon_{1,t} \qquad \varepsilon_{1,t} \sim i.i.d.\ \mathsf{N}(0,1) \tag{6}$$

$$y_{2,t} = y_{2,t-1} + \varepsilon_{2,t} \qquad \varepsilon_{2,t} \sim i.i.d.\ \mathsf{N}(0,1) \tag{7}$$

$$\beta_t = \begin{cases} \beta_1 & \text{if } t \leq \tau T \\ \beta_2 & \text{if } t > \tau T \end{cases} \tag{8}$$

where the innovation series $\{\epsilon_{1t}\}$ and $\{\epsilon_{2t}\}$ are generated using pseudo i.i.d. $\mathsf{N}(0,1)$ random numbers from the RNDNS procedure in the GAUSS. All experiments are performed over 25,000 replications using a sample size of $T = 100$, with a further initial 50 observations created and discarded to minimise the influence of the initial conditions $y_{1,0} = y_{2,0} = 0$.[2] A break in the cointegrating relationship is generated by $\beta_1 \neq \beta_2$ with both increases and decreases in the cointegrating coefficient considered. For an increase in the cointegrating coefficient, the values $(\beta_1, \beta_2) = (1, 2)$ are imposed, while $(\beta_1, \beta_2) = (2, 1)$

[2] In the interests of brevity, and following the power experiments of Enders and Siklos (2001), results are presented for a single sample size of 100 observations.

results in a decreasing break.[3] In contrast to previous studies which examine the properties of the EG test alone, the break in the cointegrating relationship is imposed at all points in the sample with $\tau = 0.01, 0.02, ..., 0.99$.

Given the above DGP, the ability of the TAR, MTAR and EG tests to reject the null of no cointegration is examined at the 5% level of significance using the critical values provided by Enders and Siklos (2001) and MacKinnon (1991) respectively. Joint rejection of the no cointegration and symmetry hypotheses using the TAR and MTAR tests is examined using the appropriate critical value from the F-distribution, again at the 5% level of significance.

3.2 Monte Carlo results

To ease interpretation, the results of the simulation analysis are presented graphically. Figure One contains empirical rejections of the null of no cointegration by the alternative tests in the presence of a decrease in the cointegrating coefficient ($\beta_1 > \beta_2$). From inspection of the results, it is clear that all tests experience a decrease in rejection of the null when a break in the cointegrating relationship occurs. Two further points can be noted. First, the tests experience minimum power when the break is imposed around the mid-point of the sample period. More precisely, the TAR, MTAR and EG tests exhibit minimum power when $\tau = 0.54$, 0.52 and 0.64 respectively, with empirical powers of 29.6%, 31.6% and 40.2%. Second, it is apparent that the TAR and MTAR exhibit a similar level of power which is lower than that of the EG test over the majority of breakpoints. However, this is to be expected intuitively, as the EG test is known to have greater power than the TAR and MTAR tests in the presence of symmetric adjustment as considered here.

In Figure Two, corresponding results are presented for an increase in the cointegrating coefficient. Maximum reduction in the power of the tests now occurs for breaks towards the end of the sample. The minimum powers of the TAR, MTAR and EG tests are now observed when $\tau = 0.75$, 0.7 and 0.78 resulting in empirical powers of 35%, 35.1% and 46.4% respectively. However, while these results show

[3] Throughout, α is set equal to one without loss of generality.

increasing and decreasing breaks to both reduce rejection of no cointegration, it is clear that decreasing breaks have a more significant impact on the properties of the tests. This finding has not been noted previously, as the literature has only considered breaks in which the size of the cointegrating coefficient is increased.

In Figures Three and Four results are presented for joint rejections of the no cointegration and symmetry hypotheses. From inspection of Figure Three it can be seen that a decreasing break induces spurious asymmetric stationarity when the TAR and MTAR tests are employed. While the MTAR test experience severe size distortion for very early and very late breaks, size distortion of the TAR test is more apparent for very late breaks only. Considering the exact values, the maximum rejection frequencies are 34.1% and 38.6% for the MTAR and TAR tests respectively, these occurring when $\tau = 0.01$ and 0.98. Given the nominal level of significance of 5%, these values represent severe size distortion. Results for an increasing are presented in Figure Four. Again the properties of the tests differ with the MTAR test experiencing much greater size distortion than the TAR test.

4 Conclusion

In this paper the literature on testing for cointegration in the presence of structural change in cointegrating relationships has been extended by considering the finite-sample properties of asymmetric cointegration tests based upon the use of threshold autoregression. It was shown that although the presence of a break results in a decrease in the rejection of the null of no cointegration, it does increase detection of spurious asymmetric cointegration.

References

[1] Enders, W. and Siklos, P. (2001) 'Cointegration and threshold adjustment', *Journal of Business and Economic Statistics*, **19**, 166-176.

[2] Engle, R. and Granger, C. (1987) 'Cointegration and error correction: representation, estimation and testing', *Econometrica*, **55**, 251-276.

[3] Gregory, A. and Hansen, B. (1996) 'Residual-based tests for cointegration in models with regime shifts', *Journal of Econometrics*, **70**, 99-126.

[4] Gregory, A., Nason, J. and Watt, D. (1996) 'Testing for structural breaks in cointegrated relationships', *Journal of Econometrics*, **71**, 321-341.

[5] MacKinnon, J. (1991) 'Critical values for cointegration tests', in Engle, R. and Granger, C. (eds) *Long-run economic relationships: Readings in cointegration*, Oxford: Oxford University Press.

[6] Tong, H. (1983) *Threshold models in non-linear time-series analysis*, New York: Springer-Verlag.

[7] Tong, H. (1990) *Non-linear time-series: a dynamical approach*, Oxford: Oxford University Press.

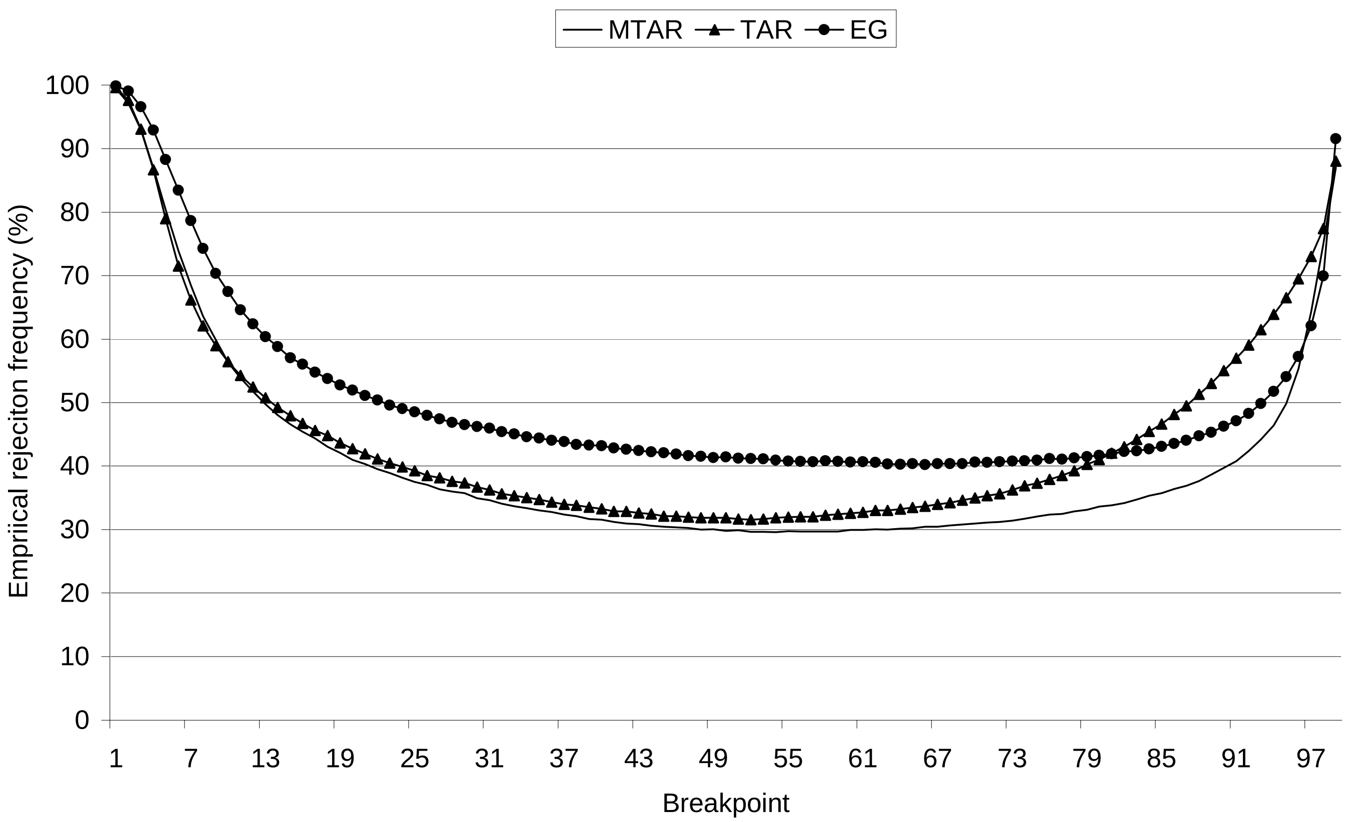

Figure 1: Cointegration testing in the presence of a decreasing break in the cointegrating relationship

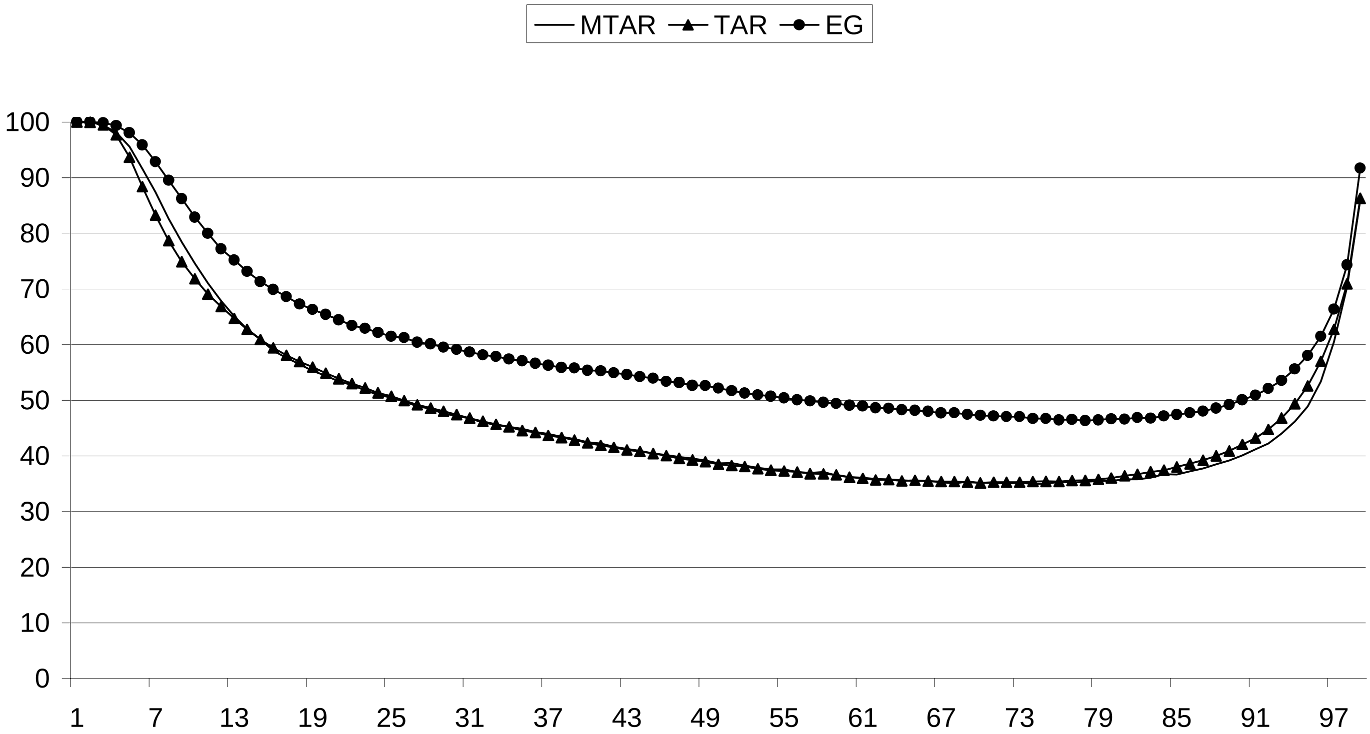

Figure 2: Cointegration testing in the presence of an increasing break in the cointegrating relationship

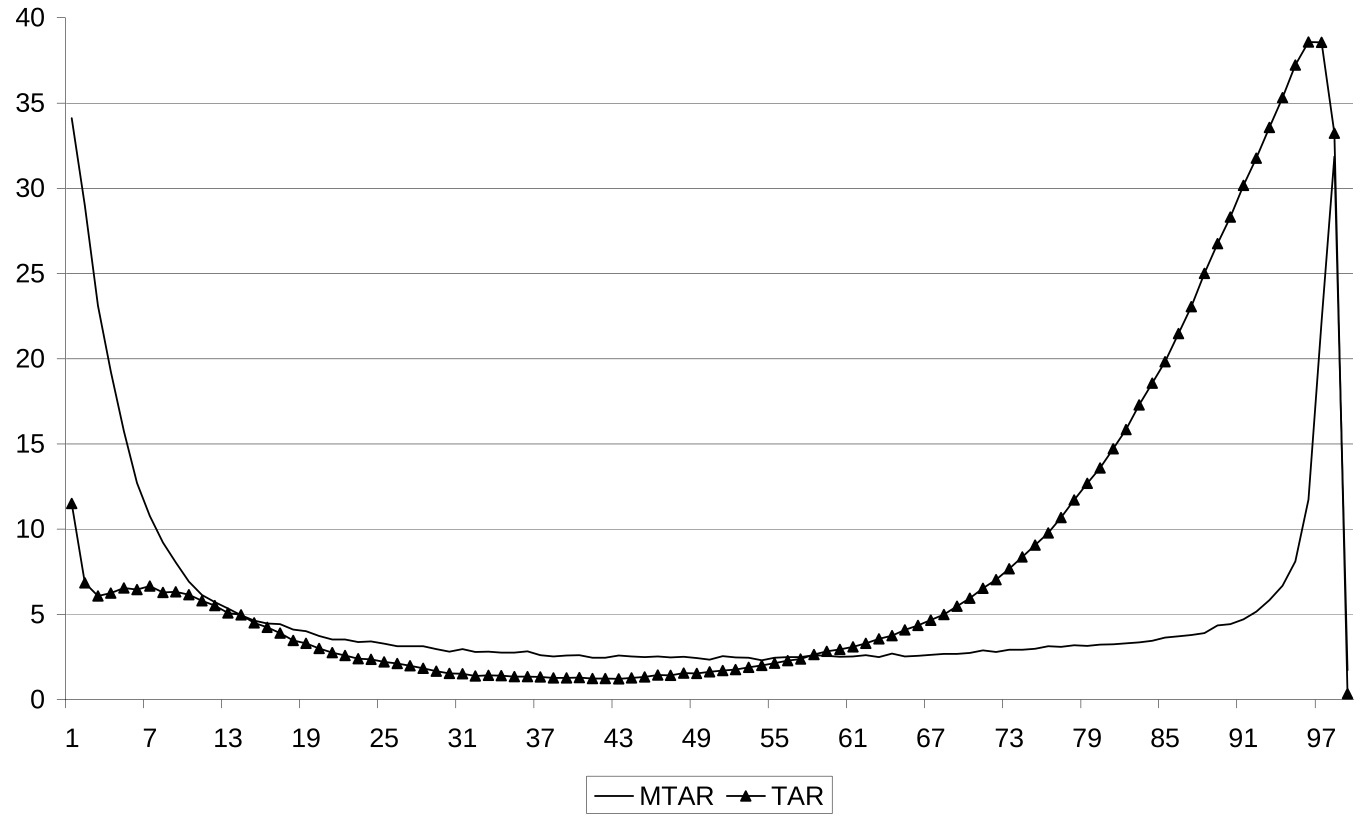

Figure 3: Testing asymmetric cointegration in the presence of a decreasing break in the cointegrating relationship

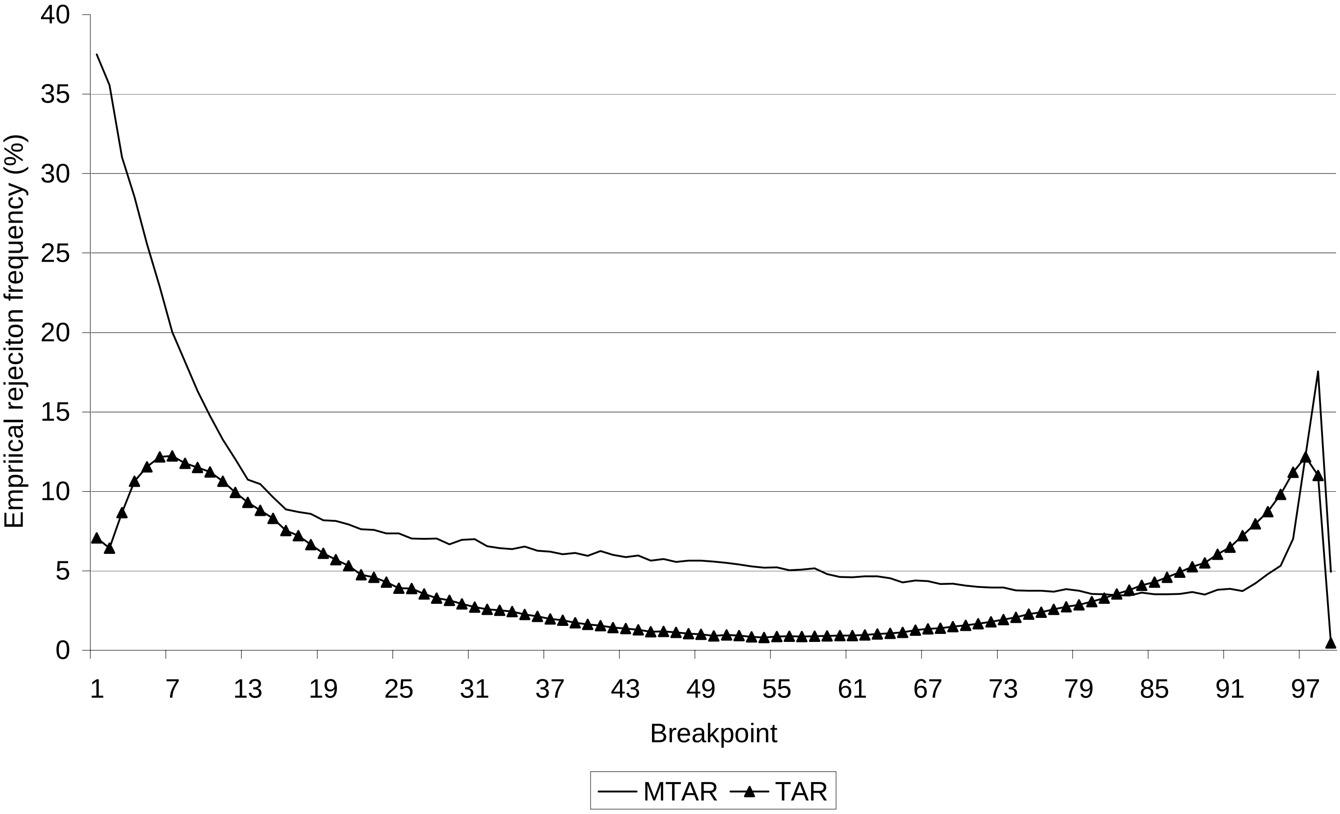

Figure 4: Testing asymmetric cointegration in the presence of an increasing break in the cointegrating relationship