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# A complementary test for the KPSS test with an application to the US Dollar/Euro exchange rate

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**Abstract**

This paper shows by simulation experiments some failures of the KPSS test when the source of the nonstationarity is explained by an unconditional volatility shift. So, a complementary test is proposed. An application to the US Dollar/Euro exchange rate reveals an instability in the unconditional volatility.

**Key-words**: KPSS test; Unconditional volatility shift.
**JEL classification**: C12; C22.

## 1 Introduction

The KPSS test (Kwiatkowski, Phillips, Schmidt and Shin, 1992) is often used for testing the null hypothesis of stationarity against the alternative of unit root. This test is well known among the most powerful. In this paper we show by simulation experiments that the non-rejection of the null hypothesis does not necessarily imply the stationarity of the data. When the source of the nonstationarity of the data is concerned with a shift in the unconditional volatility instead of unit root, then the KPSS test fails to detect this form of instability, the null is not rejected while the process is not really stationary. Then we propose a complementary test when the null hypothesis is not rejected by the KPSS test. This enables us to test the null hypothesis of the homogeneity of the unconditional variance against the alternative of time varying of the unconditional volatility. Our approach does not compete with the KPSS test but it can be used as a complementary test. In the first section we define our complementary test for the KPSS test and its asymptotic distribution. The second section gives some simulation experiments. In the last section we apply the KPSS test and the complementary approach to the return series of the US Dollar/Euro exchange rate. Some comments of the obtained results are given before concluding and remarks.

## 2 Definition of the complementary test

Let us consider the following process $\{y_t\}$:

$$y_t = r_t + \varepsilon_t, \quad t = 1, ..., T, \tag{1}$$

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where $r_t = r_{t-1} + u_t$ is a random walk, $\{u_t\}$ is an $i.i.d(0, \sigma_u^2)$ and $\varepsilon_t$ a zero mean stationary process with $E(\varepsilon_t^2) = \sigma_\varepsilon^2 > 0$. We are concerned with the null hypothesis of stationarity of $\{y_t\}$. The KPSS stationarity test is based on the null hypothesis of the absence of random walk, i.e. $H_0 : \sigma_u^2 = 0$. Under the null hypothesis, $\{y_t\}$ is stationary around the level $r_0$:

$$y_t = r_0 + \varepsilon_t, \quad t = 1, ..., T. \tag{2}$$

We consider the KPSS statistic $\widehat{\eta}$ defined as follows:

$$\widehat{\eta} = T^{-2} \sum_{t=1}^{T} [S_t^2 / s^2(l)], \tag{3}$$

where $S_t = \sum_{i=1}^{t} e_i$, $\{e_i\}$ the ols residuals from the regression (2) and $s^2(l)$ a long term variance estimator of $\varepsilon_t$. In this paper we calculate $s^2(l)$ as follows: $s^2(l) = T^{-1} \sum_{t=1}^{T} e_t^2 + 2T^{-1} \sum_{s=1}^{l} w(s,l) \sum_{t=s+1}^{T} e_t e_{t-s}$ where[1] $w(s,l) = (1 - \frac{s}{(l+1)})$. Under the null hypothesis, $\widehat{\eta}$ is asymptotically distributed as $\int_0^1 V_1(r)^2 dr$, where $V_1(r) = W(r) - rW(r)$ is a standard Brownian bridge. Now suppose that in the model (2), $\varepsilon_t$ is a variance shift process, i.e. $E(\varepsilon_t^2) = \sigma_t^2$ takes many values in successive interval, thus we have a nonstationary variance process. The next section shows by simulations that in this case the KPSS stationarity test fails to detect this form of nonstationarity. This implies that the non-rejection of the null hypothesis by the KPSS test must be completed by a jump variance test to be sure that the data are completely covariance stationary. Thus we are concerned with a test of the null hypothesis of variance constancy in the model (2): $H_0^{(2)}$ : $E(\varepsilon_t^2) =$ constant (i.e. $E(e_t^2) =$ constant). Let us consider the statistic $\tau$ defined as follows:

$$\tau = \max_{k=1,\ldots,T} \sqrt{\frac{T}{2}} \left| D_k \right|, \tag{4}$$

where $D_k = \frac{C_k}{C_T} - \frac{k}{T}$, $C_k = \sum_{t=1}^{k} e_t^2$, $k = 1, ..., T$.

**Proposition 1** *Under the null hypothesis of variance constancy,.i.e. $H_0^{(2)}$, and supposing that $\{e_t\}$ is independent and identically distributed, then the limiting distribution of $\tau$ is given by the one of $sup(W_t^0)$ where $W_t^0$ is a standard Brownian Bridge.*

**Proof.** Under the null hypothesis of variance constancy,.i.e. $H_0^{(2)}$, and supposing that $\{e_t\}$ is independent and identically distributed then the condition of the theorem of Inclan and Tiao (1994) is obviously satisfied. Then the limiting distribution of $\sqrt{\frac{T}{2}} \left| D_k \right|$ is given by the one of $W_t^0$, where $W_t^0$ is a standard Brownian Bridge. So the $\max_{k=1,\ldots,T} \sqrt{\frac{T}{2}} \left| D_k \right|$ is asymptotically distributed as $sup(W_t^0)$ and the desired conclusion holds.

■

This proposition gives some critical values of the statistic $\tau$. From Inclan and Tiao (1994), $C_{0.05} = 1.36$ with $Pr(sup(W_t^0) > C_{0.05}) = 0.05$. The statistic $\tau$ must be applied as follows: First, apply the KPSS test. If the null hypothesis is rejected, then conclude that the data contain a unit root, i.e. there is nonstationarity. If the null hypothesis is not rejected, then there is no unit root but a shift in the variance is possible. Then apply the statistic $\tau$. If the statistic $\tau$ does not reject the null hypothesis, then there is a complete covariance stationarity. Else, if the null is rejected by $\tau$, then conclude that there is no unit root but the data have variance shift and the process is not covariance stationary.

[1] In this paper we take $l = 4$. For more precisions about the values of this parameter $l$, the readers are referred to Kwiatkowski *et al.* (1992).

# 3 Simulation experiments

We consider the followings data-generating process (DGP):

$$DGP_{H_0}: \quad x_t = 0.01 + \varepsilon_t, \ \ t = 1, ..., 200, \quad \text{and} \quad \varepsilon_t \tilde{} IIN(0,1), \tag{5}$$

and

$$DGP_{H_1}: \ \ y_t = 0.01 + \varepsilon_t' \ \ \text{where } \varepsilon_t' \tilde{} IIN(0, \sigma_t^2), \ \sigma_t^2 = 1 \text{ if } t \leq 100 \text{ and } \sigma_t^2 = 1.5 \text{ if } t > 100. \tag{6}$$

While the process $\{x_t\}$ is stationary around level 0.01, the process $\{y_t\}$ is nonstationary since the variance is not constant. The following Table.1 gives the proportion of the rejection of the null hypothesis at the 5% level for both $\{x_t\}$ and $\{y_t\}$ for 1000 replications of each DGP. Asymptotic critical values of $\widehat{\eta}$ and $\tau$ at the 5% level are given in table.2.

Table 1. Proportion of rejecting of the null hypothesis of stationarity

| | $KPSS$ test: $\widehat{\eta}$ | Complementary test: $\tau$ |
|---|---|---|
| $DGP_{H_0}: \{x_t\}$ | 0.010 | 0.010 |
| $DGP_{H_1}: \{y_t\}$ | 0.013 | 0.980 |

The table clearly points out the failure of the KPSS test to detect the nonstationarity characteristic of $\{y_t\}$. The KPSS test indicates similar behavior for both DGP while the complementary test $\tau$ rejects without ambiguousness the null hypothesis for the DGP$_{H_1}$. This means that if the null hypothesis is not rejected by the KPSS test, the data are not necessary stationary. A shift variance can be present in the process. So, the complementary test $\tau$ can be applied to be sure that the process is variance constancy.

# 4 Application to the US Dollar/Euro exchange rate

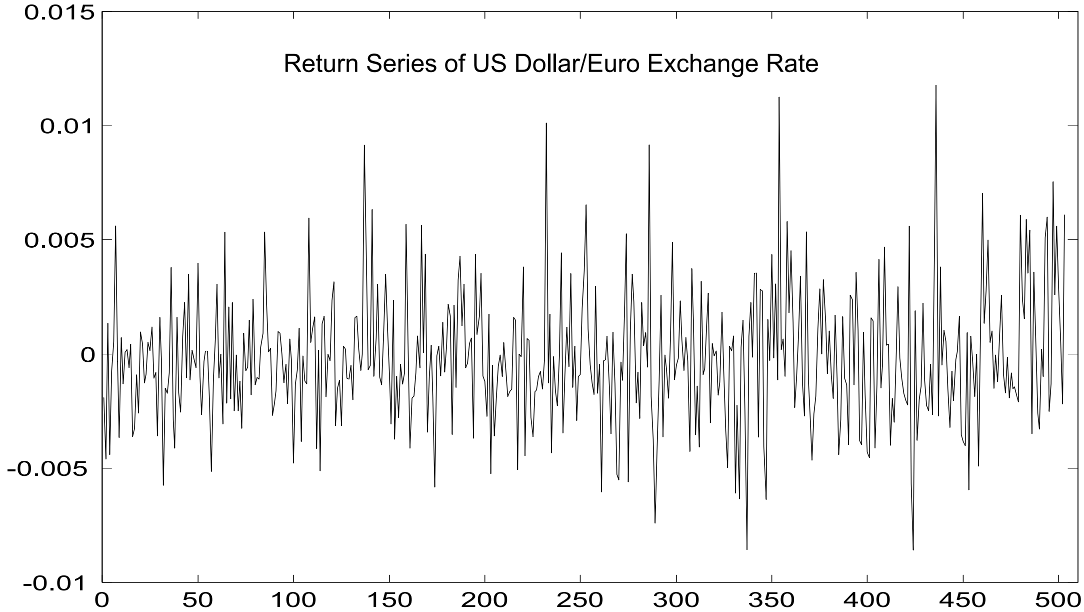
The problem of the covariance stationarity for financial data was investigated by many authors. The covariance stationarity is an important hypothesis since traditional models of financial data require such hypothesis. Thus, the unconditional volatility of traditional ARCH model family is supposed stationary (constant), the long-memory concept requires the covariance stationarity. Loretan and Phillips (1994) concluded for a rejection of the null of the constancy of the unconditional volatility for a set of financial data. Starica and Mikosch (1999) observed that the nonstationarity of the unconditional volatility can explain many stylized facts always observed in financial data. We consider the data $X_t = \log(S_t/S_{t-1})$ where $S_t$ is the daily US Dollar/Euro exchange rate from 04.01.1999 to 29.12.2000, (yielding $T = 504$ observations). Note that $X_t = \log(S_t/S_{t-1})$ is the return series. The results of the KPSS test applied to $X_t$ are given in Table 3. The results clearly show the non-rejection of the null hypothesis by the KPSS test which means that the data $X_t$ do not contain a unit root but as noted previously the data are not necessary covariance stationary. Unconditional variance shift can be masked in the data. So, we apply the complementary test $\tau$ to $X_t$ and the results are reported in Table 3. They indicate that the null hypothesis of the constancy of the unconditional variance is rejected by the statistic $\tau$. This result confirms the conclusion obtained by Loretan and Phillips (1994). Similar conclusions are obtained by Ahamada and Boutahar (2002). So the description of the series $X_t$ by the traditional approaches requiring the stationary hypothesis (as long-memory, ARCH stationary model...) is not necessarily adapted. Nonstationary tools can be applied as GARCH model with time varying parameters (Starica and Mikosch; 1999).

Table 2. Asymptotic critical values of $\widehat{\eta}$ and $\tau$ at the 5% level

| $\widehat{\eta}$ | $\tau$ |
|---|---|
| 0.463 | 1.360 |

Table 3. KPSS and complementary statistic applied to $X_t = \log(S_t/S_{t-1})$

| $\widehat{\eta}$ | $\tau$ |
|---|---|
| 0.1997 | 2.1853628 |




## 5 Conclusion

In this paper we showed by simulation experiments the failure of the KPSS test to detect the non stationarity explained by variance shift. A complementery test is then proposed. It's not compete the KPSS test but it can be used to detect a possible variance shift when the null is not rejected by the KPSS test. An applicaion of both tests to the US Dollar/Euro exchange rate is given. While the KPSS test does not reject the null of stationarity, the complementery test reject the null of the constancy of the unconditionaly variance. That confirm the already results obtained by Loretan an Philips(1994), Starica and Micosh(1999). The data can be descripted in this case by a non stationary model like GARCH model with time varying parametrs as it is proposed by Starica and Micosh(1999).

## References

[1] Ahamada, I. and Boutaha, M.(2002). Tests for covariance stationarity and white noise with an application to euro/US dollar exchange rate. An approach based on the evolutionary spectral density, Economics Letters, 77, 177-186.

[2] Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. and Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of unit root, Journal of Econometrics, 54, 159-178.

[3] Loretan, M. and Phillips, P. C.(1994). Testing the covariance stationarity of heavy-tailed time series: An overview of the theory with applications to several financial data sets, Journal of Empirical Finance, 1, 211-248.

[4] Inclan, C. and Tiao, C. G. (1994). Use of cumulative sums of squares for retrospective detection of changes of variance, Journal of the American Statiscal Association, vol. 89, n° 427.

[5] Starica, C. and Mikosch,T.(1999). Change of structure in financial time series, long range dependence and the GARCH model, Departement of mathematical statistics, Chalmers University of Technology, Gothenbourg, Sweden. *www.math.chalmers.se/starica.*