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Nonsuperneutrality of Money in the Sidrauski Model with Heterogenous Agents

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Abstract

Superneutrality is demonstrated to no longer hold in the Sidrauski model as soon as agents are heterogenous with regard to their productivity. However, quantitative effects of inflation on the capital stock are found to be rather small.

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1 Introduction

As a well-known result, money is superneutral in the Sidrauski (1967) money-in-the-utility model. The steady-state values of capital, consumption, and output are all independent of the inflation rate and changes of the rate of money growth have no real effects. If the labor supply is endogenous, money is still superneutral if the utility is separable in utility from consumption and leisure on the one hand and utility from money on the other hand. In the present paper, we extend the analysis of the Sidrauski model and consider heterogenous households rather than one representative household.[1] Heterogeneity is introduced in the form of stochastic idiosyncratic labor productivity. We show that the result of superneutrality of money does not hold any longer in this case.

2 The Sidrauski Model with Heterogenous Agents

As the representative-agent Sidrauski model is well-known, we keep the exposition as brief as possible. For an extensive description of the model, the reader is referred to Sidrauski (1967) or Walsh (1998, Ch. 2.3).

2.1 Households

The household $j \in [0,1]$ lives infinitely and is characterized by her productivity ϵ_t^j and her wealth a_t^j in period t. Wealth a_t^j is composed of capital k_t^j and real money $m_t^j \equiv \frac{M_t^j}{P_t}$, where M_t^j and P_t denote the nominal money holdings of agent j and the aggregate price level, respectively. Individual productivity ϵ_t^j is assumed to follow a first-order Markov chain with conditional probabilities given by:

$$\Gamma(\epsilon'|\epsilon) = Pr\left\{\epsilon_{t+1} = \epsilon'|\epsilon_t = \epsilon\right\}, \tag{1}$$

where $\epsilon, \epsilon' \in \mathcal{E} = \{\epsilon_1, \ldots, \epsilon_n\}$.

The household faces a budget constraint. She receives income from labor l_t^j, capital k_t^j, and lump-sum transfers tr_t which she either consumes at the amount of c_t^j or accumulates in the form of capital or money:

$$k_{t+1}^j + (1+\pi_{t+1})m_{t+1} = (1+r)k_t^j + m_t + w_t\epsilon_t^j l_t^j + tr_t - c_t^j, \tag{2}$$

[1] The study of heterogenous-agent economies has received increasing attention in the recent literature. A survey of computable general equilibrium studies that analyze the distribution of income and wealth is provided by Quadrini and Ríos-Rull (1997).

where $\pi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$, r_t, and w_t denote the inflation rate, the real interest rate, and the wage rate in period t.

The household j maximizes life-time utility:

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t^j, m_t^j, 1 - l_t^j) \tag{3}$$

subject to (2). The functional form of instantaneous utility $u(.)$ is chosen from the following three cases:

$$u(c, m, 1-l) = \begin{cases} \gamma \ln c + (1-\gamma) \ln m & \text{case I} \\ \frac{(c^\gamma m^{1-\gamma})^{1-\sigma}}{1-\sigma} & \text{case II} \\ \gamma \ln c + (1-\gamma) \ln m + \eta_0 \frac{(1-l)^{1-\eta_1}}{1-\eta_1} & \text{case III} \end{cases} \tag{4}$$

In cases I and II, the labor supply is exogenous, $l = \bar{l}$. In all three cases, money is superneutral in the representative-agent Sidrauski model.

2.2 Production

Firms are of measure one and produce output with effective labor N and capital K. Let $l(k, m, \epsilon)$ and $\phi_t(k, m, \epsilon)$ denote the labor supply and the period-t measure of the household with wealth $a = k + m$ and idiosyncratic productivity ϵ, respectively. Effective labor N_t is given by:

$$N_t = \sum_{\epsilon \in \mathcal{E}} \int_k \int_m l(k, m, \epsilon) \cdot \epsilon \cdot \phi_t(k, m, \epsilon) \; dm \; dk. \tag{5}$$

Effective labor N is paid the wage w. Capital K is hired at rate r and depreciates at rate δ. Production Y is characterized by constant returns to scale and assumed to be Cobb-Douglas:

$$Y_t = F(K_t, N_t) = K_t^\alpha N_t^{1-\alpha}. \tag{6}$$

In a factor market equilibrium, factors are rewarded with their marginal product:

$$w_t = (1-\alpha) K_t^\alpha N_t^{-\alpha}, \tag{7}$$
$$r_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha} - \delta. \tag{8}$$

2.3 Monetary Authority

Nominal money grows at the exogenous rate θ_t:

$$\frac{M_t - M_{t-1}}{M_{t-1}} = \theta_t. \tag{9}$$

The seignorage is transferred lump-sum to the households:

$$tr_t = \frac{M_t - M_{t-1}}{P_t}. \tag{10}$$

2.4 Stationary Equilibrium

The concept of equilibrium applied in this paper uses a recursive representation of the consumer's problem following Stokey et al. (1989). We will concentrate on the analysis of a stationary equilibrium with constant money growth $\theta_t = \theta$ that is characterized by a stationary distribution of wealth and constant aggregate capital stock and effective labor. As a consequence, factor prices and inflation are also constant. In the following, we drop time subscripts.

The household's state variable is denoted by $x = (k, m, \epsilon) \in \mathcal{X}$. Let $V(k, m, \epsilon)$ be the value of the objective function of a household characterized by wealth $a = k + m$ and productivity ϵ. $V(k, m, \epsilon)$ is defined as the solution to the dynamic program:

$$V(k, m, \epsilon) = \max_{c,l,k',m'} \left[u(c, m, 1 - l) + \beta E \left\{ V(k', m', \epsilon') \right\} \right], \tag{11}$$

subject to the budget constraint (2), the monetary policy θ and the transition matrix $\Gamma(\epsilon'|\epsilon)$. k', m', and ϵ' denote next-period capital stock, money, and productivity, respectively.

Let $(\mathcal{X}, \mathcal{B}, \psi)$ be a probability space where $\mathcal{B}$ is a suitable σ-algebra on $\mathcal{X}$ and ϕ a probability measure. We will define a stationary equilibrium for given government monetary policy θ and stationary measure ϕ.

Definition

A stationary equilibrium for a given set of government policy parameters is a value function $V(k, m, \epsilon)$, individual policy rules $c(k, m, \epsilon)$, $l(k, m, \epsilon)$, $k'(k, m, \epsilon)$, and $m'(k, m, \epsilon)$ for consumption, labor supply, and next-period capital and real money balances, respectively, a time-invariant distribution ϕ of the state variable $x = (k, m, \epsilon) \in \mathcal{X}$, time-invariant relative prices of labor and capital $\{w, r\}$, and a vector of aggregates K, N such that:

1. Factor inputs, aggregate consumption C, and real money M/P are obtained aggregating over households:

$$K = \sum_{\epsilon \in \mathcal{E}} \int_k \int_m k \cdot \phi(k, m, \epsilon) \, dm \, dk, \tag{12}$$

$$N = \sum_{\epsilon \in \mathcal{E}} \int_k \int_m l(k, m, \epsilon) \cdot \epsilon \cdot \phi(k, m, \epsilon) \, dm \, dk, \tag{13}$$

$$C = \sum_{\epsilon \in \mathcal{E}} \int_k \int_m c(k, m, \epsilon) \cdot \phi(k, m, \epsilon) \, dm \, dk, \tag{14}$$

$$\frac{M}{P} = \sum_{\epsilon \in \mathcal{E}} \int_k \int_m m \cdot \phi(k, m, \epsilon) \, dm \, dk. \tag{15}$$

2. $c(k, m, \epsilon)$, $l(k, m, \epsilon)$, $k'(k, m, \epsilon)$, and $m'(k, m, \epsilon)$ are optimal decision rules and solve the household decision problem described in (11).

3. Factor prices (7) and (8) are equal to the factors' marginal productivities, respectively.

4. The goods market clears:

$$F(K, N) + (1 - \delta)K = C + K' = C + K. \tag{16}$$

5. Seignorage tr is transferred lump-sum to households.

6. The measure of households is stationary:

$$\phi(B) = \sum_{\epsilon \in \mathcal{E}} \int_k \int_m 1_{(k'(k,m,\epsilon), m'(k,m,\epsilon), \epsilon') \in B} \cdot \Gamma(\epsilon' | \epsilon) \cdot \phi(k, m, \epsilon) \, dm \, dk \tag{17}$$

   for all $B \in \mathcal{B}$. 1_x is an index function that takes the value one if x is true and zero otherwise.

Since the household's decision problem is a finite-state, discounted dynamic program, an optimal stationary Markov solution to this problem always exists.

2.5 Calibration

In order to compute the quantitative effect of a change in the steady-state money growth rate on real variables, the model has to be calibrated. The model parameters are chosen with respect to the characteristics of the German economy during 1995-96.[2] Periods correspond to years. The number of productivities is set to $n = 5$ with $\mathcal{E} = \{0.2327, 0.4476, 0.7851, 1.0544, 1.7129\}$. The transition matrix is given by:

[2]If not stated otherwise, the parameters are taken from Heer/Trede (2003).

$$\pi(\epsilon'|\epsilon) = \begin{pmatrix} 0.3500 & 0.6500 & 0.0000 & 0.0000 & 0.0000 \\ 0.0800 & 0.6751 & 0.1702 & 0.0364 & 0.0383 \\ 0.0800 & 0.1651 & 0.5162 & 0.2003 & 0.0384 \\ 0.0800 & 0.0422 & 0.1995 & 0.5224 & 0.1559 \\ 0.0800 & 0.0371 & 0.0345 & 0.1606 & 0.6879 \end{pmatrix} \tag{18}$$

The parameters γ, η_0, and η_1 presented in table 1 are chosen in order to imply i) an average working time $\bar{l}$ equal to approximately 1/3, ii) a coefficient of variation of workers' labor supply equal to the empirical value of 0.385 in Germany during 1995-96 (in case III), and iii) a velocity of money $M1$ equal to the average value during 1995-96, $PY/M = 4.55$. The remaining parameters are set equal to $\sigma \in \{1, 2\}$, $\beta = 0.96$, $\alpha = 0.36$, and $\delta = 0.04$. Furthermore, we set the inelastic labor supply $\bar{l}$ equal to 0.33 in cases I and II. The computation of the stationary equilibrium is described in the appendix.

3 Results

Table 1 reports our results for a change of the money growth rate from 0% to 10% in the three cases considered. Notice that in all cases, money is no longer superneutral. For higher inflation, the capital stock increases. The reason is straightforward: Seignorage is transferred lump-sum to the households. The richer households hold higher real money balances than the poorer households and, therefore, pay a higher inflation tax, even though they all receive equal amounts of seignorage. As a consequence, the income of the wealth-rich households declines relative to the one of the poor households. As the former, however, have a lower propensity to save out of income (for precautionary reasons) than the latter, total savings increase. For $\sigma = 1$, aggregate capital K and output Y rise by 0.45% and 0.16% for a 10 percentage point increase of the inflation rate, respectively. This effect is even more pronounced for a higher curvature of the utility curve (case II with $\sigma = 2$), even though still quantitatively small. In addition, endogenous labor supply, at least if utility is additively separable in consumption, leisure, and money, does not have an economically significant effect (see case III).

4 Conclusion

The Sidrauski (1967) model has been used widely to study many diverse issues in monetary economics and is still one of the most prominent approaches to the theory of money demand besides the transaction costs approach as in Baumol (1952), Tobin (1956) or Clower (1989), the barter approach (Kiyotaki and Wright, 1989) or the use of money for intertemporal consumption smoothing (Samuelson, 1958). One important finding in this money-in-the-utility model is that money is superneutral in the representative-agent

Table 1: The money growth rate and real variables

θ	case	Calibration	K	M/P	Y	N	$\bar{l}$
0%	I	$\{\sigma, \gamma\} = \{1, 0.990\}$	2.902	0.1354	0.6200	0.2603	0.330
10%	I		2.915	0.0357	0.6210	0.2603	0.330
0%	II	$\{\sigma, \gamma\} = \{2, 0.9912\}$	3.245	0.1430	0.6455	0.2603	0.330
10%	II		3.263	0.0336	0.6468	0.2603	0.330
0%	III	$\{\sigma, \gamma, \eta_0, \eta_1\} = \{1, 0.991, 0.57, 2.70\}$	3.786	0.168	0.758	0.307	0.333
10%	III		3.810	0.0400	0.760	0.308	0.334

economy. We demonstrate that money is no longer superneutral in the presence of idiosyncratic productivity heterogeneity. Changes in the money growth rate are found to affect the capital stock. However, quantitative effects are small.

References

Baumol, W., 1952, The Transactions Demand for Cash, *Quarterly Journal of Economics*, vol. 67, 545-56.

Clower, R.W., 1967, A Reconsideration of the Microfoundations of Monetary Theory, *Western Economic Journal*, vol. 6, 1-9.

Erosa, A., and G. Ventura, 2002, On inflation as a regressive consumption tax, *Journal of Monetary Economics*, vol. 49, 761-95.

Heer, B., and M. Trede, 2003, Efficiency and distribution effects of a revenue-neutral income tax reform, *Journal of Macroeconomics*, vol. 25, 87-107.

Huggett, M., 1993, The risk-free rate in heterogenous-agent incomplete-insurance economies, *Journal of Economic Dynamics and Control*, vol. 17, 953-69.

Judd, K.L., 1998, *Numerical Methods in Economics*, MIT Press, Cambridge, MA.

Kiyotaki, N., and R. Wright, 1989, On Money as a Medium of Exchange, *Journal of Political Economy*, vol. 97, 927-54.

Krussell, P. and Smith, A.A. (1998), Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy 106*, 867-96.

Quadrini, V., and J.V. Ríos-Rull, 1997, Understanding the U.S. Distribution of Wealth, *Federal Reserve Bank of Minneapolis Quarterly Review*, vol. 21, 22-36.

Samuelson, P.A., 1958, An Exact Consumption-Loan Market Model of Interest with or without the Social Contrivance of Money, *Journal of Political Economy*, vol. 66, 467-82.

Sidrauski, M., 1967, Rational Choice and Patterns of Growth in a Monetary Economy, *American Economic Review*, vol. 57, 534-44.

Stokey, N., J.R. Lucas, and E.C. Prescott, 1989, *Recursive methods in economic dynamics*, Harvard University Press, Cambridge, Ma.

Tobin, J., 1956, The Interest Elasticity of the Transactions Demand for Cash, *The Review of Economics and Statistics*, vol. 38, 241-47.

Walsh, C.E., 1998, *Monetary theory and policy*, MIT Press, Cambridge, Ma.

Appendix

Computation

The solution algorithm is described by the following steps:

1. Make initial guesses of the aggregate capital stock K, aggregate effective labor N, and aggregate real money M/P.
2. Compute the values of w and r that solve the firm's Euler equations. Compute the transfers tr.
3. Compute the household's decision functions.
4. Compute the steady-state distribution of the state variable $\{k, m, \epsilon\}$.
5. Update K, N, and M/P, and return to step 1 until convergence.

In step 3, an optimization problem is to be solved. One possible solution method consists in the computation of the individual policy function as functions of the individual state space $\{\epsilon_t, k_t, m_t\}$. A much easier way to solve this problem, however, consists in a consideration of a different state space, as has been proposed by Krusell and Smith (1998) and as has also recently been applied by Erosa and Ventura (2002). For this reason, we reformulate the individual optimization problem. Let $\omega_t = k_t + (1+\pi)m_t$ denote current wealth. The optimization problem of the household can be separated into two individual optimization problems: i) the choice of next-period wealth ω_{t+1} and ii) the portfolio allocation problem, i.e. the allocation of ω_t on k_t and m_t, respectively.

Accordingly, the dynamic program problem (11) can also be stated in the individual state space $\{\omega_t, \epsilon_t, \epsilon_{t-1}\}$:

$$V(\omega_t, \epsilon_t, \epsilon_{t-1}) = \max_{c_t, \omega_{t+1}} \left[u(c_t, m_t, 1-l_t) + \beta E_t \left\{ V(\omega_{t+1}, \epsilon_{t+1}, \epsilon_t) \right\} \right], \tag{19}$$

subject to the budget constraint:

$$(1+r)k_t + m_t + w_t \epsilon_t l_t = c_t + \omega_{t+1}. \tag{20}$$

The capital stock k_t and the real money balances m_t are functions of ω_t and ϵ_{t-1} (ϵ_t is not known when the households decides upon k_t and m_t in period $t-1$). The solution is a function $\omega_{t+1} = g_\omega(\omega_t, \epsilon_t, \epsilon_{t-1})$.

The optimal portfolio of capital $k_t = g_k(\omega_t, \epsilon_{t-1})$ and money $m_t = g_m(\omega_t, \epsilon_{t-1})$ is obtained from the following problem:

$$(g_k(\omega_t, \epsilon_{t-1}), g_m(\omega_t, \epsilon_{t-1})) = \text{argmax}_{k,m} \beta E_{t-1} u(c_t, m_t, 1 - l_t), \tag{21}$$

subject to $\omega_t = k_t + (1 + \pi)m_t$ and $\omega_{t+1} = g_\omega(\omega_t, \epsilon_t, \epsilon_{t-1})$.

In our algorithm, we started with an initial guess for $\omega_{t+1} = g_\omega(\omega_t, \epsilon_t, \epsilon_{t-1})$. We then computed the portfolio allocation. In particular, for given ϵ_{t-1} and ω_t, the optimal capital stock k_t solves the Euler equation:

$$E_{t-1}\left\{(1 + r)u_c(c_t, m_t, 1 - l_t)\right\} = E_{t-1}\left\{\frac{u_c(c_t, m_t, 1 - l_t) + u_m(c_t, m_t, 1 - l_t)}{1 + \pi}\right\}, \tag{22}$$

subject to $m_t = (\omega_t - k_t)/(1+\pi)$ and the first-order condition of the household with respect to leisure in period t:

$$u_l(c_t, m_t, 1 - l_t) = u_c(c_t, m_t, 1 - l_t)w\epsilon_t. \tag{23}$$

The optimal k_t was computed with a nonlinear-equations solver.

Given the optimal portfolio allocation functions g_k and g_m as well as the labor supply $l(\omega_t, \epsilon_t)$, we solved the intertemporal optimization problem of the household. In particular, for every $\{\omega_t, \epsilon_t, \epsilon_{t-1}\}$, we solved:

$$u_c(c_t, m_t, 1 - l_t) = \beta E_t\left\{(1 + r)u_c(c_{t+1}, m_{t+1}, 1 - l_{t+1})\right\}, \tag{24}$$

subject to $k_t = g_k(\omega_t, \epsilon_{t-1})$, $k_{t+1} = g_k(\omega_{t+1}, \epsilon_t)$, $m_t = g_m(\omega_t, \epsilon_{t-1})$, and $m_{t+1} = g_m(\omega_{t+1}, \epsilon_t)$. The solution is given by $\omega_{t+1} = g_\omega(\omega_t, \epsilon_t, \epsilon_{t-1})$. We then updated g_ω and continued to compute g_ω, g_k, and g_m until they converged.

Finally, in step 4, the stationary distribution is computed as described in Huggett (1993).

Accuracy of the Computation

The basic criterion applied in the CGE literature in order to check for accuracy of the computation is the violation of the Euler equations:[3]

[3]See, e.g., Judd (1998).

$$
\begin{aligned}
R_1(k,m,\epsilon) &= 1 - \frac{u_c(c_t, m_t, 1-l_t)w\epsilon}{u_{1-l}(c_t, m_t, 1-l_t)}, && (25)\\
R_2(k,m,\epsilon) &= 1 - \beta E_t \left\{ \frac{u_c(c_{t+1}, m_{t+1}, 1-l_{t+1})(1+r)}{u_c(c_t, m_t, 1-l_t)} \right\}, && (26)\\
R_3(k,m,\epsilon) &= 1 - \frac{\beta}{u_c(c_t, m_t, 1-l_t)} E_t \left\{ \frac{u_c(c_{t+1}, m_{t+1}, 1-l_{t+1}) + u_m(c_{t+1}, m_{t+1}, 1-l_{t+1})}{(1+\pi)} \right\} && (27)
\end{aligned}
$$

As our algorithm, however, is designed to solve these equations, the accuracy, of course, depends on the accuracy of our non-linear equations solver, which is equal to 10^{-8}.

Accuracy of the Calibration

In case I-III, the velocity of money PY/M is equal to 4.58, 4.51, and 4.51, respectively. In case III, the variational coefficient of labor supply amounts to 0.381.