E C O N O M I C S   B U L L E T I N

When envy helps explain coordination

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Abstract

This paper identifies a class of symmetric coordination games in which the presence of envious people helps players to coordinate on a particular strict Nash equilibrium. In these games, the selected equilibrium is always risk−dominant. We also find that envious preferences are evolutionary stable when they lead to Pareto−efficiency.

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1 Introduction

Coordination games are representative of many interesting economic situations, including for instance macroeconomic coordination failure, cooperation in teams and arms races.[1] However, due to the presence of strategic uncertainty, these games exhibit multiple (strict) Nash equilibria which leaves economic analysis without sharp predictions.

This equilibrium selection problem has been addressed in several and various ways, using the *risk-dominance* criterion (Harsanyi and Selten (1988)), *global games* (Carlsson and van Damme (1993)) and *evolutionary processes*.[2] Nevertheless, game theorists have still not reached a consensus regarding the predicted issue. In particular, in coordination games with Pareto-ranked Nash equilibria Harsanyi and Selten consider risk-dominance to be irrelevant to equilibrium selection, partly because Pareto-optimality and risk dominance may diverge and partly because the authors argue that payoff dominance is a crucial aspect of their intuition. On the other hand, stochastic evolutionary models favor either risk or Pareto dominance depending on the adaptive rule (imitation or best-reply) as well as the number of rounds of matching per period.[3]

The present paper considers another approach to investigating equilibrium selection problem in symmetric coordination games with Pareto-ranked Nash equilibria. In a common knowledge framework, it is assumed that players may experience *envy*. This negative emotion is incorporated into the framework by constructing a *psychological game* in the sense of Geanakoplos et al. (1989). In such a game utility is a function of a player's own payoff and his relative payoff. An envious player suffers if his opponent earns a higher payoff and has some pleasure in the opposite situation. Thus, players care about their relative position in a given outcome. Motivations for studying envy in coordination games rely on several empirical and theoretical studies which have emphasized the importance of envy and spitefulness as a motive for Pareto-efficiency rejection.[4]

We identify a class of symmetric *coordination* games for which the presence of *envious* people generates coordination on a particular strict Nash equilibrium. This happens when the magnitude of asymmetry in payoff in *out-of-coordination* outcomes is sufficiently large. We also establish a link with risk dominance, showing that the equilibrium selected by envious players is always *risk-dominant*, but may be either *Pareto*-efficient or not.

The model predicts Pareto-gains rejection in some classes of symmetric coordination games. In these games, the psychological gain of deviation from the Pareto-inferior equilibrium overcomes its cost. This is because the deviating player obtains a higher status (relative to his opponent) in the out-of-coordination outcome than in the Pareto-inferior equilibrium. Moreover, and perhaps surprisingly, the reverse also holds in some other classes of coordination games, meaning that Pareto-optimality may be

[1]For more details see Milgrom and Roberts (1990) as well as Cooper and John (1988).

[2]See Kim (1996) for a survey.

[3]Wei-Torng (2002) provides an interesting comparison of the Kandori *et al.* (1993) and Robson and Vega-Redondo (1996) models.

[4]see Beckman *et al.* (2002), Cason and Muy (2002) and Mui (1995).

sustained by envious players. We then investigate the *evolutionary* stability of envious preferences in coordination games, and find that they are evolutionary stable when they favor Pareto-efficiency.

The next section provides the general analysis. Evolutionary stability of envious preferences is examined in Section 3, and Section 4 concludes the paper.

2 Symmetric coordination games with envious agents

Consider two players, $i = 1,2$, who interact in the 2×2 symmetric game G described in the table below. Let $s_i \in \{A,B\}$, π_i and u_i respectively denote player i's strategy, *material* payoff and utility.

	A	B
A	a,a	b,c
B	c,b	d,d

Assuming that $a > c$ and $d > b$, G represents a coordination game with two strict Nash equilibria (A,A) and (B,B). In strict Nash equilibria, players have no alternative best reply so that they are particularly robust (for instance, they are subgame-perfect). Coordination games describe perfectly contexts in which game theory is confronted with an equilibrium selection problem.[5] For our purpose, we consider situations in which Nash-equilibria are Pareto-ranked. This is obtained by letting $a > d$, so that (A,A) Pareto dominates (B,B).

Beside Pareto-dominance, *risk-dominance* is another solution concept proposed to resolve the multiplicity problem of Nash equilibria.[6] Equilibrium (A,A) is said *risk dominant* if $(a-c)^2 > (d-b)^2$. Similarly, (B,B) risk dominates (A,A) if the reverse inequality holds.

Incorporating *envy* into the analysis amounts to constructing a *psychological game* as defined in Geanakoplos et al. (1989). In this game, u_i is not only governed by material payoff π_i but also integrates opponent's payoff π_j. Concerning envy, such *subjective* preferences may be simply defined as follows.[7]

$$u_i = \pi_i + \alpha_i(\pi_i - \pi_j). \tag{1}$$

It is assumed that $0 \leq \alpha_i < 1$ meaning that players are not too envious, in particular none has a higher regard for his opponents than for himself. For $\alpha_i = 0$, players have selfish preferences and only maximize their material payoffs. The (psychological) Nash equilibrium of G, denoted by (s_1^*, s_2^*), is solution of the pair of programs

$$s_1^* \in \ arg\max_{s_1} \ u_1(s_1, s_2^*), \ s_2^* \in \ arg\max_{s_2} \ u_2(s_2, s_1^*),$$

in which each player seeks to maximize his subjective preferences, taking the strategy of the other as given.

[5] See Section 1.

[6] See Harsanyi and Selten (1988).

[7] See Bolle (2000), or Kirchsteiger (1994) or Bethwaite and Tompkinson (1996).

The main objective here is to show that, in presence of envy, i.e., for $0 < \alpha_i < 1$, one may restrict the set of Nash equilibria in some classes of coordination games. This is stated in the following theorem.

Theorem 1 *Consider any symmetric* 2×2 *coordination games with Pareto-ranked Nash equilibria. If there is some* $b > c$ *so that* $(d-b)/(b-c) < \alpha_i$*, then* (A,A) *is the unique (psychological) Nash equilibrium of the game. On the other hand,* (B,B) *constitutes the unique Nash outcome if there is some* $b < c$ *so that* $(a-c)/(c-b) < \alpha_i$*.*

Proof. Let $b > c$ and consider $u_i(A,B) > u_i(B,B)$. This inequality holds for $b + \alpha_i(b-c) > d$, that is when

$$0 < \frac{d-b}{b-c} < \alpha_i.$$

In that case, player i chooses A when player j's strategy is B, for all $i \neq j$. On the other hand, $b > c$ does not prevent player i from playing strategy A when his opponent plays A (as $u_i(B,A) < u_i(A,A)$). Thus, $\forall \alpha_j$ and $j \neq i$, player j's best reply is A. As a result, (A,A) is the only Nash equilibrium of the game.

Consider now $b < c$. In that case, player $i = 1,2$ has no (psychological) incentive to play A against B, but may experience sufficient envy to play B against A. This happens when $u_i(B,A) > u_i(A,A)$, that is when

$$0 < \frac{a-c}{c-b} < \alpha_i.$$

At the same time, $b < c$ does not involve $u_i(A,B) > u_i(B,B)$ and thus (B,B) is here the only Nash equilibrium of the game.

Finally, in games with $b = c$, envious preferences are equivalent to selfish ones (as payoffs of both players are equal in all possible outcomes), so that coordination games remain with two strict Nash equilibria. □

Theorem 1 identifies a class of coordination games in which the presence of envious agents generates coordination.[8] This depends on payoffs players can earn in out-of-coordination outcomes. In particular, these outcomes have to present some sufficient *asymmetries* in payoffs ($b < c$ or $b > c$) to generate coordination. Notice that this process does not necessary induce efficiency, as envy may lead to the Pareto-dominated Nash equilibrium.

Conditions in Theorem 1 inform us that coordination happens when the psychological *gain* of deviation, from a particular Nash equilibrium, overcomes the *cost* of this deviation. This does not mean that the *material* gain from deviation has to be higher and, in fact, it turns out to be lower in coordination games (by assumption). Thus, an envious player is willing to incur a cost provided that his strategy sufficiently degrades the payoff of his opponent. For instance, coordination on Pareto-efficiency requires that the gain of deviation from (B,B), $\alpha_i(b-c) > 0$, is higher than its cost $d-b$. The psychological gain comes from the fact that the player who deviates may

[8]Notice that it is sufficient that *one* player fulfills conditions in Theorem 1 to ensure this result, meaning that only one *envious* player is sufficient.

reach a higher status, relative to his opponent in outcome (A,B), even if he incurs a cost in term of material payoff. In that case A becomes a *dominant* strategy (under envious preferences), and thus (A,A) constitutes the only issue of the game. This mechanism is also at work when (B,B) represents the unique issue of the coordination game.

Notice that coordination through envy favors outcomes *equalizing* payoffs, which in some cases may reveal to be *fairer*[9] than coordination failures (A,B) or (B,A) (whose occurrence has a positive probability under selfish preferences). This is clearly true for (A,A) but may also hold when envy selects the Pareto-dominated equilibrium (B,B). This happens when $c < d$ since here both players earn a higher payoff in (B,B) than in (A,B) or (B,A).

Finally one can establish a connexion between theorem 1 and the risk-dominance criterion. This is stated in the following corollary.

Corollary 1 *In symmetric 2×2 coordination games with Pareto-ranked Nash equilibria, if a (psychological) Nash equilibrium constitutes the unique outcome of the game, then it is also risk-dominant.*

Proof. Assume that (A,A) is the unique psychological Nash equilibrium and (B,B) risk-dominates (A,A). From theorem 1, we know that $b > c$ so that $(d-b)/(b-c) < \alpha_i$. By definition of risk-dominance, we also have $a-c < d-b$ which contradicts $(d-b) < \alpha_i(b-c)$ as $a > b$ (recall that $a > d$ but $d > b$). The same reasoning applies when (B,B) is the unique psychological Nash equilibrium and (A,A) risk-dominates (B,B), which completes the proof. □

The intuition behind Corollary 1 is as follows. The occurrence of an unique equilibrium requires (as a necessary condition) some asymmetries in payoffs, $b < c$ or $b > c$ (Theorem 1). These asymmetries have to be sufficiently high to reduce the cost of deviation from the *discarded* Nash equilibrium, which amounts to say that it is less risky to deviate from this equilibrium. As a result, the *selected* Nash equilibrium has to be risk-dominant (under restrictions defined in Theorem 1).

3 The evolution of envious preferences

In the previous section, we have investigated when envious preferences could favor coordination assuming envy as given. To complete the analysis, one may ask what sustains the presence of such preferences in coordination games. In particular, the question is whether envy can prevail and dominate in a (polymorphic) population composed of both selfish and envious preferences.

One way for analyzing the latter question is to resort to the *indirect* evolutionary approach.[10] Here, selfish and envious preferences compete and evolve through an evolutionary process selecting in the long-run preferences giving higher expected success. In this framework, the common knowledge assumption is maintained and evolutionary stability applies to preferences not to strategies, as defined in the traditional evolutionary game theory.

[9] In the egalitarian sense.

[10] See for instance Bester and Güth (1998).

Let $R(\alpha,\beta)$ represent a player's success when he has the envious parameter α and his opponent has the parameter β. A preference parameter $\alpha^* \in [0,1]$ is said to be *evolutionary stable* if it satisfies
(1) $R(\alpha^*,\alpha^*) \geq R(\alpha,\alpha^*)\ \forall\alpha$,
(2) if $R(\alpha^*,\alpha^*) = R(\alpha,\alpha^*)$ for $\alpha \neq \alpha^*$, then $R(\alpha^*,\alpha) > R(\alpha,\alpha)$.
Let $k = (d-b)/(b-c)$ and consider two types of preferences, $0 \leq \underline{\alpha} < k$ and $k < \overline{\alpha} < 1$. Call these types respectively the non-envious and the envious types. We can then state the following result.

Theorem 2 *In coordination games where (A,A) is the unique psychological Nash equilibrium, $\overline{\alpha}$ is the unique evolutionary stable type of preferences.*

Proof. Let (A,A) be the unique psychological Nash equilibrium. By Theorem 1, $b > c$ so that $(d-b)/(b-c) < 1$ which implies that $R(\overline{\alpha},\overline{\alpha}) = a$. However, we also have $R(\underline{\alpha},\overline{\alpha}) = a$ since common knowledge ensures that a non-envious player ($\underline{\alpha}$) perfectly knows that his opponent (who is of the envious-type $\overline{\alpha}$) always play A.

It remains to show that $R(\overline{\alpha},\underline{\alpha}) > R(\underline{\alpha},\underline{\alpha})$. Due to uncertainty concerning the opponent's strategy when both players are of type $\underline{\alpha}$, coordination failure occurs with a non negative probability. Then, $R(\underline{\alpha},\underline{\alpha})$ has to be lower than $R(\overline{\alpha},\underline{\alpha})$, that is $R(\underline{\alpha},\underline{\alpha}) < a$, because
(1) $a > b,c,d$ and,
(2) $R(\underline{\alpha},\underline{\alpha})$ represents the expected payoff of a $\underline{\alpha}$-type when confronted to a large population of $\underline{\alpha}$-players, and then coordination failure will occur with a non-zero probability. □

This result indicates that the envious-type may prevail in the long-run in a population playing a coordination game in which Pareto-efficiency is the unique psychological Nash equilibrium. The reason why is the uncertainty non-envious types generate on coordination which, in turn, gives a positive probability to the occurrence of coordination failure and then limit success of these preferences. A similar result cannot be establish in coordination games where (B,B) is the unique psychological Nash equilibrium. To see why, observe that although $R(\overline{\alpha},\overline{\alpha}) = R(\underline{\alpha},\overline{\alpha}) = d$, one may have $R(\underline{\alpha},\underline{\alpha}) \geq d$ as $a > d$. As a result, a population of envious-players may be invaded by non-envious agents. Obviously, one could find a subclass of coordination games selecting (B,B) in which envy can survive. However, this would correspond to a very restricted subclass of games.

4 Conclusion

This paper has shown that incorporating envy into the analysis allows to solve the problem of multiple Nash equilibria in some classes of coordination games. This happens in games where the magnitude of asymmetry in payoff in *out-of-coordination* outcomes is sufficiently important to ensure that psychological gains from deviation overcomes its cost. One interesting result here is that the selected Nash equilibria is also risk-dominant.

For convenience we have restricted our attention to 2×2 symmetric coordination games. It would be interesting to extend the analysis to *n*-person generalized coordination games. One of the main difficulties in carrying such an extension resides in the choice of the reference payoff to be used in the formalization of envious preferences: do agents compare themselves with all others or only with the average payoff of the group? One way to solve this selection problem would be to use the indirect evolutionary approach to evaluate the survival of envious preferences when both types of reference payoff are present in the population.

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