E C O N O M I C S   B U L L E T I N

The Egalitarian sharing rule in provision of public goods

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Abstract

In this note we consider a society that partitions itself into disjoint jurisdictions, each choosing a location of its public project and a taxation scheme to finance it. The set of public project is multi−dimensional, and their costs could vary from jurisdiction to jurisdiction. We impose two principles, egalitarianism, that requires the equalization of the total cost for all agents in the same jurisdiction, and efficiency, that implies the minimization of the aggregate total cost within jurisdiction. We show that these two principles always yield a core−stable partition but a Nash stable partition may fail to exist. We demonstrate moreover that stable partitions are not necessarily consecutive.

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1 Introduction

In this note we consider a model of jurisdiction formation, where the entire population has to be partitioned into several jurisdictions, each deciding on the public project. Whereas the existing literature deals with unidimensional universe of public projects, we consider a set of feasible projects to be imbedded in the set of an arbitrary dimension. Agents are assumed to have Euclidean preferences over possible project locations, and, thus, can be identified by their peaks (best preferred locations) in the multi-dimensional space.

When a jurisdiction is formed and a public project is selected, the jurisdiction must choose the way to distribute the project cost among its members. Part of the literature (Le Breton and Weber (2003), (2004), Haimanko et al. (2004a), Le Breton et al. (2004)) assumes the transferable utility framework with the unrestricted set of redistribution schemes within a jurisdiction. However, in many situations the degree of freedom in selecting a taxation scheme and degree of compensation to disadvantaged agents could be limited by customs, law, feasibility or variety of other constraints. Thus, a large number of papers (Alesina and Spolaore (1997), Casella (2001), Jéhiel and Scotchmer (2001), Haimanko et al. (2004b)) consider an *equal share* taxation scheme where all members of jurisdiction make an equal monetary contribution This cost sharing mechanism has some appealing properties, such as simplicity and anonymity. However, it rules out any degree of equalization between agents who enjoy vastly different benefits from the public project.

In this note we consider the case where every jurisdiction applies *egalitarian rule* of full equalization, when the total burden of taxation cost and disutility from having a project different from agent's ideal choice is equally divided among the members of the jurisdiction. This rule implements the Rawlsian principle by equalizing the total costs of all agents and, therefore, minimizing the total cost of the most disadvantaged agent. Agents are not hold accountable for their preferences and the burden of total disutility is shifted to the entire jurisdiction.

This is the *hedonic* games framework (Banerjee et al. (2001), Bogomolnaia and Jackson (2002)), where once a coalition is formed, one can uniquely determine the (equal) payoff of all of its members. Thus, agents form well-defined preferences over possible coalitions they could join. This feature of the hedonic model allows us to consider not only traditional cooperative stability notions (like core), but also non-cooperative stability, when only one agent can migrate to another jurisdiction or create a new one. We show that whereas the core stable partitions always exist in our multi-dimensional framework, Nash stable partition may fail to exist even in the case where the set of projects is represented by a unidimensional space.

Further, the preceding literature, working with the unidimensional set of public projects, consistently reaffirms the consecutivity property of stable partitions. Whenever the set of core or Nash stable partitions is nonempty it consists of (or at least includes) consecutive partitions, where each jurisdiction is an interval. In our model though, the core can fail to contain consecutive partitions, and Nash stable partitions can be non-consecutive as well.

2 The Model

We consider a finite society $N = \{1, \ldots, n\}$ of agents. The set of public projects is given by the compact subset I of k-dimensional Euclidean space $\Re^k$. Each agent has Euclidean preferences over I, which allows us to identify an agent i with his ideal point $p^i \in I$. Slightly abusing the terminology, we will refer to p^i as the location of the agent i.

The society faces a task of partitioning itself into disjoint jurisdictions, where each jurisdiction selects a location of the public project and a taxation scheme to finance it. The cost of a public project in jurisdiction S is given by a positive value $g(S)$, which may depend on the size and composition of a jurisdiction. When the location of the project p and a taxation scheme within jurisdiction S are chosen, every member i of S incurs two types of cost: the monetary contribution towards provision of the public project and transportation cost $d\left(||p^i - p||\right)$, which is assumed to be continuous and increasing on $\Re$ with $d(0) = 0$. We assume a voluntary participation: the benefits from public projects exceed their costs. Given the location p of the public project, the aggregate cost of members of jurisdiction S will be the sum of the cost of the public project $g(S)$ and the aggregate transportation cost $D(S,p) = \sum_{i \in S} d\left(||p - p^i||\right)$.

The locational and redistributional choices within each jurisdiction are guided by two principles, *egalitarianism* and *efficiency*. Efficiency requires that the project location is chosen in such a way as to minimize $D(S,p)$. Let $D(S) = \min_{p \in I} D(S,p)$, and the location $p(S)$ (not necessarily uniquely defined) is determined by $D(S,p(S)) = D(S)$.

Egalitarianism demands that all members of S equally share the aggregated total cost (tax plus transportation cost) $g(S) + D(S)$. Thus, every member i of S will incur the same cost $c(S) = \frac{(g+D(S))}{|S|}$, where $|S|$ is the cardinality of jurisdiction S. Given that i's transportation cost is $d(||p^i - p(S)||)$, her tax share (that can actually be a subsidy) under the *egalitarian rule* is

$$\frac{(g(S) + D(S))}{|S|} - d\left(||p^i - p(S)||\right).$$

Efficiency and egalitarianism completely determine the choices for each potential jurisdiction. Thus, these requirements lead to hedonic cooperative coalition formation game, where once a jurisdiction is formed, the payoff or cost for each its member is uniquely determined. We will examine the existence of stable partitions in our hedonic game where all jurisdiction adopt the efficiency and the egalitarian rule. We consider two notions of stability:

Definition (I) Let $\pi = \{S_1, \ldots, S_K\}$ be a jurisdictional structure. We say that a jurisdiction $S \subset N$ *blocks* π if $c(S^i) > c(S)$ for all $i \in S$, where S^i is the jurisdiction in π that contains i. A jurisdictional structure π is called *core-stable* if no jurisdiction blocks π.

(II) A jurisdictional structure $\pi = \{S_k\}_{k=1,\ldots,K}$ is *Nash stable* if

$$c(S^i) \leq g(\{i\}) \text{ and } c(S^i) \leq c(S_k \sqcup \{i\}) \text{ for every } i \in S \text{ and every } S_k \in \pi.$$

The first is the standard notion of the core. Nash stable jurisdictional structure can be viewed as a *free mobility equilibrium*: no agent has an incentive to move from his current jurisdiction to either "empty" jurisdiction or to another existing jurisdiction. Note also that the set of Nash stable jurisdictional structures is the set of pure Nash equilibria of the non-cooperative game, where each agent announces his "address" and all agents with the same address form a jurisdiction.

While the definition of Nash stability allows only for deviations by a single agent, an agent can move to a jurisdiction without the consent of its members. Thus, some or even all members of that jurisdiction could be worse off. This observation shows that there is no logical connection between the notions of core- and Nash stability. And, indeed, the next section indicates sharply different stability implications with regard to these two notions.

3 The Results

We now assume that the efficiency and the egalitarian rule are imposed throughout this section. Under the the egalitarian rule, all members of the same coalition bear the same total cost (or total disutility). Each coalition S is assigned a number $c(S)$ that represents the total cost of each of its members. Thus, all agents derive their preferences over coalitions from the common ordering by comparing jurisdictions on the basis of their assigned contributions. In hedonic games, such common ordering guarantees the existence of a stable partition. Our first proposition indeed states that, without any further qualification, stable jurisdictional structures always exist:

Proposition 1: There is always a core-stable jurisdictional structure.

Proof: Let us construct the partition π as follows. Take a coalition T^1 that minimizes the contribution of its members across all coalitions:

$$c(T^1) = \min_{S \subset N} c(S).$$

(obviously, the choice of T^1 as well as of all subsequent elements of π is not necessarily unique.)

If $T^1 = N$, we are done. Otherwise, choose $T^2 \subset N \setminus T^1$ to minimize the cost over all coalitions that have an empty intersection with T^1:

$$c(T^2) = \min_{S \subset N \setminus T^1} c(S).$$

If $T^1 \sqcup T^2 = N$, we are done. Otherwise, choose $T^3 \subset N \setminus (T^1 \sqcup T^2))$ to minimize the cost over all coalitions that have an empty intersection with $T^1 \sqcup T^2$:

$$c(T^3) = \min_{S \subset N \setminus (T^1 \sqcup T^2)} c(S).$$

By continuing this process, after at most n iterations, we obtain a partition $\pi = \{T^1, \dots, T^K\}$ of N. We show that π is stable. Indeed, consider an arbitrary coalition $T \subset N$, and let T^k be the first coalition in π that has a nonempty intersection with T, i.e., $k = \min\{j | T^j \bigcap T \neq \emptyset\}$. Then the choice of T^k implies that $c(T^k) \leq c(T)$. Thus, no agent that belongs to both T^k and T, is better off at T as compared to T^k, which means that T could not block π.$\square$[1]

It is worthwhile pointing out that if all values $c(S)$ are different, our construction yields a unique outcome π. It is easy to demonstrate that π is the unique core stable partition. Indeed, T^1 has to be a part of every stable partition, otherwise this very coalition will block the latter one; given that, T^2 must belong to it as well. By extending the argument, we obtain the uniqueness of the core stable partition in this case.

While we can guarantee core stability in the egalitarian game, such general statement does not hold for Nash stability:

Proposition 2: A Nash stable jurisdictional structure may fail to exist even if the space of public projects is unidimensional.

Proof: Let space of public projects I be given by the unidimensional interval $[0, 4.1]$ and consider a society with six agents, located at points $p^1 = 0$, $p^2 = p^3 = 1.9$, $p^4 = p^5 = p^6 = 4.1$. The transportation costs are linear, i.e., $d(|p, p'|) = |p - p'|$ for all $p, p' \in I$, and project costs $g(S)$ be equal to 1 for all jurisdictions S. We will show that under the Egalitarian rule, this society does not admit a Nash stable partition. Suppose, in negation, that π is a Nash stable partition. Denote $T = \{4, 5, 6\}$. It is easy to verify that if there is $S \in \pi$ that contains one or two agents from T in addition to some of agents $1, 2, 3$, then $c(S) > 1$. Thus, if agents $4, 5, 6$ are separated in at least two groups, they cannot be joined by other agents, implying that the members of T should belong to the same jurisdiction in a Nash stable partition.

[1]Since the egalitarian game satisfies the *common ranking* property of Farrell and Scotchmer (1988), our result could be derived from theirs. However, we opted here to offer a direct proof.

Since the members' cost exceeds one in every jurisdiction with more than four members, and agent 1 contributes strictly more than one in all multi-agent coalitions, except $\{1,2,3\}$, it follows that only candidates for Nash stable jurisdiction structures are:
$\{\{1,2,3\},T\}$, $\{\{1\},\{2\},\{3\}\sqcup T\}$, $\{\{1\},\{3\},\{2\}\sqcup T\}$, $\{\{1\},\{2,3\},T\}$.

Consider the following cycle
$\{\{1,2,3\},T\} \to \{\{1,2\},\{3\}\sqcup T\} \to \{\{1\},\{2\},\{3\}\sqcup T\} \to \{\{1\},\{2,3\},T\} \to \{\{1,2,3\},T\}$, in which every partition is obtained from the previous one by moving one agent to a jurisdiction that offers her a lower cost. Thus, no partition in the cycle is Nash stable, and due to the symmetry between agents 2 and 3, it rules out all candidates to constitute a Nash stable partition. $\square$

Let us notice that the only core-stable partition in this example is $\{\{1\},\{2,3\},T\}$.

One could expect that stable partitions would consist of *connected* jurisdictions. Recall that if the projects' space is unidimensional, the connectedness is reduced to *consecutivity*: whenever agents i and j, with $p^i < p^j$, belong to the same jurisdiction S, all agents k with $p^i < p^k < p^j$ should also be members of S. Unlike in the existing literature (e.g, Greenberg-Weber (1986)), this property is not generally satisfied in our framework. In the next two examples we consider the unidimensional set of projects $I = [0,4]$, the project cost $g(S) = 1$ for all S, and linear transportation cost: $d(|p,p'|) = |p - p'|$

Proposition 3: The set of consecutive core stable partitions could be empty.

Proof: Let $I = [0, 0.8]$. Consider 5 agents whose locations are given by $p^1 = 0$, $p^2 = p^3 = p^4 = 0.4$ and $p^5 = 0.8$. Indeed, The jurisdiction $\{2,3,4\}$ is the best choice for its members (in any other jurisdiction, either there are at most three agents, or agents pay 0.35, 0.36, or 0.45, so that total cost is always more then $\frac{1}{3}$). Hence, any core stable partition should contain $\{2,3,4\}$. Then two remaining agents 1 and 5 will form the second coalition, that results in the only core stable, but, obviously, nonconsecutive partition $(\{1,5\},\{2,3,4\})$. $\square$

Proposition 4: There is a nonconsecutive Nash stable partition.

Proof: Let $I = [0, 3.4]$. Consider 7 agents, whose locations are given by $p^1 = 0$, $p^2 = p^3 = 1.7$, and $p^4 = p^5 = p^6 = p^7 = 3.4$. We will show that the nonconsecutive partition (S,T), where $S = \{1,4,5,6,7\}$, $T = \{2,3\}$ is Nash stable.

Indeed, no agent $i \in S$ would benefit by joining T, since $c(T \cup \{i\}) = 0.9 > c(S) = 0.88$. Moreover, no agent $j \in T$ would benefit by joining S, since $c(S \cup \{j\}) = 1.02 > c(T) = 0.5$.

Finally, since both values, $c(S)$ and $c(T)$ are less than 1, no player would leave her current jurisdiction to stay alone. $\square$

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