ECONOMICS BULLETIN

On bribing and balking in a simple queuing model of resource allocation

Amitrajeet Batabyal
Department of Economics, Rochester Institute of Technology

Abstract

We analyze two versions of a simple queuing model of resource allocation in which an impatient citizen of a transition economy or a developing country wishes to purchase a scarce good, namely, bread. In the first version of the model, our citizen must pay a bribe to obtain bread immediately from a government shop. In the second version of the model, when the government shop is too crowded, our citizen refuses to join the crowd. In other words, he balks and goes instead to a private shop to purchase bread. In this setting, we study three questions from the standpoint of resource allocation. First, in the presence of bribery, what is the expected monetary benefit per citizen to a corrupt government official? Second, in the presence of balking, what proportion of all citizens eventually get bread? Finally, and once again in the presence of balking, what is the expected amount of time a citizen spends waiting to get bread?

Batabyal thanks the Associate Editor Josef Brada and an anonymous referee for their helpful comments on a previous version of this paper. In addition, he acknowledges financial support from the Gosnell endowment at RIT. The usual disclaimer applies.
Citation: Batabyal, Amitrajeet, (2005) "On bribing and balking in a simple queuing model of resource allocation." *Economics Bulletin*, Vol. 16, No. 1 pp. 1−10
Submitted: February 7, 2005. **Accepted:** July 14, 2005.
URL: http://www.economicsbulletin.com/2005/volume16/EB−05P30001A.pdf

1. Introduction

It is quite common in transition economies and in many developing countries to allocate goods that are in short supply and goods deemed essential for human survival to citizens by means of queuing mechanisms. Examples of goods that are often allocated in this manner include bread,[1] rice,[2] and groundwater.[3] An essential aspect of these goods allocation mechanisms is that they involve waiting in line by citizens. Put differently, as a matter of fact, citizens have to wait in queue to purchase the good that is being allocated by a government shop.

In addition to government shops, it is often the case that a good in short supply, being allocated to citizens by means of a queuing mechanism at a government shop, can also be purchased—though often at a higher price—at one or more private shops. Here are two examples of this phenomenon. As the *Belarussian Chronicle* (Anonymous 1998) has pointed out, in Minsk, as recently as 1998, citizens frequently had to wait in long queues to buy eggs and other food products from government shops. However, Belarussians who were both willing and able to pay between three and eight times the mandated government price could purchase all kinds of food products in private shops. As a second example, consider the case of Cuba. According to the *Economist* (Anonymous 2003), citizens in Cuba can buy basic foods in "peso shops" or, if they have the means, they can purchase higher quality but usually more expensive food items in the so called "dollar shops."

These two examples point to the fact that in many countries, government and private shops, often selling the same goods, frequently coexist. Given this state of affairs, it is reasonable to focus attention on the properties of queue based resource allocation mechanisms in the presence of government and private shops. In this regard, a number of specific questions come to mind from the standpoint of resource allocation. First, in the presence of bribery, what is the expected monetary benefit per citizen to a corrupt government official? Second, in the presence of balking, what proportion of all citizens eventually get the good under study? Finally, and once again in the presence of balking, what is the expected amount of time a customer spends waiting to get the good?

Several papers in the economics literature have analyzed questions related to the distribution of scarce goods[4] by queuing, by rationing, and the related issue of corruption. Sah (1987) and Shleifer and Vishny (1993) are representative. Sah (1987) formally analyzes the pros and cons of resource

[1] See Goodman (1995) for a description of Romania.

[2] See Gunawardana (2000) for a discussion of Sri Lanka.

[3] See Wood (1999) for an account of the state of Bihar in India.

[4] The classic work on shortages in socialist systems is Kornai (1980). In this book Kornai showed that chronic shortages are not necessarily the outcome of wrong prices or incorrect planning but instead a systemic feature of socialist economies.

allocation by means of queues, rations, and the market, for poor and wealthy citizens. Shleifer and Vishny (1993) study corruption. Although both these papers are interesting, neither paper addresses the three questions that we have identified in the previous paragraph. In addition, neither of these two papers make use of queuing theory even though the issues being analyzed in both these papers are very amenable to analysis using queuing theoretic methods.

Lui (1985) and more recently Batabyal and Nijkamp (2004) and Batabyal and Yoo (2004, 2005) have used queuing models to investigate the nexuses between bribery and corruption. Although all four of these papers have studied aspects of the public provision of goods by means of queuing mechanisms, these papers have *not* concentrated on the properties of queue based resource allocation mechanisms in the presence of government and private shops. This (non-exhaustive) review of the pertinent literature leads to the following conclusion. Despite the obvious relevance of the three questions that we stated a little while ago, to the best of our knowledge, these questions have *not* been analyzed previously in the economics literature. Therefore, the purpose of this paper is to provide a queuing theoretic perspective on these three questions.

The remainder of this paper is organized as follows. Section 2 describes two versions of a Markovian queuing model.[5] The first version of this model is discussed in greater detail in section 3 and this version contains an explicit accounting of *bribery*. The particular question of interest in this section is this: In the presence of bribery, what is the expected monetary benefit per citizen to the corrupt government official? The second version of the model is elaborated on in section 4 and this version models the phenomenon of *balking*. Two specific questions are analyzed in this section: First, what proportion of all citizens eventually get the good under study? Second, what is the expected amount of time a citizen spends waiting to get the good? Section 5 concludes and then discusses two ways in which the research of this paper might be extended.

2. The Queuing Model

Consider a particular region within a transition economy or a developing country. An essential commodity, which we suppose without loss of generality is bread, is in short supply. An impatient citizen of this region can obtain bread in one of two ways. In the first version of our queuing model, this citizen can either get in queue at the government shop and wait in line until it is his turn to buy one unit of this bread at a predetermined price[6] from a corrupt public official. However, if this impatient citizen wants his bread immediately, that is without waiting in line, then he must pay a *bribe* to the public official. The nature of this bribe is described in greater detail in section 3.

In the second version of our model, we suppose that our impatient citizen encounters too

5

See Ross (2003, chapter 8) and Hillier and Lieberman (2005, chapter 17) for textbook accounts of Markovian queuing models.

6

Because our focus is not on this predetermined price, in the rest of this paper, with minimal implications for the analysis, we shall abstract away from this price.

large a crowd at the government shop. In this case, instead of joining the crowd and/or bribing the corrupt government official, this citizen *balks*. In other words, he refuses to join the crowd. Instead, he goes to a private shop to purchase his bread, possibly at a higher price. The details of this balking behavior are provided in section 4.

Citizens arrive at the government shop in accordance with a Poisson process with rate λ. This tells us that the time between successive arrivals of citizens is exponentially distributed and, as is well known, the exponential distribution is memoryless or *Markovian*. The corrupt public official takes a random amount of time to sell bread to citizens and we suppose that this random time is exponentially distributed with rate μ. Therefore, like the interarrival times of the citizens, the bread provision times are also memoryless or *Markovian*. Finally, in both versions of our model, there is a single government official who sells bread. In the language of queuing theory, we are using a prominent Markovian model, namely the $M/M/1$ model, to analyze the three specific questions that we mentioned in section 1.

3. The Model With Bribing

Suppose that our impatient citizen has just arrived at the government shop to buy bread. In the typical case, there will already be a queue of citizens waiting in line to get their bread. Without loss of generality, we suppose that when our citizen gets to the government shop, he has $n \in \mathbb{N}$ citizens ahead of him in line. Our citizen always has the choice of becoming the $(n+1)th$ person in the queue. However, because this citizen is impatient *and* because he really wants his bread, he would like to get his bread immediately. Now, to completely eliminate the wait time in queue, our citizen must bribe the public official selling bread.

The reader may be wondering why there is a queue in the first place. In the cases that we are analyzing in this paper, there is a queue because the good in question is believed to be in short supply. This means that if our citizen does not pay a bribe and instead waits in line then after the requisite wait, he may or *may not* get his bread. Put differently, the situation we are describing is not one of slow distribution only. It is one of slow distribution along with the potential non-receipt of bread. In this paper, we are interested in obtaining answers to the three specific questions that we have discussed in the last paragraph of section 1. This is why we do not explicitly account for the possibility that the citizen may not get his bread. Having said this, we note that it is certainly possible in our $M/M/1$ queuing framework to model this probabilistic receipt of bread explicitly. One way to do this would be to analyze a $M/M/1$ queuing model with *finite capacity*. Following Batabyal (2005), one could let an apposite public authority choose the capacity optimally so as to keep the likelihood of the non-receipt of bread below an exogenously given level.

Now, to keep the subsequent analysis straightforward, we suppose that the magnitude of the bribe depends on the number of citizens ahead of our citizen in a very simple way. Specifically, when our impatient citizen sees n citizens already in queue, to obtain his one unit of bread immediately, he must pay a bribe of n monetary units to the public official. The reader will note that with this bribing function, the only way in which our impatient citizen can get bread immediately and costlessly is if he is the first person to arrive at the government shop. Put differently, when $n=0$ the bribe is zero, and bread is purchased by our impatient citizen without any wait.

From the standpoint of resource or bread allocation, the key question before us now is this: What is the expected benefit to the corrupt government official in monetary units per citizen? Answering this question essentially involves determining the mean number of citizens in the government shop. Let us denote this mean number by $E[C]$. Further, let P_n be the long run or stationary probability that there are n citizens in the government shop. In words, P_n tells us the probability, as time approaches infinity, that there are exactly n citizens in the government shop. Now, if $Y(t)$ represents the number of citizens in the government shop at some arbitrary time t, then mathematically we have

$$P_n = \lim_{t\to\infty} Prob\{Y(t)=n\},\ \ n=0,1,2,... \tag{1}$$

Given the discussion in the previous paragraph, equation 8.7 in Ross (2003, p. 483) tells us that

$$E[C] = \Sigma_{n=0}^{\infty} nP_n = \Sigma_{n=0}^{\infty} n(\lambda/\mu)^n\{(\mu-\lambda)/\mu\} = \lambda/(\mu-\lambda). \tag{2}$$

Inspection of equation (2) tells us that for this equation to make sense, we must have $\lambda<\mu$. In words, the arrival rate of citizens at the government shop cannot be larger than the rate at which bread is sold to citizens by the corrupt public official. Equation (2) also provides us with the answer to the question we posed in the beginning of the paragraph before equation (1). Specifically, the expected benefit to the corrupt government official in monetary units per citizen—or the benefit from bribery—equals the ratio of the citizen arrival rate λ to the difference between the bread provision rate and the citizen arrival rate $(\mu-\lambda)$.

Simple comparative statics exercises tell us that $\partial E[C]/\partial\lambda>0$ and that $\partial E[C]/\partial\mu<0$. These results conform well with our intuition about the actual impact of bribery. Specifically, the first result says that if the rate λ at which citizens come to the government shop increases, then the average monetary benefit to our corrupt government official goes up. In contrast, raising the rate μ at which the venal public official sells bread has a negative impact on this official's average monetary benefit from bribery per citizen. We now proceed to analyze balking behavior in the context of the model described in section 2.

4. The Model With Balking

As in section 3, citizens arrive at the government shop in accordance with a Poisson process with rate λ and the relevant public official sells bread to citizens at rate μ. Obviously, citizens who upon arrival find no queue in the government shop proceed to purchase their bread right away. Now, to focus on the balking aspect of the story, we suppose that an arriving citizen who finds the public official busy does not linger in the shop but, instead, roams around the government shop for an exponentially distributed amount of time with rate θ. He then returns to the shop. If the public official is still busy upon his return then our citizen repeats the above course of action and then returns to the shop again, and so on and so forth. An impatient citizen wanting bread who finds the government official busy and n other citizens roaming around the shop *balks*. In other words, he refuses to also roam around the government shop. As an alternative, this citizen leaves the government shop and goes instead to a private shop to purchase his bread.

Following the explanation in the second paragraph of section 3, we remind the reader that in this balking model, the issue from the standpoint of our citizen is not one of speed of service only. It is one of speed of service *combined* with the possible non-receipt of bread. In addition, although it is possible that the government shop may end up with unsold bread, we do not consider this possibility very likely. After all, only very impatient and relatively more wealthy citizens will want to purchase bread at the generally more expensive private shop. Finally, note that we *have* modeled, albeit in a preliminary way, the situation in which our citizen does *not* wait in the government shop. As explained in the previous paragraph, when this citizen finds the government official busy and n other citizens roaming around the shop, he balks. That is, he refuses to also roam around the government shop and he goes instead to the private shop to purchase bread.[7]

Given the description in the previous two paragraphs, our task now is to answer the two remaining questions that we identified in section 1: First, what proportion of all citizens are eventually able to buy bread? Second, what is the expected amount of time a citizen spends roaming around the government shop, waiting to buy bread? Let us begin our analysis of these two questions by first defining the states of the model with balking. To this end, let i denote the number of citizens who are being sold bread by the government official and let j denote the number of citizens who are roaming around the government shop. Then, from the delineation of events in the previous two paragraphs, it follows that i=0,1, and that $0 \leq j \leq n$. With this information, we can now think of the state of our model as the pair (i,j).

To answer the question about the proportion of all citizens who are eventually able to buy bread, we will have to specify the so called balance equations.[8] These equations are

$$(\lambda + j\theta)P_{(0,j)} = \mu P_{(1,j)},\ j=0,1,...,n, \tag{3}$$

$$(\lambda + \mu)P_{(1,j)} = \lambda P_{(0,j)} + (j+1)\theta P_{(0,j+1)},\ j=0,...,n-1, \tag{4}$$

and

7 In a more general model, one could endogenize our citizen's decision regarding whether to wait in queue or to go instead to a private shop. Such a general model would involve the analysis of a priority queuing model—such as the *M/G/*1 model—with at least two (more and less impatient) types of citizens. Batabyal and Yoo (2004, 2005) have analyzed such priority queuing models and it should be possible to use the modeling techniques employed in these two papers to shed light on the questions that we are analyzing in the present paper.

8 For more on these balance equations, see Ross (2003, pp. 481-483) and Hillier and Lieberman (2005, pp. 782-784).

$$\mu P_{(1,n)} = \lambda P_{(0,n)}. \quad (5)$$

Inspecting equations (3)-(5) carefully, the reader will note that $P_{(1,n)}$ is the stationary probability that the government official is busy selling bread and that n citizens are roaming around the government shop. In other words, $P_{(1,n)}$ is the probability of balking or, alternately, the proportion of all citizens who are unable to buy bread from the government shop. Therefore, the proportion of all citizens who are eventually able to buy bread from the government shop is given by

$$1 - P_{(1,n)} = \{\mu - \lambda P_{(0,n)}\}/\mu. \quad (6)$$

We have just answered the second question of this paper. As one would expect, equation (6) tells us that an increase in the citizen arrival rate λ and the public official's bread provision rate μ have opposite impacts on the proportion of all citizens who are eventually able to buy bread from the government shop. In particular, as λ goes up, the proportion of interest $1 - P_{(1,n)}$ goes down. Conversely, when μ goes up, the same proportion $1 - P_{(1,n)}$ goes up. We now proceed to answer the third and final question of this paper.

To reiterate, we would now like to ascertain the expected amount of time a citizen spends roaming around the government shop, waiting to buy bread. To undertake this exercise, let us use the notation of section 3 and first compute the mean number of citizens in the government shop or $E[C]$. Some thought tells us that in this section's model with balking

$$E[C] = \Sigma_{(\forall i,j)}(i+j)P_{(i,j)}. \quad (7)$$

Now, using Little's formula or equation 8.2 in Ross (2003, p. 478), we can deduce that the average time spent by a citizen in the government shop or $E[T]$ is given by

$$E[T] = E[C]/[\lambda\{1 - P_{(1,n)}\}] = [\Sigma_{(\forall i,j)}(i+j)P_{(i,j)}]/[\lambda\{1 - P_{(1,n)}\}]. \quad (8)$$

With equation (8) in place, let us denote the expected amount of time a citizen spends roaming around the government shop by $E[T_{RA}]$. Then, from Hillier and Lieberman (2005, p. 771) we get

$$E[T_{RA}] = E[T] - 1/\mu, \quad (9)$$

where $E[T]$ is given by equation (8).

Equation (9) tells us that if one is concerned about reducing the amount of time citizens spend, presumably unproductively, roaming around the government shop then the most obvious way to do so would be to decrease the rate μ at which the public official sells bread to citizens. Alternately, one could also attain the above result by increasing the number of public officials working in the government shop under study. This completes our analysis of the third and final question of this paper.

5. Conclusions

In this paper, we provided a theoretical perspective on the properties of queue based resource allocation mechanisms in the presence of government and private shops. In particular, we accomplished three objectives. First, in the presence of bribery, we determined the expected monetary benefit per citizen to the corrupt government official. Second, in the presence of balking, we ascertained the proportion of all citizens who are eventually able to buy bread from the government shop. Finally, and once again in the presence of balking, we computed the average amount of time a citizen spends roaming around the government shop, waiting to buy bread.

The preliminary analysis of this paper can be extended in a number of different directions. In what follows, we propose two plausible extensions. First, we analyzed three specific questions in the context of a Markovian queuing model. Therefore, it would be useful to explore how robust our findings are by studying the kinds of resource allocation questions that we have examined in this paper with more general priority queuing models of the sort analyzed in Batabyal and Yoo (2004, 2005). Second, it would be useful to investigate queuing models in which there is interaction between citizens while they wait in queue to purchase bread. One way to model this interaction would be to specify that citizens arrive at the government shop in accordance with a nonhomogeneous Poisson process with a particular intensity function. By modeling the arrival process in this way, we would allow for the possibility that events—pertaining to the purchase of bread—may be more likely to occur at certain times than at other times. Studies that analyze these aspects of the problem will increase our understanding of the properties of queue based resource allocation mechanisms.

References

Anonymous. 1998. Queues Build Up in Minsk as Food Shortages Bite, *Belarussian Chronicle*, December 17. http://chronicle.home.by/9812/9812172.htm

Anonymous. 2003. The Dollarised Revolution, *The Economist*, July 31.

Batabyal, A.A. 2005. On the Allocation of Commodities by Queuing and the Prevention of Violence, *Economics Bulletin*, 15, 1-7.

Batabyal, A.A., and Nijkamp, P. 2004. Favoritism in the Public Provision of Goods in Developing Countries, *Economics Bulletin*, 15, 1-10.

Batabyal, A.A., and Yoo, S.J. 2004. Corruption, Bribery, and Wait Times in the Public Allocation of Goods in Developing Countries. Forthcoming, *Review of Development Economics*. http://ssrn.com/abstract=545022

Batabyal, A.A., and Yoo, S.J. 2005. A Complete Characterization of Mean Wait Times for Citizens in the Nonpreemptive Corruption Regime. Forthcoming, *Applied Economics Letters*. http://ssrn.com/abstract=654662

Goodman, S.L. 1995. Lessons From a Year in Romania, *The Freeman*, 45, June.

Gunawardana, P.J. 2000. Concessional Sales, Open Market Demand, and the Consumption of Rice in Sri Lanka, *International Journal of Social Economics*, 27, 847-861.

Hillier, F.S., and Lieberman, G.J. 2005. *Introduction to Operations Research*, 8th edition. McGraw Hill Higher Education, Boston, Massachusetts.

Kornai, J. 1980. *Economics of Shortage*. Elsevier, Amsterdam, The Netherlands.

Lui, F.T. 1985. An Equilibrium Queuing Model of Bribery, *Journal of Political Economy*, 93, 760-781.

Ross, S.M. 2003. *Introduction to Probability Models*, 8th edition. Academic Press, San Diego, California.

Sah, R.K. 1987. Queues, Rations, and Market: Comparisons of Outcomes for the Poor and Rich, *American Economic Review*, 77, 69-77.

Shleifer, A., and Vishny, R. 1993. Corruption, *Quarterly Journal of Economics*, 108, 599-617.

Wood, G. 1999. Private Provision after Public Neglect: Bending Irrigation Markets in North Bihar, *Development and Change*, 30, 775-794.