E C O N O M I C S   B U L L E T I N

The properties of asymmetric unit root tests in the presence of mis−specified asymmetry

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Abstract

The seminal analysis of Enders and Granger (1998) is extended to examine the properties of asymmetric unit root tests when the nature of the actual asymmetric adjustment process underlying the observed data is unknown. The analysis is further extended by considering joint testing for asymmetric stationarity in addition to unit root testing. It is shown that the momentum−threshold autoregressive (MTAR) test outperforms the threshold autoregressive (TAR) test. The results indicate that when employing asymmetric unit root tests, practitioners will tend to detect asymmetry of an MTAR rather TAR nature, irrespective of the form of asymmetry actually present in the data.

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1 Introduction

In recent research, Enders and Granger (1998) have extended the Dickey-Fuller (1979) testing procedure to allow the unit root hypothesis to be tested against an alternative of asymmetric stationarity. This development is to be welcomed as a large literature now exists suggesting the presence of asymmetric or non-linear behaviour in a range of economic time series.[1] Consider the Dickey-Fuller (DF) test in its simplest form:

$$\Delta y_t = \rho y_{t-1} + \xi_t \tag{1}$$

It is apparent that this is an explicitly symmetric specification, with (asymptotic) stationarity occurring when $|\rho| < 1$. To allow for the possibility of *asymmetric* stationarity, Enders and Granger (1998), hereafter referred to as EG, extend (1) by drawing upon the threshold autoregressive methods of Tong (1990). Following this approach, the resulting generalisation of (1) is given as:

$$\Delta y_t = I_t \rho_1 y_{t-1} + (1 - I_t)\, \rho_2 y_{t-1} + \varepsilon_t \tag{2}$$

where I_t is the zero-one Heaviside indicator function. EG consider two specifications for the Heaviside indicator function based upon the sign and difference of y_{t-1}. These rival specifications are given as:

$$I_t = \begin{cases} 1 & \text{if } y_{t-1} \geq 0 \\ 0 & \text{if } y_{t-1} < 0 \end{cases} \tag{3}$$

and:

$$I_t = \begin{cases} 1 & \text{if } \Delta y_{t-1} \geq 0 \\ 0 & \text{if } \Delta y_{t-1} < 0 \end{cases} \tag{4}$$

EG refer to the model defined by (2) and (3) as threshold autoregressive (TAR), while a model combining (2) and (4) is referred to as momentum-threshold autoregressive (MTAR). Under both models the unit root hypothesis ($\mathsf{H}_0 : \rho_1 = \rho_2 = 0$) is tested using specifically derived critical values provided by EG. To incorporate deterministic terms within the TAR and MTAR models, a preliminary regression of the variable of interest upon the appropriate deterministic terms is undertaken. The resulting revised series (either demeaned or detrended) then replaces y_t in (2) and the appropriate Heaviside indicator function. The results of Tong (1983) show that if the unit root hypothesis is rejected, the adjustment parameters (ρ_1, ρ_2) converge to a multivariate normal distribution. EG therefore suggest that following rejection of the unit root hypothesis, asymmetric stationarity can be tested formally via an F-test of the symmetry hypothesis $\mathsf{H}_0 : \rho_1 \neq \rho_2$.

[1] See, *inter alia*, Ball and Mankiw (1994), Dixit (1992), Gale (1996) and Krane (1994).

To examine the properties of the TAR and MTAR unit root tests, EG performed a number of power experiments for the 'with intercept' specification. That is, rejections of the unit root hypothesis by the TAR test were considered in the presence of TAR adjustment for a range of values of (ρ_1, ρ_2). A similar approach was then followed to examine the MTAR test under MTAR adjustment. However, in practice a practitioner will not know the data generation process. It is therefore of interest to examine the properties of the TAR (MTAR) test when the data possess asymmetry in the form of MTAR (TAR) adjustment. This allows the behaviour of the asymmetric unit root tests to be examined when the nature of asymmetry present is mis-specified. In addition to considering this possibility, the present paper further extends the seminal study by considering joint rejections of the unit root and symmetry hypotheses rather than rejections of the unit root hypothesis alone, as considered by EG.

2 Simulation results

To examine the powers of the rival asymmetric unit root tests, the analysis of EG is followed with the TAR and MTAR tests considered in their 'with intercept' forms. The TAR and MTAR tests, denoted as Φ_μ and Φ_μ^* respectively, are derived via application of (2) and (3) or (4) as appropriate, following the regression of y_t upon a constant. In addition to the asymmetric tests, the familiar DF τ_μ test, given as the t-ratio for ϕ in the following regression, is also considered:

$$\Delta y_t = \alpha + \phi y_{t-1} + \eta_t \tag{5}$$

To examine the properties of the tests under TAR adjustment, a data generation process given by (2) and (3) above is employed. For MTAR adjustment, the data generation process is given by (2) and (4). In each case the properties of the Φ_μ, Φ_μ^* and τ_μ tests are considered for a range of values of the adjustment parameters (ρ_1, ρ_2), the specific values being reported in each set of tabulated results below. Again following EG, all experiments are performed for an effective sample size of 100 observations.[2] Empirical rejection frequencies of the unit root hypothesis for the Φ_μ, Φ_μ^* and τ_μ tests are calculated at the 5% level of significance using critical values presented by EG and Fuller (1996) respectively. Joint rejections of the unit

[2] The error series $\{\varepsilon_t\}$ is generated as pseudo *i.i.d.* $\mathsf{N}(0,1)$ random numbers using the RNDNS procedure in the Gauss programming language version 3.2.13. For all experiments, an additional, initial 100 observations are created and discarded to remove the effects of initial condition $y_0 = 0$. All experiments are performed over 10,000 replications.

root and symmetry hypotheses by the Φ_μ and Φ^*_μ tests at the 5% level of significance employ the appropriate critical value from the F-distribution for the secondary test of symmetry.

The results for rejection of the unit root hypothesis under TAR adjustment are reported in Table One. From inspection of this table it can be seen that the τ_μ test exhibits the greatest power of the tests in all experiments apart from where $(\rho_1, \rho_2) = (-0.1, -0.8)$ in which case the Φ_μ test has highest power. The relatively poor performance of the Φ_μ test in comparison to the τ_μ test under TAR adjustment was noted by EG. When comparing the Φ^*_μ and Φ_μ tests, the former has greater power apart from cases where there is a high degree of both asymmetry and stationarity. Generally, the results therefore suggest a ranking in which the τ_μ test is preferred to the Φ^*_μ test, which is in turn preferred to the Φ_μ test. However, it must be noted that the differences between the tests are slight and the results obtained for the tests are qualitatively very similar. The results for MTAR adjustment in Table Two are in sharp contrast to this as the Φ^*_μ test can be seen to possess much greater power than the rival Φ_μ and τ_μ tests. As an example of this, consider the results for $(\rho_1, \rho_2) = (-0.025, -0.2)$ where the empirical rejection frequency of the Φ^*_μ test is 62.56% compared to 38.59% and 33.97% respectively for the τ_μ and Φ_μ tests. This example also illustrates the slightly higher power generally exhibited by the τ_μ test in comparison to the Φ_μ test. In summary, the results for testing of the unit root hypothesis show that under TAR adjustment all three tests possess similar properties, while under MTAR adjustment, the Φ^*_μ test is clearly preferred, as might be expected.

As noted above, testing the unit root hypothesis can be considered to be the first stage in the application of asymmetric unit root tests, as the hypothesis of symmetry can be tested explicitly should the unit root be rejected. The results for the joint rejection of the unit root and symmetry are presented in Tables Three and Four. Intuitively it is to be expected that the rival Φ_μ and Φ^*_μ tests would perform better in the presence of their own form of asymmetry. However, from inspection of Table Three it is clear that this is not the case as the Φ^*_μ test outperforms the Φ_μ test in the presence of TAR asymmetry. It should also be noted that neither of the tests performs well in the presence of TAR adjustment, as joint rejection of non-stationarity and symmetry is rarely observed under either test. In particular, it can be seen that low rejection frequencies correspond to undersizing of the test of symmetry, a phenomenon noted and discussed by Cook and Manning (2003). This undersizing therefore questions the applicability of the (asymptotically justified) F-test of symmetry for finite samples. Considering the results for MTAR adjustment presented in Table Four, the Φ^*_μ test clearly outperforms the Φ_μ test, with the joint hypothesis frequently rejected by the former, but seldom by the latter. The results therefore indicate that should asymmetric stationarity of an MTAR form exist, it is likely that it will be detected by the Φ_μ^* test, but unlikely that it will be uncovered by the Φ_μ test. In contrast, if asymmetric stationarity is of a TAR form, it is unlikely to be detected by either of the asymmetric unit root tests, although it is more probable that it will be uncovered by the Φ_μ^* test than the Φ_μ test.

3 Conclusion

In this paper the analysis of asymmetric unit root tests presented by Enders and Granger (1998) has been extended. In addition to examining the properties of the TAR and MTAR tests under mis-specification of the asymmetric adjustment process, the ability of the tests to jointly reject non-stationarity and symmetry were also considered. The results obtained showed that when considering rejection of the unit root hypothesis, the TAR and MTAR tests have similar power in the presence of TAR adjustment. However, when asymmetry is of an MTAR nature, the MTAR clearly outperforms the TAR test. The superior performance of the MTAR test was further emphasised by results of joint testing of the non-stationarity and symmetry hypotheses. Analysis of the joint hypothesis produced two interesting results. First, the MTAR test was found to have greater power than the TAR test irrespective of the form of asymmetry considered. Second, it was found that substantial rejection of the joint hypothesis occurred when the MTAR test was employed in the presence of MTAR asymmetry. The results suggest that when data generation process is unknown, as will be the case in practice, application of the asymmetric unit root tests of Enders and Granger (1998) will lead to a tendency for a practitioner to uncover asymmetry of an MTAR form alone, irrespective of the true nature of the asymmetric adjustment process present. The superiority of the MTAR test might be viewed as advantageous as this is arguably the form of asymmetry more commonly expected in economic time series. When considering asymmetry in either the level (TAR) or change of a series (MTAR), it is often the latter that attracts more interest as it can be related to turning points and growth rates. In addition, early interest in business cycle asymmetry focussed on the speed at which peaks and troughs were approached, with recessions thought to be shorter and sharper than recovery periods. However, the attractive properties of the MTAR test may not persist when the underlying mechanism giving rise to the data is neither TAR nor MTAR in form. The analysis of alternative asymmetric adjustment mechanisms is one potentially fruitful area of future research.

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Table One

Unit root testing under TAR adjustment

ρ_1	ρ_2	Φ_μ	Φ^*_μ	τ_μ
0.00	0.00	4.97	5.05	5.03
−0.025	−0.05	7.80	8.72	9.49
−0.025	−0.075	9.30	10.43	11.34
−0.025	−0.1	10.64	11.89	12.97
−0.025	−0.125	11.98	13.27	14.49
−0.025	−0.15	13.42	14.45	15.87
−0.025	−0.2	15.80	16.86	17.83
−0.05	−0.1	16.39	17.48	19.51
−0.05	−0.15	21.57	22.76	25.25
−0.05	−0.2	26.60	27.15	30.41
−0.05	−0.25	30.66	30.75	34.56
−0.05	−0.3	34.26	34.18	37.77
−0.05	−0.4	39.94	39.30	42.78
−0.1	−0.2	51.48	49.83	57.13
−0.1	−0.3	66.02	63.52	69.98
−0.1	−0.4	74.88	71.38	77.39
−0.1	−0.5	80.65	76.95	81.59
−0.1	−0.6	84.40	81.05	84.68
−0.1	−0.8	89.13	86.38	88.88

Notes: Percentage rejection frequencies of the unit root hypothesis using the DGP of (2) and (3). All experiments performed using a sample size of 100 observations and 10,000 replications.

Table Two

Unit root testing under MTAR adjustment

ρ_1	ρ_2	Φ_μ	Φ^*_μ	τ_μ
0.00	0.00	4.99	5.01	5.02
−0.025	−0.05	7.87	9.55	9.31
−0.025	−0.075	10.09	14.39	11.98
−0.025	−0.1	13.17	20.44	16.05
−0.025	−0.125	17.34	28.93	20.51
−0.025	−0.15	21.95	39.83	25.99
−0.025	−0.2	33.97	62.56	38.59
−0.05	−0.1	17.56	20.97	21.12
−0.05	−0.15	28.47	38.96	33.00
−0.05	−0.2	41.90	60.54	47.28
−0.05	−0.25	56.57	79.22	61.95
−0.05	−0.3	70.26	91.13	74.64
−0.05	−0.4	89.35	99.14	91.35
−0.1	−0.2	58.01	63.70	63.91
−0.1	−0.3	82.97	91.50	87.19
−0.1	−0.4	95.55	99.30	96.87
−0.1	−0.5	99.26	99.95	99.31
−0.1	−0.6	99.88	100.00	99.90
−0.1	−0.8	100.00	100.00	99.99

Notes: Percentage rejection frequencies of the unit root hypothesis using the DGP of (2) and (4). All experiments performed using a sample size of 100 observations and 10,000 replications.

Table Three

Joint testing of the unit root and symmetry hypotheses testing under TAR adjustment

ρ_1	ρ_2	Φ_μ	Φ_μ^*
−0.025	−0.025	0.10	1.92
−0.025	−0.05	0.07	2.13
−0.025	−0.075	0.05	2.44
−0.025	−0.1	0.07	2.65
−0.025	−0.125	0.06	2.78
−0.025	−0.15	0.08	2.86
−0.025	−0.2	0.09	3.10
−0.05	−0.05	0.08	2.66
−0.05	−0.1	0.06	3.10
−0.05	−0.15	0.07	3.44
−0.05	−0.2	0.12	3.67
−0.05	−0.25	0.16	4.03
−0.05	−0.3	0.23	4.35
−0.05	−0.4	0.60	4.89
−0.1	−0.1	0.05	3.83
−0.1	−0.2	0.13	4.31
−0.1	−0.3	0.27	4.77
−0.1	−0.4	0.70	5.65
−0.1	−0.5	1.15	6.73
−0.1	−0.6	2.07	8.17
−0.1	−0.8	4.28	11.50
−0.2	−0.5	0.72	5.56
−0.3	−0.75	2.22	8.11
−0.5	−0.9	1.58	6.94

Notes: Percentage rejection frequencies of the joint symmetry and unit root hypothesis using the DGP of (2) and (3). All experiments performed using a sample size of 100 observations and 10,000 replications.

Table Four

Joint testing of the unit root and symmetry hypotheses testing under MTAR adjustment

ρ_1	ρ_2	Φ_μ	Φ_μ^*
-0.025	-0.025	0.17	1.85
-0.025	-0.05	0.14	2.74
-0.025	-0.075	0.13	4.84
-0.025	-0.1	0.18	8.28
-0.025	-0.125	0.20	12.77
-0.025	-0.15	0.17	19.30
-0.025	-0.2	0.23	34.07
-0.05	-0.05	0.13	2.61
-0.05	-0.1	0.15	5.16
-0.05	-0.15	0.15	12.71
-0.05	-0.2	0.17	24.21
-0.05	-0.25	0.25	37.39
-0.05	-0.3	0.29	48.67
-0.05	-0.4	0.65	68.46
-0.1	-0.1	0.13	3.45
-0.1	-0.2	0.13	11.71
-0.1	-0.3	0.24	30.61
-0.1	-0.4	0.49	50.62
-0.1	-0.5	1.03	68.84
-0.1	-0.6	1.82	82.03
-0.1	-0.8	4.80	95.19
-0.2	-0.5	0.45	37.78
-0.3	-0.75	1.33	54.39
-0.5	-0.9	1.07	33.06

Notes: Percentage rejection frequencies of the joint symmetry and unit root hypothesis using the DGP of (2) and (4). All experiments performed using a sample size of 100 observations and 10,000 replications.