E C O N O M I C S B U L L E T I N

On cooperation structures resulting from simultaneous proposals

Rod Garratt
University of California, Santa Barbara

Cheng−Zhong Qin
University of California, Santa Barbara

Abstract

This paper looks at cooperation structures that result from a strategic game where players make simultaneous proposals for cooperation. We identify cooperation structures that maximize the potential of the game, and show how the outcome of potential maximization depends on the players' Shapley values. We do not assume superadditivity and hence, potential−maximizing strategy profiles do not always involve full cooperation. In cases where full cooperation does result from potential maximization it can be inefficient. An example provides intuition.

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1. Introduction

This paper considers cooperation structures that emerge in an environment where players make simultaneous proposals for cooperation. The players can be thought of as leaders of political parties looking to form a coalition government, CEO's of firms contemplating a merger, or producers of a product looking to form a cartel. Such examples have often been formulated as sequential processes. However, actions taken early in negotiations are inherently irreversible and determine at least to some extent the ultimate outcome.[1] The selection of these initial positions is well modelled as a static game. We characterize the potential-maximizing solutions to the game, and provide an example that explains the occurrence of inefficient, potential-maximizing outcomes.

The games we consider are called cooperation-formation games (Qin 1996). These are strategic games that determine both a coalitional structure (a partition of the players into coalitions) and how members within each coalition are connected. A connection can be thought of as a communication channel. Players within a coalition may be connected directly, meaning they can communicate directly with each other, or indirectly, meaning that they can communicate only through others. The reason for focusing on communication channels (as opposed to simply looking at coalitional structures) is that we wish to understand situations where the payoffs to members of a coalition depend on their "power" within the coalition.[2]

A cooperation-formation game is built upon a coalitional game, which specifies the maximum value members of each coalition can create by acting together. The coalitional game does not specify how players decide which coalitions to form. This choice problem is dealt with by the cooperation-formation game. In a cooperation-formation game, each player independently identifies whom she wishes to cooperate with. A link forms between two players if and only if each proposes to cooperate with the other. Players' payoff functions are computed by applying the Myerson value to the coalitional game and the cooperation structure determined by the players' strategy profile.

The Myerson value has a natural interpretation in many contexts. Moreover, under this method of assigning payoffs the cooperation-formation game is a potential game, as defined by Monderer and Shapley (1996).[3] In general, only a subset of the Nash equilibria of potential games coincide with the set of strategy profiles that maximize the potential,

[1] In the words of Omar Khayyam (fl. 11th cent.), " The Moving Finger writes; and having writ, Moves on; nor all your Piety nor Wit Shall lure it back to cancel half a Line, Nor all your Tears wash out a Word of it." (Rubáiyát. Stanza lxxi.)

[2] An alternative view is that links represent favorable relationships between players. See Garratt *et. al.* (2001) for more on this interpretation in the context of coalition government formation.

[3] In fact, Qin (1996) establishes that a cooperation formation game is a potential game if and only if payoffs determined using the Myerson value. Hence, any rule for dividing surplus to coalitions that does not correspond to the Myerson value, for example equal division, will produce a game that does not have a potential.

and hence potential maximization can be regarded as a refinement device. For three-player cooperation-formation games, it turns out that, except in some knife-edge cases, all of the Nash equilibria that maximize the potential correspond to a single cooperation structure. Thus, in almost all cases, potential maximization provides a unique prediction of the cooperation structure.[4]

In the three-player case, the cooperation structure that survives potential maximization includes either two linked players and an isolated player, or the cooperation structure in which every pair of players is linked. Cooperation structures in which players hold special positions do not survive potential maximization. We provide conditions for either of the two possible outcomes. These conditions are summarized in terms of restrictions on the players' Shapley values.

Qin (1996), Dutta, van den Nouweland, and Tijs (1998), Slikker, Dutta, van den Nouweland (2000), Tijs (2000), and Slikker and van den Nouweland (2002) also consider cooperation formation in simultaneous-play games and characterize potential-maximizing strategy profiles. However, these papers assume superadditivity of the coalitional games when characterizing potential-maximizing cooperation structures.

2. Three-player cooperation-formation game

Players are denoted by $i \in \{1, 2, 3\} = N$. The strategy set of player i is $\Pi_i = \{S \subseteq N \mid i \in S\}$. A strategy $\pi_i \in \Pi_i$ is a set of players with whom player i wishes to form links. Let $\Pi = \times_{i \in N} \Pi_i$. Given $\pi = (\pi_1, \pi_2, \pi_3) \in \Pi$, a link between players i and j forms if $i \in \pi_j$ and $j \in \pi_i$.[5] The undirected bilateral link between players i and j is denoted $i : j$. The set of all (undirected) bilateral links between players is $L = \{i : j \mid i, j \in N\}$. A cooperation structure is a list of undirected bilateral links in L. Let g be the mapping that maps strategy profiles in Π into cooperation structures in L. Then, given $\pi \in \Pi$, $g(\pi) = \{i : j \mid i \in \pi_j \text{ and } j \in \pi_i\}$.

Given a strategy profile and hence a cooperation structure, the payoffs are determined as follows. First, it is assumed that any coalition has a value that is expressed by a characteristic function $v : 2^N \to \mathbb{R}$, with $v(\emptyset) = 0$. Second, those coalitions whose members are either directly or indirectly connected by the cooperation structure resulting from the strategy profile are formed. Players' payoffs are then determined by how the values of these coalitions are divided among the respective members. Two properties are imposed on the divisions. One is that it be feasible in the sense that individual payoffs to the players in a coalition add up exactly to the value of the coalition. The other is that the division pattern allows the cooperation-formation game to have a potential. A

[4] Potential maximization is a solution concept that by definition only applies to potential games. See Jackson and Wolinski (1996) for a general analysis of stability of link structures.

[5] This method of determining a cooperation structure follows Myerson (1991, pp. 448).

potential for a game with player i's strategy set Π_i and payoff function U_i for $i \in N$ is a function $P : \Pi \to \mathbb{R}$ such that for any $i \in N$, $\pi \in \Pi$, and $\pi'_i \in \Pi_i$,

$$U_i(\pi'_i, \pi_{-i}) - U_i(\pi) = P(\pi'_i, \pi_{-i}) - P(\pi).$$

A game is a *potential game* if it has a potential. Qin (1996) shows that the only division rule for the cooperation-formation game that meets these requirements is the Myerson value (see Myerson, 1977).

The Myerson value for the game (N, v) with cooperation structure g is denoted $\psi(v, g) = (\psi_i(v, g))_{i \in N}$.[6] It can be constructed using the Shapley value (see Shapley, 1953). The Shapley value of a game (N, v) is denoted $\phi(v) = (\phi_i(v))_{i \in N}$, where

$$\phi_i(v) = \sum_{S:S \ni i} \frac{(|S|-1)!(n-|S|)!}{n!}[v(S) - v(S \backslash i)].$$

Let $\phi(v) = (\phi_i(v))_{i \in N}$. To construct the Myerson value, let S/g denote the partition of $S \subseteq N$ into subsets of players that are connected by cooperation structure g in S. Let v^g denote the characteristic function determined according to

$$v^g(S) = \sum_{R \in S/g(\pi)} v(R), \ S \subseteq N.$$

Myerson (1977) shows that $\psi(v, g) = \phi(v^g)$. The cooperation-formation game is the strategic-form game $\Gamma = \{\Pi_i, U_i\}_{i \in N}$, where Π_i is as before and $U_i(\pi) = \psi_i(v, g(\pi))$ for $\pi \in \Pi$.

Remark 1 $U_i(\pi) = U_i(\pi')$ if and only if $g(\pi) = g(\pi')$.

Remark 1 says that the payoffs depend only on the resulting cooperation structure. It is important to emphasize this because multiple strategy profiles may yield the same cooperation structure. For example consider the strategy profiles $\pi = (\{1, 2\}, \{1, 2\}, \{1, 3\})$ and $\pi' = (N, \{1, 2\}, \{2, 3\})$. Both produce the same cooperation structure with a single link, namely, $g(\pi) = g(\pi') = \{1 : 2\}$. In the strategy profile π player 3 is willing to cooperate with player 1 but not vice versa. In the strategy profile π' player 1 is willing to cooperate with player 3 but not vice versa. Links only form if the desire to cooperate is mutual. It does not matter who vetoes a proposal for cooperation.

Definition 1 *(Monderer and Shapley 1996, pp. 128) A path in* Π *is a sequence* $\gamma = (\pi^0, \pi^1, ...)$ *of strategy profiles such that for every* $\ell \geq 1$ *there exists a unique player denoted* i_ℓ *such that* $\pi^\ell = (\pi^\ell_{i_\ell}, \pi^{\ell-1}_{-i_\ell})$ *for some* $\pi_{i_\ell} \in \Pi_{i_\ell}$ *with* $\pi_{i_\ell} \neq \pi^{\ell-1}_{i_\ell}$ *(Player* i_ℓ *is the only deviator from* $\pi^{\ell-1}$ *in* π^ℓ*.)*

[6] We use g both as a mapping and a generic cooperation structure.

Fix $\pi^0 = (\pi_1^0, \pi_2^0, ..., \pi_n^0)$ with $\pi_i^0 = \{i\}$. For $\pi \in \Pi$, let $\gamma(\pi) = (\pi^0, \pi^1, ..., \pi^m)$ denote a path such that $\pi^m = \pi$. That is, $\gamma(\pi)$ is a path that connects π^0 with π. Given $\pi \in \Pi$ and given a path $\gamma(\pi)$, let $I(\gamma(\pi)) = \sum_\ell (U_{i_\ell}(\pi^\ell) - U_{i_\ell}(\pi^{\ell-1}))$.

Remark 2 By Theorem 2.8 of Monderer and Shapley (1996), $I(\gamma(\pi)) = I(\gamma'(\pi))$ if Γ is a potential game and if $\gamma(\pi)$ and $\gamma'(\pi)$ are paths connecting π^0 with π.

3. Potential-maximizing cooperation structures

In this section we characterize the potential-maximizing cooperation structures for three-player cooperation-formation games.

Theorem 1 *Suppose $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$, and $v(S) > 0$ for $S \subseteq N$ with $|S| = 2$. Let $\pi^* \in \Pi$ be a strategy profile that maximizes the potential of the cooperation-formation game Γ. Then, either $g(\pi^*) = \{i : j\}$ for some $i, j \in N$ with $i \neq j$ or $g(\pi^*) = \{1 : 2,\ 1 : 3,\ 2 : 3\}$.*

Proof. For $\pi \in \Pi$, let $P(\pi) = I(\gamma(\pi))$, where $I(\gamma(\pi))$ is as defined at the end of section 2.. Then, since Γ is a potential game, $P(\pi)$ is a potential function for the game (see (2.1) of Monderer and Shapley (1996)). Fix $\pi \in \Pi$.

Case 1: $g(\pi) = \emptyset$.

In this case, by Remark 2 we can assume $\gamma(\pi)$ satisfies $g(\pi^\ell) = g(\pi^{\ell-1})$ for $1 \leq \ell \leq m$. Since $v(\{i\}) = 0$, $U_i(\pi) = \psi(v, g(\pi)) = 0$ for $i \in N$. By Remark 1, $I(\gamma(\pi)) = 0$, and thus

$$P(\pi) = 0. \tag{1}$$

Case 2: $g(\pi) = \{i : j\}$ for some $i,\ j \in N$ with $i \neq j$.

In this case, by Remarks 1 and 2, we may assume that $\pi_i = \{i, j\}$, $\pi_j = \{i, j\}$, and $\gamma(\pi) = \{\pi^0, \pi^1, \pi^2\}$ with $\pi^1 = (\pi_i, \pi_{-i}^0)$, and $\pi^2 = (\pi_j, \pi_{-j}^1)$. Since $g(\pi^1) = g(\pi^0) = \emptyset$ and $g(\pi^2) = \{i : j\}$, we have $P(\pi) = U_j(\pi^2) - U_j(\pi^1)$. Because $v(S) = 0$ for $|S| = 1$, $U_j(\pi^1) = 0$ and $U_j(\pi^2) = \frac{1}{2}v(\{i, j\})$, and hence

$$P(\pi) = \frac{1}{2}v(\{i, j\}). \tag{2}$$

Case 3: $g(\pi) = \{i : j,\ i : k\}$ for $i,\ j, k \in N$ with $i \neq j,\ j \neq k,\ i \neq k$.

In this case, by Remarks 1 and 2, we may assume $\pi_i = N$, $\pi_j = \{i, j\}$, $\pi_k = \{i, k\}$, and $\gamma(\pi) = (\pi^0, \pi^1, \pi^2, \pi^3)$, where $\pi^1 = (\pi_j, \pi_{-j}^0)$, and $\pi^2 = (\pi_k, \pi_{-k}^1)$ and $\pi^3 = (\pi_i, \pi_{-i}^2)$. Since $g(\pi^1) = g(\pi^2) = \emptyset$ and $g(\pi^3) = \{i : j,\ i : k\}$, we have $U_j(\pi^0) = U_j(\pi^1) = 0$, $U_k(\pi^1) = U_k(\pi^2) = 0$, $U_i(\pi^2) = 0$, and $U_i(\pi^3) = \frac{1}{6}v(\{i, j\}) + \frac{1}{6}v(\{i, k\}) + \frac{1}{3}v(N)$. Thus,

$$P(\pi) = \frac{1}{6}v(\{i, j\}) + \frac{1}{6}v(\{i, k\}) + \frac{1}{3}v(N). \tag{3}$$

Case 4: $g(\pi) = \{1:2,\ 1:3,\ 2:3\}$

In this case it must be true that $\pi_1 = \pi_2 = \pi_3 = N$. As before, we may assume that $\gamma(\pi) = (\pi^0, \pi^1, \pi^2, \pi^3)$ where $\pi^1 = (\pi_i, \pi^0_{-i})$, $\pi^2 = (\pi_j, \pi^1_{-j})$, and $\pi^3 = (\pi_k, \pi^2_{-k})$. Then, $g(\pi^1) = \emptyset$, $g(\pi^2) = \{i : j\}$, and $g(\pi^3) = \{i : j,\ i : k,\ j : k\}$. This implies that $U_i(\pi^0) = U_i(\pi^1) = 0, U_j(\pi^1) = 0$, $U_j(\pi^2) = \frac{1}{2}v(\{i,j\})$, $U_k(\pi^2) = 0$, and $U_k(\pi^3) = \frac{1}{6}v(\{i,k\}) + \frac{1}{6}v(\{j,k\}) + \frac{1}{3}v(N) - \frac{1}{3}v(\{i,j\})$. We therefore have

$$P(\pi) = \frac{1}{6}[v(\{i,j\}) + v(\{i,k\}) + v(\{j,k\})] + \frac{1}{3}v(N) \tag{4}$$

Now let π^* denote a strategy profile that maximizes the potential. Then, since $v(S) > 0$ for $S \subseteq N$ with $|S| = 2$, by (1) - (4), $g(\pi^*) \neq \emptyset$ and $g(\pi^*) \neq \{i : j,\ i : k\}$ for any $i \neq j$, $i \neq k$, $j \neq k$. Thus, either $g(\pi^*) = \{i : j\}$ for some $i, j \in N$ with $i \neq j$ or $g(\pi^*) = \{1:2,\ 1:3,\ 2:3\}$. ■

Additional details follow from the proof of Theorem 1. They are presented in the following remarks.

Remark 3(i) Let π^* maximize P. By (2) and (4), $g(\pi^*) = \{i : j\}$ implies that $v(\{i,j\}) = \max\{v(S) : |S| = 2\}$ and $v(\{i,j\}) \geq v(N) + \frac{1}{2}[v(\{i,k\}) + v(\{j,k\})]$. Moreover, $v(\{i,j\}) = \max\{v(S) : |S| = 2\}$ and $v(\{i,j\}) \geq v(N) + \frac{1}{2}[v(\{i,k\}) + v(\{j,k\})]$ imply that $\pi \in \Pi$ with $g(\pi) = \{i : j\}$ is a potential maximizer. Similarly, $g(\pi^*) = \{1:2,\ 1:3,\ 2:3\}$ implies that $v(\{i,j\}) \leq v(N) + \frac{1}{2}[v(\{i,k\}) + v(\{j,k\})]$ for $i \neq j, i \neq k$, $j \neq k$; and $v(\{i,j\}) \leq v(N) + \frac{1}{2}[v(\{i,k\}) + v(\{j,k\})]$ for $i \neq j, i \neq k$, $j \neq k$ implies that (N,N,N), the unique strategy profile that yields the cooperation structure $\{1:2,\ 1:3,\ 2:3\}$, is a potential maximizer. In cases where $v(\{i,j\}) > \max\{v(\{i,k\}),\ v(\{j,k\})\}$ and $v(\{i,j\}) > v(N) + \frac{1}{2}[v(\{i,k\}) + v(\{j,k\})]$, it must be true that $g(\pi^*) = \{i : j\}$ is the unique potential-maximizing cooperation structure. In cases where $v(\{i,j\}) < v(N) + \frac{1}{2}[v(\{i,k\}) + v(\{j,k\})]$ for $i \neq j$, $i \neq k$, $j \neq k$, it must be true that $g(\pi^*) = \{1:2,\ 1:3,\ 2:3\}$ is the unique potential-maximizing cooperation structure.

Remark 3(ii) The Shapley value for player $k \in N$ in the full cooperation game is $\phi_k = \frac{1}{6}v(\{i,k\}) + \frac{1}{6}v(\{j,k\}) + \frac{1}{3}v(N) - \frac{1}{3}v(\{i,j\})$. It follows from Remark 3(i) that $g(\pi^*) = \{i : j\}$ implies that $v(\{i,j\}) = \max\{v(S) : |S| = 2\}$ and $\phi_k(v) \leq 0$; and $v(\{i,j\}) > \max\{v(\{i,k\}),\ v(\{j,k\})\}$ and $\phi_k(v) < 0$ implies that $g(\pi^*) = \{i : j\}$ is the unique potential-maximizing cooperation structure. Likewise, it follows from Remark 3(i) that $g(\pi^*) = \{1:2, 1:3, 2:3\}$ implies that $\phi_1(v), \phi_2(v), \phi_3(v) \geq 0$; and $\phi_1(v), \phi_2(v), \phi_3(v) > 0$ implies $g(\pi^*) = \{1:2,\ 1:3,\ 2:3\}$ is the unique potential-maximizing cooperation structure.

The cases described in Remark 3(ii) and the remaining, intermediate cases (mixtures of zero and nonzero Shapley values) are summarized in the tables below. By Theorem 1, the possible graphs are either pairs or full cooperation. In cases where a pair will form, but the actual pair depends on additional information, the word "pair" appears in the tables. In such cases, the pair can be determined by Remark 3(i). If full cooperation (i.e., the complete graph) is potential maximizing for any combination of the players' Shapley values, the word "full" appears.

	$\phi_2(v) < 0$	$\phi_2(v) = 0$	$\phi_2(v) > 0$
$\phi_1(v) < 0$	pair	pair	pair
$\phi_1(v) = 0$	pair	{1:2}	{1:2}
$\phi_1(v) > 0$	pair	{1:2}	{1:2}

Table 1a: $\phi_3(v) < 0$.

	$\phi_2(v) < 0$	$\phi_2(v) = 0$	$\phi_2(v) > 0$
$\phi_1(v) < 0$	pair	{2:3}	pair
$\phi_1(v) = 0$	{1:3}	{1:2}, {1:3}, {2:3}, full	{1:2}, {2:3}, full
$\phi_1(v) > 0$	pair	{1:2}, {1:3}, full	{1:2}, full

Table 1b: $\phi_3(v) = 0$.

	$\phi_2(v) < 0$	$\phi_2(v) = 0$	$\phi_2(v) > 0$
$\phi_1(v) < 0$	pair	pair	pair
$\phi_1(v) = 0$	pair	{1:3}, {2:3}, full	{2:3}, full
$\phi_1(v) > 0$	pair	{1:3}, full	full

Table 1c: $\phi_3(v) > 0$.

Tables 1a-c illustrate an important and intuitive fact: Full cooperation is the unique potential-maximizing outcome if and only if all three players' Shapley values are strictly positive.

4. An example

One of the interesting aspects of potential maximization in the context of non superadditive games is that inefficient outcomes are sometimes selected. To demonstrate this possibility, and to provide insight into why such outcomes are reasonable we specify an example. Consider a three-player coalitional game, with characteristic function v satisfying $v(1) = v(2) = v(3) = 0$, $v(\{1,2\}) = 12$, $v(\{1,3\}) = v(\{2,3\}) = 9$, and $v(N) = 8$. The game is not superadditive and the efficient outcome is for the two-player coalition $\{1,2\}$ to form. However, the unique, potential-maximizing strategy profile is (N, N, N). The resulting cooperation structure is $g = \{1:2,\ 1:3,\ 2:3\}$ and the payoffs are $(3\frac{1}{6}, 3\frac{1}{6}, 1\frac{2}{3})$. The total payoff from this strategy is 8, which is less than the total payoff that could be achieved by any pair of players.

The strategy profile $\pi_1 = \{1,2\}$, $\pi_2 = \{1,2\}$, and $\pi_3 = \{3\}$ produces the efficient outcome and is a Nash equilibrium. Insight into why this Nash equilibrium is not likely to be selected in practice is obtained by switching player 3's strategy to N, her weakly dominant strategy, and assuming players 1 and 2 each focus on the choice between connecting only with each other and connecting with everyone.[7] The resulting game is as follows

$\pi_1 \backslash \pi_2$	$\{1,2\}$	N
$\{1,2\}$	$6,6$	$1\frac{2}{3}, 6\frac{1}{6}$
N	$6\frac{1}{6}, 1\frac{2}{3}$	$3\frac{1}{6}, 3\frac{1}{6}$

The game has the structure of the prisoner's dilemma. Playing N is a dominant strategy for both player 1 and 2 and this is inefficient.[8]

5. More than three players

For an arbitrary number of players, the statement of Theorem 1 would be as follows: *Whenever any two players are indirectly connected in the graph that results from a potential maximizing strategy profile, they are also directly connected.* However, consider a 4-player game (N, v) where (i) $v(k) = 0$ for $k \in N$; (ii) $v(\{1,2\}) = v(\{2,3\}) = v(\{3,4\}) = 2$ and $v(\{1,3\}) = v(\{1,4\}) = v(\{2,4\}) = 10$; (iii) $v(S) = 1$ for $|S| = 3$; and (iv) $v(N) = 100$. The potential of the resulting cooperation-formation game is maximized at $\pi^* = (\{1,3,4\}, \{2,4\}, \{1,3\}, \{1,2,4\})$.[9] All four players are indirectly connected in the corresponding graph, $g(\pi^*) = \{1:3, 1:4, 2:4\}$, but players 1 and 2, 3 and 4, and 2 and 3 are not directly connected. This example shows that further (more restrictive) assumptions on the form of the characteristic function are required to generalize Theorem 1.

[7] This is arguably the natural choice for players 1 and 2 because they are the value-maximizing pair. Moreover, experimental evidence supports this restriction. Garratt *et. al.* (2001, Table 4, Game 4) reports frequencies of strategy choices played by paid subjects who participated in a three-player, cooperation-formation game with the same payoff structure as this example. In 90 games, subjects acting as player 1 played $\{1,2\}$ 16 times and N 72 times. Subjects acting as player 2 played $\{1,2\}$ 13 times and N 75 times.

[8] Note that a sequence of (weakly) self-improving, unilateral deviations leads from the efficient strategy profile $(\{1,2\}, \{1,2\}, \{3\})$ to the potential-maximizing profile (N, N, N). Potential-maximizing outcomes can be interpreted as the limit of an adjustment process based on such deviations (see Garratt and Qin, 2002).

[9] This is true because at π^* no player is willing to establish missing links with other players or break existing links. A player who forms missing links loses more from having negative marginal contributions to the additional connected 3-player coalitions than she gains from her low marginal contributions to the additional connected 2-player coalitions. Similarily, each player loses by breaking existing links.

References

Dutta, B., A. van den Nouweland, and S. Tijs (1998) "Link formation in cooperative situations" *International Journal of Game Theory* **27**, 245-256.

Garratt, R., J. Parco, C.-Z. Qin, and Am. Rapoport (2001) "Potential maximization and coalition government formation" unpublished paper.

Garratt, R., and C.-Z. Qin (2002) "On potential maximization as a refinement of Nash equilibrium" unpublished paper.

Jackson, M. O., and A. Wolinsky (1996) "A strategic model of social and economic networks" *Journal of Economic Theory* **71**, 44-74.

Monderer, D., and L. Shapley (1996) "Potential games" *Games and Economic Behavior* **14**, 124-143.

Myerson, R. (1977) "Graphs and cooperation in games" *Mathematics of Operations Research* **2**, 225-229.

Myerson, R. (1991) *Game Theory: Analysis of Conflict,* Harvard University Press: Cambridge.

Qin, C.-Z. (1996) "Endogenous formation of cooperation structures" *Journal of Economic Theory* **69**, 218-226.

Shapley, L. S. (1953) "A Value for n-person Games" in *Contributions to the Theory of Games, Volume II, Annals of Mathematics Studies No. 28* by H. W. Kuhn and A. W. Tucker, Eds., Princeton University Press: Princeton, 307-317.

Slikker, M., B. Dutta, A. van den Nouweland, and S. Tijs (2000) "Potential maximizers and network formation" *Mathematical Social Sciences* **39**, 55-70.

Slikker, M., and A. van den Nouweland (2002) "Network formation, costs, and potential games" in *Chapters in Game Theory (In honor of Stef Tijs)* by P. Borm and H. Peters Eds., Kluwer Academic Publishers: Dordrecht, 223-246.